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Mathematical Modeling in Enhanced Index Tracking with Optimization Model

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Abstract

In portfolio management, the fund managers and investors desire to determine the optimal portfolio that can generate higher return at the minimum risk. Enhanced index tracking is a popular type of portfolio management which aims to construct the optimal portfolio in order to generate higher portfolio mean return than the benchmark index mean return. The fund managers can achieve this purpose by using the optimization model as a decision-making tool. The objective of this paper is to apply the optimization model with weighted approach in constructing the optimal portfolio to track the Technology Index in Malaysia. In this study, the data consists of weekly return of the companies from technology sector in Malaysia Main Market. The results of this study indicate that the optimal portfolio is able to outperform Technology Index by generating weekly excess mean return 0.3168% at minimum tracking error 1.8282%. The significance of this study is to identify and apply the optimization model with weighted approach as a strategic decision-making tool for the fund managers to track the benchmark Technology Index effectively in Malaysia stock market.

Keywords

Weighted Model, Enhanced Index Tracking, Non-Linear Programming, Optimal Portfolio, Mean Return, Tracking Error

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1. Introduction

Index tracking is a portfolio management which aims to reproduce the performance of a stock market index without purchasing all of the stocks that make up the index [1]. This objective can be achieved by constructing an optimal portfolio to minimize the tracking error. Tracking error is a risk measure of how closely a portfolio follows the index [2]. Enhanced index tracking is a popular type of portfolio management which aims to construct the optimal portfolio in order to generate higher portfolio mean return than the benchmark index mean return [3]. In enhanced index tracking, the optimal portfolio is constructed by maximizing the portfolio mean return besides minimizing the tracking error.

Tracking the sectorial index is important because the index represents the overall performance of the economic sectors in a country such as technology sector and industrial product sector. In order for the fund managers and investors to track the benchmark index effectively, the optimization model with weighted approach has been developed to determine the trade-off between minimizing the tracking error and maximizing the mean return of the portfolio [3]. The optimization model has been studied by the past researchers as a strategic decision-making tool in portfolio management [4-7]. Tracking error and mean return of the optimal portfolio are two main elements in enhanced index tracking problem [8, 9]. The objective of this paper is to apply the optimization model with weighted approach in constructing the optimal

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portfolio to track the Technology Index in Malaysia. The performance of the optimal portfolio is then compared with the benchmark Technology Index. The rest of the paper is organized as follows. The next section describes the materials and methods used in this study. Section 3 presents the empirical results of this study. Section 4 concludes the paper.

2. Materials and Methods

2.1. Data

In this study, the data consists of weekly return of 17 stocks from technology sector which are listed on the Malaysia Main Market as shown in Table 1.

Table 1. List of Stocks from Technology Sector on Malaysia Main Market.

Stocks
CUSCAPI
D&O
DATAPRP
DIGISTAR
DNEX
GHLSYS
GPACKET
GTRONIC
MPI
MSNIAGA
NOTION
OMESTI
PANPAGE
PENTA
THETA
TRIVE
UNISEM

The study period is from January 2012 until December 2015. This data is applied in the optimization model with weighted approach for portfolio construction to track the Technology Index in Malaysia. An optimization model is a mathematical model which aims to find the values of decision variables that optimize an objective function among the set of all values of the decision variables that satisfy the given constraints [10]. In portfolio construction with the optimization model, the decision variables represents the optimal portfolio composition that can be determined by solving the model. The return of the stocks is determined as below [3].

$$R_{jt} = \ln\left(\frac{P_{j,t}}{P_{j,t-1}}\right) \quad (1)$$

R_{jt} is the return of stock j at time t,

$P_{j,t}$ is the closing price of stock j at time t,

$P_{j,t-1}$ is the closing price of stock j at time t-1.

The return of the benchmark index is determined as below [3].

$$R_{It} = \ln\left(\frac{I_t}{I_{t-1}}\right) \quad (2)$$

R_{It} is the return of index at time t,

I_t is the index value at time t,

I_{t-1} is the index value at time t-1.

The mean return of the stock j is calculated as below [11].

$$r_j = \frac{1}{T}\sum_{t=1}^{T} R_{jt} \quad (3)$$

r_j is the mean return of stock j,

R_{jt} is the return of stock j at time t,

T is the number of observations.

Figure 1 shows the construction process of the optimal portfolio in tracking the benchmark Technology Index with an optimization model.

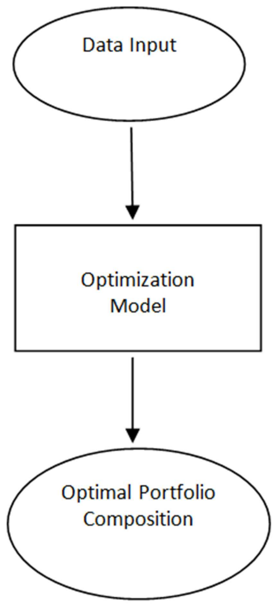

Figure 1. Construction Process of the Optimal Portfolio Composition with Optimization Model.

In this study, the optimization model is solved using LINGO software.

2.2. Optimization Model with Weighted Approach

An optimization model is a decision-making tool which aims to find the values of decision variables that optimize an objective function among the set of all values of the decision variables that satisfy the given constraints [12]. The optimization model with weighted approach has been introduced to construct the optimal portfolio for tracking the benchmark index [3]. This model is a non-linear programming model which aims to determine the trade-off between minimizing the tracking error and maximizing the excess mean return of the optimal portfolio over the mean return of benchmark index. The optimization model is formulated as follow.

$$\text{Minimize } z = \lambda(TE) - (1-\lambda)\alpha \tag{4}$$

subject to

$$TE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(R_{Pt} - R_{It}\right)^2} \tag{5}$$

$$\alpha = \sum_{i=1}^{T}\left(\frac{R_{Pt} - R_{It}}{T}\right) \tag{6}$$

$$\sum_{j=1}^{N} y_j = K \tag{7}$$

$$w_j = \frac{V_{jT}x_j}{C} \tag{8}$$

$$\varepsilon_j y_j \le w_j \le \delta_j y_j \tag{9}$$

$$y_j \in [0,1] \tag{10}$$

$$G \le \gamma C \tag{11}$$

$$\sum_{j=1}^{N} V_{jT}x_j = C - G \tag{12}$$

$$x_j, w_j \ge 0 \tag{13}$$

K is the number of stocks in tracking the benchmark index,

R_{Pt} is the mean return of the optimal portfolio at time *t*,

R_{It} is the mean return of the benchmark index at time *t*,

ε_j is the lower bounds of the fund proportion respectively on stock *j*,

δ_j is the upper bounds of the fund proportion respectively on stock *j*,

V_{jT} is the price of one unit of stock *j* at time *T*,

x_j is the number of units of stock *j* in the optimal portfolio,

w_j is the weight of stock *j* in the optimal portfolio,

C is the total amount of fund,

y_j is the binary integer,

G is the total transaction cost,

TE is the tracking error with non-linear function,

α is the excess mean return of the optimal portfolio over the mean return of the benchmark index,

γ is the limit on the proportion of *C* that can be consumed by the transaction cost ($0 \le \gamma \le 1$)

λ represents an implicit trade-off between the minimizing the tracking error and maximizing the excess mean return ($0 \le \lambda \le 1$).

Equation (4) is the objective function of the model which determines the trade-off between minimizing the tracking error and maximizing the excess return based on parameter λ . $\lambda = 1$ corresponds to minimize the tracking error whereas $\lambda = 0$ corresponds to maximize the excess return. Constraint (7) ensures that the number of stocks in the optimal portfolio is equal to *K*. Constraint (8) is the weight of stock *j* in the optimal portfolio. Constraint (9) indicates that the value of w_j is limited in the interval $\left[\varepsilon_j, \delta_j\right]$ where $0 \le \varepsilon_j \le 1$ and $0 \le \delta_j \le 1$. Both constraint (9) and (10) show that if stock *j* is not selected in the optimal portfolio (i.e., $y_j = 0$), then $w_j = 0$, and if stock *j* is selected in the optimal portfolio (i.e., $y_j = 1$), then $w_j \ne 0$. Constraint (11) limits the total transaction cost incurred. Constraint (12) is the amount of fund allocated for the optimal portfolio. Constraint (13) states that the weight of each stock and the number of units of stock *j* in the optimal portfolio are positive.

2.3. Portfolio Performance

Tracking error and mean return of the optimal portfolio are the main elements in enhanced index tracking problem [8, 9]. Tracking error is a risk measure of how closely the optimal portfolio follows the benchmark index [13, 14]. Tracking error is the standard deviation of the difference between the returns of the portfolio and the returns of the benchmark index [15]. The formula for tracking error is as follows.

$$TE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(R_{Pt} - R_{It}\right)^2} \tag{14}$$

TE is the tracking error,

T is the number of periods,

R_{Pt} is the mean return of the optimal portfolio at time *t*,

R_{It} is the mean return of the benchmark index at time *t*.

The mean return of the optimal portfolio is formulated as follow [16].

$$r_P = \sum_{i=1}^{N} R_j w_j \tag{15}$$

r_P is the mean return of the optimal portfolio,

w_j is the weight of stock j in the optimal portfolio,

R_j is the mean return of stock j in the optimal portfolio. Excess return is defined as the difference between the portfolio mean return and benchmark index mean return which is formulated as follow [8, 9].

$$\alpha = r_P - r_I \tag{16}$$

α is the excess return,

r_P is the mean return of the optimal portfolio,

r_I is the mean return of the benchmark index.

The performance of the optimal portfolio is measured with information ratio [17]. The information ratio is defined as the ratio of portfolio's excess mean return to the portfolio's tracking error which is formulated as below.

$$IR = \frac{\alpha}{TE} \tag{17}$$

IR is the information ratio,

α is the excess mean return of the optimal portfolio over the mean return of the benchmark index return,

TE is the tracking error.

Higher information ratio indicates higher performance of the optimal portfolio.

3. Empirical Results

Table 2 presents the optimal portfolio composition which is constructed using the optimization model with weighted approach.

As shown in Table 2, the list of stocks with positive weights indicate that those stocks are selected by the optimization model in constructing the optimal portfolio to track the Technology Index in Malaysia. The optimal portfolio consists of D&O, DATAPRP, GHLSYS, GPACKET, GTRONIC, MPI, PENTA, THETA, TRIVE and UNISEM. This optimal portfolio is constructed by the optimization model which can minimize the tracking error and maximize the portfolio mean return. GTRONIC is the most dominant stock in the optimal portfolio with 34.54% of the allocated fund. On the other hand, GHLSYS is the smallest stock in the optimal portfolio with 1.57% of the allocated fund. The rest of the components in the optimal portfolio are D&O (4.91%), DATAPRP (2.45%), GPACKET (7.82%), MPI (18.65%), PENTA (9.08%), THETA (1.67%), TRIVE (3.23%) and UNISEM (15.08%). CUSCAPI, DIGISTAR, DNEX, MSNIAGA, NOTION, OMESTI and PANPAGE are not selected by the optimization model in constructing the optimal portfolio due to zero weights. Table 3 displays the performance of the optimal portfolio constructed using the optimization model with weighted approach.

Table 2. Optimal Portfolio Composition.

Stocks	Weights (%)
CUSCAPI	0.00
D&O	4.91
DATAPRP	2.45
DIGISTAR	0.00
DNEX	0.00
GHLSYS	1.57
GPACKET	7.82
GTRONIC	34.54
MPI	18.65
MSNIAGA	0.00
NOTION	0.00
OMESTI	0.00
PANPAGE	0.00
PENTA	9.08
THETA	1.67
TRIVE	3.23
UNISEM	15.08

Table 3. Performance of the Optimal Portfolio.

Portfolio	Technology Index	Optimization Model
Mean Return (%)	0.2270	0.5438
Excess Return (%)	-	0.3168
Tracking Error (%)	-	1.8282
Information Ratio	-	0.1733

As shown in Table 3, the weekly mean return for Technology Index is 0.2270% based on the study period. The optimal portfolio tracks the Technology Index with weekly mean return 0.5438% which is higher than the mean return of Technology Index. This implies that the optimal portfolio constructed by the optimization model with weighted approach is able to outperform the Technology Index with weekly excess mean return 0.3168% at minimum tracking error 1.8282%. Besides that, the information ratio 0.1733 indicates that the optimal portfolio can generate weekly excess mean return 0.1733% over the mean return of Technology Index at 1% tracking error. Therefore, the optimization model with weighted approach is suitable to be used as a strategic decision-making tool for the fund managers and investors in Malaysia.

4. Conclusion

This paper presents the strategic decision-making tool in portfolio management by applying the optimization model with weighted approach to track the Technology Index in Malaysia. In conclusion, the optimal portfolio constructed by the optimization model is able to outperform Technology Index by generating weekly excess mean return 0.3168% at minimum tracking error 1.8282%. The significance of this study is to identify and apply the optimization model with weighted approach as a strategic decision-making tool for the fund managers and investors to track the benchmark Technology Index effectively in Malaysia.

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