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Certain General Zagreb Indices and Zagreb Polynomials of Molecular Graphs

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Abstract

Chemical compounds and drugs are often modelled as graphs where each vertex represents an atom of molecule, and covalent bounds between atoms are represented by edges between the corresponding vertices. This graph derived from a chemical compounds is often called its molecular graph, and can be different structures. In this paper, by virtue of mathematical derivation, we determine the fourth, fifth and sixth general Zagreb indices and Zagreb polynomials of fan molecular graph, wheel molecular graph, and gear fan molecular graph, gear wheel molecular graph, and their *r*-corona molecular graphs. The results obtained in our paper illustrate the promising application prospects for chemistry and pharmacy science.

Keywords

Chemical Graph Theory, General Zagreb Index, Zagreb Polynomial, R-Corona Molecular Graph

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1. Introduction

Wiener index, edge Wiener index, Hyper-wiener index, Geometric-arithmetic index and Zagreb indices are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the distance-based index of special molecular graphs (See Yan et al., [1-2], Gao et al., [3-4], Gao and Shi [5], Gao and Wang [6], Xi and Gao [7-8], Xi et al., [9], Gao et al., [10] for more detail). The notation and terminology used but undefined in this paper can be found in [11].

Let P_n and C_n be path and cycle with n vertices. The molecular graph $F_n=\{v\} \vee P_n$ is called a fan molecular graph and the molecular graph $W_n=\{v\} \vee C_n$ is called a wheel molecular graph. Molecular graph $I_r(G)$ is called r- crown molecular graph of G which splicing r hang edges for every vertex in G. By adding one vertex in every two adjacent vertices of the fan path P_n of fan molecular graph F_n, the resulting molecular graph is a subdivision molecular graph called gear fan molecular graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle C_n of wheel molecular graph W_n, The resulting molecular graph is a subdivision molecular graph, called gear wheel molecular graph, denoted as $\tilde{W}_n$.

The eccentricity $ec(u)$ of vertex $u\in V(G)$ is the maximum distance between u and any other vertex in G. In [12], Gao introduced the following general indices and polynomials:

The general fourth Zagreb index: $Zg_4^k(G) = \sum_{uv\in E(G)} (ec(u)+ec(v))^k$

The general fifth Zagreb index: $Zg_5^k(G)= \sum_{v\in V(G)} ec(v)^k$

The general sixth Zagreb index: $Zg_6^k(G) = \sum_{uv\in E(G)} (ec(u)ec(v))^k$

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The fourth Zagreb polynomial: $Zg_4(G,x)=\sum_{uv\in E(G)} x^{ec(u)+ec(v)}$

The fifth Zagreb polynomial: $Zg_5(G,x)=\sum_{v\in V(G)} x^{ec(v)^2}$

The sixth Zagreb polynomial: $Zg_6(G,x)=\sum_{uv\in E(G)} x^{ec(u)ec(v)}$

Here,k is a real number.

In this paper, we present these indices and polynomials of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$.

2. Main Results and Proofs

Theorem 1.

$$Zg_4^k(I_r(F_n))=(n+r)5^k+(n-1)6^k+nr7^k,$$

$$Zg_5^k(I_r(F_n))=2^k+(n+r)3^k+nr4^k,$$

$$Zg_6^k(I_r(F_n))=(n+r)6^k+(n-1)9^k+nr\cdot 12^k,$$

$$Zg_4(I_r(F_n),x)=(n+r)x^5+(n-1)x^6+nrx^7,$$

$$Zg_5(I_r(F_n),x)=x^4+(n+r)x^9+nrx^{16},$$

$$Zg_6(I_r(F_n),x)=(n+r)x^6+(n-1)x^9+nrx^{12}.$$

Proof. Let $P_n=v_1v_2\dots v_n$ and the r hanging vertices of v_i be v_i^1, $v_i^2,\dots, v_i^r$ $(1\le i\le n)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By the definition of general Zagreb indices and Zagreb polynomials, we have

$$Zg_4^k(I_r(F_n))=\sum_{i=1}^{r}(ec(v)+ec(v^i))^k+\sum_{i=1}^{n}(ec(v)+ec(v_i))^k+\sum_{i=1}^{n-1}(ec(v_i)+ec(v_{i+1}))^k+\sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)+ec(v_i^j))^k$$

$$=\sum_{i=1}^{r}5^k+\sum_{i=1}^{n}5^k+\sum_{i=1}^{n-1}6^k+\sum_{i=1}^{n}\sum_{j=1}^{r}7^k$$

$$=(n+r)5^k+(n-1)6^k+nr7^k.$$

$$Zg_5^k(I_r(F_n))=ec(v)^k+\sum_{i=1}^{n}ec(v_i)^k+\sum_{i=1}^{r}ec(v^i)^k+\sum_{i=1}^{n}\sum_{j=1}^{r}ec(v_i^j)^k=2^k+(n+r)3^k+nr4^k.$$

$$Zg_6^k(I_r(F_n))=\sum_{i=1}^{r}(ec(v)ec(v^i))^k+\sum_{i=1}^{n}(ec(v)ec(v_i))^k+\sum_{i=1}^{n-1}(ec(v_i)ec(v_{i+1}))^k+\sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)ec(v_i^j))^k$$

$$=\sum_{i=1}^{r}6^k+\sum_{i=1}^{n}6^k+\sum_{i=1}^{n-1}9^k+\sum_{i=1}^{n}\sum_{j=1}^{r}12^k$$

$$=(n+r)6^k+(n-1)9^k+nr\cdot 12^k$$

$$Zg_4(I_r(F_n),x)=\sum_{i=1}^{r}x^{ec(v)+ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)+ec(v_i)}+\sum_{i=1}^{n-1}x^{ec(v_i)+ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)+ec(v_i^j)}$$

$$=\sum_{i=1}^{r}x^5+\sum_{i=1}^{n}x^5+\sum_{i=1}^{n-1}x^6+\sum_{i=1}^{n}\sum_{j=1}^{r}x^7$$

$$=(n+r)x^5+(n-1)x^6+nrx^7.$$

$$Zg_5(I_r(F_n),x)=x^{ec(v)^2}+\sum_{i=1}^{n}x^{ec(v_i)^2}+\sum_{i=1}^{r}x^{ec(v^i)^2}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i^j)^2}=x^4+(n+r)x^9+nrx^{16}.$$

$$Zg_6(I_r(F_n),x)=\sum_{i=1}^{r}x^{ec(v)ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)ec(v_i)}+\sum_{i=1}^{n-1}x^{ec(v_i)ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)ec(v_i^j)}=\sum_{i=1}^{r}x^6+\sum_{i=1}^{n}x^6+\sum_{i=1}^{n-1}x^9+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{12}$$

$$=(n+r)x^6+(n-1)x^9+nrx^{12}.$$

Corollary 1.

$$Zg_4^k(F_n)=n5^k+(n-1)6^k,$$

$$Zg_5^k(F_n)=2^k+n3^k,$$

$$Zg_6^k(F_n)=n6^k+(n-1)9^k,$$

$$Zg_4(F_n,x)=nx^5+(n-1)x^6,$$

$$Zg_5(F_n,x)=x^4+nx^9,$$

$$Zg_6(F_n,x)=nx^6+(n-1)x^9.$$

Theorem 2.

$$Zg_4^k(I_r(W_n))=(n+r)5^k+n6^k+nr7^k,$$

$$Zg_5^k(I_r(W_n))=2^k+(n+r)3^k+nr4^k,$$

$$Zg_6^k(I_r(W_n))=(n+r)6^k+n9^k+nr\cdot 12^k,$$

$$Zg_4(I_r(W_n),x)=(n+r)x^5+nx^6+nrx^7,$$

$$Zg_5(I_r(W_n),x)=x^4+(n+r)x^9+nrx^{16},$$

$$Zg_6(I_r(W_n),x)=(n+r)x^6+nx^9+nrx^{12}.$$

Proof. Let $C_n=v_1v_2\dots v_n$ and v_i^1, $v_i^2,\dots, v_i^r$ be the r hanging vertices of v_i ($1\le i\le n$). Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. By the definition of general Zagreb indices and Zagreb polynomials, we have

$$Zg_4^k(I_r(W_n))=\sum_{i=1}^{r}(ec(v)+ec(v^i))^k+\sum_{i=1}^{n}(ec(v)+ec(v_i))^k+\sum_{i=1}^{n}(ec(v_i)+ec(v_{i+1}))^k+\sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)+ec(v_i^j))^k$$

$$=\sum_{i=1}^{r}5^k+\sum_{i=1}^{n}5^k+\sum_{i=1}^{n}6^k+\sum_{i=1}^{n}\sum_{j=1}^{r}7^k$$

$$=(n+r)5^k+n6^k+nr7^k .$$

$$Zg_5^k(I_r(W_n))=ec(v)^k+\sum_{i=1}^{n}ec(v_i)^k+\sum_{i=1}^{r}ec(v^i)^k+\sum_{i=1}^{n}\sum_{j=1}^{r}ec(v_i^j)^k=2^k+(n+r)3^k+nr4^k .$$

$$Zg_6^k(I_r(W_n))=\sum_{i=1}^{r}(ec(v)ec(v^i))^k+\sum_{i=1}^{n}(ec(v)ec(v_i))^k+\sum_{i=1}^{n}(ec(v_i)ec(v_{i+1}))^k+\sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)ec(v_i^j))^k$$

$$=\sum_{i=1}^{r}6^k+\sum_{i=1}^{n}6^k+\sum_{i=1}^{n}9^k+\sum_{i=1}^{n}\sum_{j=1}^{r}12^k$$

$$=(n+r)6^k+n9^k+nr\cdot 12^k .$$

$$Zg_4(I_r(W_n),x)=\sum_{i=1}^{r}x^{ec(v)+ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)+ec(v_i)}+\sum_{i=1}^{n}x^{ec(v_i)+ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)+ec(v_i^j)}=\sum_{i=1}^{r}x^5+\sum_{i=1}^{n}x^5+\sum_{i=1}^{n}x^6+\sum_{i=1}^{n}\sum_{j=1}^{r}x^7=(n+r)x^5+nx^6+nrx^7 .$$

$$Zg_5(I_r(W_n),x)=x^{ec(v)^2}+\sum_{i=1}^{n}x^{ec(v_i)^2}+\sum_{i=1}^{r}x^{ec(v^i)^2}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i^j)^2}$$

$$=x^4+(n+r)x^9+nrx^{16} .$$

$$Zg_6(I_r(W_n),x)=\sum_{i=1}^{r}x^{ec(v)ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)ec(v_i)}+\sum_{i=1}^{n}x^{ec(v_i)ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)ec(v_i^j)}$$

$$=\sum_{i=1}^{r}x^6+\sum_{i=1}^{n}x^6+\sum_{i=1}^{n}x^9+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{12}$$

$$=(n+r)x^6+nx^9+nrx^{12} .$$

Corollary 2.

$$Zg_4^k(W_n)=n5^k+n6^k ,$$

$$Zg_5^k(W_n)=2^k+n3^k ,$$

$$Zg_6^k(W_n)=n6^k+n9^k ,$$

$$Zg_4(W_n,x)=nx^5+nx^6 ,$$

$$Zg_5(W_n,x)=x^4+nx^9 ,$$

$$Zg_6(W_n,x)=nx^6+nx^9 .$$

Theorem 3.

$$Zg_4^k(I_r(\tilde{F}_n))=(r+n)7^k+(nr+2n-2)9^k+(n-1)r\cdot 11^k ,$$

$$Zg_5^k(I_r(\tilde{F}_n))=3^k+(n+r)4^k+(nr+n-1)5^k+(n-1)r6^k ,$$

$$Zg_6^k(I_r(\tilde{F}_n))=(r+n)\cdot 12^k+(nr+2n-2)\cdot 20^k+(n-1)r\cdot 30^k ,$$

$$Zg_4(I_r(\tilde{F}_n),x)=(r+n)x^7+(nr+2n-2)x^9+(n-1)rx^{11} ,$$

$$Zg_5(I_r(\tilde{F}_n),x)=x^9+(n+r)x^{16}+(nr+n-1)x^{25}+(n-1)rx^{36} ,$$

$$Zg_6(I_r(\tilde{F}_n),x)=(r+n)x^{12}+(nr+2n-2)x^{20}+(n-1)rx^{30} .$$

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i ($1\le i\le n$). Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i\le n$-1). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By virtue of the definition of general Zagreb indices and Zagreb polynomials, we get

$$Zg_4^k(I_r(\tilde{F}_n))=\sum_{i=1}^{r}(ec(v)+ec(v^i))^k+\sum_{i=1}^{n}(ec(v)+ec(v_i))^k+\sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)+ec(v_i^j))^k+\sum_{i=1}^{n-1}(ec(v_i)+ec(v_{i,i+1}))^k+\sum_{i=1}^{n-1}(ec(v_{i,i+1})+ec(v_{i+1}))^k+\sum_{i=1}^{n-1}\sum_{j=1}^{r}(ec(v_{i,i+1})+ec(v_{i,i+1}^j))^k$$

$$=\sum_{i=1}^{r}7^k+\sum_{i=1}^{n}7^k+\sum_{i=1}^{n}\sum_{j=1}^{r}9^k+\sum_{i=1}^{n-1}9^k+\sum_{i=1}^{n-1}9^k+\sum_{i=1}^{n-1}\sum_{j=1}^{r}11^k$$

$$=(r+n)7^k+(nr+2n-2)9^k+(n-1)r\cdot 11^k .$$

$$Zg_5^k(I_r(\tilde{F}_n))=ec(v)^k+\sum_{i=1}^{n}ec(v_i)^k+\sum_{i=1}^{r}ec(v^i)^k+\sum_{i=1}^{n}\sum_{j=1}^{r}ec(v_i^j)^k+\sum_{i=1}^{n-1}ec(v_{i,i+1})^k+\sum_{i=1}^{n-1}\sum_{j=1}^{r}ec(v_{i,i+1}^j)^k$$

$$=3^k+(n+r)4^k+(nr+n-1)5^k+(n-1)r6^k .$$

$$Zg_6^k(I_r(\tilde{F}_n)) = \sum_{i=1}^{r}(ec(v)ec(v^i))^k + \sum_{i=1}^{n}(ec(v)ec(v_i))^k + \sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)ec(v_i^j))^k + \sum_{i=1}^{n-1}(ec(v_i)ec(v_{i,i+1}))^k + \sum_{i=1}^{n-1}(ec(v_{i,i+1})ec(v_{i+1}))^k + \sum_{i=1}^{n-1}\sum_{j=1}^{r}(ec(v_{i,i+1})ec(v_{i,i+1}^j))^k$$

$$= \sum_{i=1}^{r}12^k + \sum_{i=1}^{n}12^k + \sum_{i=1}^{n}\sum_{j=1}^{r}20^k + \sum_{i=1}^{n-1}20^k + \sum_{i=1}^{n-1}20^k + \sum_{i=1}^{n-1}\sum_{j=1}^{r}30^k$$

$$= (r+n)\cdot 12^k + (nr+2n-2)\cdot 20^k + (n-1)r\cdot 30^k .$$

$$Zg_4(I_r(\tilde{F}_n),x) = \sum_{i=1}^{r}x^{ec(v)+ec(v^i)} + \sum_{i=1}^{n}x^{ec(v)+ec(v_i)} + \sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)+ec(v_i^j)} + \sum_{i=1}^{n-1}x^{ec(v_i)+ec(v_{i,i+1})} + \sum_{i=1}^{n-1}x^{ec(v_{i,i+1})+ec(v_{i+1})} + \sum_{i=1}^{n-1}\sum_{j=1}^{r}x^{ec(v_{i,i+1})+ec(v_{i,i+1}^j)}$$

$$= \sum_{i=1}^{r}x^7 + \sum_{i=1}^{n}x^7 + \sum_{i=1}^{n}\sum_{j=1}^{r}x^9 + \sum_{i=1}^{n-1}x^9 + \sum_{i=1}^{n-1}x^9 + \sum_{i=1}^{n-1}\sum_{j=1}^{r}x^{11}$$

$$= (r+n)x^7 + (nr+2n-2)x^9 + (n-1)rx^{11} .$$

$$Zg_5(I_r(\tilde{F}_n),x) = x^{ec(v)^2} + \sum_{i=1}^{n}x^{ec(v_i)^2} + \sum_{i=1}^{r}x^{ec(v^i)^2} + \sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i^j)^2} + \sum_{i=1}^{n-1}x^{ec(v_{i,i+1})^2} + \sum_{i=1}^{n-1}\sum_{j=1}^{r}x^{ec(v_{i,i+1}^j)^2}$$

$$= x^9 + (n+r)x^{16} + (nr+n-1)x^{25} + (n-1)rx^{36} .$$

$$Zg_6(I_r(\tilde{F}_n),x) = \sum_{i=1}^{r}x^{ec(v)ec(v^i)} + \sum_{i=1}^{n}x^{ec(v)ec(v_i)} + \sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)ec(v_i^j)} + \sum_{i=1}^{n-1}x^{ec(v_i)ec(v_{i,i+1})} + \sum_{i=1}^{n-1}x^{ec(v_{i,i+1})ec(v_{i+1})} + \sum_{i=1}^{n-1}\sum_{j=1}^{r}x^{ec(v_{i,i+1})ec(v_{i,i+1}^j)}$$

$$= \sum_{i=1}^{r}x^{12} + \sum_{i=1}^{n}x^{12} + \sum_{i=1}^{n}\sum_{j=1}^{r}x^{20} + \sum_{i=1}^{n-1}x^{20} + \sum_{i=1}^{n-1}x^{20} + \sum_{i=1}^{n-1}\sum_{j=1}^{r}x^{30}$$

$$= (r+n)x^{12} + (nr+2n-2)x^{20} + (n-1)rx^{30} .$$

Corollary 3.

$$Zg_4^k(\tilde{F}_n) = n7^k + (2n-2)9^k ,$$

$$Zg_5^k(\tilde{F}_n) = 3^k + n4^k + (n-1)5^k ,$$

$$Zg_6^k(\tilde{F}_n) = n12^k + (2n-2)\cdot 20^k ,$$

$$Zg_4(\tilde{F}_n,x) = nx^7 + (2n-2)x^9 ,$$

$$Zg_5(\tilde{F}_n,x) = x^9 + nx^{16} + (n-1)x^{25} ,$$

$$Zg_6(\tilde{F}_n,x) = nx^{12} + (2n-2)x^{20} .$$

Theorem 4.

$$Zg_4^k(I_r(\tilde{W}_n)) = (r+n)7^k + (nr+2n)9^k + nr\cdot 11^k ,$$

$$Zg_5^k(I_r(\tilde{W}_n)) = 3^k + (n+r)4^k + (nr+n)5^k + nr6^k ,$$

$$Zg_6^k(I_r(\tilde{W}_n)) = (r+n)12^k + (nr+2n)20^k + nr\cdot 30^k ,$$

$$Zg_4(I_r(\tilde{W}_n),x) = (r+n)x^7 + (nr+2n)x^9 + nrx^{11} ,$$

$$Zg_5(I_r(\tilde{W}_n),x) = x^9 + (n+r)x^{16} + (nr+n)x^{25} + nrx^{36} ,$$

$$Zg_6(I_r(\tilde{W}_n),x) = (r+n)x^{12} + (nr+2n)x^{20} + nrx^{30} .$$

Proof. Let $C_n=v_1v_2\ldots v_n$ and v be a vertex in W_n beside C_n. $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i ($1\le i\le n$). Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i \le n$). In view of the definition of general Zagreb indices and Zagreb polynomials, we deduce

$$Zg_4^k(I_r(\tilde{W}_n)) = \sum_{i=1}^{r}(ec(v)+ec(v^i))^k + \sum_{i=1}^{n}(ec(v)+ec(v_i))^k + \sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)+ec(v_i^j))^k + \sum_{i=1}^{n}(ec(v_i)+ec(v_{i,i+1}))^k + \sum_{i=1}^{n}(ec(v_{i,i+1})+ec(v_{i+1}))^k + \sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_{i,i+1})+ec(v_{i,i+1}^j))^k$$

$$= \sum_{i=1}^{r}7^k + \sum_{i=1}^{n}7^k + \sum_{i=1}^{n}\sum_{j=1}^{r}9^k + \sum_{i=1}^{n}9^k + \sum_{i=1}^{n}9^k + \sum_{i=1}^{n}\sum_{j=1}^{r}11^k$$

$$= (r+n)7^k + (nr+2n)9^k + nr\cdot 11^k .$$

$$Zg_5^k(I_r(\tilde{W}_n)) = ec(v)^k + \sum_{i=1}^{n}ec(v_i)^k + \sum_{i=1}^{r}ec(v^i)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}ec(v_i^j)^k + \sum_{i=1}^{n}ec(v_{i,i+1})^k + \sum_{i=1}^{n}\sum_{j=1}^{r}ec(v_{i,i+1}^j)^k$$

$$= 3^k + (n+r)4^k + (nr+n)5^k + nr6^k .$$

$$Zg_6^k(I_r(\tilde{W}_n)) = \sum_{i=1}^{r}(ec(v)ec(v^i))^k + \sum_{i=1}^{n}(ec(v)ec(v_i))^k + \sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_i)ec(v_i^j))^k + \sum_{i=1}^{n}(ec(v_i)ec(v_{i,i+1}))^k + \sum_{i=1}^{n}(ec(v_{i,i+1})ec(v_{i+1}))^k + \sum_{i=1}^{n}\sum_{j=1}^{r}(ec(v_{i,i+1})ec(v_{i,i+1}^j))^k$$

$$=\sum_{i=1}^{r}12^k+\sum_{i=1}^{n}12^k+\sum_{i=1}^{n}\sum_{j=1}^{r}20^k+\sum_{i=1}^{n}20^k+\sum_{i=1}^{n}20^k+\sum_{i=1}^{n}\sum_{j=1}^{r}30^k$$

$$=(r+n)12^k+(nr+2n)20^k+nr\cdot 30^k .$$

$$Zg_4(I_r(\tilde{W}_n),x)=\sum_{i=1}^{r}x^{ec(v)+ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)+ec(v_i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)+ec(v_i^j)}+\sum_{i=1}^{n}x^{ec(v_i)+ec(v_{i,i+1})}+\sum_{i=1}^{n}x^{ec(v_{i,i+1})+ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_{i,i+1})+ec(v_{i,i+1}^j)}$$

$$=\sum_{i=1}^{r}x^7+\sum_{i=1}^{n}x^7+\sum_{i=1}^{n}\sum_{j=1}^{r}x^9+\sum_{i=1}^{n}x^9+\sum_{i=1}^{n}x^9+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{11}$$

$$=(r+n)x^7+(nr+2n)x^9+nrx^{11} .$$

$$Zg_5(I_r(\tilde{W}_n),x)=x^{ec(v)^2}+\sum_{i=1}^{n}x^{ec(v_i)^2}+\sum_{i=1}^{r}x^{ec(v^i)^2}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i^j)^2}+\sum_{i=1}^{n}x^{ec(v_{i,i+1})^2}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_{i,i+1}^j)^2}$$

$$=x^9+(n+r)x^{16}+(nr+n)x^{25}+nrx^{36} .$$

$$Zg_6(I_r(\tilde{W}_n),x)=\sum_{i=1}^{r}x^{ec(v)ec(v^i)}+\sum_{i=1}^{n}x^{ec(v)ec(v_i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_i)ec(v_i^j)}+\sum_{i=1}^{n}x^{ec(v_i)ec(v_{i,i+1})}+\sum_{i=1}^{n}x^{ec(v_{i,i+1})ec(v_{i+1})}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{ec(v_{i,i+1})ec(v_{i,i+1}^j)}$$

$$=\sum_{i=1}^{r}x^{12}+\sum_{i=1}^{n}x^{12}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{20}+\sum_{i=1}^{n}x^{20}+\sum_{i=1}^{n}x^{20}+\sum_{i=1}^{n}\sum_{j=1}^{r}x^{30}$$

$$=(r+n)x^{12}+(nr+2n)x^{20}+nrx^{30} .$$

Corollary 4.

$$Zg_4^k(\tilde{W}_n)=n7^k+2n9^k ,$$

$$Zg_5^k(\tilde{W}_n)=3^k+n4^k+n5^k ,$$

$$Zg_6^k(\tilde{W}_n)=n12^k+2n20^k ,$$

$$Zg_4(\tilde{W}_n,x)=nx^7+2nx^9 ,$$

$$Zg_5(\tilde{W}_n,x)=x^9+nx^{16}+nx^{25} ,$$

$$Zg_6(\tilde{W}_n,x)=nx^{12}+2nx^{20} .$$

3. Conclusions

In this paper, we mainly present the fourth, fifth and sixth Zagreb index and Zagreb polynomial of fan molecular graph, wheel molecular graph, and gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs.

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