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Approximate Solution of Single Spring Moving System for Damped and Undamped Simple Harmonic Motion by Using Homotopy Perturbation Method

Mohammad Masud Rana, Ahammodullah Hasan*

Department of Mathematics, Faculty of Science, Islamic University, Kushtia, Bangladesh

Abstract

In this paper, Homotopy perturbation method (HPM) is applied to find the approximate solution of simple harmonic motion of single spring equation, which is known a well-known nonlinear ordinary differential equation. The Homotopy Perturbation Method deforms a difficult problem into a simple problem which can be easily solved. Firstly, the approximate solution of single spring equation is developed using initial conditions. Then the results are compared with the results obtained by Numerical solutions. Finally, Homotopy Perturbation Method (HPM) is applied to find the approximate solution of single spring equation with initial conditions. The results reveal that the HPM is very effective, convenient and quite accurate to systems of nonlinear equations. Some examples are presented to show the ability of the method for moving single spring equation.

Keywords

Homotopy Perturbation Method (HPM), Simple Harmonic Motion, Nonlinear Differential Equation, Damped, Undamped, Approximate Solution, Numerical Solution

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1. Introduction

In the last three decades with the rapid development of nonlinear science, there has appeared ever increasing interest of Mathematics, Physicists and Engineers in the analytical techniques for nonlinear problems. However, in many cases it is possible to replace a nonlinear differential equation with a related linear differential equation that approximates the actual nonlinear equation closely enough to given useful results. Often such linearization is not possible or feasible, when it is not, the original nonlinear equation itself must be trickled. The general theory and methods of dealing with linear equations constitute a highly developed branch of mathematics, whereas very little of a general nature is known about nonlinear equation. Generally the study of nonlinear equations is confined to a variety of very special cases, and the method of solution usually involves one or more of a limited number of differential method of approximation, in this study we are used for approximation result using Homotopy perturbation method and comparing numerical solution. In the last three decades with the rapid development of nonlinear science, there has appeared ever increasing interest of Mathematics, Physicists and Engineers in the analytical techniques for nonlinear problems. It is well known, that perturbation methods provide the most versatile tools available in nonlinear analysis of engineering problems [1-3]. The perturbation methods, like other nonlinear analytical techniques, have their own limitations. At first, almost all perturbation methods are based on the assumption

* Corresponding author
E-mail address: ahammodullah.iu@gmail.com (A. Hasan)

that a small parameter must exist in the equation. This so-called small parameter assumption greatly restricts applications of perturbation techniques. As is well known, an overwhelming majority of nonlinear problems have no small parameters at all. Secondly, the determination of small parameters seems to be a special art requiring special techniques. An appropriate choice of small parameters leads to the ideal results, but an unsuitable choice may create serious problems. Furthermore, the approximate solutions solved by perturbation methods are valid, in most cases, only for the small values of the parameters. It is obvious that all these limitations come from the small parameter assumption. These facts have motivated to suggest alternate techniques, such as variational iteration [4-5] decomposition [6], exp-function [7], variation of parameters [8] and iterative [9]. In order to overcome these drawbacks, combining the standard Homotopy and perturbation method, which is called the Homotopy perturbation, modifies the Homotopy method. Many problems in natural and engineering sciences are modeled by partial differential equations (PDEs). These equations arise in a number of scientific models such as the propagation of shallow water waves, long wave and chemical reaction-diffusion models [10-11]. A substantial amount of work has been invested for solving such models. Several techniques including the method of characteristic, Riemann invariants, combination of waveform relaxation and multi-grid, periodic multi-grid wave form, variational iteration, Homotopy perturbation and Adomian's decomposition [10-11] have been used for the solutions of such problems. Most of these techniques encounter the inbuilt deficiencies and involve huge computational work. He [2] developed the Homotopy perturbation method for solving linear, nonlinear, initial, and boundary value problems by merging two techniques, the standard Homotopy and the perturbation technique. The Homotopy perturbation method was formulated by taking the full advantage of the standard Homotopy and perturbation methods and has been applied to a wide class of functional equations [1-2]. The HPM gives the solution in the form of a convergent series with easily computable components. Unlike the method of separation of variables which requires both initial and boundary conditions, the HPM gives the solution by using the initial conditions only [12-16]. In a series of papers He [17-20], has outlined and refined the HPM, showing its usefulness by nonlinear differential equations. As a rule, HPM tends to produce much more elegant solutions as compared to the other competing techniques such as Homotopy analysis method (HAM), regular perturbation methods etc., yet it is not at the cost of accuracy In particular the proposed Homotopy perturbation method (HPM) is tested on single degree of freedom system and spring loaded inverted pendulum, linear and nonlinear single degree of freedom system equations, with damping and without forcing and spring loaded inverted pendulum equation, with damping and adding forcing. The proposed iterative scheme finds the solution without any discretization, linearization or restrictive assumptions and is free from round off errors. The solutions obtained by using Mathematica program 9.0.

2. Formulation of HPM

To illustrate the Homotopy perturbation method, we consider a general equation of the type,

$$M(u(x)) - f(r) = 0, \quad r \in \Omega \tag{1}$$

with the boundary conditions

$$N(u, \frac{du}{dx}) = 0, \quad r \in \xi \tag{2}$$

Where M is a general differential operator, N is a boundary operator, ξ is the boundary domain of Ω and $f(r)$ is a known analytical function. Generally speaking, the operator M can be divided into a linear part Ln and a nonlinear part Nl. Now equation (1) can be rewritten as:

$$Ln(u(x)) + Nl(u(x)) - f(r) = 0 \tag{3}$$

By the homotopy perturbation method, we construct a Homotopy as $v(r, p) : \Omega \times [0,1] \to \mathbb{R}$ which satisfies

$$H(v, p) = (1-p)[Ln(v) - Ln(u_0)] + p[M(v) - f(r)] = 0 \tag{4}$$

or

$$H(v, p) = Ln(v) - Ln(u_0) + pLn(u_0) + p[Nl(v) - f(r)] = 0 \tag{5}$$

where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of equation (1) which satisfies the boundary conditions. Considering equation (4), we will have

$$H(v, 0) = Ln(v) - Ln(u_0) = 0 \tag{6}$$

And,

$$H(v, 1) = M(v) - f(r) = 0 \tag{7}$$

The changing process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In topology this is called deformation and $Ln(v) - Ln(u_0)$ *and* $M(v) - f(r)$ are called homotopy.

According to the homotopy perturbation theory, we can first use the embedding parameter p as a small parameter and assume that the solution of equation (4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \dots\dots\dots\dots\dots\quad (8)$$

Setting $p = 1$ one has the approximation solution of equation (1) as the following if

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \dots\dots\dots\dots\dots\quad (9)$$

The series (9) is convergent for most cases. However, the convergent rate depends on the nonlinear operator $A(v)$.

3. Absolute Error in Percentage

Absolute error is the numerical difference between the true value of a quantity and its approximate value. Let X_H is Homotopy results and X_N is Numerical results value, then the absolute error in percentage is given by

$$A_E = \left| \frac{X_H - X_N}{X_H} \right| \times 100\% \quad (10)$$

4. An Example of Simple Harmonic Motion for Single Spring

The example of a system that demonstrates simple harmonic motion is a spring object system on a frictionless surface, shown in Figure 1.

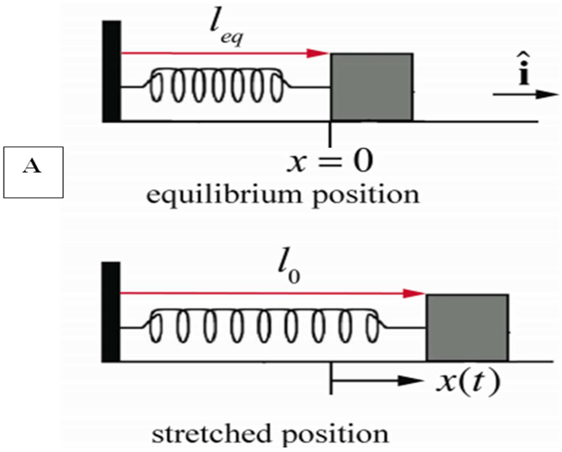

Figure 1. Spring-object system.

The object is attached to one end of a spring. The other end of the spring is attached to a wall at the left in Figure 1. Assume that the object undergoes one-dimensional motion. The spring has a spring constant k and equilibrium length l_{eq}. Choose the origin at the equilibrium position and choose the positive x- direction to the right in the Figure 1. In the figure, $x > 0$ corresponds to an extended spring, and $x < 0$ to a compressed spring. Define $x(t)$ to be the position of the object with respect to the equilibrium position. The force acting on the spring is a linear restoring force, $F_x = -kx$ (Figure 2). The initial conditions are as follows. The spring is initially stretched a distance l_0 and given some initial speed v_0 to the right away from the equilibrium position. The initial position of the stretched spring from the equilibrium position (our choice of origin) is $x_0 = l_0 - l_{eq} > 0$ and its initial x-component of the velocity is $v_{x,0} = v_0 > 0$.

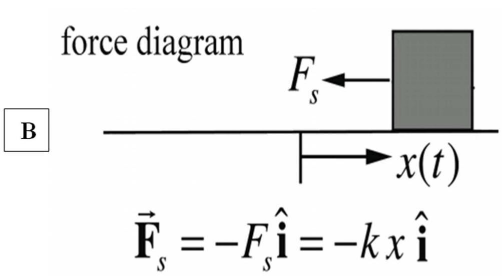

Figure 2. Free-body force diagram for spring-object system.

Based on the governing equation for this system, we would be able to list up all the component force acting upon to the system as shown fellow:

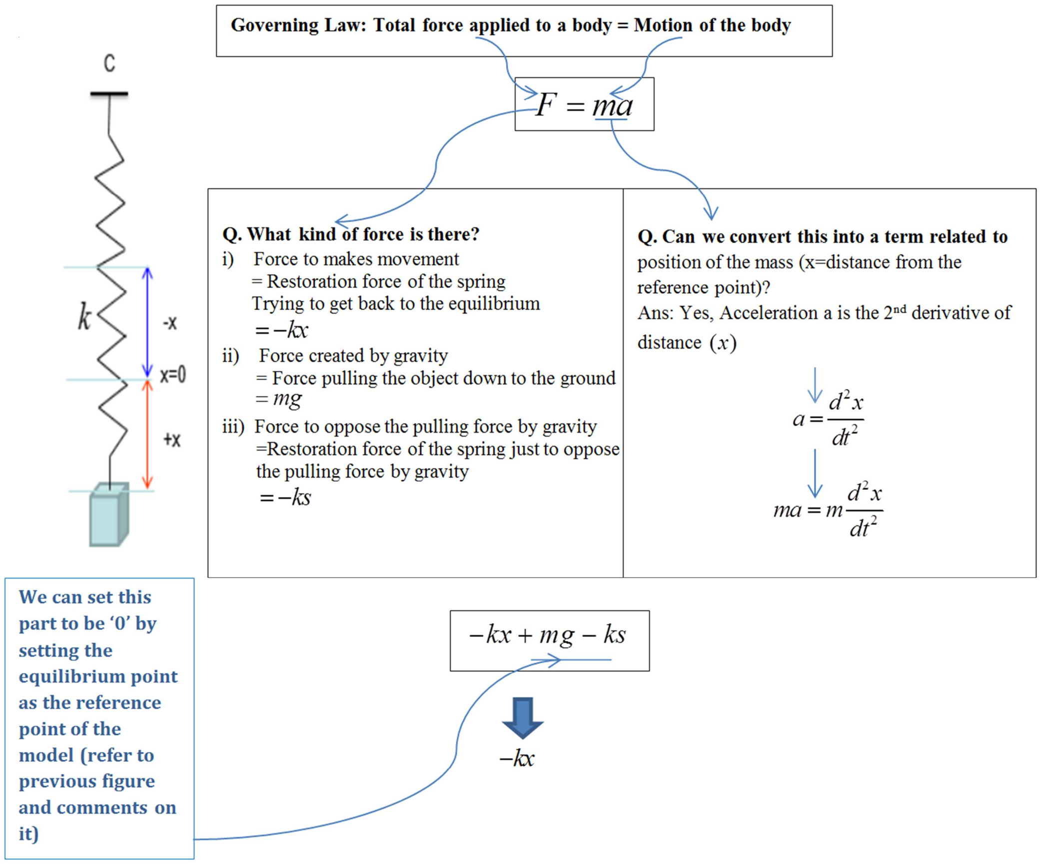

Figure 3. Mechanism of governing equation.

Combining these two parts we will get the equation (11) and with a little bit of rearrangement we will get the equation as (12). We may ask 'Do we have to rearrange (11) always? The answer is No. The equation (11) and (12) are completely same. The physical interpretation of (11) and (12) may vary a little bit, but mathematically they are same. We will see both forms for the exactly same physical model in various materials (e.g., textbook, internet etc.)

$$-kx = m\frac{d^2x}{dt^2} \tag{11}$$

This equation of motion, Eq. (11), is called the simple harmonic oscillator equation (SHO). Because the spring force depends on the distance x, the acceleration is not constant. Eq. (11) is a second order linear differential equation, in which the second derivative of the dependent variable is proportional to the negative of the dependent variable,

$$m\frac{d^2x}{dt^2} + kx = 0 \tag{12}$$

4.1. Damped Harmonic Motion of Single Spring

The differential equation of single spring is

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{13}$$

where, x is the angular displacement, t is the time, $\omega^2 = \frac{k}{m}$ is the natural frequency of the small oscillations of the spring, k spring constant and m is the mass of the spring.

The oscillations of the spring are subjected to the initial conditions

$$x(0) = a, x'(0) = 0 \tag{14}$$

where, a being the amplitude of the oscillations.

The spring equation with damped force is

$$x'' = -2hx' - \omega^2 x - \alpha x^3 \tag{15}$$

According to the Equation (4), we consider the following homotopy for the Equation (15) is

$$x''+2hx'+\omega^2x+\alpha x^3=0 \tag{16}$$

As above, the basic assumption is that the solutions of Equations (15) and (14) can be written as power series in p:

$$x=x_0+px_1+p^2x_2+\ldots\ldots \tag{17}$$

Therefore, substituting (17) into (16), also equating the terms with identical powers of *p,* we can obtain the following set of linear partial differential equations:

$$\left.\begin{aligned} p^0 &: \ \omega^2x_0+2hx_0'+x_0''=0 \quad & x_0(0)=a, x_0'(0)=0 \\ p^1 &: \ \alpha x_0^3+\omega^2x_1+2hx_1'+x_1''=0 \quad & x_1(0)=0, x_1'(0)=0 \end{aligned}\right\}$$

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Consequently, the first few components of the Homotopy perturbation solution for Eq. (15) are derived in the following form:

$$x_0=\frac{ae^{-ht}(\lambda Cos\lambda t+kSin\lambda t)}{\lambda}$$

$$x_1=\frac{3a^3e^{-3ht}\alpha Sin\lambda t}{4\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(h+\frac{\lambda^4}{h}-\frac{h^4}{\lambda}-\frac{13h^3}{2}\right)$$
$$+\frac{a^3e^{-3ht}\alpha Cos\lambda t}{2(h^2+\lambda^2)(h^2+4\lambda^2)}\left(\frac{11\lambda^2}{4}-\frac{15h^2}{4}-3\lambda^2-\frac{3}{4\lambda^2}+\frac{3h^4}{4\lambda^2}\right)$$
$$-\frac{a^3e^{-3ht}\alpha Cos3\lambda t}{(h^2+\lambda^2)(h^2+4\lambda^2)}\left(h^2-\frac{\lambda^2}{8}\right)+\frac{a^3e^{-kt}h\alpha Sin\lambda t}{\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(1-\frac{19h^2}{8}-\frac{37\lambda^2}{8}\right)$$
$$+\frac{23a^3e^{-kt}h^2\alpha Cos\lambda t}{8(h^2+\lambda^2)(h^2+4\lambda^2)}+\frac{a^3e^{-3kt}hCos2\lambda tSin\lambda t}{8\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(\frac{h^4}{2\lambda^2}-\frac{7h^2}{4}+\frac{9\lambda^2}{2}\right)$$

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Therefore, the solution of the equation (15) is

$$x(t)=x_0+x_1+x_2+\cdots\cdots=\frac{ae^{-kt}(\lambda Cos\lambda t+kSin\lambda t)}{\lambda}+\frac{3a^3e^{-3ht}\alpha Sin\lambda t}{4\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(h+\frac{\lambda^4}{h}-\frac{h^4}{\lambda}-\frac{13h^3}{2}\right)$$
$$+\frac{a^3e^{-3ht}\alpha Cos\lambda t}{2(h^2+\lambda^2)(h^2+4\lambda^2)}\left(\frac{11\lambda^2}{4}-\frac{15h^2}{4}-3\lambda^2-\frac{3}{4\lambda^2}+\frac{3h^4}{4\lambda^2}\right)$$
$$-\frac{a^3e^{-3ht}\alpha Cos3\lambda t}{(h^2+\lambda^2)(h^2+4\lambda^2)}\left(h^2-\frac{\lambda^2}{8}\right)+\frac{a^3e^{-kt}h\alpha Sin\lambda t}{\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(1-\frac{19h^2}{8}-\frac{37\lambda^2}{8}\right)$$
$$+\frac{23a^3e^{-kt}h^2\alpha Cos\lambda t}{8(h^2+\lambda^2)(h^2+4\lambda^2)}+\frac{a^3e^{-3kt}hCos2\lambda tSin\lambda t}{8\lambda(h^2+\lambda^2)(h^2+4\lambda^2)}\left(\frac{h^4}{2\lambda^2}-\frac{7h^2}{4}+\frac{9\lambda^2}{2}\right)$$

The figure of above solutions is given Figure 4 to Figure 11

4.2. Undamped Harmonic Motion of Single Spring

The undamped spring equation is

$$x''=-\omega^2x-\alpha x^3 \tag{18}$$

According to the Equation (4), we consider the following homotopy for the Equation (18) is

$$x''+\omega^2x+\alpha x^3=0 \tag{19}$$

As above, the basic assumption is that the solutions of Equations (18) and (19) can be written as power series in p:

$$x=x_0+px_1+p^2x_2+\ldots\ldots \tag{20}$$

Therefore, substituting (20) into (19), also equating the terms with identical powers of *p,* we can obtain the following set of linear partial differential equations:

$$p^0: \ \omega^2x_0+x_0''=0 \quad x_0(0)=a, x_0'(0)=0$$

$$p^1: \ \omega^2x_1+\alpha x_0^3+x_1''=0 \quad x_1(0)=0, x_1'(0)=0$$

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Consequently, the first few components of the homotopy perturbation solution for Eq. (18) are derived in the following form:

$$x_0 = aCos\lambda t$$

$$x_1 = -\frac{3a^3 t\alpha Sin\lambda t}{8\lambda} - \frac{a^3 \alpha Sin\lambda t Sin2\lambda t}{16\lambda^2}$$

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Therefore, the solution of the equation (18) is

$$x(t) = x_0 + x_1 + x_2 + ...$$

$$= aCos\lambda t - \frac{3a^3 t\alpha Sin\lambda t}{8\lambda} - \frac{a^3 \alpha Sin\lambda t Sin2\lambda t}{16\lambda^2}$$

The figure of above solutions is given Figure 7 to Figure 10.

5. Results and Discussions

The Homotopy perturbation method is successfully applied to solve the spring equation. To test the accuracy of our results, we match our results with the numerical results obtained by the Mathematica 9.0. The solution obtained by Homotopy perturbation method is an infinite series for appropriate initial condition. The spring damping constant h, nonlinear constant ω^2, constant of spring is k are taken the appropriate values for solving the problems and drawing the natural figure. The solutions are described in the article 4.1 and 4.2. The corresponding numerical solutions that have been computed by the Mathematica 9.0 program for various values of t and all the results are showed in their corresponding figures in Figure 3 to Figure 10 respectively. These solutions produce a wide variety of interesting motions.

Table 1. Comparison between Homotopy results and Numerical results for $k = 50, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
2	−0.0130090	−0.0130089	0.00146
4	−0.0248240	−0.0248239	0.00076
6	0.0253234	0.0253231	0.00149
8	−0.0002664	−0.0002662	0.02015
10	−0.0168009	−0.0168006	0.00205
12	0.0113862	0.0113865	0.00221
14	0.0036695	0.0036697	0.00251
16	−0.0100797	−0.0100795	0.00242
18	0.0042597	0.0042596	0.00217
20	0.0039173	0.0039174	0.00266

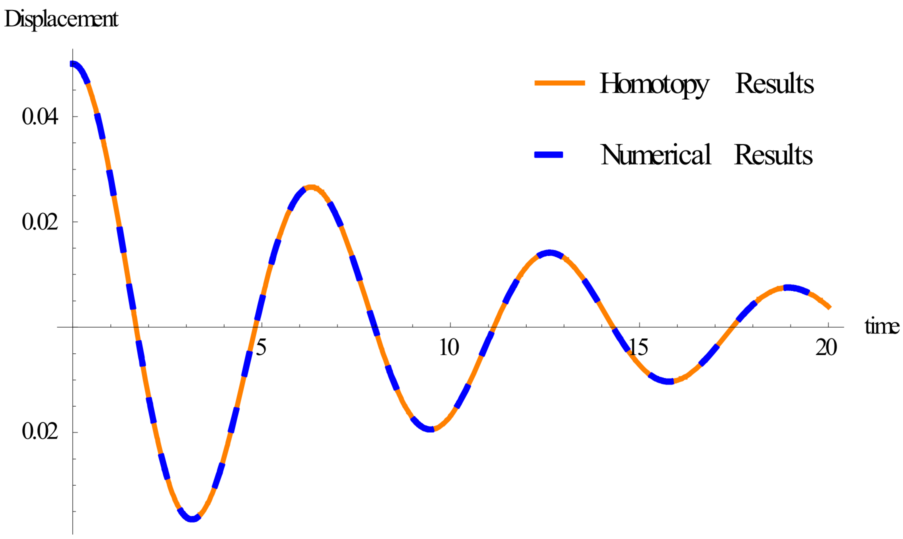

Figure 4. The Spring Equation for $k = 50, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 2. Comparison between Homotopy results and Numerical results for $k = 100, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
2	0.0412011	0.0412021	0.00230
4	0.0218352	0.0218351	0.03419
6	0.0018199	0.0018340	0.76966
8	−0.0228749	−0.0228716	0.01415
10	−0.0100513	−0.0100514	0.00760
12	0.0047522	0.0047525	0.03579
14	0.0117434	0.0117443	0.02071
16	0.0207150	0.0207152	0.01727
18	0.0017989	0.0017998	0.00033
20	−0.0042595	−0.0042598	0.02528

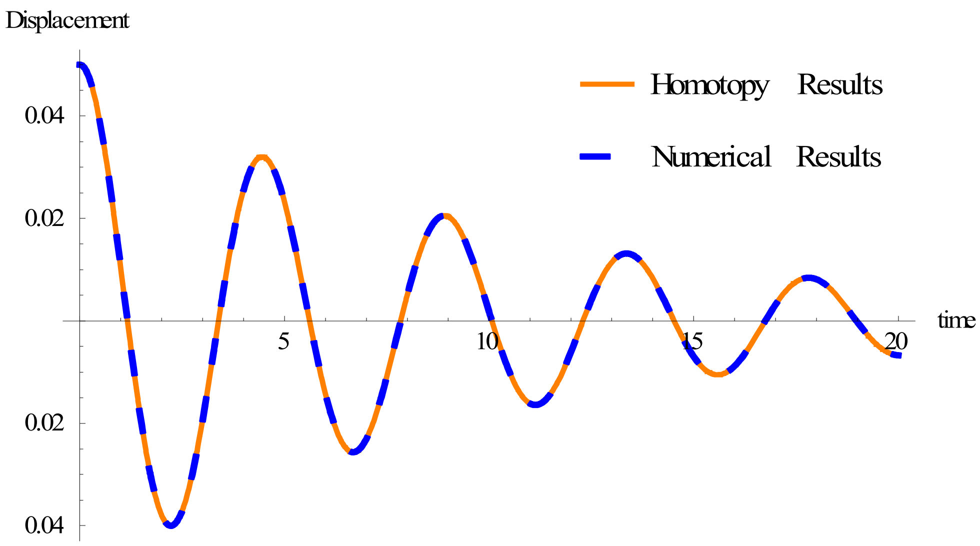

Figure 5. The Spring Equation for $k = 100, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 3. Comparison between Homotopy results and Numerical results for $k = 300, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
2	0.0258683	0.0258684	0.00243
4	−0.0135708	−0.0135688	0.01433
6	−0.0278460	−0.0278462	0.00705
8	−0.0124406	−0.0124408	0.00541
10	0.0090292	0.0090293	0.01302
12	0.0153405	0.0153420	0.01035
14	0.0058407	0.0058409	0.00868
16	0.0207150	0.0207152	0.01727
18	−0.0083787	−0.0083789	0.01108
20	−0.0026439	−0.0026440	0.00892

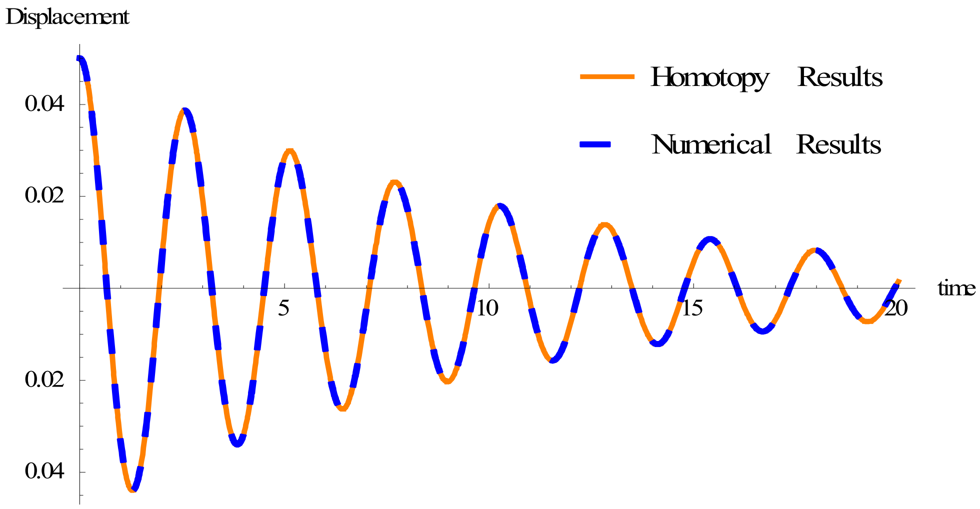

Figure 6. The Spring Equation for $k = 300, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 4. Comparison between Homotopy results and Numerical results for $k = 900, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
2	0.0065798	0.0065810	0.00541
4	−0.0331884	−0.0331886	0.00193
6	−0.0062002	−0.0062003	0.00014
8	0.0219130	0.0219123	0.00337
10	0.0053704	0.0053708	0.00335
12	−0.0143909	−0.0143910	0.00409
14	−0.0058407	−0.0058409	0.00868
16	0.0093993	0.0093939	0.00440
18	0.0034837	0.0034873	0.00336
20	−0.0061041	−0.0061062	0.00455

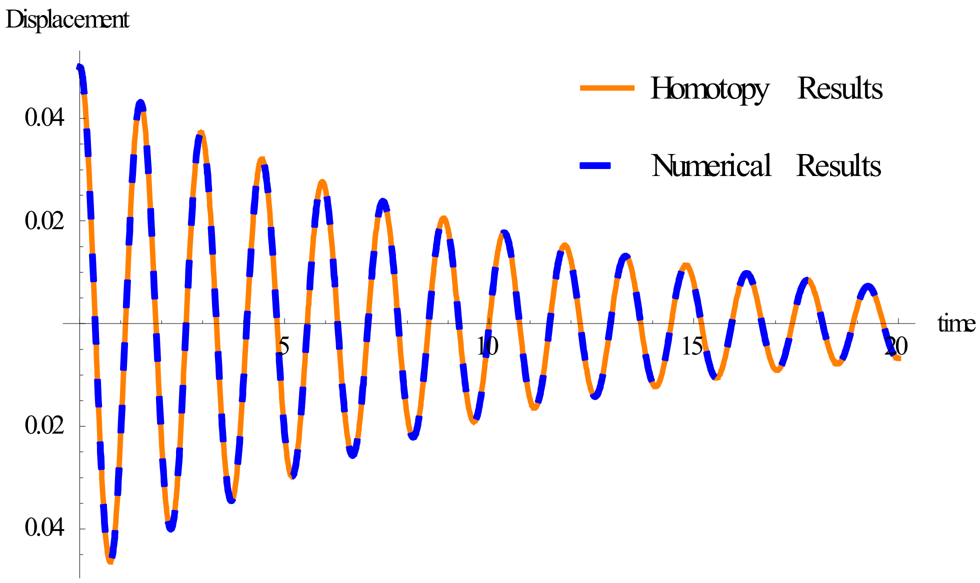

Figure 7. The Spring Equation for $k = 900, m = 50, h = 0.1, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 5. Comparison between Homotopy results and Numerical results for $k = 15, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	−0.0462354	−0.0462358	0.00830
10	0.0355189	0.0355058	0.03709
15	−0.0194772	−0.0194778	0.11105
20	0.0457693	0.0457698	0.10188
25	0.0185570	0.0185578	0.17241
30	−0.0348917	−0.0347879	0.29858
35	0.0460263	0.0460265	0.41372
40	−0.0502523	−0.0502532	0.52738
45	0.0469077	0.0466057	0.64806

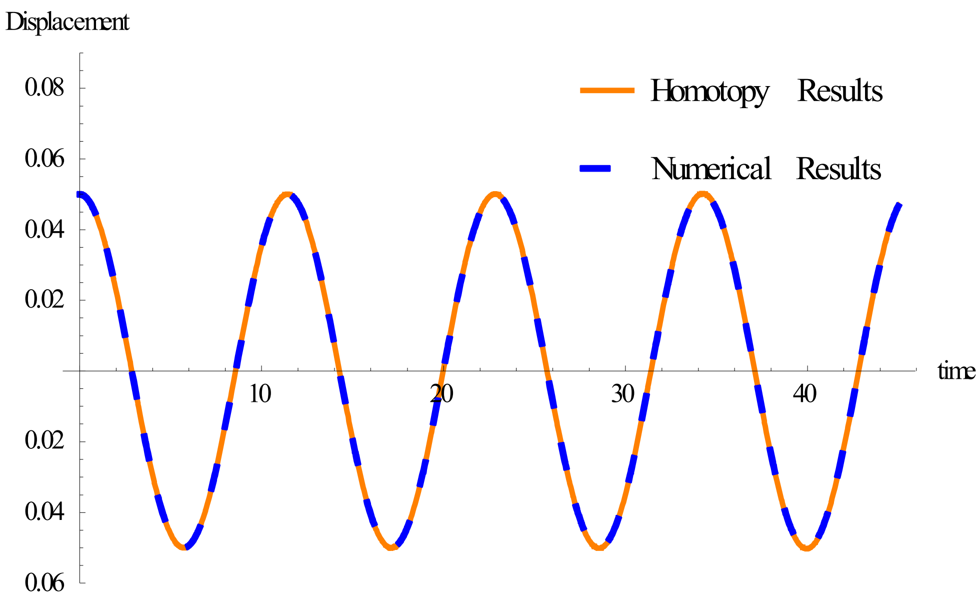

Figure 8. The Spring Equation for $k = 15, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 6. Comparison between Homotopy results and Numerical results for $k = 30, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	−0.0368962	−0.0368964	0.00333
10	0.0044836	0.0044837	0.02893
15	0.0302794	0.0302796	0.04198
20	−0.0492252	−0.0492255	0.06612
25	0.0423790	0.0423792	0.10336
30	−0.0133275	−0.0133278	0.12405
35	−0.0227127	−0.0227138	0.21089
40	0.0469194	0.0469106	0.26043
45	−0.0465689	−0.0464111	0.33986

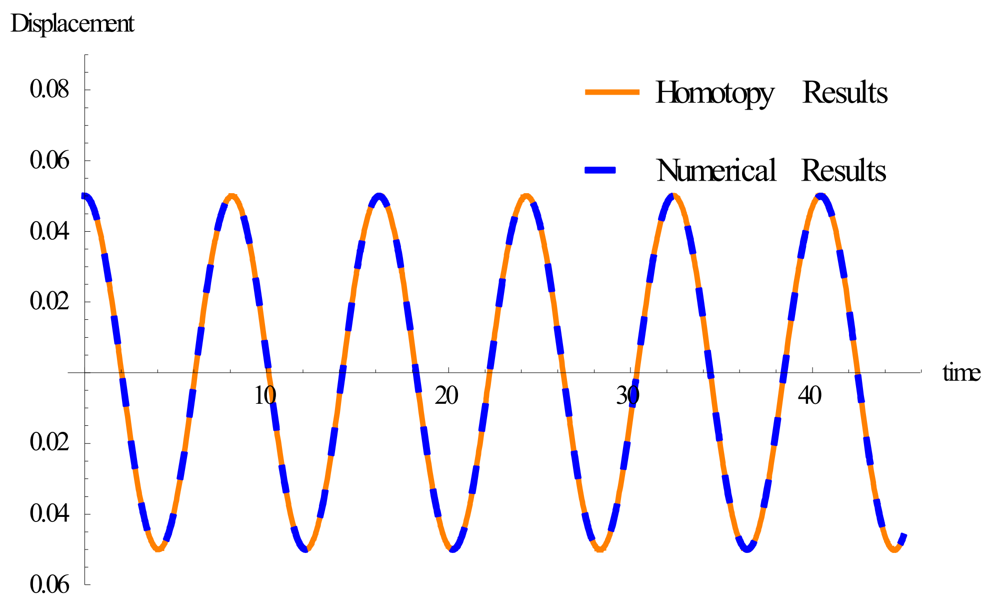

Figure 9. The Spring Equation for $k = 30, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 7. Comparison between Homotopy results and Numerical results for $k = 50, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

time	Homotopy results	Numerical results	Error in %
0	0.0499707	0.05	0.05859
5	0.0145058	0.0145132	0.05147
10	−0.0415406	−0.0415407	0.04962
15	−0.0386404	−0.0386405	0.03461
20	0.0191003	0.0191059	0.02943
25	0.0497633	0.0497636	0.00309
30	0.0097886	0.0097888	0.05814
35	−0.0441002	−0.0441005	0.06206
40	−0.0354143	−0.0354149	0.10046
45	0.0235494	0.0235497	0.13794

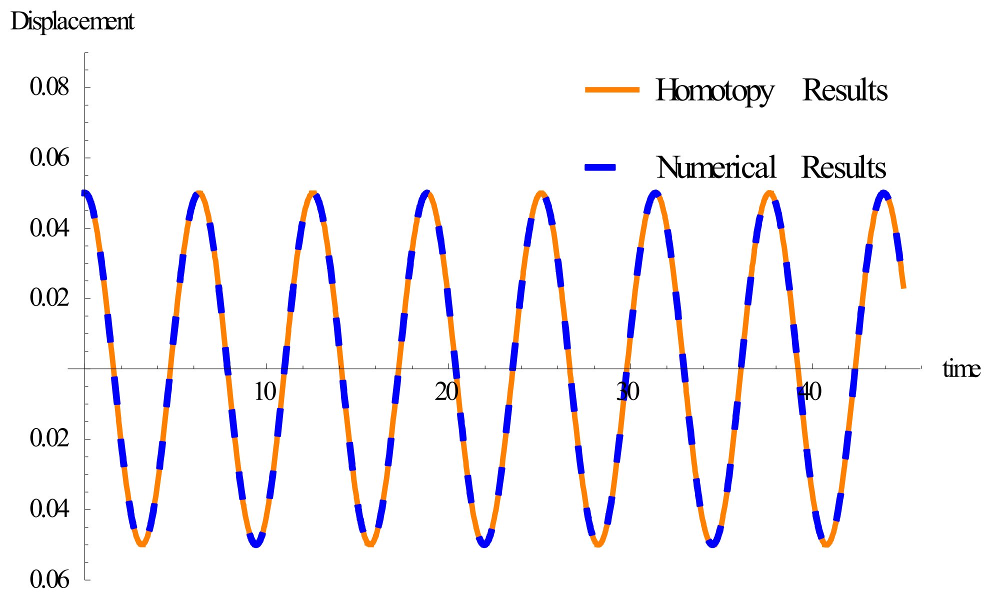

Figure 10. The Spring Equation for $k = 50, m = 50, h = 0, a = 0.05, \alpha = 1.5$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 8. Comparison between Homotopy results and Numerical results for $k=100, m=50, h=0, a=0.05, \alpha=1.5$ and the initial conditions $x(0)=0.05, x'(0)=0$.

time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	0.0350870	0.0350872	0.00097
10	−0.0007455	−0.0007456	0.09251
15	−0.0361371	−0.0361374	0.01292
20	−0.0499876	−0.0499879	0.02037
25	−0.0340195	−0.0340198	0.02936
30	0.0022367	0.0022364	0.11941
35	0.0371696	0.0371707	0.06358
40	0.0499505	0.0499522	0.08022
45	0.0329348	0.0329352	0.09920

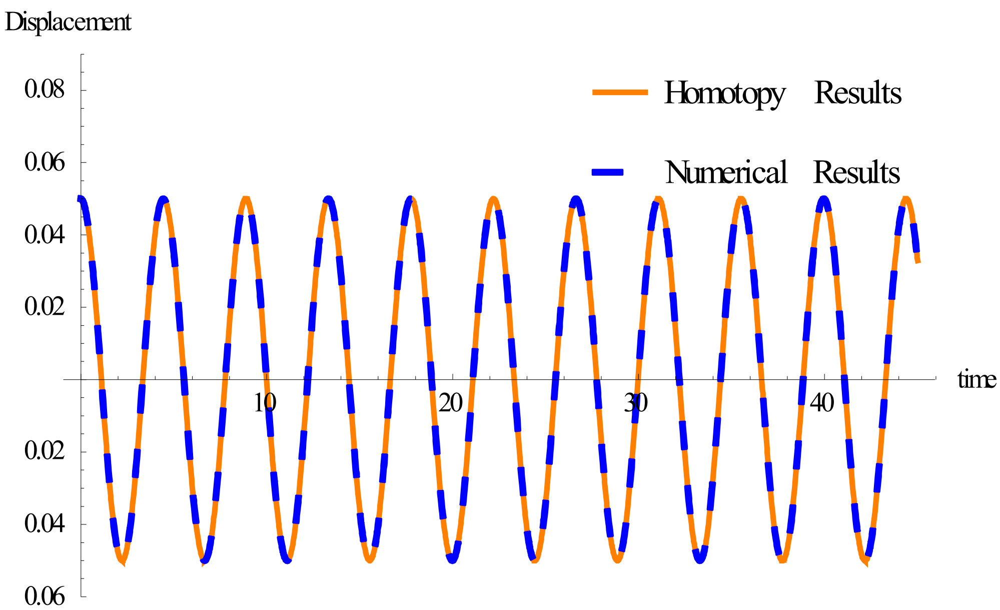

Figure 11. The Spring Equation for $k=100, m=50, h=0, a=0.05, \alpha=1.5$ and the initial conditions $x(0)=0.05, x'(0)=0$.

6. Conclusion

In this research, the homotopy perturbation method has been successfully used to solve the single spring equation. The solution obtained by means of the homotopy perturbation method is an infinite power series for appropriate initial condition, which can be intern, expressed in a closed form. The results obtained here has compared with the exact solutions and the results reported by using other method. All the figures show that the results of the present method are in excellent agreement with numerical solutions obtained by the Mathematica program. The HPM has got many merits and much more advantages. Also the HPM does not require small parameters in the equation, so that the limitations of the traditional perturbation methods can be eliminated and also the calculations in the HPM are simple and straightforward. The reliability of the method and reduction in the size of computational domain give this method a wider applicability. Therefore, it is not affected by rounding the errors computing process. Also, it may conclude that HPM is very effective technique to find the analytical solutions for highly non-linear ordinary differential equation with initial conditions or boundary conditions.

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Biography

Mohammad Masud Rana was born on October 21, 1993 in Natore, Bangladesh. He received B.Sc. in Mathematics from Islamic University, Bangladesh in 2015, Master in Mathematics, Islamic University, Bangladesh in 2016. He has been completed several project and thesis under his supervision. Currently he is doing research on Chain moving system in Homotopy perturbation method. His research interests include computational Mathematics like as, Mathematical physics, Nonlinear Oscillation, PDE, Numerical analysis and number theory.

Ahammodullah Hasan was born on October 12, 1994 in Kushtia, Bangladesh. He received B.Sc. in Mathematics from Islamic University, Bangladesh in 2015, Master in Mathematics, Islamic University, Bangladesh in 2016. He has been completed several project and thesis under his supervision, his some research paper has been published in peer-review international journal. Currently he is doing research on various iterative Method, approximation method, fuzzy logic obtaining weather forecast with neural-network. His research interests include computational Mathematics and applied Mathematics like as, Mathematical physics, Nonlinear Oscillation, PDE, Numerical analysis and number theory.