International Journal of Mathematics and Computational Science
Vol. 5, No. 2, 2019, pp. 24-35
http://www.aiscience.org/journal/ijmcs
ISSN: 2381-7011 (Print); ISSN: 2381-702X (Online)

Approximate Solution of Simple Pendulum Equation for Damped and Undamped Oscillatory Motion by Using Homotopy Perturbation Method

Ahammodullah Hasan*, Mohammad Masud Rana

Department of Mathematics, Faculty of Science, Islamic University, Kushtia, Bangladesh

Abstract

In this article, Homotopy perturbation method (HPM) is applied to find the approximate solution of free oscillation for simple pendulum equation, which is known a well-known nonlinear ordinary differential equation. The Homotopy Perturbation Method deforms a difficult problem into a simple problem which can be easily solved. Firstly, the approximate solution of simple pendulum equation is developed using initial conditions, and then results are compared with the results obtained by numerical solutions. Finally, Homotopy Perturbation Method (HPM) is applied to find the approximate solution of simple pendulum equation with initial conditions. The model absolute error was found below the 5% significance level. The absolute error between homotopy result and numerical result was average 0.0003%. In this study, we have found the Homotopy result and numerical result makes a good agreement. The results reveal that the HPM is very effective, convenient and quite accurate to systems of nonlinear equations. Some examples are presented to show the ability of the method for free oscillations of simple Pendulum equation.

Keywords

Homotopy Perturbation Method (HPM), Simple Pendulum Equation (SPE), Nonlinear Differential Equation, Damped, Undamped, Approximate Solution, Numerical Solution

Received: March 22, 2019 / Accepted: May 20, 2019 / Published online: June 6, 2019

1. Introduction

Mathematical formulations of many physical problems often result in differential equations that are nonlinear. However, in many cases it is possible to replace a nonlinear differential equation with a related linear differential equation that approximates the actual nonlinear equation closely enough to given useful results. Often such linearization is not possible or feasible, when it is not, the original nonlinear equation itself must be trickled. The general theory and methods of dealing with linear equations constitute a highly developed branch of mathematics, whereas very little of a general nature is known about nonlinear equation. Generally the study of nonlinear equations is confined to a variety of very special cases, and the method of solution usually involves one or more of a limited number of differential method of approximation, in this study we are used for approximation result using Homotopy perturbation method and comparing numerical solution. In the last three decades with the rapid development of nonlinear science, there has appeared ever increasing interest of Mathematics, Physicists and Engineers in the analytical techniques for nonlinear problems. It is well known, that perturbation methods provide the most versatile tools available in nonlinear analysis of engineering problems [1-3]. The perturbation methods, like other nonlinear analytical techniques, have their own limitations. At first, almost all perturbation methods are based on the assumption that a small parameter must exist in the equation. This so-called small parameter assumption greatly restricts applications of perturbation techniques. As is well known, an

* Corresponding author
E-mail address: ahammodullah.iu@gmail.com (A. Hasan)

overwhelming majority of nonlinear problems have no small parameters at all. Secondly, the determination of small parameters seems to be a special art requiring special techniques. An appropriate choice of small parameters leads to the ideal results, but an unsuitable choice may create serious problems. Furthermore, the approximate solutions solved by perturbation methods are valid, in most cases, only for the small values of the parameters [30-32]. It is obvious that all these limitations come from the small parameter assumption. These facts have motivated to suggest alternate techniques, such as variational iteration [4-5] decomposition [6], exp-function [7], variation of parameters [8] and iterative [9]. In order to overcome these drawbacks, combining the standard Homotopy and perturbation method, which is called the Homotopy perturbation, modifies the Homotopy method. Many problems in natural and engineering sciences are modeled by partial differential equations (PDEs) [33-35]. These equations arise in a number of scientific models such as the propagation of shallow water waves, long wave and chemical reaction-diffusion models [10-11]. A substantial amount of work has been invested for solving such models. Several techniques including the method of characteristic, Riemann invariants, combination of waveform relaxation and multi-grid, periodic multi-grid wave form, variational iteration, Homotopy perturbation and Adomian's decomposition [10, 11] have been used for the solutions of such problems. Most of these techniques encounter the inbuilt deficiencies and involve huge computational work. He [2] developed the Homotopy perturbation method for solving linear, nonlinear, initial, and boundary value problems by merging two techniques, the standard Homotopy and the perturbation technique. The Homotopy perturbation method was formulated by taking the full advantage of the standard Homotopy and perturbation methods and has been applied to a wide class of functional equations [1, 2]. The HPM gives the solution in the form of a convergent series with easily computable components. Unlike the method of separation of variables which requires both initial and boundary conditions, the HPM gives the solution by using the initial conditions only [12-16]. In a series of papers He [17, 18, 19, 20], has outlined and refined the HPM, showing its usefulness by nonlinear differential equations [22-25]. As a rule, HPM tends to produce much more elegant solutions as compared to the other competing techniques such as Homotopy analysis method (HAM), regular perturbation methods etc., yet it is not at the cost of accuracy In particular the proposed Homotopy perturbation method (HPM) is tested on single degree of freedom system and spring loaded inverted pendulum, linear and nonlinear single degree of freedom system equations, with damping and without forcing and spring loaded inverted pendulum equation, with damping and adding forcing [26-29]. The proposed iterative scheme finds the solution without any discretization, linearization or restrictive assumptions and is free from round off errors. The solutions obtained by using Mathematica program 9.0. In this paper, attention is taken on the simple pendulum equation that is nonlinear systems, which can be expressed as $m\frac{d^2s}{dt^2} + mgsin(\frac{S}{L}) = 0$ with initial condition$x(0) = a, x'(0) = 0$. In section 2, the formulation of homotopy perturbation method. The above systems are proved by using the method in reference [1]. And in Section 3 gives an example for the above systems and obtains two types of analytical approximation via two different sets of base functions that is damped and undamped, and the problem is further solution by homotopy approximation and numerical approximation. In section 4 drawing the all figure and table with discuss the result part of this study and conclusions are drawn in Section 5 finally.

2. Formulation of Homotopy Perturbation Method

To illustrate the Homotopy perturbation method, we consider a general equation of the type,

$$M(u(x)) - f(r) = 0, \quad r \in \Omega \tag{1}$$

with the boundary conditions

$$N(u, \frac{du}{dx}) = 0, \quad r \in \xi \tag{2}$$

Where M is a general differential operator, N is a boundary operator, ξ is the boundary domain of Ω and $f(r)$ is a known analytical function. Generally speaking, the operator M can be divided into a linear part Ln and a nonlinear part Nl. Now equation (1) can be rewritten as:

$$Ln(u(x)) + Nl(u(x)) - f(r) = 0 \tag{3}$$

By the homotopy perturbation method, we construct a Homotopy as $v(r, p): \Omega \times [0,1] \to \mathbb{R}$ which satisfies

$$H(v, p) = (1-p)[Ln(v) - Ln(u_0)] + p[M(v) - f(r)] = 0 \tag{4}$$

or

$$H(v, p) = Ln(v) - Ln(u_0) + pLn(u_0) + p[Nl(v) - f(r)] = 0 \tag{5}$$

where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of equation (1) which satisfies the boundary conditions. Considering equation (4), we will have

$$H(v, 0) = Ln(v) - Ln(u_0) = 0 \tag{6}$$

And,

$$H(v,1) = M(v) - f(r) = 0 \quad (7)$$

The changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology this is called deformation and $Ln(v) - Ln(u_0)$ *and* $M(v) - f(r)$ are called homotopy.

According to the homotopy perturbation theory, we can first use the embedding parameter p as a small parameter and assume that the solution of equation (4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \dots\dots\dots\dots \quad (8)$$

Setting $p = 1$ one has the approximation solution of equation (1) as the following if

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \dots\dots\dots\dots \quad (9)$$

The series (9) is convergent for most cases. However, the convergent rate depends on the nonlinear operator $A(v)$.

3. Absolute Error in Percentage

Absolute error is the numerical difference between the true value of a quantity and its approximate value. Let X_H is Homotopy results and X_N is Numerical results value, then the absolute error in percentage is given by

$$A_E = \left| \frac{X_H - X_N}{X_H} \right| \times 100\% \quad (10)$$

4. An Example of Physical Explanation of Simple Pendulum

Perhaps the simplest nonlinear vibrating system is composed of the free oscillations of a pendulum. The pendulum consists of a particle of much m attached to the end of a light inextensible rod, with motion taking place in a vertical plane. As shown Figure 1 below.

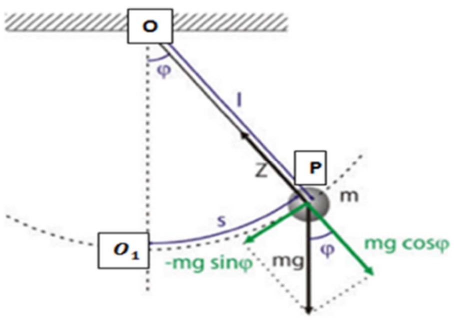

Figure 1. Physical figure of simple pendulum.

Let φ be the angle between the vertical and line OP. Where O is the centre of the circular path and P is the instantaneous position of the particle. The distance S is measured from the equilibrium position O_1. From the figure it is easy to see that the component of the force of gravity mg in the direction of S is equal to the $-mgSin\varphi$. If is the length of the pendulum, then the angle φ is equal to $\frac{S}{L}$ and the equation of motion is

$$m\frac{d^2S}{dt^2} + mgSin(\frac{S}{L}) = 0 \quad (11)$$

In terms of φ, we may write

$$\frac{d^2\varphi}{dt^2} + (\frac{g}{L})Sin\varphi = 0 \text{; where } \omega = \sqrt{\frac{g}{L}} \quad (12)$$

If we put $\varphi = x$ then we have from equation (12)

$$\frac{d^2x}{dt^2} + (\frac{g}{L})Sinx = 0 \text{; where } \omega = \sqrt{\frac{g}{L}} \quad (13)$$

4.1. Solution of Damped Pendulum Oscillatory Motion

Consider the differential equation of simple pendulum is

$$\frac{d^2x}{dt^2} + \omega^2 Sinx = 0 \quad (14)$$

where x is the angular displacement, t is the time, $\omega^2 = \frac{g}{L}$ is the natural frequency of the small oscillations of the pendulum, L is the length of the pendulum and g is the acceleration due to gravity.

The oscillations of the pendulum are subjected to the initial conditions.

$$x(0) = a, x'(0) = 0 \quad (15)$$

where a being the amplitude of the oscillations.

The periodic solution $x(t)$ of equation (14) and the period depend on the amplitude a.

For small angles, we may use the series expansion.

$$Sinx = x - \frac{x^3}{6} + \frac{x^5}{120} - \dots\dots \quad (16)$$

Equation (14) can be re-written as follows:

$$x'' + \omega^2 x - \frac{1}{6}\omega^2 x^3 = 0 \quad (17)$$

The simple pendulum equation with damping force is

$$x'' = -2kx' - \omega^2 x + \frac{1}{6}\omega^2 x^3 \tag{18}$$

According to the Equation (4), we consider the following Homotopy for the Equation (18) is

$$x'' + 2kx' + \omega^2 x - \frac{1}{6}\omega^2 x^3 = 0 \tag{19}$$

As above, the basic assumption is that the solutions of Equations (18) and (19) can be written as power series in p:

$$x = x_0 + px_1 + p^2 x_2 + \ldots\ldots\ldots\ldots \tag{20}$$

Therefore, substituting (20) into (19), also equating the terms with identical powers of p, we can obtain the following set of linear partial differential equations:

$$\left.\begin{array}{lll} p^0: & \omega^2 x_0 + 2kx_0' + x_0'' = 0 & x_0(0) = a, x_0'(0) = 0 \\ p^1: & \omega^2 x_1 - \frac{1}{6}\omega^2 x_0^3 + 2kx_1' + x_1'' = 0 & x_1(0) = 0, x_1'(0) = 0 \\ \ldots.: & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ \ldots.: & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \end{array}\right\} \tag{21}$$

Consequently, the first few components of the Homotopy perturbation solution for Eq. (18) are derived in the following form:

$$x_0 = e^{-(k+\sqrt{k^2-\lambda^2})t}\left(\frac{a\sqrt{k^2-\lambda^2} - ak}{2\sqrt{k^2-\lambda^2}} + \frac{ak + a\sqrt{k^2-\lambda^2}}{2\sqrt{k^2-\lambda^2}} e^{2\sqrt{k^2-\lambda^2}}\right) \tag{22}$$

$$\begin{aligned} x_1 = & \frac{52a^3 e^{-kt} k^4 \sqrt{\lambda^2}\, Cos(t\lambda)}{3\lambda(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} + \frac{12a^3 e^{-kt} k^4 \lambda\sqrt{\lambda^2}\, Cos(t\lambda)}{98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5} + \frac{5a^3 e^{-kt} k^4 \lambda^3 \sqrt{\lambda^2}\, Cos(t\lambda)}{3(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & + \frac{99ae^{-kt} k^3 \lambda^3 Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5} + \frac{25ae^{-kt} \lambda^3 Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5} \\ & - \frac{97a^3 e^{-3kt} \lambda^3 \sqrt{\lambda^2}\, Cos\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{48\lambda(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{9a^3 e^{-3kt} k^2 \lambda\sqrt{\lambda^2}\, Cos\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{8(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & + \frac{245ae^{-3kt} k^4 \sqrt{\lambda^2}\, Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{16\lambda(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{245ae^{-kt} k^3 \sqrt{\lambda^2}\, Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{16(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & - \frac{87a^2 e^{-3kt} k^3 \sqrt{\lambda^2}\, Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{8(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{25a^3 e^{-3kt} \lambda^3 \sqrt{\lambda^2}\, Cos\left(t\sqrt{2k^2-\lambda^2}\right)}{16(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & - \frac{5a^3 e^{-3kt} \lambda^2 \sqrt{\lambda^2}\, Cos\left(3t\sqrt{2k^2-\lambda^2}\right)}{48(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{14a^3 e^{-kt} k^5 Sin(t\lambda)}{3\sqrt{\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & - \frac{5a^3 e^{-3kt} \lambda^2 \sqrt{\lambda^2}\, Cos\left(3t\sqrt{2k^2-\lambda^2}\right)}{48(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{14a^3 e^{-kt} k^5 Sin(t\lambda)}{3\sqrt{\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & + \frac{161a^3 e^{-3kt} \lambda^2 \sqrt{\lambda^2}\, Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{16\lambda\sqrt{2k^2-\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} + \frac{137a^3 e^{-3kt} \lambda^2 \sqrt{\lambda^2}\, Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{48\lambda\sqrt{2k^2-\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & + \frac{55a^3 e^{-3kt} k^3 \lambda\sqrt{\lambda^2}\, Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{24\sqrt{2k^2-\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} - \frac{7a^3 e^{-3kt} k\lambda^3 \sqrt{\lambda^2}\, Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{16\sqrt{2k^2-\lambda^2}\,(98k^4\lambda + 99k^2\lambda^2 + 25\lambda^5)} \\ & \ldots\ \ldots\ \ldots \\ & \ldots\ \ldots\ \ldots \end{aligned}$$

Therefore, the solution of the equation (18) is

$$x(t) = x_0 + x_1 + x_2 + ...$$

$$= e^{-(k+\sqrt{-k^2-\lambda^2})t}\left(\frac{a\sqrt{-k^2-\lambda^2}-ak}{2\sqrt{-k^2-\lambda^2}}+\frac{ak+a\sqrt{-k^2-\lambda^2}}{2\sqrt{-k^2-\lambda^2}}e^{2\sqrt{-k^2-\lambda^2}}\right)$$

$$+\frac{a^3e^{-kt}k^4Cos(\lambda t)}{(98k^4\lambda+99k^2\lambda^2+25\lambda^5)}\left(\frac{52}{3}+12\lambda+\frac{5\lambda^4}{3}\right)+\frac{ae^{-kt}k^3\lambda^3Cos\left(\sqrt{2k^2-\lambda^2}\,t\right)}{98k^4\lambda+99k^2\lambda^2+25\lambda^5}\left(99k^3+25-\frac{245k^3\lambda}{16}\right)$$

$$+\frac{a^3e^{-kt}k^3\sqrt{\lambda^2}\,Sin(\lambda t)}{98k^4\lambda+99k^2\lambda^2+25\lambda^5}\left(3k^3-\frac{14}{3\lambda^2}\right)-\frac{a^2e^{-3kt}k^3\sqrt{\lambda^2}\,Cos\left(\sqrt{2k^2-\lambda^2}\,t\right)}{98k^4\lambda+99k^2\lambda^2+25\lambda^5}\left(\frac{245}{16a}-\frac{87k^2}{8}-\frac{25a\lambda^3}{16}\right)$$

$$-\frac{a^3e^{-3kt}k^3\sqrt{\lambda^2}\,Cos\left(\sqrt{2k^2-\lambda^2}\,t\right)}{98k^4\lambda+99k^2\lambda^2+25\lambda^5}\left(-\frac{97\lambda^2}{8}-\frac{9ak^2}{8}\right)-\frac{a^3e^{-3kt}\lambda^3Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{\sqrt{2k^2-\lambda^2}\,(98k^4\lambda+99k^2\lambda^2+25\lambda^5)}\left(\frac{99}{16}+\frac{161}{16\lambda^2}\right)$$

$$-\frac{a^3e^{-3kt}\lambda^3Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{\sqrt{2k^2-\lambda^2}\,(98k^4\lambda+99k^2\lambda^2+25\lambda^5)}\left(\frac{137}{48\lambda^2}\right)-\frac{a^3e^{-3kt}Sin\left(3\sqrt{2k^2-\lambda^2}\,t\right)}{\sqrt{2k^2-\lambda^2}\,(98k^4\lambda+99k^2\lambda^2+25\lambda^5)}\left(\frac{55}{24}-\frac{7k\lambda^3}{16}\right)$$

The figure of above solutions is given Figure 2 - Figure 5. The Figure 2- Figure 5 depicts the phase portrait curves of [0, 45] homotopy approximation and that of numerical approximation of HMP method, in which, the matched interval is at least expanded into [0, 45], compared with Table 1-Table 4 are good agreement of the solution of simple pendulum equation by using equation (10).

4.2. Solution of Undamped Pendulum Oscillatory Motion

The Undamped pendulum equation is

$$x'' = -\omega^2\,x + \frac{1}{6}\omega^2 x^3 \tag{23}$$

According to the Equation (4), we consider the following Homotopy for the Equation (23) is

$$x'' + \omega^2\,x - \frac{1}{6}\omega^2 x^3 = 0 \tag{24}$$

As above, the basic assumption is that the solutions of Equations (23) and (24) can be written as power series in p:

$$x = x_0 + px_1 + p^2x_2 + ... \tag{25}$$

Therefore, substituting (25) into (24), also equating the terms with identical powers of *p,* we can obtain the following set of linear partial differential equations:

$$\left.\begin{array}{lll} p^0: & \omega^2 x_0 + x_0'' = 0 & x_0(0) = a, x_0'(0) = 0 \\ p^1: & \omega^2 x_1 - \frac{1}{6}\omega^2 x_0^3 + x_1'' = 0 & x_1(0) = 0, x_1'(0) = 0 \\ ....: & .... & .... \\ ....: & .... & .... \end{array}\right\} \tag{26}$$

Consequently, the first few components of the Homotopy perturbation solution for Eq. (17) are derived in the following form:

$$x_0 = aCos(\lambda t) \tag{27}$$

$$x_1 = \left(a\cos\omega t - \frac{a^3\alpha\sqrt{\omega^2}\cos\omega t}{32\omega^4} + \frac{a^3\alpha\sqrt{\omega^2}\cos^3\omega t}{32\omega^4} - \frac{3a^3\alpha t\sqrt{\omega^2}\sin\omega t}{8\omega^3} - \frac{3a^3\alpha\sqrt{\omega^2}\cos\omega t\sin^2\omega t}{32\omega^4} \right)$$

$$= \left(-\frac{a^3\alpha\sqrt{\lambda^2}Cos\lambda t}{32\lambda^4} + \frac{a^3\alpha\sqrt{\lambda^2}Cos^3\lambda t}{32\lambda^4} - \frac{3a^3\alpha t\sqrt{\lambda^2}Sin\lambda t}{8\lambda^3} - \frac{3a^3\alpha\sqrt{\lambda^2}Cos\lambda tSin^2\lambda t}{32\lambda^4} \right)$$

.....

.....

Therefore, the solution of the equation (17) is

$$x(t) = x_0 + x_1 + x_2 + \ldots\ldots$$

$$= aCos(\lambda t) - \frac{a^3\alpha\sqrt{\lambda^2}Cos(\lambda t)}{32\lambda^4} + \frac{a^3\alpha\sqrt{\lambda^2}Cos^3(\lambda t)}{32\lambda^4} - \frac{3a^3\alpha t\sqrt{\lambda^2}Sin(\lambda t)}{8\lambda^3} - \frac{3a^3\alpha\sqrt{\lambda^2}Cos(\lambda t)Sin^2(\lambda t)}{32\lambda^4}$$

$$= aCos(\lambda t) - \frac{a^3\alpha\sqrt{\lambda^2}}{32\lambda^4}\left(Cos(\lambda t) - Cos^3(\lambda t)\right) - \frac{3a^3\alpha t\sqrt{\lambda^2}Sin(\lambda t)}{8\lambda^3}\left(Cos(\lambda t) + Sin(\lambda t)\right)$$

The figure of above solutions is given Figure 6 - Figure 9.

5. Results and Discussions

The Homotopy perturbation method is successfully applied to solve the simple pendulum equation. To test the accuracy of our results, we match our results with the numerical results obtained by the mathematica 9.0. The solution obtained by Homotopy perturbation method is an infinite series for appropriate initial condition. The Pendulum damping constant k, nonlinear constant ω^2, forcing constant ω are taken the appropriate values for solving the problems and drawing the natural figure. The solutions are described in the article 4.1 and 4.2. The corresponding numerical solutions that have been computed by the Mathematica 9.0 program for various values of t and all the results are showed in their corresponding figures in Figure 2 to Figure 9 respectively. The Table 1 to Table 8 reveals that the Homotopy results, Numerical results associated with absolute error, our research has received a very small number of errors that are acceptable in the study. These solutions produce a wide variety of interesting motions.

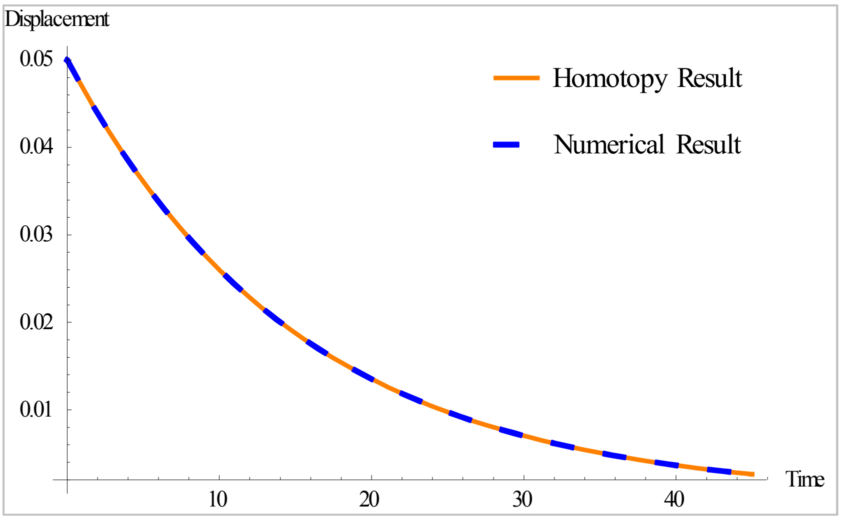

Figure 2. The Simple Pendulum Equation for $k = 30, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 1. Comparison between Homotopy results and Numerical results for $k = 30, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	0.0360843	0.0360844	0.00002

Time	Homotopy results	Numerical results	Error in %
10	0.0260119	0.0260120	0.00008
15	0.0187506	0.0187507	0.00003
20	0.0135161	0.0135162	0.00001
25	0.0097428	0.0097429	0.00017
30	0.0070229	0.0070230	0.00031
35	0.0050623	0.0050624	0.00047
40	0.0036490	0.0036491	0.00062
45	0.0026303	0.0026304	0.00098

Table 2. Comparison between Homotopy results and Numerical results for $k = 0.1, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	0.0457072	0.0457073	0.00005
10	0.0335756	0.0335757	0.00012
15	0.0157008	0.0157007	0.00058
20	-0.0048383	-0.0048384	0.00147
25	-0.0245101	-0.0245102	0.00018
30	-0.0399384	-0.0399385	0.00004
35	-0.0484827	-0.0484826	0.00008
40	-0.048689	-0.0486890	0.00018
45	-0.040537	-0.0405369	0.00030

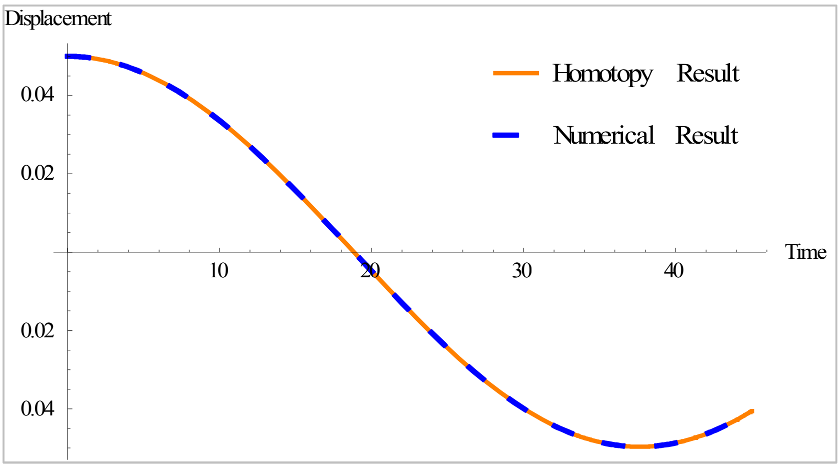

Figure 3. The Simple Pendulum Equation for $k = 0.1, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

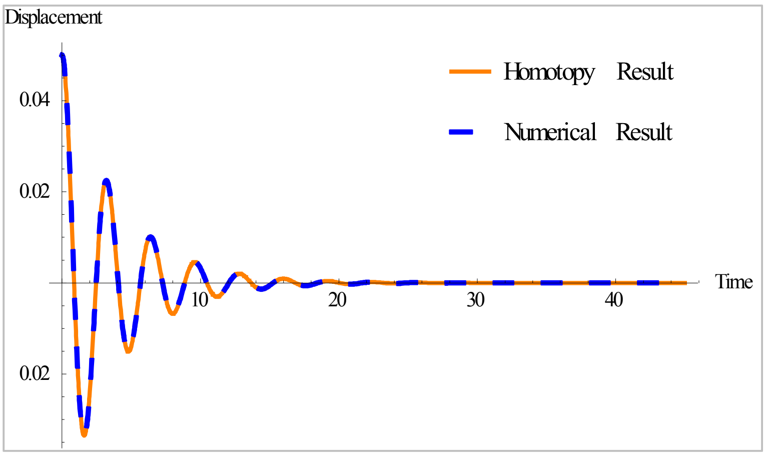

Figure 4. The Simple Pendulum Equation for $k = 0.25, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 3. Comparison between Homotopy results and Numerical results for $k = 0.25, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	-0.0139036	-0.0139037	0.00009
10	0.0032326	0.0032327	0.00021
15	-0.0005644	-0.0005645	0.00612
20	0.0000324	0.0000325	0.09178
25	0.0000292	0.0000291	0.08700
30	-0.0000180	-0.0000180	0.15745
35	0.0000071	0.0000072	0.06937
40	-0.0000022	-0.0000023	0.52248
45	0.0000061	0.0000064	4.62433

Table 4. Comparison between Homotopy results and Numerical results for $k = 0.15, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	-0.0220419	-0.0220420	0.00001
10	0.0075847	0.0075848	0.00024
15	-0.0015297	-0.0015298	0.00367
20	-0.0003919	-0.0003918	0.01360
25	0.0006745	0.0006744	0.00917
30	-0.0004860	-0.0004859	0.01272
35	0.0002626	0.0002627	0.01495
40	-0.0001148	-0.0001149	0.03520
45	0.0000390	0.0000389	0.15139

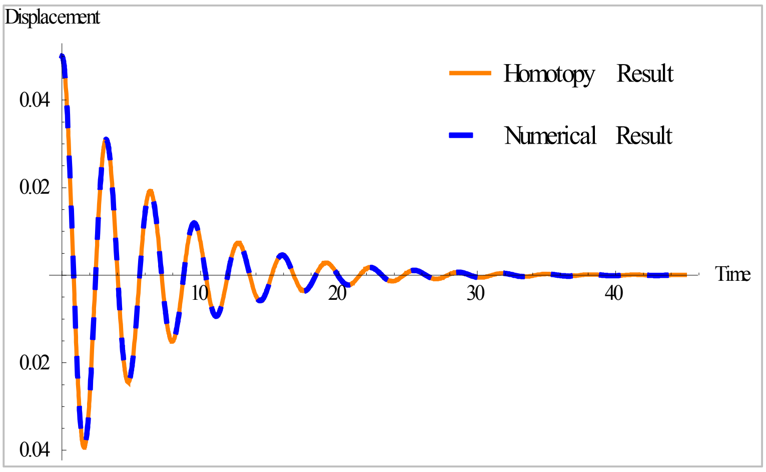

Figure 5. The Simple Pendulum Equation for $k = 0.15, m = 50, g = 9.81, L = 2.5, a = 0.05$ and the initial conditions. $x(0) = 0.05, x'(0) = 0$.

Table 5. Comparison between Homotopy results and Numerical results for $k = 0, m = 50, g = 9.81, L = 35.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	-0.0435494	-0.0435493	0.00020
10	0.025861	0.0258609	0.00029
15	-0.0014981	-0.0014982	0.00186
20	-0.0232514	-0.0232515	0.00019
25	0.042	0.0419998	0.00026
30	-0.0499104	-0.0499102	0.00042
35	0.0449427	0.0002628	0.01495
40	-0.0283779	-0.0283776	0.00099
45	0.0044891	0.0044888	0.00457

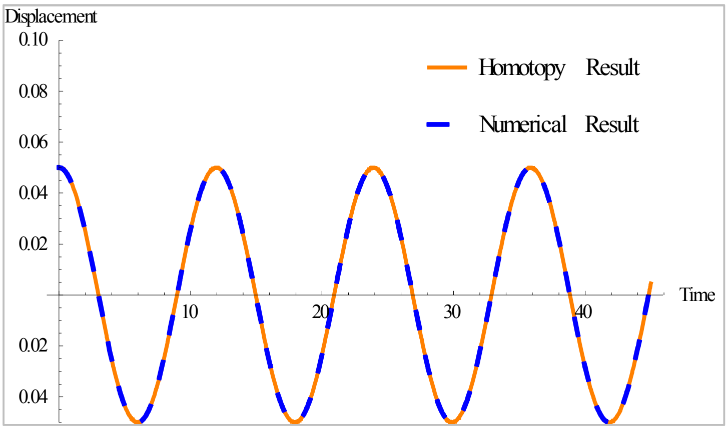

Figure 6. The Simple Pendulum Equation for $k=0, m=50, g=9.81, L=35.5, a=0.05$ and the initial conditions $x(0)=0.05, x'(0)=0$.

Table 6. Comparison between Homotopy results and Numerical results for $k=0, m=50, g=9.81, L=25.5, a=0.05$ and the initial conditions $x(0)=0.05, x'(0)=0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	-0.0499583	-0.0499584	0.00010
10	0.0498333	0.0498332	0.00018
15	-0.0496252	-0.0496251	0.00020
20	0.0493343	0.0493342	0.00033
25	0.0003382	0.0003383	0.00040
30	0.0485064	0.0485062	0.00057
35	-0.0479708	-0.0479704	0.00071
40	0.0473551	0.0473546	0.00095
45	-0.0466604	-0.0466599	0.00113

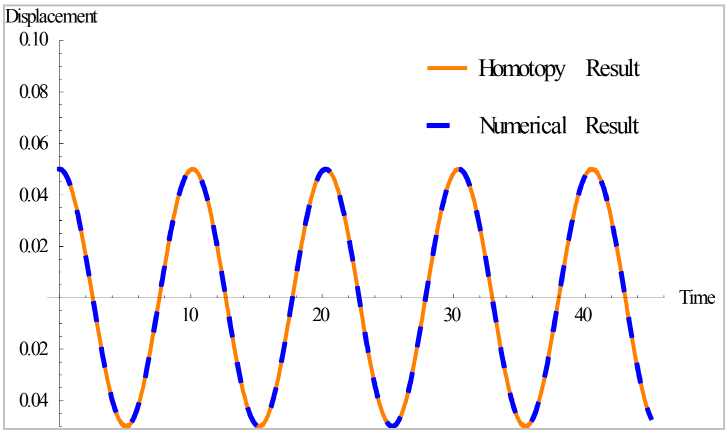

Figure 7. The Simple Pendulum Equation for $k=0, m=50, g=9.81, L=25.5, a=0.05$ and the initial conditions $x(0)=0.05, x'(0)=0$.

Table 7. Comparison between Homotopy results and Numerical results for $k=0, m=50, g=9.81, L=10.5, a=0.05$ and the initial conditions $x(0)=0.05, x'(0)=0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	-0.0335396	-0.0335397	0.00002
10	-0.0050067	-0.0050065	0.00243
15	0.0402558	0.0402556	0.00041
20	-0.0489977	-0.0489976	0.00015
25	0.0254784	0.0254785	0.00039
30	0.0148194	0.014819	0.00206
35	-0.0453578	-0.0453574	0.00087
40	0.0460308	0.0460306	0.00055
45	-0.0163952	-0.0163954	0.00117

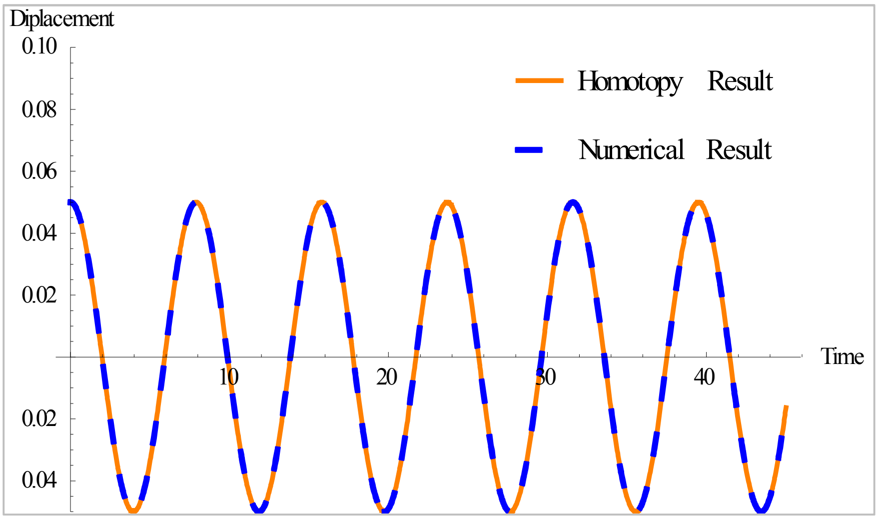

Figure 8. The Simple Pendulum Equation for $k = 0, m = 50, g = 9.81, L = 10.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

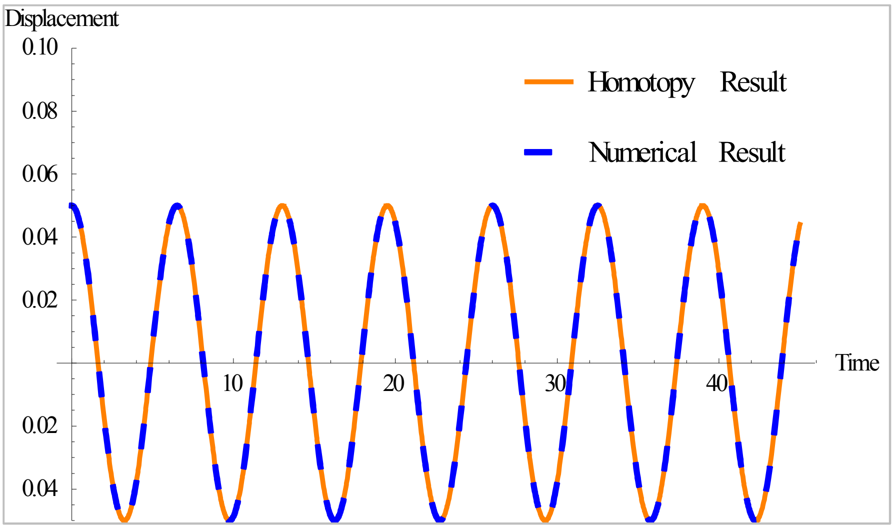

Figure 9. The Simple Pendulum Equation for $k = 0, m = 50, g = 9.81, L = 5.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Table 8. Comparison between Homotopy results and Numerical results for $k = 0, m = 50, g = 9.81, L = 5.5, a = 0.05$ and the initial conditions $x(0) = 0.05, x'(0) = 0$.

Time	Homotopy results	Numerical results	Error in %
0	0.05	0.05	0
5	0.0059749	0.0059747	0.00102
10	-0.0485724	-0.0485723	0.00011
15	-0.0175835	-0.0175834	0.00073
20	0.0443709	0.0443707	0.00027
25	0.0281876	0.0281874	0.00095
30	-0.0376353	-0.0376351	0.00058
35	-0.0371818	-0.0371813	0.00142
40	0.02875	0.0287497	0.00094
45	0.0440524	0.0440515	0.00211

6. Conclusion

In this study, the Homotopy perturbation method has been successfully used to solve the simple pendulum equation. The solution obtained by means of the Homotopy perturbation method is an infinite power series for appropriate initial condition, which can be intern, expressed in a closed form. The results obtained here has compared with the exact solutions and the results reported by using other method. All the figures show that the results of the present method are in excellent agreement with numerical solutions obtained by the Mathematica program. The HPM has got many merits and much more advantages. Also the HPM does not require small parameters in the equation, so that the limitations of the traditional perturbation methods can be eliminated and also the calculations in the HPM are simple and straightforward. The reliability of the method and reduction in the size of computational domain give this method a wider applicability.

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Biography

Ahammodullah Hasan was born on October 12, 1994 in Kushtia, Bangladesh. He received B.Sc. in Mathematics from Islamic University, Bangladesh in 2015, Master in Mathematics, Islamic University, Bangladesh in 2016. He has been completed several project and thesis under his supervision, his some research paper has been published in peer-review international journal. Currently he is doing research on various iterative Method, approximation method, fuzzy logic obtaining weather forecast with neural-network. His research interests include computational Mathematics and applied Mathematics like as, Mathematical physics, Nonlinear Oscillation, PDE, Numerical analysis and number theory.

Mohammad Masud Rana was born on October 21, 1993 in Natore, Bangladesh. He received B.Sc. in Mathematics from Islamic University, Bangladesh in 2015, Master in Mathematics, Islamic University, Bangladesh in 2016. He has been completed several project and thesis under his supervision. Currently he is doing research on Chain moving system in Homotopy perturbation method. His research interests include computational Mathematics like as, Mathematical physics, Nonlinear Oscillation, PDE, Numerical analysis and number theory.