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Fluid-Structure Interaction Modeling of Aortic Blood Flow Behavior in the Human Cardiovascular System Using Differential Equations

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Abstract

Blood flow dynamics in arteries is an underlying factor for much vascular pathology. A better understanding of these dynamics could improve the prediction and diagnosis in both healthy and pathological situations. The study of the mechanical properties of aorta will help to understand the normal and pathological situation and to predict their adaptation to changing circumstances. The aorta is continuously exposed to blood pressure caused by the pulsatile effect of the heart. These stresses are noticed by elastic deformation of the organ. In the present study, we have developed a mathematical model that allowed us to evaluate the blood-aorta interaction by resolving differential equations that describe biomechanical characteristics of aorta. The present study revealed that the stress exerted by the blood flow in aorta causes its elastic deformation. However, we reported for the first time that the resulting aortic deformation threshold is higher than the standard commonly values (7-10% vs 12%). The experimental data acquired with the help of Doppler echo device and the computer simulation with MATLAB® software package permitted to confirm the perfect agreement between the experiments and the theoretical predictions.

Keywords

Blood Dynamics, Blood Pressure, Aorta, Deformation, Simulation, Biomechanics

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1. Introduction

The human body is a set of the anatomical structures forming the organism that through the circulatory system (heart and vessels) assures the circulation of blood. Blood being considered as a Newtonian fluid and the vessels like the elastic ducts (solids), the blood circulation in the vessels constitutes a permanent exercise of the constraints within them. As result, the solids react to the constraints by a distortion [1-6]. For the vessels, it has been reported that the distortion threshold does not pass 10% according to the diameter and the normal length of the vessel. Besides, all solids do not distort themselves in the same way. This arise the question on the behavior of the elastic conditions in terms

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of the cardiovascular system biomechanics and therefore on the distortion threshold of different vessels [7-10]. The present study was carried out with the aim of developing a mathematical model that will allow evaluating the blood-aorta interaction by resolving differential equations that describe biomechanical characteristics of aorta in order to determine it real distortion threshold in a healthy aorta. A better understanding of these dynamics could improve the prediction and diagnosis in both healthy and pathological situations. The mathematical modeling study of the mechanical properties of aorta will also help to understand the normal and pathological situation and to predict their adaptation to changing circumstances.

2. Material and Methods

The documentary method was usein order to carry out this work. Data used for the simulations were obtained from the departments of Medical imaging, Radiology, Ultrasound, Mammography, Scanner of the General Hospital of Kinshasa, Kinshasa city, Democratic Republic of the Congo. The manipulation of the Doppler echo device and the computer simulations with the help ofMATLAB® software package permittedto confirm the perfect agreement between the experimental data and the theoretical predictions.

2.1. Biomechanical Description of Cardiovascular System

The cardiovascular system (figure 1) is consisted by the heart which acts as a four-chambered pumpthat propels blood around the circulatory system. Cardiac impulse transmission of the depolarizationwave determines the contraction of the heart muscle while the valves between the atriaand ventricles, respectively, the ventricles and the main arteries take care of unidirectionalblood flow. The systemic circulation consists of an arterial system branching in many arteriesof decreasing diameter, a capillary micro-circulatory system in the tissues and a venous systemwith venules and veins merging into larger vessels transporting blood back to the heart [12].

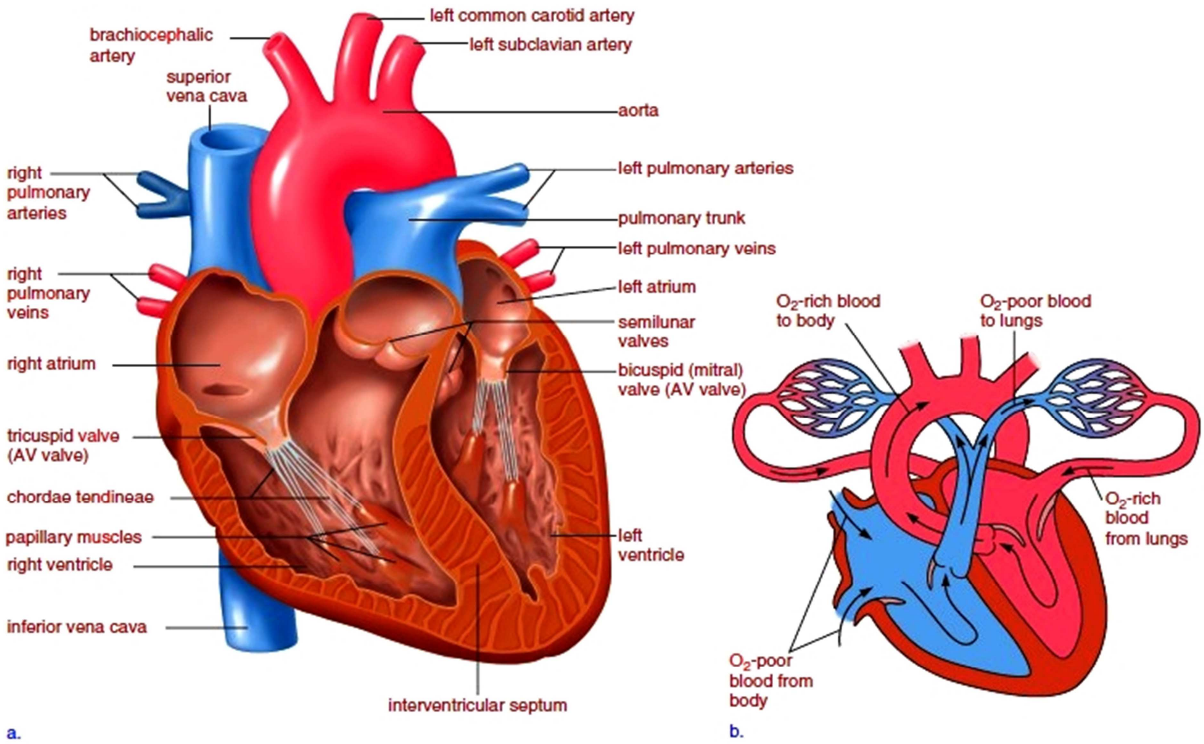

Figure 1. Schematic representation of the cardiovascular system [13].

Fluid-structure interaction modeling offers the possibility to perform simulations of fluid flow in interaction with distensible structure like aorta and can be used for a variety of medical applications, especially when combined with patient-specific data fromimaging techniques.

2.2. Mathematical Modeling

In this part of the work, we basically base ourselves on formulating a general model of the fluid circulating in an elastic tube.

Position of Problem

The tube below represents the blood-filled aorta made with the SolidWoks® software package. It has the following characteristics:

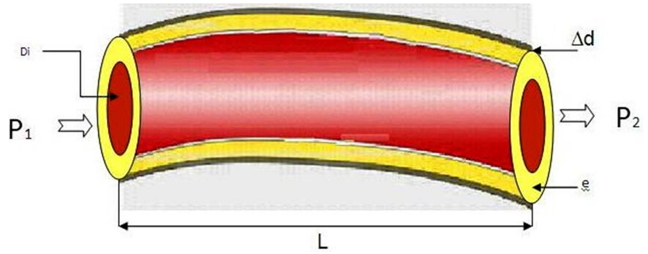

Figure 2. Aorta in the physiological state.

The figure 2 shows the flow of blood into the aorta. This model refers to a situation where there is no pathology. But in order to make a decision about the results of our research, we consider the case where the blood pressure increases and there is deformation of the aorta taking into account the different physical quantities and others that are part of the blood flow that the Model indicates.

These different quantities are:

1. Input pressure (P1): 160mmHg = 0.21280MPa;
2. Output pressure (P2): 100mmHg = 0.13300 MPa;
3. Internal diameter (Di): 25.4 mm;
4. Deformation (Δd): 10%;
5. Length (L): 120 mm;
6. Thickness: 2 mm.

Modeling blood pressure as a function of length

In the cardiovascular system, the arterial pressure at the inlet is higher than that at the outlet, i.e. it decreases according to the length.

This situation can be expressed by the following differential equation:

$$\frac{dp}{dx} = -kx \quad (1)$$

Wherep: refers to blood pressure; x: is the length of the aortic tube; k: refers the coefficient of variation of the pressure. The above differential equation being of the first order, its resolution is simple according to the following steps:

$$\frac{dp}{dx} = -kx \Rightarrow dp = -kxdx. \int dp = \int -kxdx \Rightarrow p + c1 = -k\frac{x^2}{2} + C2.$$

$$p = \frac{-k}{2}x^2 + C2 - C1 \quad (2)$$

Where $C2 - C1 = C$

The expression (2) takes the form:

$$p = \frac{-k}{2}x^2 + C \quad (3).$$

Determination of the constants of the solution of the differential equation

The constants will be determined on the basis of the following boundary conditions:

$x = 0 \rightarrow p = p1$ (With p_1 the pressure at the input),

$x = L \rightarrow p = p2$ (With p_2 the pressure at the outlet),

That is to say:

$$p_1 = \frac{-k}{2}0^2 + C \Rightarrow c = p_1 \quad (4)$$

$$p_2 = \frac{-k}{2}L^2 + p_1 \Rightarrow \frac{-k}{2}L^2 = p_2 - p_1$$

$$k = \frac{2\,(p_1 - p_2)}{L^2} \quad (5)$$

$$p = -\frac{2\,(p_1 - p_2)}{L^2}\frac{x^2}{2} + p_1 \quad (6)$$

Mechanical characteristics of the aorta

In general, the blood vessels are subjected to mechanical stresses during the pumping of blood by the heart. This makes them have mechanical properties that give them the ability to resist these efforts. As the mechanical properties of blood vessels are functions of the basic structure of the tissue, we can approximate the behavior of the aorta under load as biomaterials, dividing its length into equal parts.

Relation (5) allows us to know the value of the arterial pressure at each step along the length of the aorta. We must remember that the aorta as all blood vessels live in a range of stress. Hence, the expression (5) of the blood pressure, will allow us once more to determine the characteristics of the aorta that define its behavior, such as:

1. The normal force (N);
2. The normal stress (бN);
3. The deformation Δr.

To do this, consider the two figures below which are an orientation for the determination of the mathematical expressions of the parameters cited above.

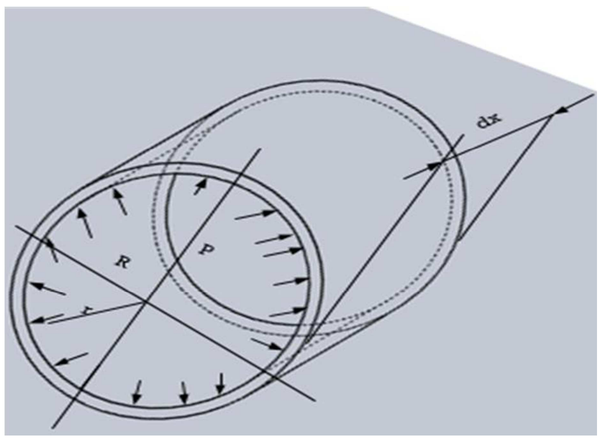

Figure 3. The physical model of the diameter of the aorta.

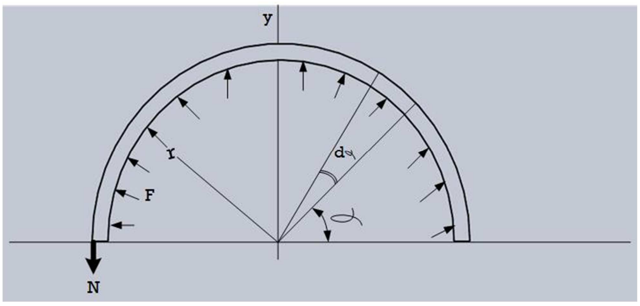

Figure 4. The aorta radius.

Expression of normal stress

By considering the ring (Figure 3) of inner radius (r) and outer radius (R), of length (L), subjected to the action of an internal radial pressure p, we can determine the longitudinal tensile forces which appear in the wall of the ring. By considering a section along a diametral plan (Figure 4) and the equilibrium condition for a half ring by projecting all the forces on the ring (Y axis), we will have:

$$\int_0^{\frac{\pi}{2}} \mathrm{pr}\,\Delta \mathrm{x}\,\sin\vartheta\,\mathrm{d}\vartheta = \mathrm{pr}\Delta \mathrm{x} \quad (7)$$

With P: arterial pressure (MPa); R: internal radius (mm); Δx: variation step (mm).

Normal stress

As mentioned above, the aorta is a large artery which starts from the heart and brings blood to the organs that make up the human body through other vessels. Its wall is continually exposed to dynamic mechanical stresses due to the pulsatile effect of the heart with minimal energy loss due to its unique biomechanical properties. The normal stress in the aortic wall is given by the relation:

$$бN = \frac{N}{AL} = \frac{pr\Delta x}{(R-r)\Delta x} = \frac{pr}{(R-r)} \quad (8).$$

With БN: Normal stress (Mpa); N: Normal effort; AL: Side wall section (mm^2).

Deformation Δr

To determine the deformation, we must know well before, the value of its Modulus of longitudinal elasticity E. According to the law of Hooke, we find the absolute increase of the internal radius of the ring by the expression:

$$2\pi(r + \Delta r) - 2\pi r = \frac{N2\pi r}{E(R-r)\Delta x} \quad (9)$$

After development, we obtain:

$$\Delta r = \frac{pr^2}{E(R-r)\Delta x} \quad (10)$$

2.3. Experimental Data Analysis

The data analysis was carried out through an algorithm established from above equations with the help of MATLAB® software package on a personal computer.

3. Results and Discussion

The figure 5 gives the variation of the arterial pressure as a function of the length of the artery.

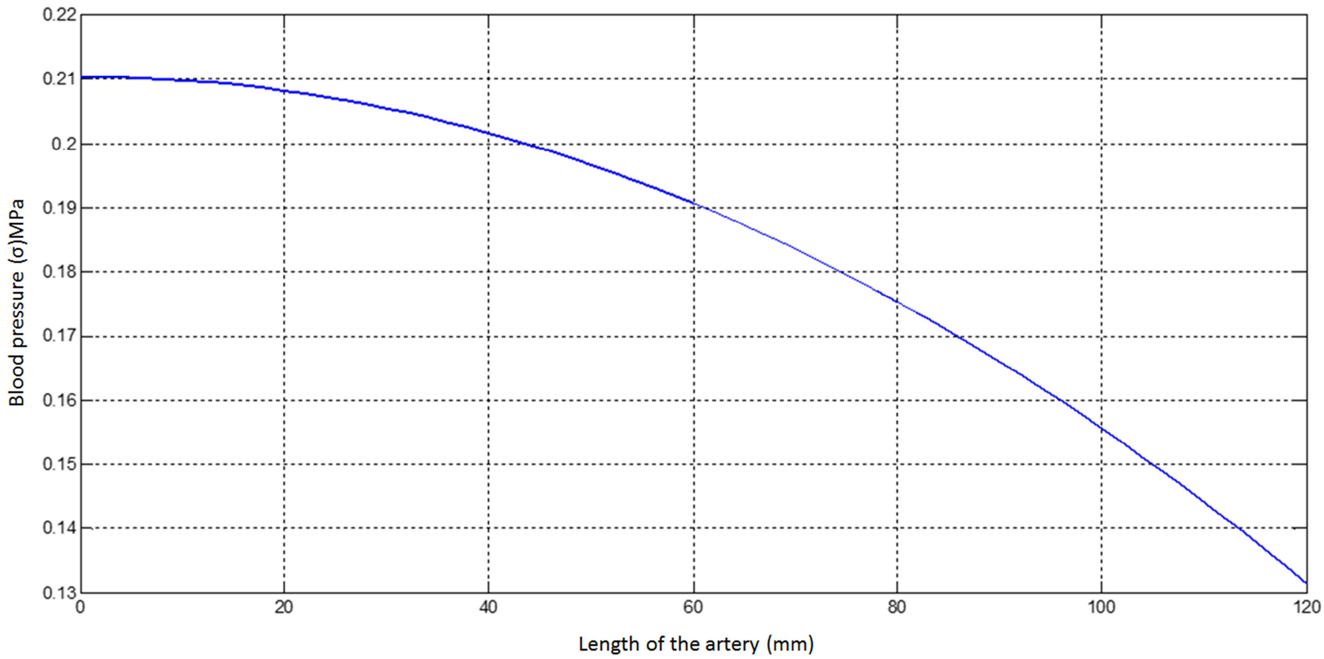

Figure 5. Evolution of blood pressure in function of artery length.

The figure 5 revealed that the blood pressure decreases from 0.21280 MPa to 0.13300 MPa. This means that there is a loss of pressure along the artery.

The figure 6 gives the evolution of the deformation of the artery as a function of its length (120 mm).

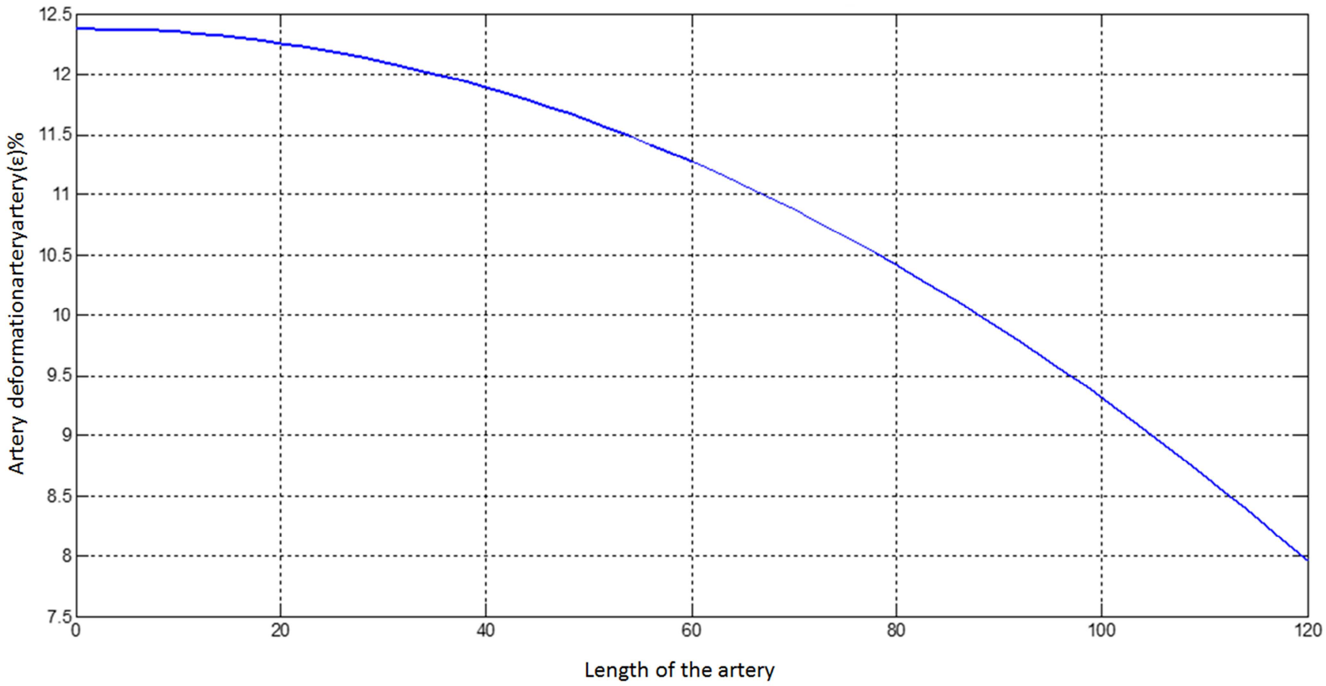

Figure 6. Variation of the deformation of the artery as a function of the length.

The figure 6 revealed that the deformation of the artery decreases from 12.4% to 7.9% depending on the length of the artery.

Figure 7 shows the variation in blood pressure as a function of normal stress.

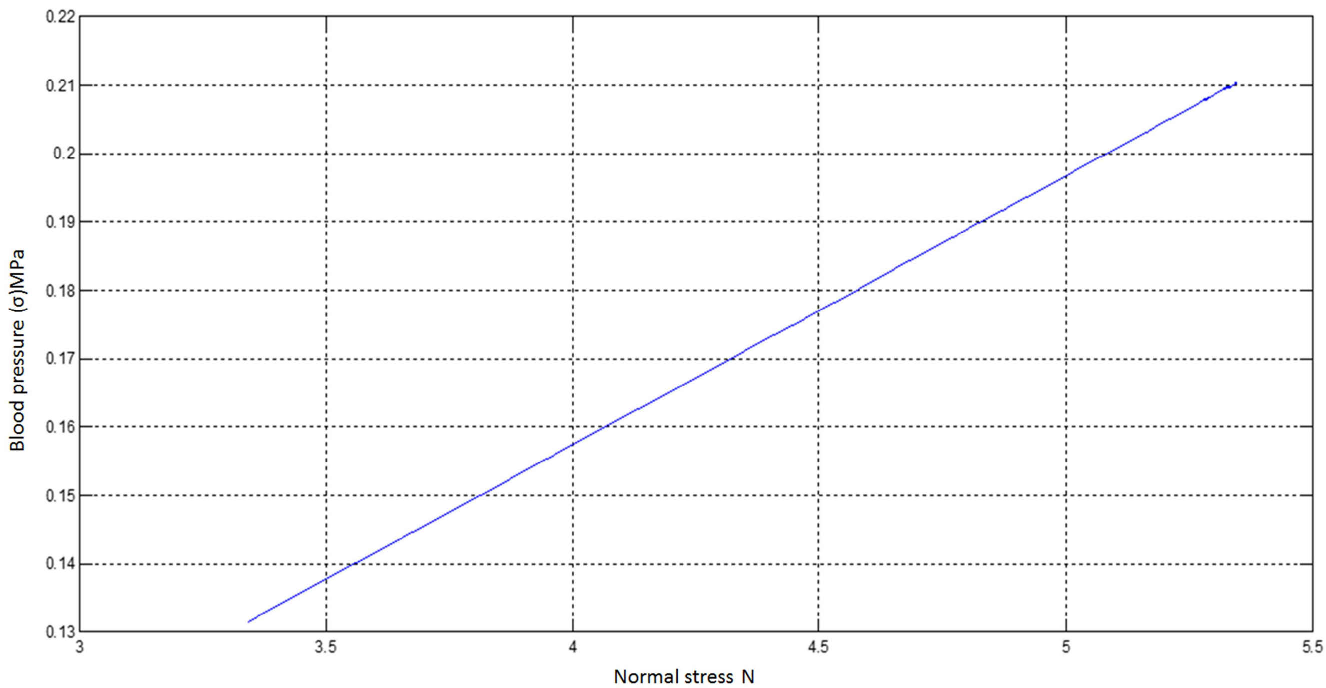

Figure 7. Change in blood pressure as a function of normal stress.

This figure revealed that the arterial pressure leaves from 0.21280 MPa (high pressure or entry pressure) at the point 5.3848508 N of the normal force to 0.13300 MPa (low pressure or exit pressure) to the point 3.51107587 N of the normal force.

The present study confirmsthat pressure is a stress, which is manifested essentially by a deformation of the structure. The pulsatile effect of blood on the heart in the aorta is accompanied by a normal stress varying from 7 to 10% as reported by Amblard (2006) and Nguyen (1997) [14, 15]. This deformation is modeled by a pulsation of the diameter of the vessel synchronized with the blood flow. By subjecting the aorta to blood pressure above the normal, the resulting aortic deformation is higher than those of the above authors (12%). Also, the normal stress varies with blood pressure.

4. Conclusion

The aim of the present study was to analyzeby mathematical modeling the behavior of aorta submitted to blood stress. Several factors were taken into account (blood pressure, diameter, length, thickness and deformation of the aorta) using differential equations and based on biomechanical characteristics that the aorta is a structure and the blood is a fluid (fluid-structure interaction). The stress exerted by the blood flow in this structure causes its elastic deformation. However, we reported for the first time that the resulting aortic deformation threshold is higher than the standard commonly values (7-10% vs 12%). The Doppler echo device, MATMAB® and SOLIDWOKS® software packages were used during experimental phase and simulation.

The present study indicated that mathematical predictive modeling, by supporting the aorta behavior, significantly contributes to both fundamental and applied cardiovascular research.

References

[1] S. Annerel, T. Claessens, L. Taelman, J. Degroote, G. Van Nooten, P. Verdonck, P. Segers, J. Vierendeels. Influence of valve size, orientation and downstream geometry of an aortic BMHV on leaflet motion and clinically used valve performance parameters. Ann Biomed Eng., 43 (6): 1370-84, 2015.

[2] M. M. Denn, J. F. Morris, D. Bonn. Shear thickening in concentrated suspensions of smooth spheres in Newtonian suspending fluids. Soft Matter., 14 (2): 170-184, 2018.

[3] S. A. Schulz, A. Wöhler, D. Beutner, D. N. Angelov. Microsurgical anatomy of the human carotid body (glomuscaroticum): Features of its detailed topography, syntopy and morphology. Ann Anat., 204: 106-113, 2016.

[4] M. G. Nestola, E. Faggiano, C. Vergara, R. M. Lancellotti, S. Ippolito, C. Antona, S. Filippi, A. Quarteroni, R. Scrofani. Computational comparison of aortic root stresses in presence of stentless and stented aortic valve bio-prostheses. Comput Methods Biomech Biomed Engin., 20 (2): 171-181, 2017.

[5] A. Aggarwal, A. M. Pouch, E. Lai, J. Lesicko, P. A. Yushkevich, J. H. Gorman Iii, R. C. Gorman, M. S. Sacks. In-vivo heterogeneous functional and residual strains in human aortic valve leaflets. J Biomech., 16; 49 (12): 2481-90, 2016.

[6] D. R. Subramaniam, D. J. Gee. Shape oscillations of elastic particles in shear flow. J Mech Behav Biomed Mater., 62: 534-544, 2016.

[7] M. F. Ashby, D. R. H Jones. Matériaux: propriétés et applications, Tome 1, Dunod: Paris, 1996.

[8] Y. Kim, M.-C. Lai. Simulating the dynamics of inextensible vesicles by the penalty immersed boundary method. J. Comput. Phys. 229: 4840–4853, 2010.

[9] P. Crosetto, P. Reymond, S. Deparis, D. Kontaxakis, N. Stergiopulosb, A. Quarteroni. Fluid–structure interaction simulation of aortic blood flow. Computers & Fluids 43: 46-57, 2011.

[10] F. N. van de Vosse. Mathematical modeling of the cardiovascular system. Journal of Engineering Mathematics 47: 175–183, 2003.

[11] J. Sigüenza, D. Pott, S. Mendez, S. J. Sonntag, T. A. S. Kaufmann, U. Steinseifer, F. Nicoud. Fluid-structure interaction of a pulsatile flow with an aortic valve model: A combined experimental and numerical study. Int J Numer Method Biomed Eng., 34 (4): e2945A. 2018.

[12] F. N. van de Vosse. Mathematical modeling of the cardiovascular system. Journal of Engineering Mathematics 47: 175–183, 2003.

[13] J. G. Betts, P. Desaix, E. Johnson, J. E. Johnson, O. Korol, D. Kruse, B. Poe, J. A. Wise, M. Womble, K. A. Young. Anatomy & Physiology, 2013.

[14] A. Amblard. Contribution à l'étude du comportement d'une endoprothèse aortique abdominale. Analyse des endofuites de type I. Thèse de Doctorat, Institut National des Sciences Appliquées de Lyon: France, 2006.

[15] S. Nguyen. Etude de l'interface stent-collet aortique: rapport de Projet de Fin d'Etudes, Laboratoire de Mécanique des Contacts et des Solides, INSA, Lyon: France, 1997