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Effects of Magnetic Field and Slip Condition on an Unsteady Viscous Fluid in a Non-Darcy Porous Medium

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Abstract

In the present context, the effects of magnetic field and wall slip condition is considered on an unsteady, Newtonian, incompressible, viscous fluid flowing through a non-Darcy porous medium. The governing momentum and energy equations have been solved by using Crank Nicolson's finite technique which is fast converging and unconditionally stable. Numerical results have been presented by showing the influence of physical parameters on velocity, temperature and concentration profiles. In all studied cases, velocity, temperature and concentration profiles are affected by all the considered parameters.

Keywords

Viscous Fluid, Wall Slip, Newtonian Fluid, Thermal Radiation, Magnetic Field

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1. Introduction

The study of heat and mass transfer with magnetic field and slip condition is very important to engineers and scientists due to its great importance in nature. Heat and mass transfer problem arises in many processes like thermal processing, energy utilization and thermal control. In our day to day activities, we are directly or indirectly exposed to heat and mass transfer problems. As a result of this emissions, proper control measure is needed for emission of thermal radiation from Newtonian fluid into the environment. The study of flows through non-Darcy porous media has become of great interest in many sciences and engineering applications, such as in the utilization of geothermal energy, high-performance building insulation; heat storage, crude oil extraction, petroleum industries, solid matrix exchangers, chemical catalytic reactor, underground disposal of nuclear waste material and so on.

Several investigations have been carried out by early researchers on the transfer of heat and mass to or from Newtonian fluid. Adomian decomposition approach was used by Makinde [1] *et al.* to studied the boundary layer flow of a viscous fluid with thermal radiation past a moving vertical porous plate. The unsteady free convection flow of an incompressible viscous fluid near a vertical plate with ramped wall temperature and compared the results with those of the plate with constant temperature was investigated by Chandran et al. [2].

Many engineering problems are subjected to MHD analysis. The study of MHD flow problems has received outstanding interest as a result of its application in MHD generators, MHD pumps and MHD flow meters, just to mention a few. Makinde and Osalusi [3] considered the effect of slip condition on MHD steady flow in a channel with permeable boundaries. Mehmood and Ali [4] also considered the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Considering a closed form solution, Khaled and Vafai [5] studied a steady periodic MHD and transient velocity field under slip condition.

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Samiulhaq et al. [6] analysed the influence of radiation and porosity on the unsteady magnetohydrodynamic flow past an infinite vertical oscillating plate with uniform heat flux in a porous medium. Mazumdar and Deka [7] analysed MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. A mathematical introduction to incompressible flow was carried out by Sibanda and Makinde [8]. Yurusoy and Pakdemirili [9] considered the Exact solutions of boundary layer equations of a special non-Newtonian fluid over a stretching sheet. The effect of the fluid slippage at the wall for couette flow are considered by Marques et al. [10] under steady state conditions and only for gases.

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. In many chemical engineering processes, the chemical reaction mostly occur between a mass and fluid in which plate is moving. Possible applications of this type of flow can be found in many industries like power industry and chemical process industries. Numerous temperature process in the industrial designs, combustion and fire science involve thermal radiation heat transfer in combination with conduction convection and mass transfer. For example, radiative convective heat transfer flows arise in industrial furnace systems, astrophysical flows, forest fire dynamics, fire spread in buildings and so on. Some of the importance of mass transfer with chemical reaction was investigated by Astarita [11]. Problems involving the first order chemical reaction effect of axial diffusion with mass transfer was studied by Apelblat [12]. Chang and Kang [13] used a radiative flux diffusion approximation to model the interaction of convective and radiative heat transfer in two dimensional complex enclosures. Muthucumaraswamy and Ganesan [14] examined the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Mohammed [15] studied double-diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects.

The aim of this work is to investigate the effects of magnetic field and wall slip condition which has not received much attention by early researchers.

2. Problem Formulation

Consider an unsteady, Newtonian, viscous incompressible and electrically conducting fluid bounded by two fixed parallel plates and separated apart by distance l in a non-Darcy porous medium. Magnetic field of uniform strength B_o is applied perpendicular to the plates. It is assumed that the interaction of the induced magnetic with the flow is considered to be negligible compared to the interaction of the applied magnetic field with the flow. The fluid properties are assumed to be constant except for the body forces terms in the momentum equation which is approximated by Boussinesq relations. Thermal radiation is assumed to be present in the form of a unidirectional flux in the $y*$ direction. The governing equations for the unsteady flow of the Newtonian fluid under the usual Boussinesq approximation for incompressible fluid model is given for momentum, energy and concentration equations respectively as follows:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\left(\frac{\partial P^*}{\partial x^*}\right) + \nu\frac{\partial^2 u^*}{\partial {y^*}^2} + g\beta(T^* - T_0^*) + g\beta^*(C^* - C_0^*) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\mu u^*}{k} - \frac{b u^{*2}}{\rho k} \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial {y^*}^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial {y^*}^2} \tag{3}$$

With the following initial and boundary conditions

$$u^* - \lambda\frac{\partial u^*}{\partial y^*} = 0, T^* = T_0^*, C^* = C_0^* \text{ at } y^* = 0$$

$$u^* = 0, T^* = T_w^*, C^* = C_w^* \text{ at } y^* = l \tag{4}$$

Where u^* is the velocity of the fluid in the x-direction, t^* is the time, ρ is the density of the fluid, g is the acceleration due to gravity, T^* is the fluid temperature, C^* is the fluid concentration, D is the mass diffusivity, P^* is the fluid pressure, C_p is the specific heat at constant pressure, β is the coefficient of thermal expansion, $\beta^*$is the coefficient of mass expansion, k is the porosity parameter, b is the Forchheimer parameter, T_w^* is the temperature of the fluid at $y^* = l$, T_0^* is the temperature of the fluid at $y^* = 0$, q_r is the radiative heat flux, σ is the electrical conductivity, υ is the kinematic viscosity, B_0 is the magnetic field, l is the distance between two plates.

The radiation parameter, according to Mazumdar and Deka [2] is given as

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T_0^*)\int_0^\infty k\lambda_w \left(\frac{de_{b\lambda}}{dT^*}\right)_w d\lambda \quad (5)$$

Where $k\lambda_w$ signifies the absorption coefficient and $e_{b\lambda}$ is the plank function.

Defining the non-dimensional quantities as below

$$x = \frac{x^*}{l}, y = \frac{y^*}{l}, u = \frac{u^*}{u_0^*}, t = \frac{t^* u_0^*}{l}, \gamma = \frac{K_c \nu}{u_0^{*2}}, P = \frac{P^*}{\rho u_0^{*2}}, T = \frac{T^* - T_0^*}{T_w^* - T_0^*}, C = \frac{C^* - C_0^*}{C_w^* - C_0^*}, Re = \frac{u_0^* l}{\nu},$$

$$Da = \frac{\nu l}{k u_o^*}, M = \frac{\sigma B_0^2 l}{\rho u_0^*}, Pr = \frac{\nu}{\alpha}, G_r = \frac{g\beta T(T_w^* - T_0^*) l}{u_0^{*2}}, F_s = \frac{bl}{\rho k}\ \gamma = \frac{\lambda}{h}, G_r = \frac{g\beta^* C(C_w^* - C_0^*) l}{u_0^{*2}}, N = \frac{4Ll}{\rho C_p u_o^*} \quad (6)$$

Introducing the non-dimensional quantities into equations (1), (2) and (3) gives equations (7), (8) and (9) below

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - Mu + G_r T + G_m C - Dau - F_s u^2 \ (7)\ \frac{\partial T}{\partial t} = \frac{1}{PrRe}\frac{\partial^2 T}{\partial y^2} - NT \quad (8)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (9)$$

with the dimensionless boundary conditions

$$u = \gamma\frac{\partial u}{\partial y}, T = 0, C = 0, \text{at } y = 0$$

$$u = 0, T = 1, C = 1 \text{ at } y = 1 \quad (10)$$

Where Re is the Reynold number, Da is the Darcy number, F_s is the Forchheimer number, G_r is the thermal Grashof number, G_m is the modified Grashof number, M is the Magnetic field parameter, γ is the slip parameter, T is the dimensionless temperature, C is the dimensionless concentration, N is the thermal radiation parameter and Pr is the Prandtl number.

3. Numerical Solution

These nonlinear partial differential equations on equations (7), (8) and (9) under the boundary conditions in (10) are solved by employing Crank Nicolson finite difference scheme. This method has been extensively developed in recent years and remains one of the best reliable methods for solving partial differential equation. The partial differential equations are converted to difference equation. The Crank-Nicolson method converges fast and is unconditionally stable. The equations discretized as follows:

$$\frac{u_j^{k+1} - u_j^k}{\Delta t} = -Q_p + \frac{1}{Re}\left[\frac{u_{j-1}^{k+1} - 2u_j^{k+1} + u_{j+1}^{k+1} + u_{j-1}^k - 2u_j^k + u_{j+1}^k}{2(\Delta y)^2}\right] - M\left(\frac{u_j^{k+1} + u_j^k}{2}\right) + G_r\left(\frac{T_j^{k+1} + T_j^K}{2}\right) + G_m\left(\frac{C_j^{k+1} + C_j^K}{2}\right) - Da\left(\frac{u_j^{k+1} + u_j^k}{2}\right) - F_s u_j^k\left(\frac{u_j^{k+1} + u_j^k}{2}\right) \quad (11)$$

$$\frac{T_j^{k+1} - T_j^k}{\Delta t} = \frac{1}{prRe}\left[\frac{T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1} + T_{j-1}^k - 2T_j^k + T_{j+1}^k}{2(\Delta y)^2}\right] N\left[\frac{T_i^{k+1} + T_i^k}{2}\right] \quad (12)$$

$$\frac{C_j^{k+1} - C_j^k}{\Delta t} = \frac{1}{Sc}\left[\frac{C_{j-1}^{k+1} - 2C_j^{k+1} + C_{j+1}^{k+1} + C_{j-1}^k - 2C_j^k + C_{j+1}^k}{2(\Delta y)^2}\right] \quad (13)$$

With the following boundary conditions

$$u_j^{k+1} = 0, u_j^k = 0, T_j^{k+1} = 0, T_j^k = 0, C_j^{k+1} = 0, C_j^k = 0\ \forall\, j, t = 0$$

$$u_{j-1}^{k+1} = \frac{-2\Delta Y u_j^{k+1}}{\lambda} + u_{j+1}^{k+1}, T_{j-1}^{k+1} = 0, C_{j-1}^{k+1} = 0\ j = 1, t > 0$$

$$u_{j-1}^k = \frac{-2\Delta Y u_j^k}{\lambda} + u_{j+1}^k, T_{j-1}^k = 0, C_{j-1}^k = 0\ j = 1, t > 0$$

$$u_j^{k+1} = 0, T_j^{k+1} = 1, C_j^{k+1} = 1, u_j^k = 0, T_j^k = 1, C_j^k = 1\ j = n, t > 0 \quad (14)$$

Where Q_p (pressure gradient) is constant, u_0, T_0 and C_0 are the velocity, temperature and concentration at $Y = 0$ respectively, u_n, T_n and C_n are the velocity, temperature and concentration at $Y = 1$ respectively and the interval $\Delta Y = \frac{1}{n}$.

For a tridiagonal matrix system, we have the following for equations (11), (12) and (13) respectively.

$$-u_{j-1}^{k+1}\frac{\Delta t}{2Re(\Delta y)^2}+u_j^{k+1}\left[1+\frac{2\Delta t}{2Re(\Delta y)^2}+\frac{\Delta tDa}{2}+\frac{\Delta tGr}{2}+\frac{\Delta tM}{2}+\frac{\Delta tF_s u_j^k}{2}\right]u_{j+1}^{k+1}\frac{\Delta t}{2Re(\Delta y)^2}$$

$$=u_{j-1}^{k}\frac{\Delta t}{2Re(\Delta y)^2}+u_j^{k}\left[1-\frac{2\Delta t}{2Re(\Delta y)^2}-\frac{\Delta tDa}{2}-\frac{\Delta tGr}{2}-\frac{\Delta tM}{2}-\frac{\Delta tF_s u_j^k}{2}\right]+u_{j+1}^{k+1}\frac{\Delta t}{2Re(\Delta y)^2}+\frac{\Delta tGr}{2}\left[T_j^{k+1}+T_j^k\right] \tag{15}$$

$$-T_{j-1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2}+T_j^{k+1}\left[1+\frac{2\Delta t}{2PrRe(\Delta y)^2}+\frac{\Delta tN}{2}\right]-T_{j+1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2}$$

$$=T_{j-1}^{k}\frac{\Delta t}{2PrRe(\Delta y)^2}+T_j^{k}\left[1-\frac{2\Delta t}{2PrRe(\Delta y)^2}-\frac{\Delta tN}{2}\right]+T_{j+1}^{k+1}\frac{\Delta t}{2PrRe(\Delta y)^2} \tag{16}$$

$$-C_{j-1}^{k+1}\frac{\Delta t}{2Sc(\Delta y)^2}+C_j^{k+1}\left[1+\frac{\Delta t}{Sc(\Delta y)^2}\right]-C_{j+1}^{k+1}\frac{\Delta t}{2Sc(\Delta y)^2}=C_{j-1}^{k}\frac{\Delta t}{2Sc(\Delta y)^2}+C_j^{k}\left[1-\frac{\Delta t}{Sc(\Delta y)^2}\right]+C_{j+1}^{k+1}\frac{\Delta t}{2Sc(\Delta y)^2} \tag{17}$$

Subscripts j and k stands for grid points along y and t directions. From the initial condition when t=0, the values of u and T are known at all grid points. Calculations of u and T at their next time step (k+1) *th* is been done using the known values of u and T at (k) *th* time step following the procedure stated below:

Equations (15), (16) and (17) are arranged such that the equations form tri-diagonal system of equations. Thomas algorithm is used to solve this tri-diagonal system of equations with the help of MATLAB programming package due to the large output involved. In equation (16) and (17), the values of C and T for (k+1) *th* time step at every nodal point are evaluated using the (k) *th* time step known values. With these known values, the values of C and T at a particular time at every nodal point.

To calculate u in equation (15), the known values of C and T at every nodal point at a particular time are used in equation (15) to calculate u at a particular time at every nodal point. These steps were repeated several times until a steady state is reached.

4. Discussion of Result

In order to get a physical view of the problem, computation is carried out to account for the impact of the physical parameters on the velocity, temperature and concentration fields graphically. The following parameter values have been used except otherwise indicated on the profiles:

Pr=0.76, N=1, Re=1, Da=0.1, F_s=0.1, M=1, Gr=2.

Figures 1, 2 and 3 represent the influence of magnetic field on the on the dimensionless velocity, dimensionless temperature and dimensionless concentration profiles. Due to the presence of Lorentz force, magnetic field presents a damping effect on the velocity profile by creating drag force that opposes the fluid motion, thereby causing the velocity to decrease for increasing values of magnetic field parameter while temperature and concentration profiles increases as magnetic field parameter increases.

Figures 4 and 5 illustrates the effects of thermal radiation on velocity and temperature profiles. Increase in the values of thermal radiation causes reduction in both the velocity and temperature of the fluid. This experience is evident because, radiation is known to cause emission of heat from the body. Figure 6 shows that an increase in the wall slip parameter causes an increase in the fluid velocity at the lower plate.

Figures 7 is plotted for different values of thermal Grashof number. It is seen from this figure that increases in thermal Grash of number causes an increase on velocity profile. This result signifies that there is a rise in the velocity due to enhancement of thermal buoyancy force

Figures 8 and 9 shows the influence of Prandtl number on velocity and temperature profiles. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Increase in Prandtl number decreases both the velocity and temperature field to decrease.

Figures 10 and 11 shows the influence of Schmidt number on velocity and concentration profiles. Increase in Schmidt number causes both the velocity and concentration field to decrease.

This result causes the concentration buoyancy effects to decrease resulting to a reduction in the fluid velocity. The reductions experienced in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers as seen on the profiles.

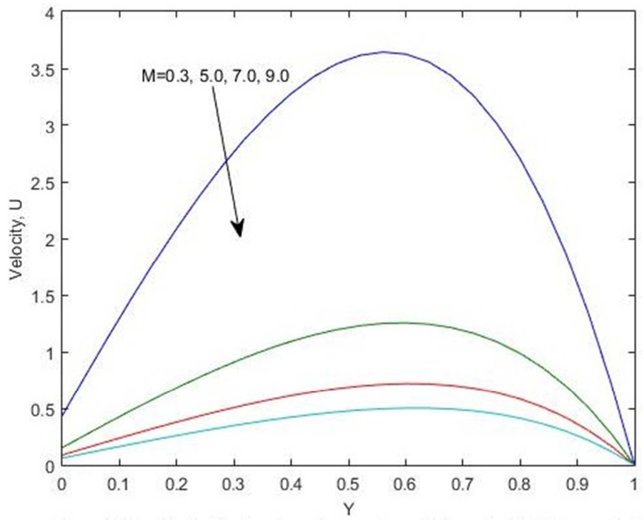

Figure 1. Velocity distribution for various values of magnetic field parameter M.

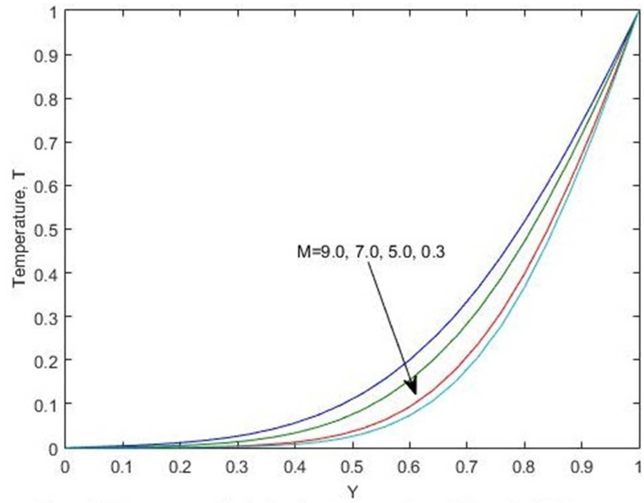

Figure 2. Temperature distribution for various values of magnetic field parameter M.

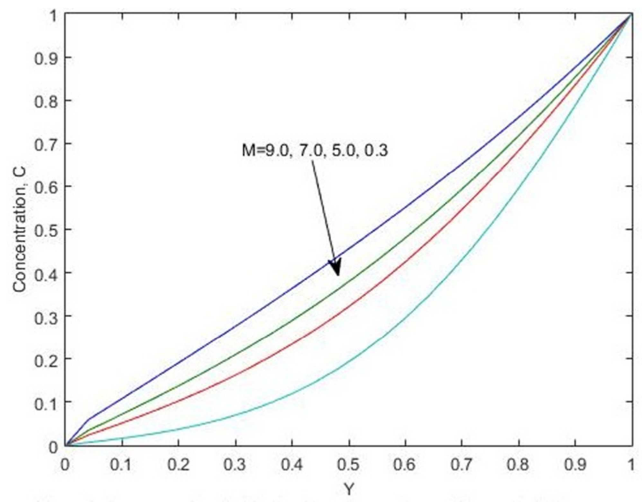

Figure 3. Concentration distribution for various values of magnetic field parameter M.

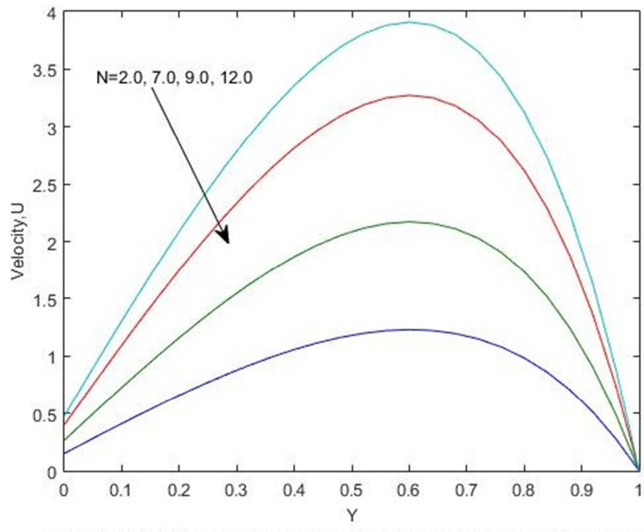

Figure 4. Velocity distribution for various values of thermal radiation parameter N.

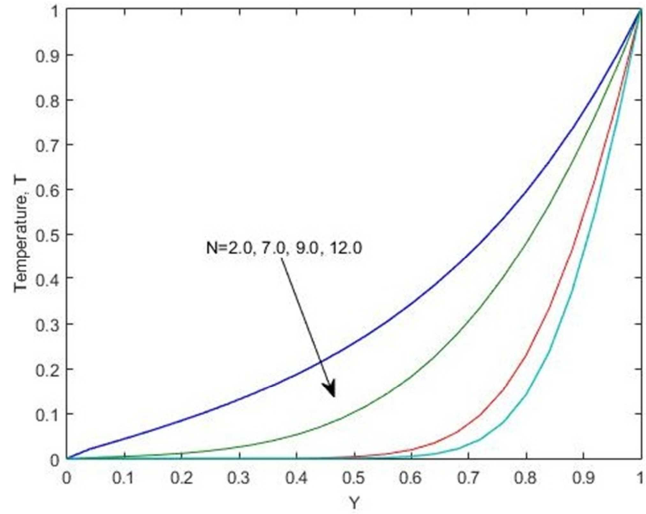

Figure 5. Temperature distribution for various values of thermal radiation parameter N.

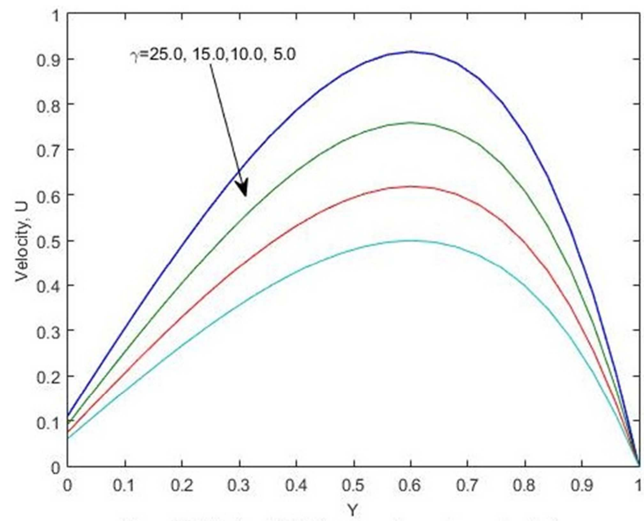

Figure 6. Velocity distribution for various values of wall slip parameter γ.

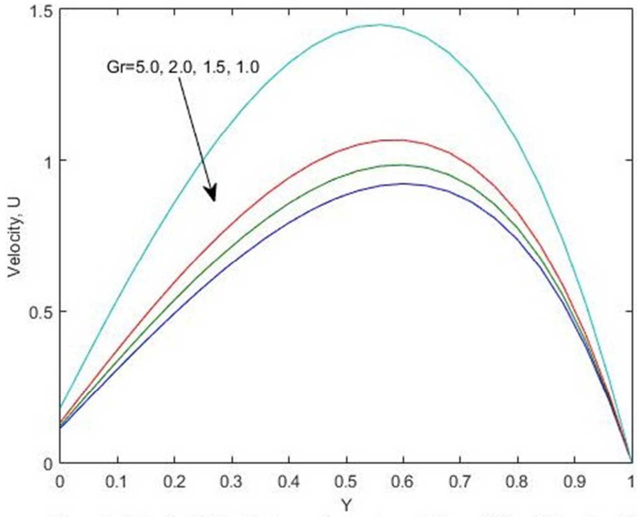

Figure 7. Velocity distribution for various values of thermal Grash of number Gr.

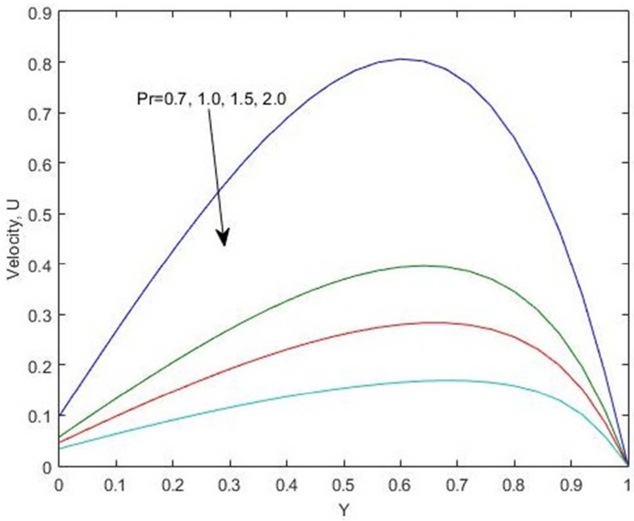

Figure 8. Velocity distribution for various values of Prandtl number Pr.

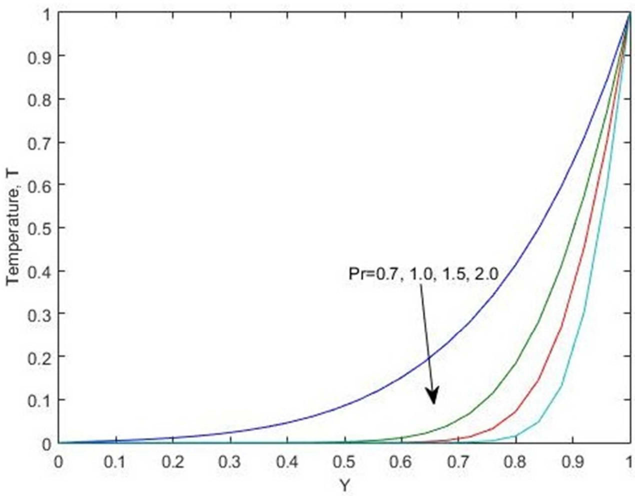

Figure 9. Temperature distribution for various values of Prandtl number Pr.

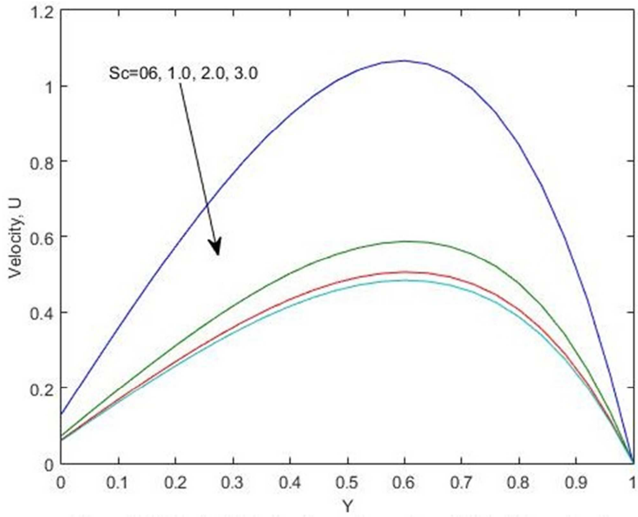

Figure 10. Velocity distribution for various values of Schmidt number Sc.

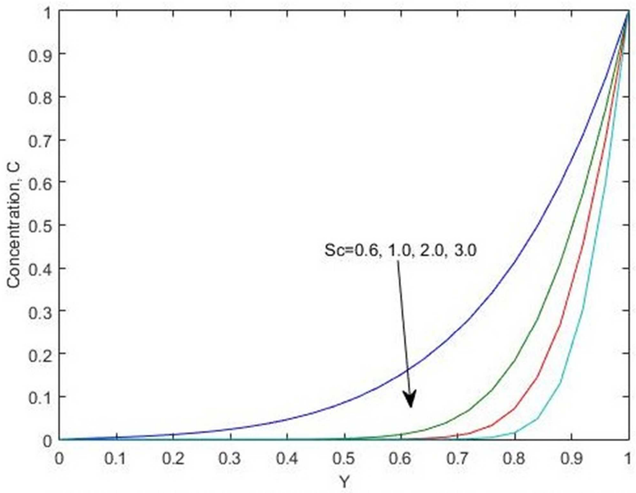

Figure 11. Temperature distribution for various values of Schmidt number Sc.

5. Conclusion

The effects of magnetic field and wall slip conditions is considered on a Newtonian, incompressible, viscous fluid flowing through a non-Darcy porous medium with appropriate boundary conditions. It was discovered that the wall can be strengthened by increasing the wall slip and thermal Grashof number while it is found to be weakened by the effects of increasing thermal radiation parameter, magnetic field, Schmidt number and Prandtl number respectively

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