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Application of the GDQ Method to Structural Analysis

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Abstract

This paper presents a generalized differential quadrature method as an accurate, efficient and simple numerical technique for structural analysis of uniform and non-uniform beams resting on fluid layer under axial force and distributed load under three sets of boundary conditions, that is, simply–simply supported (S–S), clamped–clamped supported (C–C) and clamped–simply supported (C–S) and studied the buckling of uniform and non-uniform bar resting on fluid layer under axial force and distributed load under the same three sets of boundary conditions. These problems were studied using the GDQ method. Firstly, drawbacks existing in the method of differential quadrature (DQ) are evaluated and discussed. Numerical examples have shown the super accuracy, efficiency, convenience and the great potential of this method.

Keywords

Uniform and Non-Uniform Beam and Bar, Deflection, Buckling, GDQM

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1. Introduction

Numerical approximation methods for solving partial differential equations have been widely used in various engineering fields. Most numerical simulations of engineering problems can be currently carried out by conventional low order, classical techniques, such as finite element and finite difference methods are well-developed and well known. These methods can provide very accurate results by using a large number of grid points. However, in some practical applications, the numerical solutions of partial differential equations are required at only a few specified points in the physical domain. In seeking an alternate numerical method using fewer grid points to find results with acceptable accuracy, the method of differential quadrature (DQ) was introduced by Bellman et al. [1, 2]. The DQM is a global approximate method. The DQM is an easy and efficient numerical method for the rapid solution of various linear and nonlinear differential and integro-differential equations. For more details see [3, 4]. The DQM discretizes any derivative at a point by a weighted linear sum of functional values at its neighboring points. The key to DQ is to determine the weighting coefficient for any order derivative discretization. Bellman et al. [2] suggested two methods to determine the weighting coefficients of the first order derivative. More generally, Shu and Richards [5], and Shu [6] present the generalized differential quadrature (GDQ), they applied it to solve some fluid dynamics problems. In GDQ, the weighting coefficients of the first order derivative are determined by a simple algebraic formulation without any restriction on choice of grid points, and the weighting coefficient of the second and higher order derivatives are determined by a recurrence relationship. The major advantage of GDQ over DQ is its ease of the computation of the weighting coefficients without any restriction on the choice of grid points. The pioneer works for the applications of the DQM to the general area of structural

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mechanics [7-16] and fluid mechanics [17-23]. To solve these equations, the boundary conditions have to be implemented appropriately. For the case where there is only one boundary condition at each boundary, the implementation is very simple and can be done in a straightforward way. However, in some cases, there is more than one boundary condition, which could result in difficulties in the numerical implementation of the boundary conditions. In this paper, numerical solution using a generalized differential quadrature method is applied to solve some problems in structural analysis, as determination the static deflection behaviours of uniform and non-uniform beams resting on fluid layer under axial force and the analysis of buckling behaviours of uniform and non-uniform bars resting on fluid layer.

2. Generalized Differential Quadrature Method

In order to overcome the deficiencies which appears in classical differential quadrature method *(DQM)*, Bellman et al. [2], a *generalized differential quadrature (GDQ)*, which was recently proposed by Shu and Richards [5, 6] for solving partial differential equations in fluid mechanics, will be introduced and applied to solve some problems in vibration analysis. In order to find a simple algebraic expression for calculating the weighting coefficients without restricting the choice of grid meshes, Shu chose Lagrange interpolated polynomials as the set of tests functions *g(x)*. Shu and Richards [5, 6] gave a convenient and recurrent formula for determining the derivative weighting coefficients as follows:

$$C_{ij}^{(1)} = \frac{M_N^{(1)}(x_i)}{(x_i - x_j)\, M_N^{(1)}(x_j)},$$

$$j \neq i \quad and \quad i, j = 1, \ldots, N \tag{1}$$

$$\sum_{j=1}^{M} C_{ij}^{(1)} = 0, \text{ for } i = 1, 2, \cdots, N. \tag{2}$$

Equations (1) provide simple expressions for computing $C_{ij}^{(1)}$ without any restriction in choice of the co-ordinates of the grid points x_i. It is obvious that $C_{ij}^{(1)}$ can be easily calculated for $i \neq j$. The $C_{ii}^{(m)}$ can be obtained from equation (2).

For the discretization of the second and higher order derivatives, the following linear constrained relationships are applied

$$f_x^{(m)}(x_i, t) = \sum_{j=1}^{N} C_{ij}^{(m)} \; f(x_j, t) \tag{3}$$

The recurrent formula for determining the derivative weighting coefficients as follows:

$$C_{ij}^{(m)} = m\left(C_{ij}^{(1)}\, C_{ii}^{(m-1)} - \frac{C_{ij}^{(m-1)}}{x_i - x_j} \right),$$

$$j \neq i \quad i, j = 1, 2, \ldots, N; \quad m = 2, 3, \ldots, N-1, \tag{4}$$

$$C_{ii}^{(m)} = -\sum_{j=1, j\neq i}^{N} C_{ij}^{(m)} \quad \text{for } i = 1, 2, \ldots, N \tag{5}$$

where $C_{ij}^{(m)}$ and $C_{ij}^{(m-1)}$ are the weighting coefficients of the m^{th} and the $(m-1)^{\text{th}}$ derivatives. The $C_{ii}^{(m)}$ can be obtained from a relationship similar to equation (2).

where $C_{ij}^{(m)}$ and $C_{ij}^{(m-1)}$ are the weighting coefficients of the m^{th} and the $(m-1)^{\text{th}}$ derivatives. The $C_{ii}^{(m)}$ can be obtained from a relationship similar to equation (2).

Thus equations (4) and (5) together with equation (1) and equation (2) give a convenient and general form for determining the weighting coefficients for the derivatives of orders one through $N-1$.

3. Application of GDQ to Structural Analysis

The method of GDQ using the method of directly Substitutes the Boundary Conditions into the Governing Equations is used for analyzing some static structural problems. The first problem is determining the static deflection behaviours of beams resting on fluid layer under axial force under three sets of boundary conditions, that is, simply supported–simply supported (S–S), clamped–clamped supported (C–C) and clamped–simply supported (C–S). The second problem is for the analysis of buckling behaviours of columns resting on fluid layer under the same previous boundary conditions. The formulations and programming are shown to be very straightforward and simple. The boundary conditions are easy to be implemented.

Basic Equations

(1) Fluid Back Pressure Equations

For an incompressible, irrotational, inviscid fluid of constant density γ_f the pressure of the fluid $P_f(x, y, t)$ satisfies the following equation:

$$\nabla^2 P_f + \gamma_f^2 P_f = 0 \tag{6}$$

where;

The contact conditions are:

$$\frac{\partial P_f}{\partial y} = \rho_o \frac{\partial^2 w}{\partial t^2}\ , \ at\ \ y = 0 \tag{7a}$$

$$\frac{\partial P_f}{\partial y} = 0, \ \ at\ \ y = -h \tag{7b}$$

The method of separation of variables is used to determine the fluid back pressure (P_f),

$$P_f(x, y, t) = W(x)\ P(y)\ f(t) \tag{8}$$

Substituting equation (14) in equation (6) yields;

$$P(x, 0, t) = K_f\ W(x) \tag{9}$$

where; $K_f(\rho_0, \gamma, \omega, h)$ is the fluid linear stiffness.

(2) Beam Equations

The governing equation of a Bernoulli–Euler beam in bending is:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) + p\frac{\partial^2 w}{\partial x^2} + k_f w + f(x) = 0, \ \ 0 \le x \le L \tag{10}$$

where EI is the beam's flexural rigidity, ρA is the mass per unit length; $f(x)$ the external distributed load, L is the length of the beam and $P_f = K_f\ W$ is the fluid back pressure.

Normalizing the equation (10) then the non-dimensional governing equation of a Bernoulli-Euler beam of varying cross-section resting on fluid layer under axial force may be written as:

$$S(X).\frac{d^4W}{dX^4} + 2\frac{dS(X)}{dX}.\frac{d^3W}{dX^3} + \frac{d^2S(X)}{dX^2}\frac{d^2W}{dX^2} + P\frac{d^2W}{dX^2} + K_f W + F(X) = 0, \tag{11}$$

where the non-dimensional coefficients is;

$$S(X) = \frac{EI}{EI_0},\ X = \frac{x}{L},\ W = \frac{V}{a},\ K_f = \frac{k_f L_e^4}{EI_0},\ a = \frac{f_o L^4}{EI}\ \ and\ \ F(x) = \frac{f(x)}{f_o}$$

Equation (11) is a 4^{th} order ordinary differential equation with inertia ratio $s(X) = (1 + \alpha_1 X)^{\alpha_2}$. In the case of non-uniform beam will study two cases of inertia ratio $s(X)$; the first case $\alpha_1 = 0.5$, $\alpha_2 = 1.0$ and the second case $\alpha_1 = -1.0$, $\alpha_2 = 1.0$. It requires *4* boundary conditions, two at $X = 0$, and two at $X = 1$. In the present work, the following two types of boundary conditions are considered:

Simply Supported end (S)

$$W = 0 \ \text{ and } \ \frac{d^2W(0)}{dx^2} = 0 \tag{12a}$$

Clamped Supported end (C)

$$W = 0 \ \text{ and } \ \frac{dW(0)}{dx} = 0 \tag{12b}$$

We assume that the computational domain $0 \le X \le 1$ is divided by ($N - 1$) intervals with coordinates of grid points as $X_1, X_2, \dots, X_N$. With the coordinates of grid points, the GDQ weighting coefficients can be computed through equations (1), (2), (4) and (5). Then, applying the GDQ method to the equation (11) yields;

$$S(X).\left(\sum_{j=3}^{N-2} D_{i,j}\cdot W_j\right) + 2\cdot S^{(1)}(X_i)\cdot\left(\sum_{j=3}^{N-2} C_{i,j}\cdot W_j\right) + S^{(2)}(X_i)\cdot\left(\sum_{j=3}^{N-2} B_{i,j}\cdot W_j\right) + P\sum_{j=3}^{N-2} B_{i,j}\cdot W_j + K_f\cdot W_i = -F(X_i), \ \text{for } i = 1, 2, \dots, N \tag{13}$$

where W_i , $i = 1,2,\cdots,N$, is the functional value at the gridX_i,B_{ij}, C_{ij} and D_{ij} is the weighting coefficient matrix of the sec-ond, third and forth order derivatives. $S^{(2)}(X_i)$, $S^{(1)}(X_i)$, are the second and first order derivatives of $S(X)$ at X_i . Similarly, the derivatives in the boundary conditions can be discreti-zed by the *GDQ* method. As a result, the numerical boundary conditions can be written as:

$$W_1 = 0 \tag{14a}$$

$$\sum_{k=1}^{N} C_{1,k}^{(n0)}\cdot W_k = 0 \tag{14b}$$

$$W_N = 0 \tag{14c}$$

$$\sum_{k=1}^{N} C_{N,k}^{(n1)}\cdot W_k = 0 \tag{14d}$$

where *n0, n1* may be taken as either 1 or 2. By choosing the value of *n0* and *n1*, Equation (14) can give the following four sets of boundary conditions,

n0 = 1 , n1 = 1 ——— clamped–clamped

n0 = 1 , n1 = 2 ——— clamped–simply supported

n0 = 2 , n1 = 1 ——— simply supported–clamped

n0 = 2 , n1 = 2 ——— simply supported–simply supported

Equations (14a) and (14c) can be easily substituted into the governing equation. This is not the case for Equations (14b) and (14d). However, one can couple these two equations together to give two solutions, W_2 and W_{N-1}, as

$$W_2 = \frac{1}{AXN} \sum_{k=3}^{N-2} AXK1 \cdot W_k \tag{15a}$$

$$W_{N-1} = \frac{1}{AXN} \sum_{k=3}^{N-2} AXKN \cdot W_k \tag{15b}$$

where;

$$AXK1 = C_{1,k}^{(n0)} \; C_{N,N-1}^{(n1)} - C_{1,N-1}^{(n0)} \; C_{N,k}^{(n1)}$$

$$AXKN = C_{1,2}^{(n0)} \; C_{N,k}^{(n1)} - C_{1,k}^{(n0)} \; C_{N,2}^{(n1)}$$

$$AXN = C_{N,2}^{(n1)} \; C_{1,N-1}^{(n0)} - C_{1,2}^{(n0)} \; C_{N,N-1}^{(n1)}$$

According to Equations (15), W_2 and W_{N-1} are expressed in terms of $W_3, W_4, \cdots, W_{N-2}$, and can be easily substituted into the governing equation (19). It should be noted that Equation (14) provides four boundary equations. In total we have N unknowns $W_1, W_2, \cdots, W_N$. In order to close the system, the discretized governing equation (13) has to be applied at $(N-4)$ mesh points. This can be done by applying Equation (13) at grid points $X_3, X_4, \cdots, X_{N-2}$. Substituting Equations (14a), (14c) and (15) into Equation (13) gives:

$$S(X)\left(\sum_{j=3}^{N-2} D_{i,j} \cdot W_j\right) + 2 \cdot S^{(1)}(X_i)\left(\sum_{j=3}^{N-2} C_{i,j} \cdot W_j\right) + S^{(2)}(X_i)\left(\sum_{j=3}^{N-2} B_{i,j} \cdot W_j\right) + P\sum_{j=3}^{N-2} B_{i,j}\; W_j + K_f\; W_i = -F(X_i), \;\; i = 3,4,\ldots,N-2 \tag{16}$$

Considering a uniform load with value *f(x) =fo*, then *F(X)*=1. The deflections of the beam at various points are presented together with the exact solutions, if available.

$$S(X)\left(\sum_{j=3}^{N-2} D_{i,j} \cdot W_j\right) + 2\, S^{(1)}(X_i)\left(\sum_{j=3}^{N-2} C_{i,j} \cdot W_j\right) + S^{(2)}(X_i)\left(\sum_{j=3}^{N-2} B_{i,j} \cdot W_j\right) + P\sum_{j=3}^{N-2} B_{i,j}\; W_j + K_f\; W_i = -1,,\; i = 3,4,\ldots,N-2 \tag{17}$$

It is noted that Equation (17) has $(N-4)$ equations with $(N-4)$ unknowns, which can be written in matrix form as;

$$[A]\{W\} = -1, \tag{18}$$

Where $\{W\} = \{W_3, W_4, \cdots, W_{N-2},\}^T$

The Matlab program has been used to solve this problem and get the deflection of the uniform and Non-uniform beam.

3) Buckling of Bar Resting on Fluid Layer under Axial Force and Distributed Load

For the buckling behavior of a slender elastic bar, the normalized governing differential equation can be written as:

$$S(X)\,\frac{d^4W}{dX^4} + 2\frac{dS(X)}{dX}\,\frac{d^3W}{dX^3} + \frac{d^2S(X)}{dX^2}\,\frac{d^2W}{dX^2} + K_f\, W = \lambda\,\frac{\partial^2 W}{\partial X^2} \tag{19}$$

where, X and W are defined in the same way as those in equation (10).

Applying the GDQ to equation (19) at each discrete point on the grid;

$$S(X_i).\left(\sum_{j=1}^{N} D_{i,j} \cdot W_j\right) + 2 \cdot S^{(1)}(X_i)\left(\sum_{j=1}^{N} C_{i,j} \cdot W_j\right) + S^{(2)}(X_i)\left(\sum_{j=1}^{N} B_{i,j} \cdot W_j\right) + K_f\; W_i = \lambda \sum_{j=1}^{N} B_{i,j}\; W_j \tag{20}$$

Equation (20) is a *4^th^* order ordinary differential equation with inertia ratio $s(X) = (1+\alpha_1 X)^{\alpha_2}$. In the case of non-uniform beam will study two cases of inertia ratio $s(X)$; the first case $\alpha_1 = 0.5$, $\alpha_2 = 1.0$ and the second case $\alpha_1 = -1.0$, $\alpha_2 = 1.0$. It requires *4* boundary conditions, two at $X = 0$, and two at $X = 1$. In the present work, the following two types of boundary conditions in equation (12). Similarly, the derivatives in the boundary conditions can be discretized by the *GDQ* method. As a result, the numerical boundary conditions can be written as equations (14).

Equations (14a) and (14c) can be easily substituted into the governing equation. This is not the case for Equations (14b) and (14d). However, one can couple these two equations together to give two solutions, W_2 and W_{N-1}, as equations (15).

According to Equations (15), W_2 and W_{N-1} are expressed in terms of $W_3, W_4, \cdots, W_{N-2}$, and can be easily substituted into the governing equation (20). It should be noted that Equation (14) provides four boundary equations. In total we have N unknowns $W_1, W_2, \cdots, W_N$. In order to close the system, the discretized governing equation (20) has to be applied at $(N-4)$ mesh points. This can be done by applying Equation (20) at grid points $X_3, X_4, \cdots, X_{N-2}$. Substituting Equations (14a), (14c) and (15) into Equation (20) gives:

$$S(X_i).\left(\sum_{j=3}^{N-2} D_{i,j} \cdot W_j\right) + 2 \cdot S^{(1)}(X_i) \cdot \left(\sum_{j=3}^{N-2} C_{i,j} \cdot W_j\right) + S^{(2)}(X_i) \cdot \left(\sum_{j=3}^{N-2} B_{i,j} \cdot W_j\right) + K_f W_i = \lambda \sum_{j=3}^{N-2} B_{i,j} \cdot W_j \tag{21}$$

It is noted that Equation (21) has $(N-4)$ equations with $(N-4)$ unknowns, which can be written in matrix eigen-value form as:

$$[A]\{W\}=\lambda^2[B]\{W\}, \tag{22}$$

where $\{W\}=\{W_3,W_4,\ldots,W_{N-2}\}^T$

The Matlab program has been used to solve this eigen-value problem together with appropriate boundary conditions and get the buckling load λ.

4. Results and Discussion

In this section we will analyze the determination of static deflection behaviours of uniform and non-uniform beams resting on fluid layer under axial force and the analysis of buckling behaviours of uniform and non-uniform bars resting on fluid layer. In the first case will calculating the deflection of a uniform and non-uniform beams and in the second case will calculating non-dimensional buckling loads and mode shapes of the uniform and non-uniform bars under three sets of boundary conditions, that is, simply supported–simply supported (S–S), clamped–clamped supported (C–C) and clamped–simply supported (C–S). By using the GDQ method, using the method of directly Substitutes the Boundary Conditions into the Governing Equations is referred to as (SBCGE).

In the present study, the coordinates of the grid points for the beam are chosen according to Chebyshev-Gauss-Lobatto by using *N* sampling as:

$$X(i)=\frac{1}{2}\left[1-\cos\left(\frac{i-1}{N-1}\pi\right)\right],\quad i=1,2,3,\cdots,N$$

Numerical calculations for the deflection of beam under a given distributed load f(x) have been done for both a uniform and a non-uniform beam under three sets of boundary conditions, namely simply–simply supported (S–S), clamped–clamped supported (C–C) and clamped–simply supported (C–S). The deflections of uniform beam at various points are presented in Table 1 together with the exact solutions. The exact solution of this problem for uniform beam clamped–clamped (C–C) supported is $W(X_i)=\frac{1}{24}X^2(2X-X^2-1)$, for simply–simply supported (S–S) is $W(X_i)=\frac{1}{24}X\,(2X^2-X^3-1)$, and for clamped–simply (C–S) supported (C–S) is $W(X_i)=\frac{1}{48}X^2\,(5X-2X^2-3)$. It can be seen from the table 1 that the results obtained from GDQ method (SBCGE) are very close to the exact solutions and up to 11 digits accuracy can be achieved by using 15 nodes, and are very accurate even using five grid points only. Included in Table 1 are SBCGE results for uniform beam, the SBCGE results for uniform beam resting on fluid layer under axial force, and the exact solution for uniform beam. Tables 1 and Figures 1, show that the convergence of this method is seen to be very good. Accurate results can be achieved by using very few grid points. Also we note that the deflection of beam resting on fluid decreases comparing with the deflection of uniform beam while the deflection of beam resting on fluid under axial force increases.

Table 1.a. Deflection of uniform Simple–Simple beam under uniformly distributed load.

X	W(SBCGM) for uniform beam	W (EXACT) for uniform beam	W (SBCGM) for uniform beam resting on fluid under axial force (K_f=1, P=1)
0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.012536043909088	-0.000522172019469	-0.000522172019470	-0.0005738029055314
0.049515566048790	-0.002053282235960	-0.002053282235960	-0.002256437749143
0.109084258765985	-0.004442907699535	-0.004442907699535	-0.004883446929569
0.188255099070633	-0.007340315693757	-0.007340315693758	-0.008070963881860
0.283058130441221	-0.010171639030995	-0.010171639030996	-0.011188794184760
0.388739533021843	-0.012253537486948	-0.012253537486949	-0.013483472449208
0.500000000000000	-0.013020833333332	-0.013020833333333	-0.014329660120701
0.611260466978157	-0.012253537486948	-0.012253537486949	-0.013483472449208
0.716941869558779	-0.010171639030995	-0.010171639030996	-0.011188794184760
0.811744900929367	-0.007340315693757	-0.000973019162735	-0.008070963881860
0.890915741234015	-0.004442907699535	-0.004442907699535	-0.004883446929569
0.950484433951210	-0.002053282235960	-0.002053282235960	-0.002256437749142
0.987463956090912	-0.000522172019469	-0.000522172019470	-0.000573802905531
1.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000

Table 1.b. Deflection of uniform Clamped–Clamped beam under uniformly distributed load.

X	W(SBCGM) for uniform beam	W (EXACT) for uniform beam	W (SBCGM) for uniform beam resting on fluid under axial force (K_f=1, P=1)
0.000000000000000	0.0	0.0	0.0
0.012536043909088	-6.384873128 e^{-06}	-6.384873128 e^{-06}	-6.484634965888 e^{-06}
0.049515566048790	-9.2291620641 e^{-05}	-9.2291620641 e^{-05}	-9.384013544069 e^{-05}
0.109084258765985	-3.93537563891 e^{-04}	-3.93537563891 e^{-04}	-4.0079663326029 e^{-04}
0.188255099070633	-9.73019162735 e^{-04}	-9.73019162735 e^{-04}	-9.9276756748439 e^{-04}
0.283058130441221	-0.001715962979648	-0.001715962979648	-0.0017536506516755
0.388739533021843	-0.002352657966624	-0.002352657966624	-0.0024070432078642
0.500000000000000	-0.002604166666667	-0.002604166666667	-0.0026654379817753
0.611260466978157	-0.002352657966624	-0.002352657966624	-0.0024070432078642
0.716941869558779	-0.001715962979648	-0.001715962979648	-0.0017536506516755
0.811744900929367	-9.73019162735 e^{-04}	-9.73019162735 e^{-04}	-9.927675674844 e^{-04}
0.890915741234015	-3.93537563891 e^{-04}	-3.93537563891 e^{-04}	-4.007966332603 e^{-04}
0.950484433951210	-9.2291620641 e^{-05}	-9.2291620641 e^{-05}	-9.3840135440689 e^{-05}
0.987463956090912	-6.384873128 e^{-06}	-6.384873128 e^{-06}	-6.4846349658879 e^{-06}
1.000000000000000	0.0	0.0	0.0

Table 1.c. Deflection of uniform Clamped–Simple beam under uniformly distributed load.

X	W(SBCGM) for uniform beam	W (EXACT) for uniform beam	W (SBCGM) for uniform beam resting on fluid under axial force (K_f=1, P=1)
0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.012536043909088	-0.000009617838285	-0.000009617838285	-0.000009920098516
0.049515566048790	-0.000140841400808	-0.000140841400808	-0.000145491986852
0.109084258765985	-0.000614398833749	-0.000614398833749	-0.000636152582312
0.188255099070633	-0.001572357182365	-0.001572357182365	-0.001632481926707
0.283058130441221	-0.002912686907003	-0.002912686907004	-0.003032476726389
0.388739533021843	-0.004277089607242	-0.004277089607242	-0.004464063515380
0.500000000000000	-0.005208333333333	-0.005208333333333	-0.005446596389740
0.611260466978157	-0.005378666086168	-0.005378666086168	-0.005632038957293
0.716941869558779	-0.004747077077966	-0.004747077077966	-0.004974160830139
0.811744900929367	-0.003557329408617	-0.003557329408617	-0.003728432201988
0.890915741234015	-0.002197361361855	-0.002197361361855	-0.002303081825307
0.950484433951210	-0.001024237148133	-0.001024237148133	-0.001073466060593
0.987463956090912	-0.000261045481142	-0.000261045481142	-0.000273586196411
1.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000

To demonstrate better the accuracy of the solutions, Figures 1 show the deflection which results from the GDQ method for uniform beam, uniform beam resting on Fluid Layer and uniform beam resting on Fluid Layer under axial force which compared with the exact solutions of uniform beam.

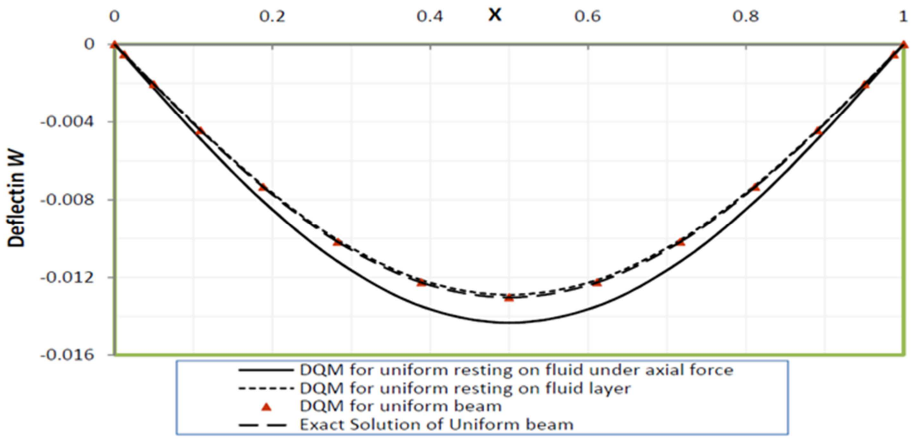

Figure 1a. Deflection of uniform Simply-Simply beam under bending (K_f=1.0, P=1.0).

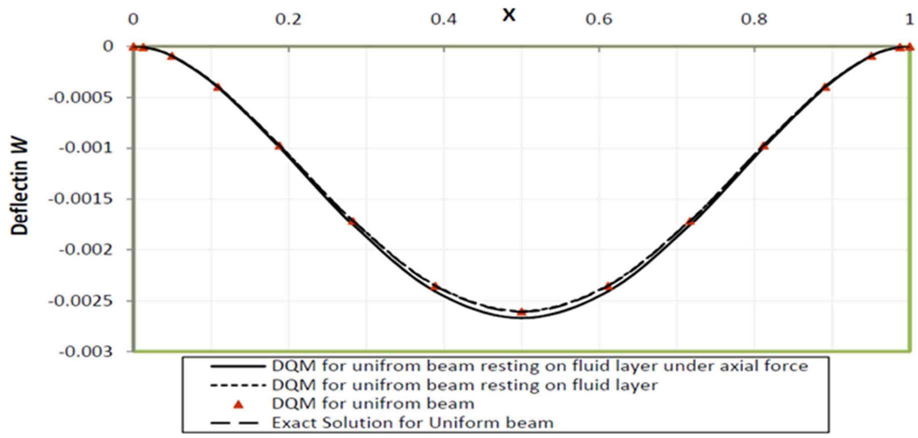

Figure 1b. Deflection of uniform Clamped-Clamped beam under bending (K_f=1.0, P=1.0).

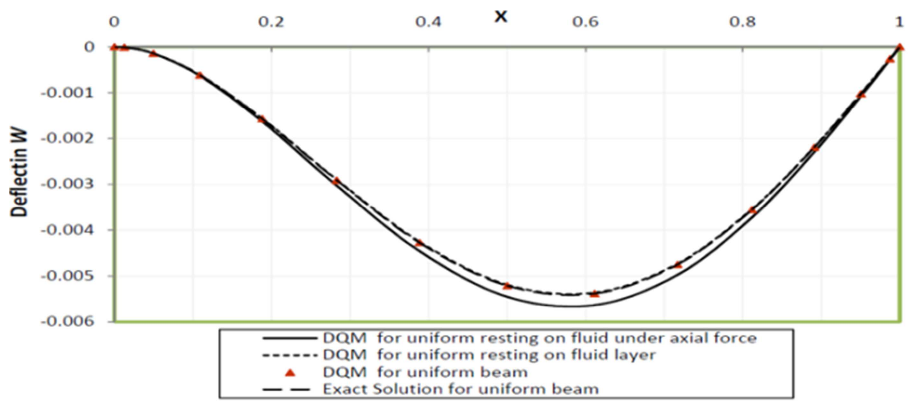

Figure 1c. Deflection of uniform Clamped-Simply beam under bending (K_f=1.0, P=1.0).

The deflection of non-uniform beam resting on fluid layer under axial force with different stiffness distributions $s(X) = (1 + \alpha_1 X)^{\alpha_2}$, the first case $\alpha_1 = 0.5$, $\alpha_2 = 1.0$ and the second case $\alpha_1 = -1.0$, $\alpha_2 = 1.0$, respectively studied under three sets of boundary conditions, (S–S, C–C and C–S). Also, the *GDQ* results are obtained using 15 non-uniformly spaced grid points.

Figures 2, 3 show the deflection of non-uniform beam, Figures 2 for the first case $\alpha_1 = 0.5$, $\alpha_2 = 1.0$, and Figures 3 for the second case $\alpha_1 = -1.0$, $\alpha_2 = 1.0$. Figures 2, 3, show that the deflections of non-uniform beam resting on fluid under axial force decreases comparing with the deflection of uniform beam. Also the deflection of non-uniform beam increases with the increase of the different stiffness distributions *S(X)*. It can be observed from Figures 2, 3 that, the deflection decreases when the beam resting on fluid layer fluid but increases when the beam resting on fluid layer under axial force.

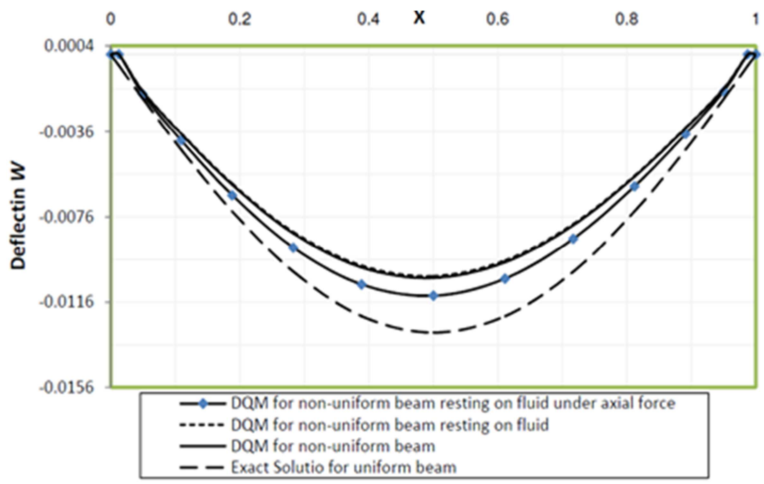

Figure 2a. Deflection of non-uniform Simply-Simply beam under bending (K_f=1.0, P=1.0).

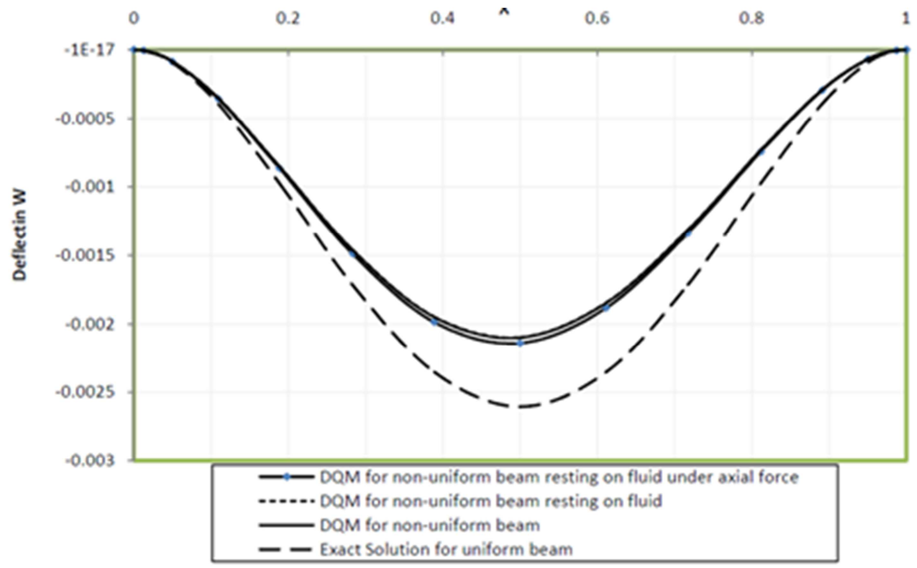

Figure 2b. Deflection of non-uniform Clamped-Clamped beam under bending (K_f=1.0, P=1.0).

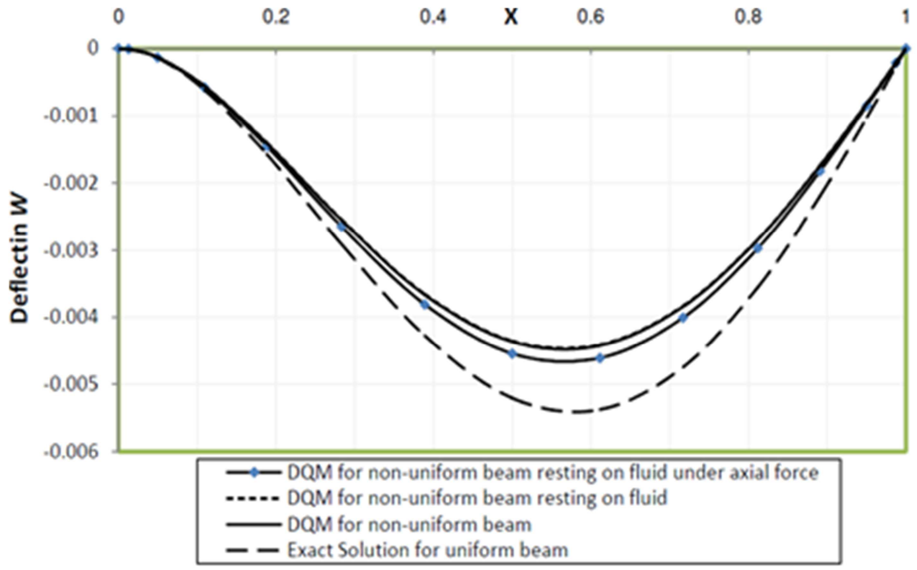

Figure 2c. Deflection of non-uniform Clamped-Simply beam under bending (K_f=1.0, P=1.0).

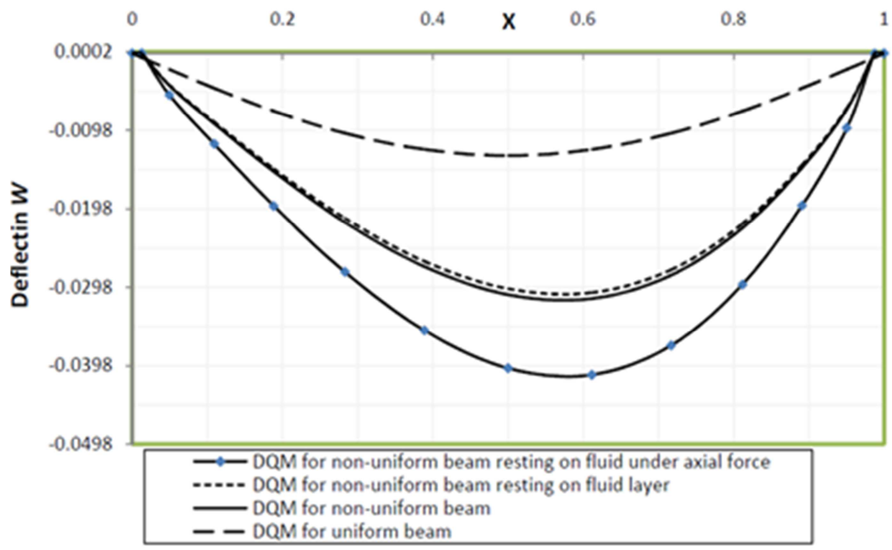

Figure 3a. Deflection of non-uniform Simply-Simply beam under bending (K_f=1.0, P=1.0).

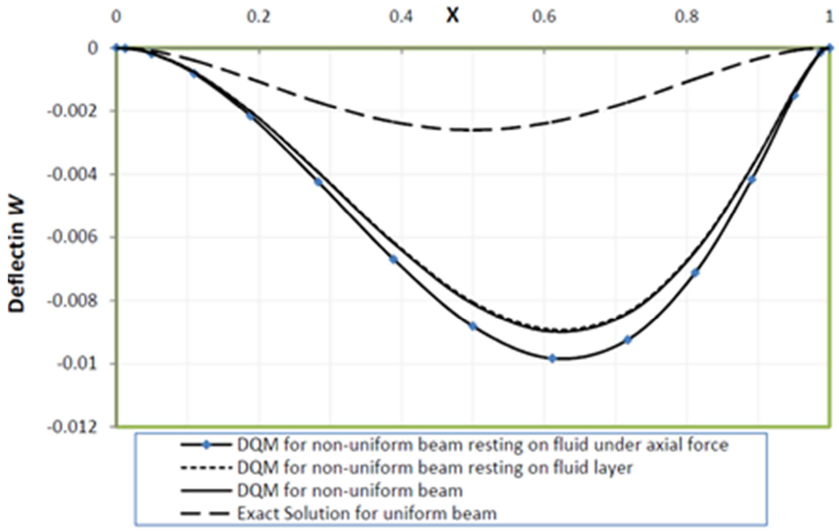

Figure 3b. Deflection of non-uniform Clamped-Clamped beam under bending (K_f=1.0, P=1.0).

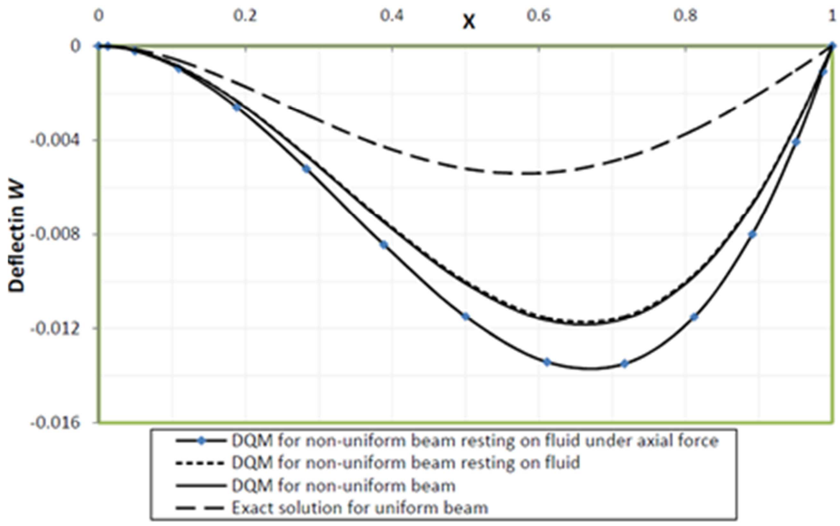

Figure 3c. Deflection of non-uniform Clamped-Simply beam under bending (K_f=1.0, P=1.0).

Numerical calculations for the computations of the non-dimensional buckling loads and mode shape of a uniform and a non-uniform bars under three sets of boundary conditions, (S–S, C–C and C–S). The buckling loads can be obtained by solving the eigen-value problem together with appropriate boundary conditions of uniform and non-uniform bars resting on fluid layer subjected to axial force by using the differential quadrature method (SBCGE). Table 2 presented *SBCGE* results for uniform bar, the *SBCGE* results for uniform bar resting on fluid layer, *GDQ* results for uniform bar Du [11] and the exact solution Chajes [24] and Newbery [25] for uniform bar. It can be seen from the Tables 2 that the convergence of the solution using GDQ is excellent comparison of the present results with the exact ones shows that the GDQ is a very accurate numerical technique by using 15 nodes.

Table 2 and Figures 4; show that the convergence of this method is seen to be very good. Accurate results can be achieved by using very few grid points. Also we note that the non-dimensional buckling loads of uniform beam resting on fluid increases comparing with the deflection of uniform beam.

Table 2. Non-dimensional buckling loads of uniform bars.

Buckling Load λ / Boundary Conditions	**Exact Chajes [8] for uniform beam**	**GDQ (N=11) Du [10] for uniform beam**	**FEM Newbery [20] for uniform beam**	**Present (SBCGM) for uniform beam**	**Present (SBCGM) for uniform beam resting on fluid, (Kf=1)**
Simple-Simple	9.8696	9.8696	9.9438	9.8696	9.9709
Clamped-Clamped	39.4784	39.4784	39.9730	39.4784	39.5544
Clamped-Simple	20.19073	20.19072	20.4972	20.19072	20.2733

To demonstrate better the accuracy of the solutions, the mode shapes of the first three modes of uniform bar resting on fluid layer were also obtained for three sets of boundary conditions by using 15 non-uniformly spaced grid points. The mode shapes are presented in Figures 4.

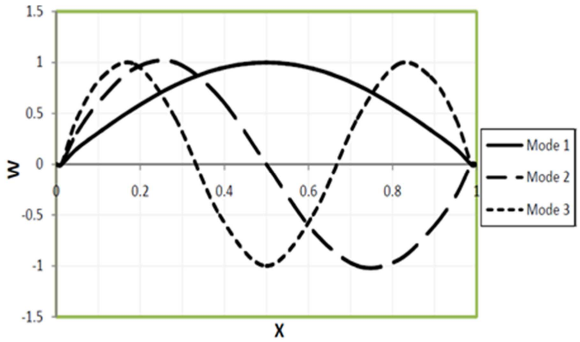

Figure 4a. The first three mode shapes of uniform Simply-Simply bar (K_f=1).

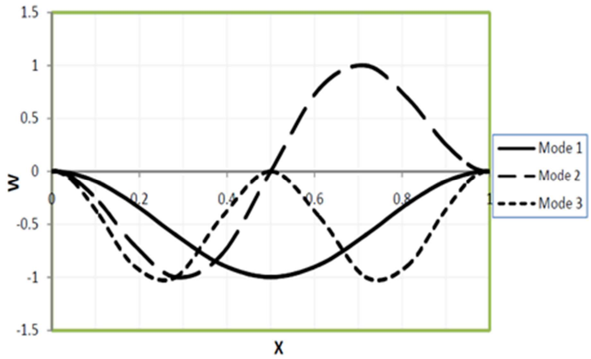

Figure 4b. The first three mode shapes of uniform Clamped- Clamped bar (K_f=1).

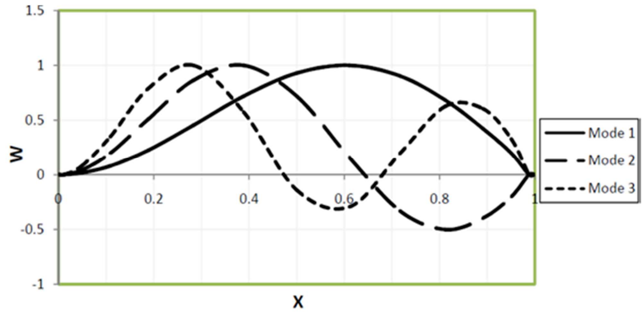

Figure 4c. The first three mode shapes of uniform Clamped- Simply bar (K_f=1).

Also, Table 3, 4 lists the non-dimensional buckling loads of a non-uniform bars resting on fluid layer with different stiffness distributions $s(X) = (1 + \alpha_1 X)^{\alpha_2}$, the first case $\alpha_1 = 1.0, \alpha_2 = 1.0$ and the second case $\alpha_1 = 1.0, \alpha_2 = 2.0$, respectively. Again, three sets of boundary conditions, (S–S, C–C and C–S), were considered for each bar. Also, the *GDQ* results are obtained using 15 non-uniformly spaced grid points. The varying cross section stiffness *S(X)* of the beams can be very easily implemented from equations (27). The computing effort is still small since one has to solve an eigen-value problem of a matrix of dimension 11 × 11 only. Tables 3, 4 presented *SBCGE* results for non-uniform bar, the *SBCGE* results for non-uniform bar resting on fluid layer, *GDQ* results for non-uniform bar Du [11],. It can be observed from Table 3, 4 that; the non-dimensional buckling loads increases when the bar resting on fluid layer.

Table 3. Non-dimensional buckling loads of non-uniform bars.

Buckling Load λ / Boundary Conditions	Reference for non-uniform beam	Reference for non-uniform beam	GDQ (N=11) Du [10] for non-uniform beam	Present (SBCGM) for non-uniform beam	Present (SBCGM) for non-uniform beam resting on fluid, (Kf=1)
Simple-Simple	15.31 [6]	14.3 [27]	14.5113	14.5112	14.6121
Clamped-Clamped	-------	-------	57.3453	57.3940	57.4697
Clamped-Simple	-------	-------	29.4406	29.4490	29.5360

Table 4. Non-dimensional buckling loads of non-uniform bars.

Buckling Load λ / Boundary Conditions	Reference for non-uniform beam	Reference for non-uniform beam	GDQ (N=11) Du [10] for non-uniform beam	Present (SBCGM) for non-uniform beam	Present (SBCGM) for non-uniform beam resting on fluid, (Kf=1)
Simple-Simple	20.7923 [10]	27.455 [26]	20.8047	19.8828	19.9810
Clamped-Clamped	-------	-------	82.1043	82.9986	83.0768
Clamped-Simple	-------	-------	41.9679	48.1054	84.2002

To demonstrate better the accuracy of the solutions, the mode shapes of the first three modes of uniform bar resting on fluid layer were also obtained for three sets of boundary conditions by using 15 non-uniformly spaced grid points. The mode shapes are presented in Figures 5, 6. Figures 5 for the first case $\alpha_1 = 1.0, \alpha_2 = 1.0$, and Figures 6 for the second case $\alpha_1 = 1.0, \alpha_2 = 2.0$,

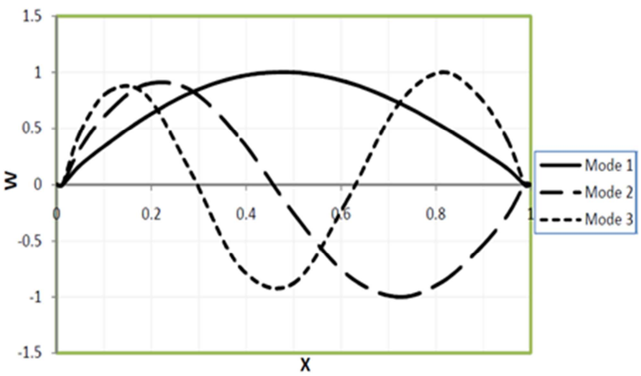

Figure 5a. The first three mode shapes of non-uniform Simply-Simply bar (K_f=1.0).

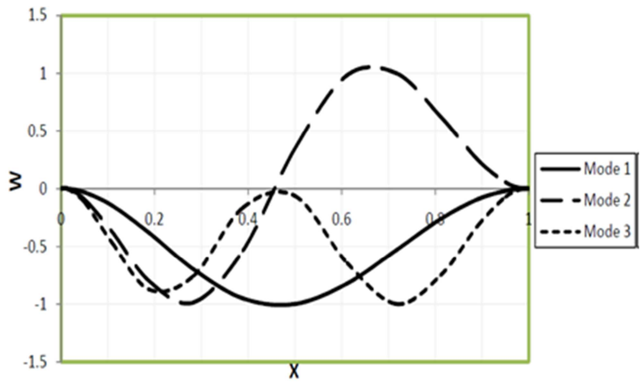

Figure 5b. The first three mode shapes of non-uniform Clamped- Clamped bar (K_f=1.0).

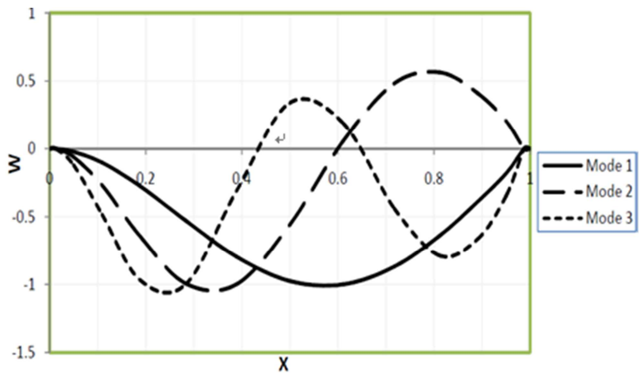

Figure 5c. The first three mode shapes of uniform Clamped- Simply bar (K_f=1.0).

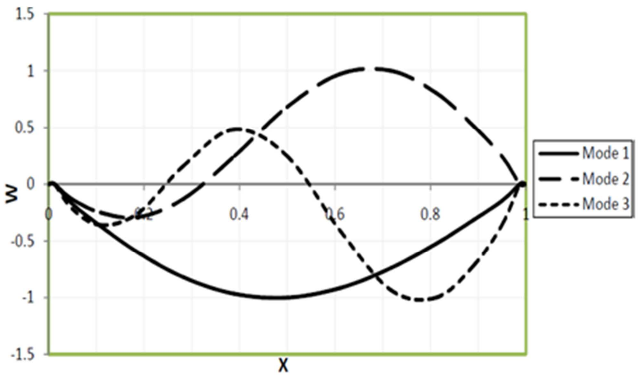

Figure 6a. The first three mode shapes of non-uniform Simply-Simply bar (K_f=1.0).

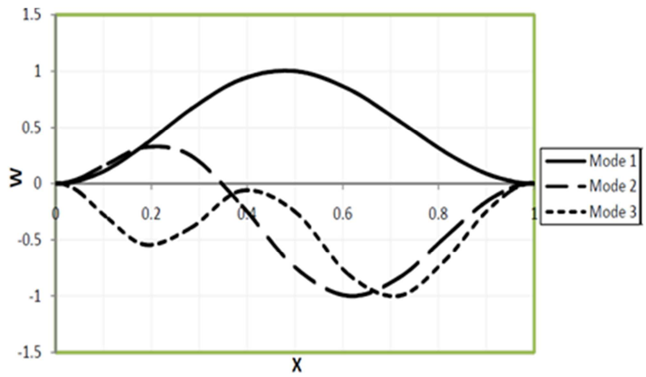

Figure 6b. The first three mode shapes of non-uniform Clamped- Clamped bar (K_f=1.0).

Figure 6c. The first three mode shapes of uniform Clamped- Simply bar (K_f=1.0).

5. Conclusions

This paper studied some problems in structural analysis. In the first case will calculating the deflection of a uniform and non-uniform beams and in the second case will calculating non-dimensional buckling loads and mode shapes of the uniform and non-uniform bars under three sets of boundary conditions. The problems is studied by using the GDQM, using the method of directly Substitutes the Boundary Conditions into the Governing Equations is referred to as (SBCGE).

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