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Variable Viscosity Effect on MHD Free Convection Flow over a Porous Plate with Suction and Injection

K. R. Jayakumar[1, *], A. T. Eswara[2]

[1]Department of Mathematics, K. S. Institute of Technology Bangalore, India
[2]Department of Mathematics, GSSS Institute of Engineering and Technology for Women KRS Road, Mysore, India

Abstract

In this study, we examine the effect of variable viscosity on steady MHD free convection flow of an electrically conducting fluid over a porous plate, in the presence of suction and injection. The system of coupled nonlinear partial differential equations governing the non-similar flow has been solved numerically using implicit finite difference scheme along with a quasilinearization technique. Computations are performed and numerical results are displayed graphically to illustrate the influence of the different physical parameters such as magnetic field parameter, viscosity variation parameter, suction and injection on the flow field and heat transfer characteristics.

Keywords

Skin Friction, Heat Transfer, Variable Viscosity, Suction, Injection

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1. Introduction

Convective boundary-layer flows are often controlled by injecting or withdrawing fluid through a porous bounding heated surface. This can lead to enhanced heating or cooling of the system and can help to delay in change over from laminar to turbulent flow. The case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [1]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [2], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection). Using the method of matched asymptotic expansions, the next order corrections to the boundary-layer solutions for this problem were obtained by Clarke [3], who extended the range of applicability of the analyses by not invoking the usual Boussinesq approximation. The effect of strong suction and blowing from general body shapes which admit a similarity solution has been studied by Merkin [4]. A transformation of the equations for general blowing (injection) and wall temperature variations has been given by Vedhanayagam et. al. [5]. The case of heated isothermal horizontal surface with transpiration has been discussed in some details first by Clarke and Riley [6] and then by Lin and Yu [7]. Kumaran and Pop [8] have studied the steady free convection boundary layer over a vertical flat plate embedded in a porous medium. Further, the effect of magnetic field on free convection flow over a plate with suction and injection is discussed by Jayakumar.et.al [9]. However, in many cases of practical interest, it is essential to study the effect variable viscosity on the momentum and transport phenomena along with an applied magnetic field. Eswara and Bommaiah [10] studied the effect of variable viscosity on laminar flow due to point sink. Recently the effect of temperature dependent viscosity on two-dimensional and axisymmetric has been discussed by Jayakumar et.al. [11].

* Corresponding author
E-mail address: krjk_maths@yahoo.com (K. R. Jayakumar)

The aim of present discussion is to study, the effects of variable viscosity on the MHD non-similar free convection flow of an electrically conducting fluid over a plate with suction and injection. The governing equation of mass, momentum and energy

were transformed into two point boundary value problem and the nonlinear equations along with proper boundary conditions are solved using an implicit finite difference scheme.

2. Problem Formulation

Consider a semi-infinite porous plate at a uniform temperature T_{w0} which is played vertical in a quiescent fluid of infinite extent maintained at constant temperature T_∞. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge in the stream wise direction and y is measured normal to the surface of the plate. The co-ordinate system and flow configuration are shown in Fig.1.

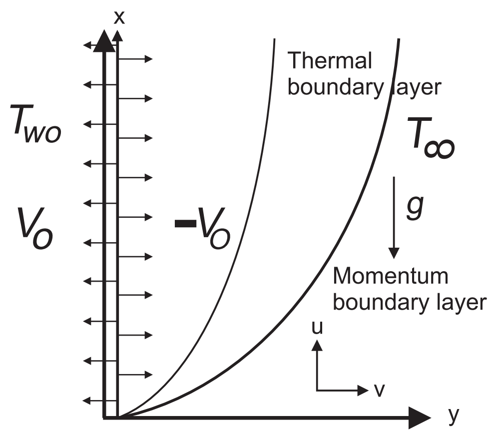


Figure 1. The coordinate system and the physical model

Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. A magnetic field B_0 is applied in y-direction normal to the body surface and it is assumed that magnetic Reynolds number is small. The Hall current and displacement current effects have been neglected. The fluid is assumed to have constant physical properties except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) (see Lai and Kulacki [12]), viz.

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right] \tag{1}$$

$$\frac{1}{\mu}=a\left(T-T_e\right) \tag{2}$$

$$where \quad a=\frac{\gamma}{\mu_\infty};\ T_e=T_\infty-\frac{1}{\gamma} \tag{3}$$

Under the aforesaid assumptions with Boussinesq's approximation, the equations governing the flow are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 u}{\rho} \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2} \tag{6}$$

The initial and boundary conditions are

$$\begin{gathered} x=0,\ \ y>0,\ \ \ u=0,\ T=T_\infty \\ y=0;\ u=0,\ v=-v_0(for\,suction), \\ v=+v_0(for\,blowing),\ T=T_{wo} \\ y\to\infty;\ u=0,\ T=T_\infty \end{gathered} \tag{7}$$

Introducing the following transformations

$$\begin{gathered} \psi=\frac{\nu^2 g\beta\left(T_{w0}-T_\infty\right)\xi^3}{V_0{}^3}\left[f(\eta,\xi)\pm\frac{\xi}{4}\right]; \\ T=T_\infty+\left(T_{wo}-T_\infty\right)G(\eta,\xi) \\ \eta=\frac{V_o y}{\nu\xi};\qquad \xi=V_o\left[\frac{4x}{\nu^2 g\beta\left(T_{wo}-T_\infty\right)}\right]^{1/4}; \\ u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x} \end{gathered} \tag{8}$$

to Eqns.(1) – (3), we see that the continuity Eq.(1) is identically satisfied and Eqns.(2) – (3) reduces, respectively, to

$$\begin{gathered} F''+G\left(1-\frac{G}{Ge}\right)+3fF'\left(1-\frac{G}{Ge}\right)-2F^2\left(1-\frac{G}{Ge}\right)\pm \\ \xi F\left(1-\frac{G}{Ge}\right)-MF\xi^2\left(1-\frac{G}{Ge}\right)-\frac{G'F'}{Ge-G}= \\ \xi\left(1-\frac{G}{Ge}\right)\left(FF_\xi-F'f_\xi\right) \end{gathered} \tag{9}$$

$$\Pr^{-1}G''+3fG'\pm\xi G'=\xi\left(FG_\xi-G'f_\xi\right) \tag{10}$$

Where

$$u=\frac{V_0^2 4x}{\nu\xi^2}F;\quad v=-\frac{V_0}{\xi}\left(3f+\xi f_\xi-\eta F\pm\xi\right)$$
$$f=\int_0^\eta F\,d\eta\ ;\quad \Pr=\frac{\nu}{\alpha}\ ;\quad M=\frac{\sigma B_0^2\upsilon}{\rho V_0^2} \tag{11}$$

It is remarked here that the upper sign in Eqns.(9) and (10) is taken throughout for suction and the lower sign for blowing (injection).

The transformed boundary conditions are

$$F = 0;\ G = 1 \text{ at } \eta = 0$$
$$F = 0;\ G = 0 \text{ as } \eta\to\infty \text{ for } \xi \ge 0 \tag{12}$$

The local skin friction parameter and heat transfer parameter can be expressed as

$$\tau_w=\frac{V_0}{g\beta\left(T_{w0}-T_\infty\right)}\left(\frac{\partial u}{\partial y}\right)_{y=0}=\xi\left(F'\right)_{\eta=0} \tag{13}$$

$$Q=\frac{\nu}{V_0\left(T_{w0}-T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\frac{1}{\xi}\left(G'\right)_{\eta=0} \tag{14}$$

Here, u and v are velocity components in x and y direction; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ,η, t* are transformed co-ordinates; ψ and f are the dimension and dimensionless stream functions respectively; Pr is the Prandtl number; ν, α are respectively kinetic viscosity and thermal diffusivity; w_0 and ∞ denote conditions at the edge of the boundary layer on the wall at time t=0 and in the free stream respectively and prime (') denotes derivatives with respect to η. The dimensionless temperature G and viscosity ratio μ/μ_∞ are redefined as follows:

$$G=\frac{T-T_e}{T_w-T_\infty}+Ge \quad \text{and hence}$$

$$\frac{\mu}{\mu_\infty}=\frac{Ge}{Ge-G} \tag{15}$$

where Ge is constant, called viscosity variation parameter, which is defined by

$$Ge=\frac{T_e-T_\infty}{T_w-T_\infty}=\frac{-1}{\gamma\left(T_w-T_\infty\right)}=\text{constant} \tag{16}$$

and its value is determined by viscosity characteristics of the fluid under consideration and operating temperature difference $\Delta T=T_w - T_\infty$. It may be remarked here that, if Ge is large (i.e.,$Ge\to\infty$) the effect of variable viscosity can be neglected. On the other hand, for a smaller value of Ge, either the fluid viscosity changes markedly with temperature or operating temperature difference is high. In either case, the variable viscosity effect is expected to become very significant. Also, it may be noted here that, liquid viscosity varies differently with temperature than that of gas and therefore, it is important to note that Ge<0 for liquids and Ge>0 for gases when the temperature difference ΔT is positive. It is worth mentioning here that when M = 0.0, $\gamma\to 0$ i.e., $\mu=\mu_\infty$ then $Ge\to\infty$ and Eqns.(9) and (10) reduces to

$$F''+G+3fF'-2F^2\pm\xi F'=\xi\left(FF_\xi-F'f_\xi\right) \tag{17}$$

$$\Pr^{-1}G''+3fG'\pm\xi G'=\xi\left(FG_\xi-G'f_\xi\right) \tag{18}$$

which are exactly same as those of Merkin [2]. Also, if M ≠ 0.0 and $Ge\to\infty$ then the Eqns. (9) and (10) reduces to

$$F''+G+3fF'-2F^2\pm\xi F'-M\xi^2F=\xi\left(FF_\xi-F'f_\xi\right) \tag{19}$$

$$\Pr^{-1}G''+3fG'\pm\xi G'=\xi\left(FG_\xi-G'f_\xi\right) \tag{20}$$

which are exactly same as those of Jayakumr et al. [9].

3. Method of Solution

The coupled non-linear partial differential Eqns.(9) and (10) under the boundary conditions (12) have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization technique [13,14]. Quasi-linearisation technique can be viewed as a generalization of the Newton-Raphson approximation technique in functional space.

Applying quasilinearzation technique, we replace the non-linear partial differential equations (9) and (10) by an iterative sequence of linear equations as follows:

$$F''^{(k+1)}+X^{(k)}{}_1F'^{(k+1)}+X^{(k)}{}_2F^{(k+1)}+X^{(k)}{}_3F_\xi^{(k+1)}+X^{(k)}{}_4G'^{(k+1)}+X^{(k)}{}_5G^{(k+1)}=U_1^{(k)} \tag{21}$$

$$G''^{(k+1)}+Y_1^{(k)}G'^{(k+1)}+Y_2^{(k)}G_\xi^{(k+1)}+Y_3^{(k)}F^{(k+1)}=U_2^{(k)} \tag{22}$$

where the coefficient functions with iterative index k are known and functions with iterative index k+1 are to be determined. The boundary conditions become

$$\begin{aligned} &F^{(k+1)}=0 \quad G^{(k+1)}=1 \quad at \quad \eta=0\\ &F^{(k+1)}=0 \quad G^{(k+1)}=0 \quad at \quad \eta=\eta_\infty \end{aligned} \tag{23}$$

The coefficients in (21) and (22) are given by

$$X_1^{(k)} = \left(1-\frac{G}{Ge}\right)\left[3f+\xi f_\xi\right] - \frac{G'}{Ge-G}$$

$$X_2^{(k)} = \left(1-\frac{G}{Ge}\right)\left[-4f \pm \xi - M\xi^2 - \xi F_\xi\right]$$

$$X_3^{(k)} = -\xi F\left(1-\frac{G}{Ge}\right)$$

$$X_4^{(k)} = -\frac{F'}{Ge-G}$$

$$X_5^{(k)} = \left(1-\frac{G}{Ge}\right) - \frac{G}{Ge} - \frac{1}{Ge}\left[3fF' - 2F^2 \pm \xi F - MF\xi^2 - \xi F F_\xi + \xi F' f_\xi\right] - \frac{G'F'}{(Ge-G)^2}$$

$$U_1^{(k)} = \left(1-\frac{G}{Ge}\right)\left[-G-2F^2-\xi F F_\xi\right] - \frac{G'F'}{Ge-G} + GX_5$$

$$Y_1^{(k)} = 3f\,\Pr + \xi\,\Pr\,f_\xi \pm \xi\,\Pr$$

$$Y_2^{(k)} = -\xi\,\Pr\,F$$

$$Y_3^{(k)} = -\xi\,\Pr\,G_\xi$$

$$U_2^{(k)} = -\xi\,\Pr\,FG_\xi$$

The equations (21) and (22) along with boundary conditions (23) were expressed in difference form, considering central difference scheme in $\eta -$direction. In each iteration step, equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is later solved using [15]. To ensure the convergence of the numerical solution to the exact solution, step size $\Delta\eta$ is optimized and taken as 0.01. The results presented here are independent of the step size in $\eta -$direction at least up to the four decimal place The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 5.0 throughout the computation. Iteration is employed to deal with the nonlinear nature of the governing equations to become linear, locally. A convergence criterion based on the relative difference between the current and the previous iteration values of the velocity and temperature gradients at wall are employed. The solution is assumed to have converged and the iterative process is terminated when

$$Max\left[\left|(F'_w)^{(k+1)} - (F'_w)^{(k)}\right|,\ \left|(G'_w)^{(k+1)} - (G'_w)^{(k)}\right|\right] < 10^{-4}$$

4. Results and Discussion

In order to assess the accuracy of the method which we have used, results were obtained for M=0.0 by solving Eqns. (12) and (13). The skin friction and heat transfer parameters (τ_w, Q) for suction [See Fig.2 (a)] and injection [See Fig.2 (b)] have been obtained and compared with those of Merkin [2]. Further, the steady state skin friction results with magnetic field $M \neq 0$ are compared with those of Jayakumar et al.[9] [See Fig.3(a) for suction] and [See Fig.3(b) for injection] by solving the Eqns.(14) and (15). Our results are found to be in excellent agreement, with the above-mentioned studies.

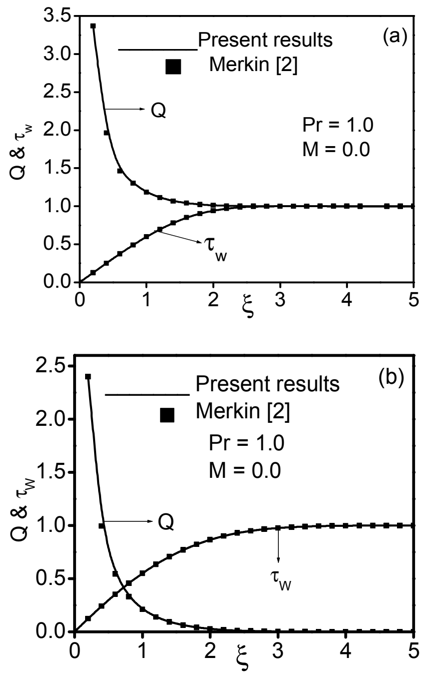
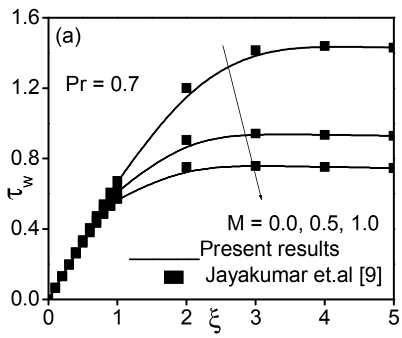


Figure 2. Comparison of skin friction and heat transfer parameters with Merkin [2] for (a) Suction (b) Blowing (injection)



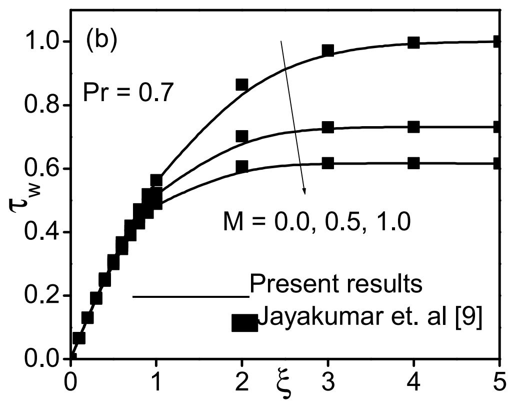


Figure 3. Comparison of skin friction parameter for (a) Suction (b) Injection with those of Jayakumar et al. [9]

4.1. Suction

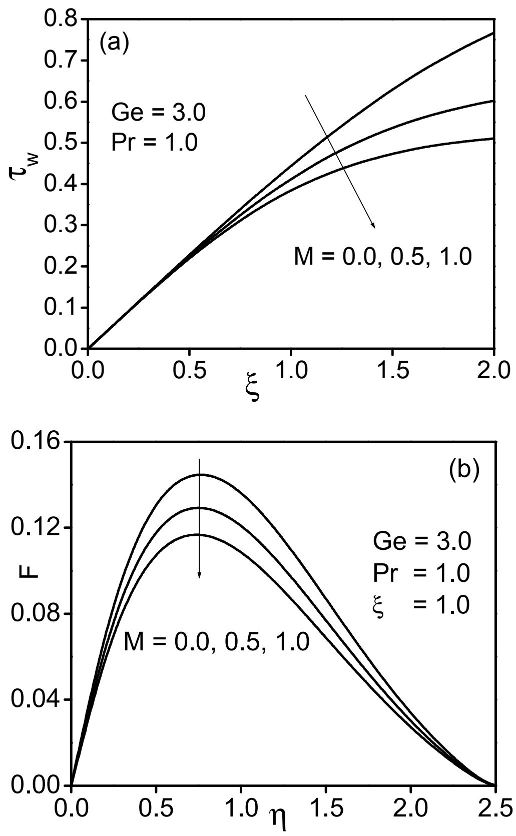


Figure 4. The effect of magnetic field (M) on (a) skin friction and (b) velocity in presence of viscosity variation parameter

The effect of magnetic field (M) on skin friction and velocity in presence of viscosity variation parameter (Ge) is displayed in Fig.4. As M increases, it is found that τ_w decreases. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to phenomena. In fact, skin friction decreases 6% at $\xi = 1.0$ in the range of M ($0.0 \leq M \leq 1.0$). Also, it is found that the thickness of momentum boundary layer decreases by 3% with the increase of magnetic parameter in the same range of M.

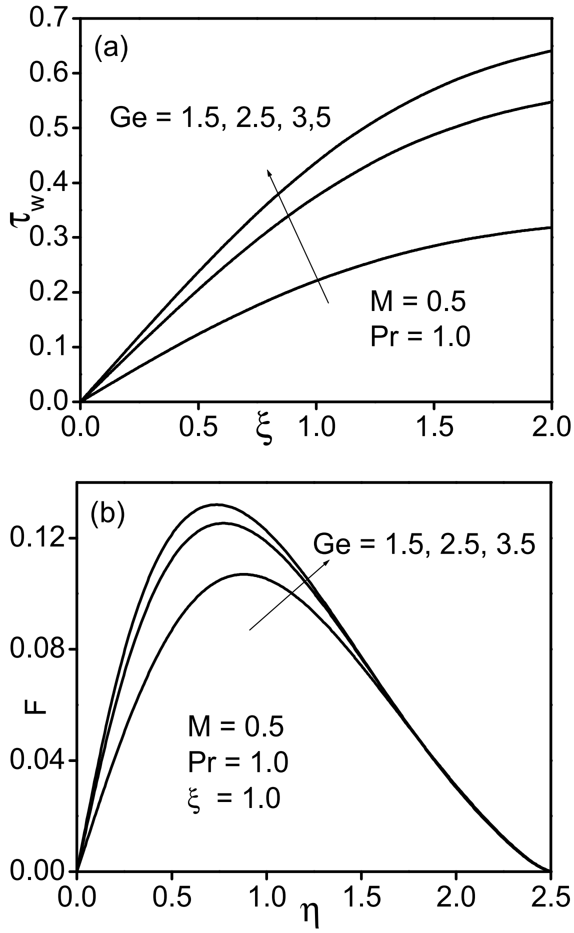
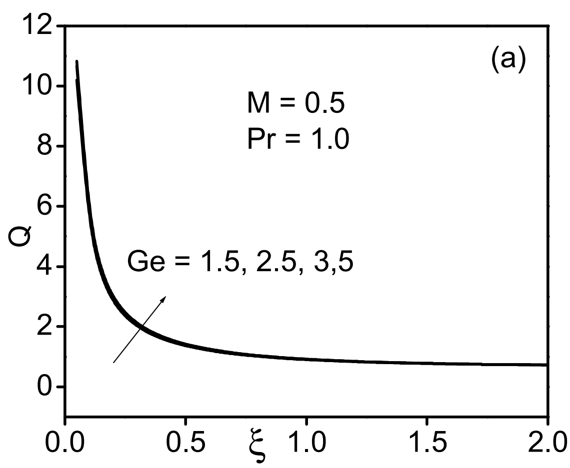


Figure 5. The effect of viscosity variation parameter (Ge) on (a) skin friction and (b) Velocity in presence of magnetic field

Fig.5 depicts the effect of viscosity variation parameter (Ge) on both skin friction parameter and velocity with magnetic parameter M. It is observed from this figure that the skin friction parameter increases with the increase of Ge. Indeed, τ_w increases 21.6 % at $\xi = 1.0$ in the range of Ge ($1.5 \leq Ge \leq 3.5$). As the parameter Ge increases the velocity inside the momentum boundary layer uniformly increases as seen in the figure5 (b).



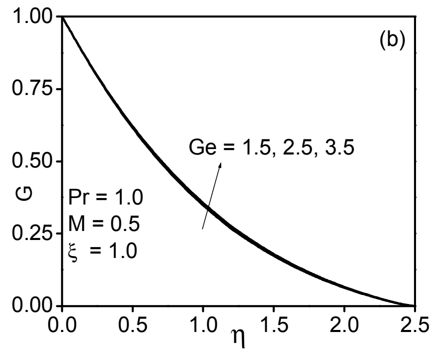


Figure 6. The effect of viscosity variation parameter (Ge) on (a) heat transfer and (b) temperature in presence of magnetic field

The influence of viscosity variation parameter (Ge) on heat transfer (Q) and temperature (G) is presented in figure 6. It is clear from the diagram that both heat transfer and temperature increases uniformly with the increase of Ge.

4.2. Injection

For the injection case, the corresponding results for skin friction parameter τ_w and velocity F are presented in Figs. 7 and 8 respectively. It is observed that the results are found to be qualitatively similar but quantitatively different as compared to suction. Actually, τ_w decreases about 5% from M=0.0 to M=1.0 at $\xi = 1.0$, while percentage of decrease in F is about 4.5% at $\eta = 1.0$ in the range $0.0 \leq M \leq 1.0$. [See Fig.7]. Also, the skin friction increases about 15.9% from Ge = 1.5 to Ge = 3.5 at $\xi = 1.0$. The corresponding increase in the thickness of the momentum boundary layer is about 5.4% in the range $1.5 \leq Ge \leq 3.5$. [See Fig.8].

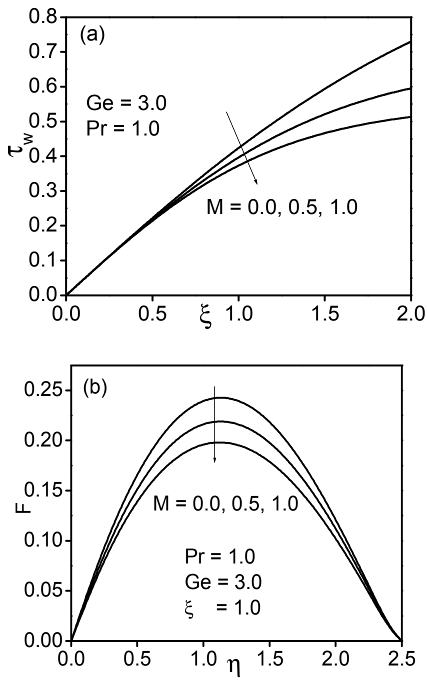


Figure 7. The effect of magnetic field (M) on (a) skin friction and (b) Velocity in presence of viscosity variation parameter

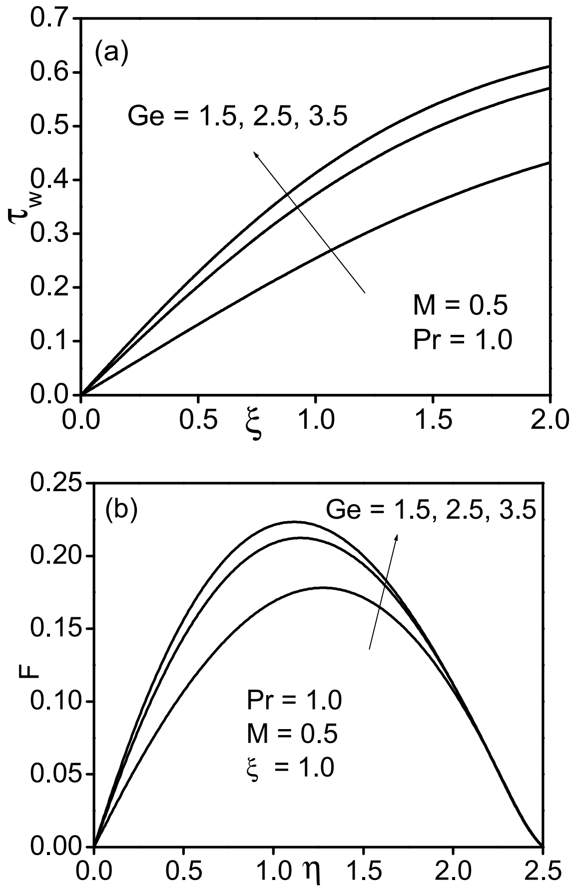


Figure 8. The effect of viscosity variation parameter on (Ge) (a) skin friction and (b) Velocity in presence of magnetic field

It is remarked here that the heat transfer parameter Q and temperature field (G) is little affected by the viscosity variation parameter (Ge) and magnetic field (M) as it is present only in the momentum equation.

5. Conclusions

From the present investigation, skin friction and heat transfer parameters are found to decrease with the increase of magnetic field both, in the presence of suction as well as injection, while the effect of viscosity variation parameter is just opposite. Also, the momentum and thermal boundary layer thicknesses are found to decrease with the increase of magnetic field and, increase with the increase of viscosity variation parameter.

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