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A Study on Syn Ecology Consisting of Two Hosts and One Commensal with Mortality Rate for the First and Second Species

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Abstract

The present investigation is a study on syn ecology with mortality rate for the first and second species. The system comprises of two hosts S_1, S_2 and one commensal S_3 i.e. S_1 and S_2 both benefit S_3, without getting themselves affected either positively or adversely. Further S_1 and S_2 are neutral. The model equations constitute a set of three first order non-linear differential equations. Criteria for the asymptotic stability of all the eight equilibrium states are established. Trajectories of the perturbations over the equilibrium states are illustrated. Further, the global stability of the system is established with the aid of suitably constructed Liapunov's function and the numerical examples for the growth rate equations are computed using Runge-Kutta fourth order method.

Keywords

Commensal, Host, Liapunov's Function, Stable, Trajectories, Unstable

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1. Introduction

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment & sustain themselves on common resources. It is a common observation that the species of same nature can not flourish is isolation without any interaction with species of different kinds. Significant researches in the area of theoretical ecology have been discussed by Kot [1] and by [2]. Several ecologists and mathematicians contributed to the growth of this area of knowledge. Mathematical ecology can be broadly divided into two main sub-divisions, Autecology and Synecology, which are described in the treatises of Arumugam [3], Sharma [4]. Syn-ecology is an ecosystem comprised of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on. Mathematical Modeling plays a vital role in providing insight into the mutual relationships (positive, negative) between the interacting species. The general concepts of Modeling in Biological Science have been initiated by several authors Ma [5], Murray [6], and Sze-Bi Hsu [7]. Recently the authors Papa Rao et al. [8], Shivareddy et al. [9], Srinivas [10] and Kumar et al. [11] discussed three species ecological models such as predation, completion and commensalism. Srinivas [12] studied the competitive ecosystem of two species and three species with limited and unlimited resources. Later, Narayan et al. [13] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Acharyulu et al. [14, 15] derived some productive results on various mathematical models of ecological Ammensalism with multifarious resources in the manifold directions. Further, Kumar [16] studied some mathematical models of ecological commensalism. The present author Prasad [17-20] investigated continuous and discrete models on the three

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species syn-ecosystems.

In this paper we study on the three species syn-eco system with mortality rate for the first and second species. The system comprises of two hosts S_1, S_2 and one commensal S_3 i.e. S_1 and S_2 both benefit S_3, without getting themselves affected either positively or adversely. Further, S_1 and S_2 are neutral. Commensalism is a symbiotic interaction between two populations where one population (S_1) gets benefit from (S_2) while the other (S_2) is neither harmed nor benefited due to the interaction with (S_1). The benefited species (S_1) is called the commensal and the other (S_2) is called the host. Some real-life examples of commensalism are presented below.

(i). A squirrel in an oak - tree gets a place to live and food for its survival, while the tree remains neither benefited nor harmed.

(ii). A flatworm attached to the horse crab and eating the crab's food, while the crab is not put to any disadvantage.

Notation Adopted

$N_i(t)$: The population strength of S_i at time t, $i=1,2,3$ t : Time instant d_1, d_2 : Natural death rates of S_1 and S_2 a_3 : Natural growth rate of S_3 a_{ii} : Self inhibition coefficients of S_i, $i=1,2,3$ a_{13}, a_{23} : Interaction coefficients of S_1 due to S_3 and S_2 due to S_3 $e_i = \dfrac{d_i}{a_{ii}}$: Extinction coefficient of S_i, $i=1,2$ $k_3 = \dfrac{a_3}{a_{33}}$: Carrying capacity of S_3 Further the variables N_1, N_2, N_3 are non-negative and the model parameters $d_1, d_2, a_3, k_3, a_{13}$, $a_{11}, a_{22}, a_{33}, a_{23}, e_1, e_2$ are assumed to be non-negative constants.

2. Equilibrium States

The model equations for syn ecosystem are given by the following system of first order non-linear ordinary differential equations.

$$\frac{dN_1}{dt} = -N_1\left(d_1 + a_{11}N_1\right) \tag{1}$$

$$\frac{dN_2}{dt} = -N_2\left(d_2 + a_{22}N_2\right) \tag{2}$$

$$\frac{dN_3}{dt} = N_3\left(a_3 - a_{33}N_3 + a_{13}N_1 + a_{23}N_2\right) \tag{3}$$

The system under investigation has eight equilibrium states given by

$$\frac{dN_i}{dt} = 0, i = 1, 2, 3$$

$$E_1\ (0,0,0),\ E_2\left(-e_1, 0, 0\right),\ E_3\ (0, -e_2, 0),\ E_4(0,0,k_3),$$

$$E_5\ (-e_1, -e_2, 0),\ E_6\ (-e_1\ , 0, k_3 - \frac{a_{13}e_1}{a_{33}}),$$

$$E_7(0\ , -e_2, k_3 - \frac{a_{23}e_2}{a_{33}}),\ E_8\left[-e_1, -e_2, k_3 - \left(\frac{a_{13}e_1 + a_{23}e_2}{a_{33}}\right)\right]$$

3. Stability of Equilibrium States

Let $N = \left(N_1, N_2, N_3\right) = \bar{N} + U$ where $U = \left(u_1, u_2, u_3\right)^T$ is a small perturbation over the equilibrium state $\bar{N} = \left(\bar{N}_1, \bar{N}_2, \bar{N}_3\right)$.

The basic equations (1), (2) and (3) are quasi-linearized to obtain the equations for the perturbed state as,

$$\frac{dU}{dt} = AU \tag{4}$$

where

$$A = \begin{bmatrix} -d_1 - 2a_{11}\bar{N}_1 & 0 & 0 \\ 0 & -d_2 - 2a_{22}\bar{N}_2 & 0 \\ a_{13}\bar{N}_3 & a_{23}\bar{N}_3 & \delta_3 \end{bmatrix}$$

with $\delta_3 = a_3 - 2a_{33}\bar{N}_3 + a_{13}\bar{N}_1 + a_{23}\bar{N}_2$

The characteristic equation for the system is

$$|A - \lambda I| = 0 \tag{5}$$

The equilibrium state is stable, if all the roots of the equation (5) are negative in case they are real or have negative real parts, in case they are complex.

3.1. Stability of E_1

In this case, we have

$$A = \begin{bmatrix} -d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

The characteristic equation is

$$\left(\lambda + d_1\right)\left(\lambda + d_2\right)\left(\lambda - a_3\right) = 0 \tag{6}$$

The characteristic roots of (6) are $-d_1, -d_2, a_3$. Since one of these three roots is positive. Hence the state is *unstable* and the solutions of the equations (4) are

$$u_1 = u_{10}\, e^{-d_1 t};\; u_2 = u_{20}\, e^{-d_2 t};\; u_3 = u_{30}\, e^{a_3 t}$$

where u_{10}, u_{20}, u_{30} are the initial values of u_1, u_2, u_3 respectively.

The trajectories in $u_1 - u_2$ and $u_2 - u_3$ planes are given by

$$\left(\frac{u_1}{u_{10}}\right)^{-\frac{1}{d_1}} = \left(\frac{u_2}{u_{20}}\right)^{-\frac{1}{d_2}} = \left(\frac{u_3}{u_{30}}\right)^{\frac{1}{a_3}}$$

3.2. Stability of E₂

In this case, we have

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

The characteristic roots are d_1, $-d_2$ and α_3. Since one of these three roots is positive, hence the state is *unstable* and the solutions are

$$u_1 = u_{10} e^{d_1 t};\; u_2 = u_{20} e^{-d_2 t};\; u_3 = u_{30} e^{\alpha_3 t}$$

where $\alpha_3 = a_3 - a_{13} e_1$

The trajectories in the $u_1 - u_2 ; u_2 - u_3$ planes are given by

$$\left(\frac{u_1}{u_{10}}\right)^{\frac{1}{d_1}} = \left(\frac{u_2}{u_{20}}\right)^{-\frac{1}{d_2}} = \left(\frac{u_3}{u_{30}}\right)^{\frac{1}{\alpha_3}}$$

3.3. Stability of E₃

In this case, we have

$$A = \begin{bmatrix} -d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix}$$

The characteristic roots are $-d_1, d_2$ and β_3. Since one of these three roots is positive, hence the state is *unstable*. The equations (4) yield the solutions,

$$u_1 = u_{10} e^{-d_1 t};\; u_2 = u_{20} e^{d_2 t};\; u_3 = u_{30} e^{\beta_3 t}$$

where $\beta_3 = a_3 - a_{23} e_2$

The trajectories in the $u_1 - u_2 ; u_2 - u_3$ planes are

$$\left(\frac{u_1}{u_{10}}\right)^{-\frac{1}{d_1}} = \left(\frac{u_2}{u_{20}}\right)^{\frac{1}{d_2}} = \left(\frac{u_3}{u_{30}}\right)^{\frac{1}{\beta_3}}$$

3.4. Stability of E₄

In this case, we get

$$A = \begin{bmatrix} -d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ a_{13} k_3 & a_{23} k_3 & -a_3 \end{bmatrix}$$

The characteristic roots are $-d_1, -d_2, -a_3$. Since all the three roots are negative, hence the state is *stable*. The equations (4) yield the solutions,

$$u_1 = u_{10} e^{-d_1 t};\, u_2 = u_{20} e^{-d_2 t};$$
$$u_3 = A_1 u_{10} e^{-d_1 t} + A_2 u_{20} e^{-d_2 t} + A_3 e^{-a_3 t}$$

where $A_1 = \dfrac{a_{13} k_3}{a_3 - d_1};\; A_2 = \dfrac{a_{23} k_3}{a_3 - d_2}$ and $A_3 = u_{30} - A_1 u_{10} - A_2 u_{20}$

with $a_3 \neq d_1 ; a_3 \neq d_2$ *and* $u_{30} \neq A_1 u_{10} + A_2 u_{20}$

The trajectories in the $u_1 - u_2$; $u_2 - u_3$ planes are given by

$$\left(\frac{u_1}{u_{10}}\right)^{-d_2} = \left(\frac{u_2}{u_{20}}\right)^{d_1} \text{ and } u_3 = A_1 u_{10}\left(\frac{u_2}{u_{20}}\right)^{\frac{d_1}{d_2}} + A_2 u_2 + A_3 \left(\frac{u_2}{u_{20}}\right)^{\frac{a_3}{d_2}}$$

3.5. Stability of E₅

In this case, we get

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & \alpha_3 - a_{23} e_2 \end{bmatrix}$$

The characteristic roots are $d_1, d_2, \alpha_3 - a_{23} e_2$. Since two of these three roots are positive, hence the state is unstable. The equations (4) yield the solutions,

$$u_1 = u_{10} e^{d_1 t};\, u_2 = u_{20} e^{d_2 t};\, u_3 = u_{30} e^{(\alpha_3 - a_{23} e_2) t}$$

The trajectories in the $u_1 - u_2 ; u_2 - u_3$ planes are given by

$$\left(\frac{u_1}{u_{10}}\right)^{\frac{1}{d_1}} = \left(\frac{u_2}{u_{20}}\right)^{\frac{1}{d_2}} = \left(\frac{u_3}{u_{30}}\right)^{\frac{1}{\alpha_3 - a_{23} e_2}}$$

3.6. Stability of E₆

In this case,

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ \dfrac{a_{13}\alpha_3}{a_{33}} & \dfrac{a_{23}\alpha_3}{a_{33}} & -\alpha_3 \end{bmatrix}$$

The characteristic roots are $d_1, -d_2, -\alpha_3$. Since one of these three roots is positive, hence the state is *unstable*. The equations (4) yield the solutions,

$$u_1 = u_{10}e^{d_1 t};\ u_2 = u_{20}e^{-d_2 t};$$
$$u_3 = B_1 u_{10}e^{d_1 t} + B_2 u_{20}e^{-d_2 t} + B_3 e^{-\alpha_3 t}$$

where $B_1 = \frac{a_{13}\alpha_3}{a_{33}(d_1+\alpha_3)}; B_2 = \frac{a_{23}\alpha_3}{a_{33}(\alpha_3 - d_2)}$; $B_3 = u_{30} - B_1 u_{10} - B_2 u_{20}$

with $\alpha_3 \neq d_2;\ u_{30} \neq B_1 u_{10} + B_2 u_{20}$

The trajectories in the $u_1 - u_2$; $u_2 - u_3$ planes are given by $\left(\frac{u_1}{u_{10}}\right)^{-d_2} = \left(\frac{u_2}{u_{20}}\right)^{d_1}$ and

$$u_3 = B_1 u_{10}\left(\frac{u_2}{u_{20}}\right)^{-\frac{d_1}{d_2}} + B_2 u_2 + B_3\left(\frac{u_2}{u_{20}}\right)^{\frac{\alpha_3}{d_2}}$$

3.7. Stability of E_7

In this case, we get $A = \begin{bmatrix} -d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ \frac{a_{13}\beta_3}{a_{33}} & \frac{a_{23}\beta_3}{a_{33}} & -\beta_3 \end{bmatrix}$

The characteristic roots are $-d_1, d_2, -\beta_3$. Since one of these three roots is positive, hence the state is *unstable*. The equations (4) yield the solutions,

$$u_1 = u_{10}e^{-d_1 t};\ u_2 = u_{20}e^{d_2 t};$$
$$u_3 = C_1 u_{10}e^{-d_1 t} + C_2 u_{20}e^{d_2 t} + C_3 e^{-\beta_3 t}$$

where $C_1 = \frac{a_{13}\beta_3}{a_{33}(\beta_3 - d_1)}; C_2 = \frac{a_{23}\beta_3}{a_{33}(\beta_3 + d_2)}$; $C_3 = u_{30} - C_1 u_{10} - C_2 u_{20}$

with $\beta_3 \neq d_1; u_{30} \neq C_1 u_{10} + C_2 u_{20}$

The trajectories in the $u_1 - u_2$; $u_2 - u_3$ planes are given by $\left(\frac{u_1}{u_{10}}\right)^{d_2} = \left(\frac{u_2}{u_{20}}\right)^{-d_1}$ and

$$u_3 = C_1 u_{10}\left(\frac{u_2}{u_{20}}\right)^{-\frac{d_1}{d_2}} + C_2 u_2 + C_3\left(\frac{u_2}{u_{20}}\right)^{\frac{-\beta_3}{d_2}}$$

3.8. Stability of E_8

In this case, we get

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ a_{13}\bar{N}_3 & a_{23}\bar{N}_3 & a_{23}e_2 - \alpha_3 \end{bmatrix}$$

The characteristic roots are $d_1, d_2, a_{23}e_2 - \alpha_3$. Since two of these three roots are positive, hence the state is *unstable*. The equations (4) yield the solutions,

$$u_1 = u_{10}e^{d_1 t};\ u_2 = u_{20}e^{d_2 t};$$
$$u_3 = D_1 u_{10}e^{d_1 t} + D_2 u_{20}e^{d_2 t} + D_3 e^{(a_{23}e_2 - \alpha_3)t}$$

where $D_1 = \frac{a_{13}(\alpha_3 - a_{23}e_2)}{a_{33}(d_1 + \alpha_3 - a_{23}e_2)}; D_2 = \frac{a_{23}(\alpha_3 - a_{23}e_2)}{a_{33}(d_2 + \alpha_3 - a_{23}e_2)}$

$D_3 = u_{30} - D_1 u_{10} - D_2 u_{20}$

with $a_{23}e_2 \neq d_1 + \alpha_3$; $a_{23}e_2 \neq d_2 + \alpha_3$ and $u_{30} \neq D_1 u_{10} + D_2 u_{20}$

The trajectories in the $u_1 - u_2$; $u_2 - u_3$ planes are given by $\left(\frac{u_1}{u_{10}}\right)^{d_2} = \left(\frac{u_2}{u_{20}}\right)^{d_1}$ and

$$u_3 = D_1 u_{10}\left(\frac{u_2}{u_{20}}\right)^{\frac{d_1}{d_2}} + D_2 u_2 + D_3\left(\frac{u_2}{u_{20}}\right)^{\frac{a_{23}e_2 - \alpha_3}{d_2}}$$

4. Liapunov's Function

In section 5 we discussed the local stability of all eight equilibrium states, from which only one state $E_4(0,0,k_3)$ is *stable* and rest of them are *unstable*. We now examine the global stability of dynamical system (1), (2) and (3) at this state by suitable Liapunov's function.

Theorem. The equilibrium state E_4 is globally asymptotically stable.

Proof. Let us consider the following Liapunov's function

$$L(N_3) = N_3 - \bar{N}_3 - \bar{N}_3 \ln\left(\frac{N_3}{\bar{N}_3}\right)$$

Now, the time derivative of L, along with solution (3) can be written as

$$\frac{dL}{dt} = \left(\frac{N_3 - \bar{N}_3}{N_3}\right)\frac{dN_3}{dt}$$
$$= \left(N_3 - \bar{N}_3\right)\left(a_3 - a_{33}N_3\right)$$
$$= \left(N_3 - \bar{N}_3\right)\left(a_{33}\bar{N}_3 - a_{33}N_3\right)$$
$$= -a_{33}\left(N_3 - \bar{N}_3\right)^2$$

which is negative definite.

Hence, the steady state is *globally asymptotically stable.*

5. Numerical Examples

The numerical solutions of the growth rate equations (1), (2) and (3) computed employing the fourth order Runge-Kutta method for specific values of the various parameters that characterize the model and the initial conditions. The results are illustrated in Figures 1, 2 and 3.

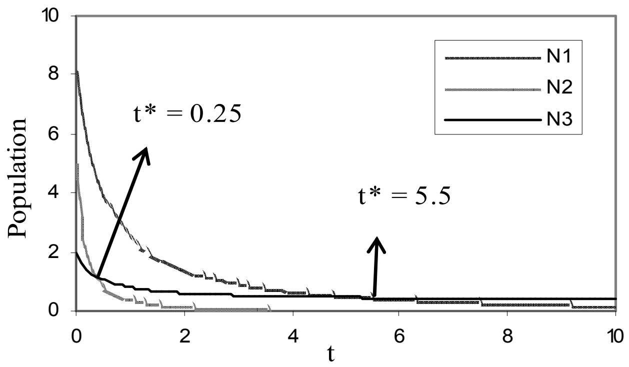

Figure 1. Variation of N_1, N_2, N_3 against time (t) for $d_1 = 0.14$, $a_{11} = 0.24$, $a_{13} = 1.04$, $d_2 = 0.66$, $a_{22} = 1.56$, $a_{23} = 0.42$, $a_3 = 2.92$, $a_{33} = 7.68$, $N_1 = 8$, $N_2 = 5$, $N_3 = 2$.

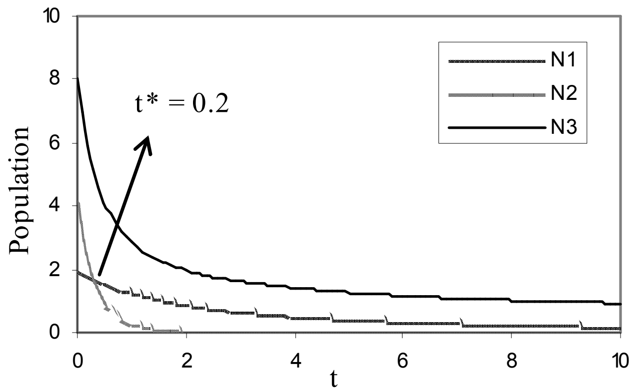

Figure 2. Variation of N_1, N_2, N_3 against time (t) for $d_1 = 0.10$, $a_{11} = 0.24$, $a_{13} = 5.74$, $d_2 = 2.22$, $a_{22} = 0.34$, $a_{23} = 4.9$, $a_3 = 2.82$, $a_{33} = 4.2$, $N_1 = 2$, $N_2 = 4$, $N_3 = 8$.

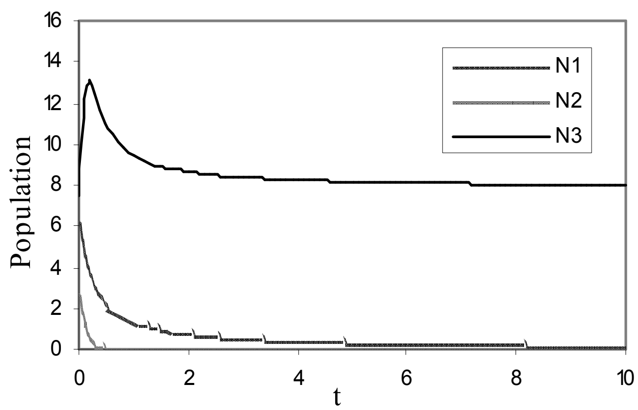

Figure 3. Variation of N_1, N_2, N_3 against time (t) for $d_1 = 0.10$, $a_{11} = 0.60$, $a_{13} = 0.60$, $d_2 = 8.1$, $a_{22} = 0.60$, $a_{23} = 2$, $a_3 = 4.7$, $a_{33} = 0.60$, $N_1 = 8$, $N_2 = 2.5$, $N_3 = 7.5$.

6. Observations

Case 1: This is a situation at the self inhibition coefficient of the third species is highest. Initially the first and second species dominates over the third till the time instant t* = 5.5 and t* = 0.25 respectively and thereafter the dominance is reversed. Further, we notice that the initial values of S1, S2, S3 are in decreasing order. This is shown in Figure 1.

Case 2: In this case the initial values of S1, S2, S3 are in increasing order. Initially the second species dominates over the first till the time instant t* = 0.2 and thereafter the dominance is reversed. Further, it is evident that all the three species asymptotically converge to the equilibrium point as shown in Figure 2.

Case 3: This is a situation at the self inhibition coefficient of all the three species are identical. The third species dominates over the other two throughout. The natural death rate of S1 is greater than of S2. In course of time we notice a steady variation with no appreciable growth rate in all the three species. (Figure 3).

7. Conclusion

The present paper deals with an investigation on the stability of a three species syn ecology consisting of two hosts and one commensal with mortality rate for the first and second species. In this paper we established all possible equilibrium states. It is conclude that, in all eight equilibrium states, only one state E_4 is stable. Further, the global stability is established with the help of suitable Liapunov's function and the growth rates of the species are numerically estimated using Runge-Kutta fourth order method.

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