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Fuzzy Methods for Student Assessment

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Abstract

In the present paper we propose three different fuzzy methods for assessing a system's performance: The measurement of its total possibilistic uncertainty, the Centre of Gravity (COG) defuzzification technique and the Trapezoidal Fuzzy Assessment Model (TRFAM). Although each one of the above methods can be applied independently, a combined utilization of them helps the user to get a more comprehensive view of the system's performance, since they compensate for each other. In fact, while the first method is focusing on the system's mean performance, the last two focus on its quality performance by assigning greater coefficients to the higher scores. Further, the TRFAM, which is a new original variation of the COG technique, treats better the ambiguous cases being at the boundaries between two successive assessment grades. An application (students' assessment)is also presented illustrating our results, in which the above three fuzzy assessment methods are compared to each other and with two traditional assessment methods based on principles of the bivalent logic (calculation of means and of the GPA index).

Keywords

System's Uncertainty, Center of Gravity (COG) Defuzzification Technique, Trapezoidal Fuzzy Assessment Model (TRFAM), Grade Point Average (GPA) Index

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1. Introduction

The assessment of a system's effectiveness (i.e. of the degree of attainment of its targets) with respect to an activity performed within the system (e.g. problem-solving, decision making, learning process, etc) is a very important task that enables the correction of the system's weaknesses resulting to the improvement of its general performance.

The assessment methods that are commonly used in practice are based on principles of the classical, bivalent logic (yes-no). However, they often appear ambiguous cases, where a crisp characterization is not probably the proper one. In Education, for example, the teacher is frequently not absolutely sure about a particular numerical grade characterizing a student's performance.Fuzzy logic, due to its nature of characterizing such cases with multiple values, offers a wider and richer field of assessment resources and it has been widely used recently to solve problems in the evaluation tasks [4, 5, 7, 14, etc].

The methods, which are used for assessing a system's performance focus on different targets; for example some of them measure the system's *mean performance*, while others focus on its *quality performance* by assigning greater coefficients (weights) to the higher scores, etc.

In this paper we propose the combined use of three compensating for each other, fuzzy methods for assessing a system's performance. The main criterion of choosing these methods is their general character that permits their

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application to almost all kinds of systems'. These methods are described in the next three Sections of the paper as follows:The measurement of the system's *total possibilistic uncertainty* in Section 2, the application of the *centre of gravity* (COG) *defuzzification technique* in Section 3, and the *trapezoidal fuzzy assessment model* (TRFAM) in Section 4. An application for student assessment is presented in Section 5 illustrating our results in practice. A discussion follows in Section 6, where the above methods are compared to each other. Finally, Section 7 presents the conclusion of the paper and some perspectives of future research.

For general facts on fuzzy sets we refer to the book [2].

2. Measuring a System's Uncertainty

An assessment method of a system's performance that we have frequently used in past (e.g. see the book [14] and its references) is based on the measurement of its existing uncertainty. In fact, from the classical information theory it is well known that the amount of information obtained by an action can be measured by the reduction of the uncertainty resulting from this action [8]. Accordingly a system's uncertainty is connected to its capacity for obtaining relevant information. Therefore a measure of uncertainty can be adopted as a measure of the system's effectiveness (performance): The lower is the uncertainty after an action within the system, the better is the system's effectiveness with respect to this action.

According to the standard probability theory a system's uncertainty (and the information connected to it) is measured by the Shannon's formula which is known as the *Shannon's entropy* [8].For use in a fuzzy environment the above formula has been expressed in the form: H= $-\frac{1}{\ln n}\sum_{s=1}^{n} m_s \ln m_s$ ([3], p. 20), where U is the universal set of the discourse, m: U $\rightarrow$ [0, 1] is the membership function of the corresponding fuzzy set, $m_s = m(s)$ denotes the membership degree of the element s of U and n denotes the total number of the elements ofU. In the above formula the sum is divided by the natural logarithm of n in order to be normalized. Thus H takes values within the real interval [0, 1].

We recall that the *fuzzy probability* of an element s of U is defined in a way analogous to the crisp probability, i.e. by $P_s = \frac{m_s}{\sum_{s \in U} m_s}$. However, according to Shackle [6] and many other researchers after him, human reasoning can be formulated more adequately by the possibility rather, than by the probability theory. The *possibility* r_s of s is defined by $r_s = \frac{m_s}{\max\{m_s\}}$, where max$\{m_s\}$ denotes the maximal value of m_s, for all s in U. In other words, the possibility of s expresses the relative membership degree of s with respect to max$\{m_s\}$.

Within the domain of possibility theory uncertainty consists of *strife* or *discord*, which expresses conflicts among the various sets of alternatives, and *non-specificity* or *imprecision*, which indicates that some alternatives are left unspecified, i.e. it expresses conflicts among the cardinalities of the various sets of alternatives ([3]; p.28). For a better intuitive understanding of the above two types of uncertainty we present the following simple example:

EXAMPLE: Let Ube the set of integers from 0 to 130 representing the humans' ages and let Y = young, A = adult and O = old be fuzzy subsets of U defined by the membership functions m_Y, m_A and m_O respectively, where people are considered as young, adult or old according to their outer appearance . Then, given x in U, there usually exists a degree of uncertainty about the reasonable values that the membership degrees $m_Y(x)$, $m_A(x)$and $m_O(x)$ could take, resulting to a conflict among the fuzzy subsets Y, A and O of U. For instance, if x = 18, values like $m_Y(x)$ = 0.8 and $m_A(x)$ = 0.3 are acceptable, but they are not the only ones. In fact, the values $m_Y(x)$ = 1 and $m_A(x)$ = 0.5 are also acceptable, etc. The existing conflict becomes even greater if x =50. In fact, is it reasonable in this case to take $m_Y(x)$ =0?Probably not, because sometimes people being 50 years old look much younger than others aged 40 or even 30 years. But, there exist also people aged 50 who look older from others aged 70, or even 80 years! So what about the acceptable values of $m_O(x)$? All the above are examples of the type of uncertainty that we have termed as strife. On the other hand, non - specificity is connected to the question: How many x in U should have non zero membership degrees in Y, A and O respectively? In other words, the existing in this case uncertainty creates a conflict among the cardinalities (sizes) of the fuzzy subsets of U. We recall that the *cardinality* of a fuzzy subset, say B, of U is defined to be the sum $\sum_{x \in U} m_B(x)$ of all membership degrees of the elements of U in B.

Strife is measured by the function $ST(r)$on the ordered possibility distribution r:$r_1=1 \geq r_2 \geq \ldots\ldots \geq r_n \geq r_{n+1}$ of the elements of U with respect to the corresponding fuzzy subset of U defined by $ST(r) = \frac{1}{\log 2}[\sum_{i=2}^{m}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}]$ ([3], p.28).

Similarly non-specificity is measured by the function $N(r)$ =

$\frac{1}{\log 2}[\sum_{i=2}^{m}(r_i - r_{i+1})\log i]$ ([3], p.28).

The sum *T(r) = ST(r) + N(r)* measures the *total possibilistic uncertainty* for ordered possibility distributions. The lower is the value of *T(r)*, which means greater reduction of the initially (before the action) existing uncertainty, the better is the system's performance with respect to this action.

3. Use of the COG Defuzzification Technique as an Assessment Method

The COG technique is a very popular in fuzzy mathematics defuzzification method(e.g. see [13]). For applying this technique we correspond to each x of the universal set U an interval of values from a prefixed numerical distribution, which actually means that we replace U with a set of real intervals. Then, we construct the graph of the corresponding membership function $y=m(x)$. There is a commonly used in fuzzy logic approach to represent the fuzzy data with the pair of numbers (x_c, y_c) as the coordinates of the COG, say F_c, of the level's section S contained between the above graph and the OX axis, which we can calculate using the following well-known (e.g. see [17]) formulas:

$$\text{xc} = \frac{\iint_S x\,dxdy}{\iint_S dxdy}, \quad \text{yc} = \frac{\iint_S y\,dxdy}{\iint_S dxdy} \qquad (1)$$

In earlier papers Voskoglou and Subbotin have properly adapted the COG technique to be used as an assessment method [9, 15, 16 etc]. In fact, let G be a group of individuals participating in a certain activity and let U={A, B, C, D, F}be a set of linguistic labels characterizing the individuals' performance with respect to this activity as follows: A=excellent, B=very good, C=good, D=fair and F= unsatisfactory. Then, we can express G as a *fuzzy set* in U in the form G = $\{(x, m(x)), x \in U\}$, where $y=m(x)$ is the corresponding membership function.

We correspond to each x in U an interval of real values as follows: F $\to$ [0, 1),D $\to$ [1, 2), C $\to$ [2, 3), B $\to$ [3,4), A $\to$ [4, 5]. Consequently, we have that $y_1 = m(x) = m(F)$ for all x in [0,1), $y_2 = m(x)$ = m(D) for all x in [1,2), $y_3 = m(x)$= m(C) for all x in [2, 3), $y_4 = m(x) = m(B)$ for all x in [3, 4) and $y_5 = m(x) = m(A)$ for all x in [4,5]. Then the graph of the membership function $y = m(x)$, takes the form of the bar graph of Figure 1, while the area of the level's section S contained between this graph and the OX axis is equal to the sum of the areas of the rectangles S_i, i=1, 2, 3, 4, 5.

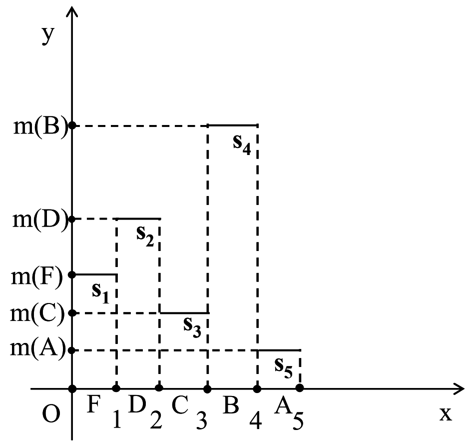

Figure 1. Bar graphical data representation

It is straightforward then to check (e.g. [16], Section 2) that in this case formulas (1) are transformed to the form:

$$x_c = \frac{1}{2}(y_1+3y_2+5y_3+7y_4+9y_5),\ y_c = \frac{1}{2}(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2) \qquad (2)$$

with $y_i = \frac{m(x_i)}{\sum_{j=1}^{5} m(x_j)}$, where x_1=F, x_2=D, x_3=C, x_4=B and x_5=A

Next, using elementary algebraic inequalities it is easy to check that there is a unique minimum for y_c corresponding to COG $F_m(\frac{5}{2}, \frac{1}{10})$ ([16], Section 2). Further, the ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (2) we get that $x_c = \frac{9}{2}$ and $y_c = \frac{1}{2}$. Therefore the COG in this case is the point $F_i\ (\frac{9}{2}, \frac{1}{2})$. On the other hand the worst case is when $y_1=1$ and $y_2=y_3=y_4= y_5=0$. Then from formulas (2) we find that the COG is the point $F_w\ (\frac{1}{2}, \frac{1}{2})$. Therefore the COG F_c of the level's section S lies in the area of the triangle $F_wF_mF_i$.

Then by elementary geometric observations ([16], Section 2) one can obtain the following criterion:

- Between two groups the group with the bigger x_c performs better.
- If the two groups have the same $x_c \geq 2.5$, then the group with the bigger y_c performs better.
- If the two groups have the same $x_c < 2.5$, then the group with the lower y_c performs better.

As it becomes evident by the above description, the application of the COG method is simple in practice and, in contrast to the measurement of the system's total possibilistic uncertainty, needs no complicated calculations to its final step. However, we must emphasize that the COG method treats differently the idea of a system's performance, than the measurement of the uncertainty does. In fact, as it can be easily observed by the above criterion andthe first of formulas (2), the weighted average plays the main role in the COG method, i.e. the result of the system's performance close to its ideal performance has much more "weight" than the one close to the lower end.In other words, while the measurement of the system's uncertainty is connected to its *mean performance*, the COG method focuses on its *quality performance*.

4. The Trapezoidal Fuzzy Assessment Model (TRFAM)

The TRFAM is a recently developed [12] variation of the COG method presented in the previous section. The novelty of this approach is in the replacement of the rectangles appearing in the graph of the membership function of the COG method (Figure 1) by isosceles trapezoids sharing common parts, so that to cover the ambiguous cases of individuals' scores being at the boundaries between two successive grades. In the TRFAM's scheme (Figure 2) we have five trapezoids, corresponding to the individuals' performance characterizations F, D, C, B and A respectively defined in the previous section. Without loss of generality and for making our calculations easier we consider isosceles trapezoids with bases of length 10 units lying on the OX axis. The height of each trapezoid is equal to the percentage of individuals who achieved the corresponding characterization for their performance, while the parallel to its base side is equal to 4 units.

We allow for any two adjacent trapezoids to have 30% of their bases (3 units) belonging to both of them. In this way we treat better the ambiguous cases of individuals' scores being at the boundaries between two successive grades. For example, in students' assessment it is a very common approach to divide the interval of the specific grades in three parts and to assign the corresponding grade using + and - . For example, we could have 75 – 77 = B-, 78 – 81 = B, 82 – 84 = B+. However, this consideration does not reflect the common situation, where the teacher is not sure about the grading of the students whose performance could be assessed as marginal between and close to two adjacent grades; for example, something like 84 - 85 being between B_+ and A_-. The TRFAM fits better than the COG technique to this kind of situations.

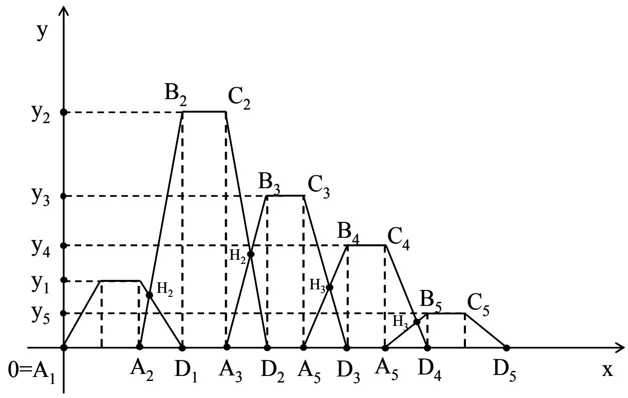

Figure 2. The TRAFM's scheme

In TRFAM an individuals' group can be represented, as in the COG method, as a fuzzy set in U, whose membership function *y=m(x)* has as graph the line $OB_1C_1H_1B_2C_2H_2B_3C_3H_3B_4C_4H_4B_5C_5D_5$ of Figure 2, which is the union of the line segments OB_1, B_1C_1, C_1H_1,…….., B_5C_5, C_5D_5. However, in case of the TRFAM the analytic form of *y = m(x)* is not needed for calculating the COG of the resulting area. In fact, since the marginal cases of the individuals' scores are considered as common parts for any pair of the adjacent trapezoids, it is logical to count these parts twice; e.g. placing the ambiguous cases B+ and A- in both regions B and A. In other words, the COG technique, which calculates the coordinates of the COG of the area between the graph of the membership function and the OX axis, thus considering the areas of the "common" triangles $A_2H_1D_1$, $A_3H_2D_2$, $A_4H_3D_3$ and $A_5H_4D_4$ only once, is not the proper method to be applied in the above situation.

Instead, in this case we represent each one of the five trapezoids of Figure 2 by its COG F_i, i=1, 2, 3, 4, 5 and we consider the entire area, i.e. the sum of the areas of the five trapezoids, as the system of these points-centers. More explicitly, the steps of the whole construction of the TRFAM are the following:

1. Let y_i, i=1, 2, 3, 4, 5 be the percentages of students whose performance was characterized by F, D, C, B, and A respectively; then $\sum_{i=1}^{5} y_i$ =1 (100%).

2. We consider the isosceles trapezoids with heights being equal to y_i, i=1, 2, 3, 4, 5, in the way that has been illustrated in Figure 2.

3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG F_i, i=1, 2, 3, 4, 5, of each trapezoid as follows: It is well known that the COG of a trapezoid lies along the line segment joining the midpoints of its parallel sides *a* and *b* at a distance *d* from the longer side *b* given by $d=\frac{h(2a+b)}{3(a+b)}$, where *h* is its height

(e.g. see [18])..Therefore in our case we have y_{c_i} == $\frac{y_i(2*4+10)}{3*(4+10)}=\frac{3y_i}{7}$. Also, since the abscissa of the COG of each trapezoid is equal to the abscissa of the midpoint of its base, it is easy to observe that $x_{ci}=7i-2$.

4. We consider the system of the COG's F_i, i=1, 2, 3, 4, 5and we calculate the coordinates (X_c, Y_c) of the COG F_c of the whole area S considered in Figure 2 by the following formulas, derived from the commonly used in such cases definition (e.g. see [19]):

$$\text{Xc}=\frac{1}{S}\sum_{i=1}^{5}S_i x_{c_i}\ ,\ \text{Yc}=\frac{1}{S}\sum_{i=1}^{5}S_i y_{c_i} \qquad (3)$$

In equations (3)Si, i= 1, 2, 3, 4, 5 denotes the area of the corresponding trapezoid. Thus, $Si=\frac{(4+10)y_i}{2}=7y_i$and S = $\sum_{i=1}^{5}S_i=7\sum_{i=1}^{5}y_i$ = 7. Therefore, from equations (3) we finally get that

$$X_c=\frac{1}{7}\sum_{i=1}^{5}7y_i(7i-2)=(7\sum_{i=1}^{5}iy_i)\ -2,$$

$$\text{Yc}=\frac{1}{7}\sum_{i=1}^{5}7y_i(\frac{3}{7}y_i)=\frac{3}{7}\sum_{i=1}^{5}y_i^2 \qquad (4)$$

5.We determine the area where the COG F_c lies as follows: For i, j=1, 2, 3, 4, 5, we have that$0 \le (y_i-y_j)^2=y_i^2+y_j^2-2y_iy_j$,therefore $y_i^2+y_j^2\ge 2y_iy_j$, with the equality holding if, and only if, $y_i=y_j$. Therefore

$$1=(\sum_{i=1}^{5}y_i)^2=\sum_{i=1}^{5}y_i^2+2\sum_{\substack{i,j=1,\\ i\neq j}}^{5}y_iy_j\ \le \sum_{i=1}^{5}y_i^2$$

$$+2\sum_{\substack{i,j=1,\\ i\neq j}}^{5}(y_i^2+y_j^2)=5\sum_{i=1}^{5}y_i^2 \text{ or } \sum_{i=1}^{5}y_i^2\ \ge\frac{1}{5} \qquad (5)$$

with the equality holding if, and only if, $y_1=y_2=y_3=y_4=y_5=\frac{1}{5}$.In the case of equality the first of equations (4) gives that X_c= $7(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+\frac{5}{5})-2=19$. Further, combining the inequality (5) with the second of equations (4) one finds that $Y_c\ge\frac{3}{35}$ Therefore the unique minimum for Y_c corresponds to the COG $F_m(19,\frac{3}{35})$.The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from equations (4) we get that X_c= 33and $Y_c=\frac{3}{7}$.Therefore the COG in this case is the point $F_i(33,\frac{3}{7})$. On the other hand, the worst case is when $y_1=1$ and $y_2=y_3=y_4=y_5=0$. Then from equations (4) we find that the COG is the point $F_w(5,\frac{3}{7})$. Therefore the area where the COG F_c lies is the area of the triangle $F_wF_mF_i$ (see Figure 3).

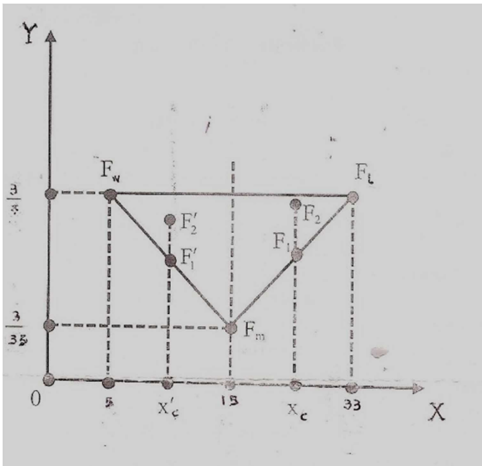

Figure 3. The area where the COG lies

6. We formulate our criterion for comparing the performances of two (or more) different student groups' as follows: From elementary geometric observations (see Figure 3) it follows that for two groups the group having the greater X_c performs better. Further, if the two groups have the same $X_c\ge 15$, then the group having the COG which is situated closer to Fi is the group with the greater Y_c. Also, if the two groups have the same $X_c<15$, then the group having the COG which is situated farther to Fw is the group with the smaller Y_c. Based on the above considerations it is logical to formulate our criterion for comparing the two groups' performance in the following form:

- Between two groups the group with the greater value of X_c demonstrates the better performance.
- If two groups have the same $X_c\ge 19$,, then the group with the greater value of Y_c demonstrates the better performance.
- If two groups have the same $X_c<19$, then the group with the smaller value of Y_c demonstrates the better performance.

The above criterion combined with the first of equations (4) shows that the TRFAM measures the system's *quality performance* by assigning higher coefficients to the greater scores.-

An alternative to the TRFAM approach is to consider isosceles triangles instead of trapezoids [10, 11]. Then applying a similar argument as for the TRAFM above one finds that the coordinates of the COG of the resulting in this case scheme are calculated by the formulas

$$Xc= (7\sum_{i=1}^{5} iy_i) -2 \text{ , } Yc=\frac{1}{5}\sum_{i=1}^{5} y_i^2 \quad (6)$$

An analogous criterion can be also obtained for comparing the performance of two (or more) different group of individuals'. We have called the above framework *Triangular Fuzzy Assessment Model* (TFAM).

5. Student Assessment

The students of two different Departments of the School of Management and Economics of the Graduate Technological Educational Institute of Western Greece achieved the following scores (in a climax from 0 to 100) at their common progress exam in the course "Mathematics for Economists I":

Department 1 (D_1):100(5 times), 99(3), 98(10), 95(15), 94(12), 93(1), 92 (8), 90(6), 89(3), 88(7), 85(13), 82(4), 80(6), 79(1), 78(1), 76(2), 75(3), 74(3), 73(1), 72(5), 70(4), 68(2), 63(2), 60(3), 59(5), 58(1), 57(2), 56(3), 55(4), 54(2), 53(1), 52(2), 51(2), 50(8), 48(7), 45(8), 42(1), 40(3), 35(1).

Department 2(D_2) :100(7), 99(2), 98(3), 97(9), 95(18), 92(11), 91(4), 90(6), 88(12), 85(36), 82(8), 80(19), 78(9), 75(6), 70(17), 64(12), 60(16), 58(19), 56(3), 55(6), 50(17), 45(9), 40(6).

The linguistic characterizations (grades) mentioned in section 3 were assigned to the above scores as follows: A (100-85), B(84-75), C (60-74), D(50-59) and F (<50). The students' results with respect to the above grades are summarized in Table 1.

Table 1. Characterization of the students' performance

Characterizations	D_1	D_2
A	60	60
B	40	90
C	20	45
D	30	45
E	20	15
Total	170	255

In order to check the effectiveness of the three fuzzy assessment methods proposed in this paper, the evaluation of the above data will be performed in two ways: I) By two very common traditional assessment methods based on principles of the bivalent logic (yes-no) and II) by applying our fuzzy methods. Then the results obtained will be compared and the proper conclusions will be drawn.

5.1. Traditional Assessment Methods

i) Calculation of the means: A straightforward calculation gives that the means of the above presented students' scores are approximately equal to 76.006 and 75.09 for D_1 and D_2respectively. This shows that the *mean performance* of both student groups was very good (on the boundary), with the performance of the group D_1 being slightly better.

(ii)Calculation of the GPA index: We recall that the *Great Point Average* (*GPA*) is a weighted mean, where more importance is given to the higher scores achieved, to which greater coefficients (weights) are attached. In other words, the GPA method focuses on the *quality performance* of a student group.For applying the GPA method on the data of our experiment let us denote by n_A, n_B, n_C, n_D and n_F the numbers of students whose performance was characterized by A, B, C, D and F respectively and by n the total number of students of each group. .Then the GPA index is calculated by the equation GPA= $\frac{n_D+2n_C+3n_B+4n_A}{n}$ (e.g. see [1]).Using the notation of section 4 the above equation can be written in the form

$$GPA= y_2+2y_3+3y_4+4y_5 \quad (7)$$

It is easy to observe that $0 \leq GPA \leq 4$. In fact, GPA=0, if $n_F= n$(worst case), while GPA=4, if $n_A= n$ (ideal case)

In our case, applying equation (7) on the data of Table 1 one finds that the GPA of both student groups' is equal to $\frac{43}{17} \approx$ 2.529. Thus, the two student groups demonstrated the same quality performance. Further, their performance can be characterized as satisfactory, since the value 2.529 of the GPA index is greater than the half of its maximal possible value, which is equal to 4.

5.2. Fuzzy Assessment Methods

(iii) Measurement of the uncertainty: We represent the two student groups as fuzzy sets in *U*. For this, we define the membership function $m: U \rightarrow [0, 1]$ for both groups D_1 and D_2 .by $y = m(x) = \frac{n_x}{n}$, for all x in U, where the notation for n_x is the same as in the above case (ii) of the GPA index. Then, from Table 1 it turns easily out that D_1 and D_2 can be written as fuzzy sets in *U* in the form

$D_1 = \{(A, \frac{6}{17}), (B, \frac{4}{17}), (C, \frac{2}{17}), (D, \frac{3}{17}), (F, \frac{2}{17})\}$ and

$D_2 = \{(A, \frac{4}{17}), (B, \frac{6}{17}), (C, \frac{3}{17}), (D, \frac{3}{17}), (F, \frac{1}{17})\}$

respectively.

From Table 1 we find that $max\{m_x\}$ = $\frac{6}{17}$ for both groups, therefore the possibilities of the elements of U are calculated by the formula $r_s = \frac{m_s}{\frac{6}{17}}$ for both groups. Performing the corresponding calculations we find that r_1=1, r_2=$\frac{2}{3}$, r_3=$\frac{1}{2}$, r_4=r_5=$\frac{1}{3}$ for D_1 and r_1=1, r_2=$\frac{2}{3}$, r_3=r_4=$\frac{1}{2}$, r_5=$\frac{1}{6}$ for D_2. Replacing the above values of possibilities to the corresponding formulas of section 2 we find for D_1 that

$$ST(r) = \frac{1}{\log 2}[\sum_{i=2}^{4}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}]$$

$$= \frac{1}{\log 2}(\frac{1}{6}\log\frac{6}{5} + \frac{1}{6}\log\frac{18}{13}) \approx 0.043, \text{N(r)=}$$

$$\frac{1}{\log 2}[\sum_{i=2}^{4}(r_i - r_{i+1})\log i] = \frac{1}{\log 2}(\frac{1}{6}\log 2 + \frac{1}{6}\log 3) \approx 0.431$$

and T(r) ≈ 0.043+0.431=0.474.

Similarly we find for D_2 that

$$ST(r) = \frac{1}{\log 2}(\frac{1}{6}\log\frac{6}{5} + \frac{2}{6}\log\frac{12}{8}) \approx 0.239,$$

$$N(r) = \frac{1}{\log 2}(\frac{1}{6}\log 2 + \frac{2}{6}\log 4) \approx 0.695 \text{ and } T(r) \approx$$

$$0.239+0.695 = 0.934.$$

Therefore D_1 demonstrates a considerably better performance than D_2.

(iv) The COG method: In our application we have $\sum_{i=1}^{5} y_i$ = m(A) + m (B) + m(C) + m(D) + m(F) = $\frac{n_A + n_B + n_C + n_D + n_F}{n}$ =1. Therefore, replacing the values of y_i's taken from the fuzzy sets D_1 and D_2 of paragraph (iii)in the first of equations (2) we find that the coordinate x_c of the COG for both D_1 and D_2 is equal to $\frac{103}{34} \approx 3.029 > 2.5$. Since the value 3.029 found for x_c is greater than the half of its value in the ideal case, which is $\frac{9}{2}$ (see section 3), both groups demonstrated a more than satisfactory performance. Further, by the second ofequations (2) one finds that the coordinate $y_c$of the COG is equal to $\frac{69}{578} \approx 0.119$ for D_1 and to $\frac{71}{578} \approx 0.122$for D_2. Therefore, according to our criterion stated in section 3, D_2 demonstrated a slightly better performance than D_1.

(v) Application of the TRFAM: Replacing the values of y_i'sin the first of equations (4) we find that X_c= $\frac{386}{17} \approx$ 22.706 >16.5 for both groups. This means that the quality performance of both groups was more than satisfactory, since the value 22.706 is greater than the half of the value of X_c in the ideal case, which is equal to 33 (see section 4)Also, the second of equations (4) gives that Y_c=$\frac{3}{7}*\frac{69}{286} \approx 0.103$ for D_1 andY_c= $\frac{3}{7}*\frac{71}{286} \approx 0.106$ for D_2. Thus, according to the criterion stated in section 4, D_2 demonstrated a slightly better performance than D_1.

REMARK: Analogous results are obtained if, instead of equations (4), we apply equations (6) of *TFAM*, the only difference being with the values of Y_c, which are approximately equal to 0.048 and 0.05 for D_1 and D_2 respectively.

6. Comparison of the Assessment Methods

In the previous Section we applied five in total methods for assessing the students' performance. The first two of them were traditional based on principles of the bivalent logic and the last three werethe fuzzy methods utilized in the proposed in this paper collaborative fuzzy assessment framework.

The application of the above methods resulted to different conclusions. However, this is not embarrassing, since, in contrast to the calculation of the means and the measurement of the system's uncertainty, which focus on the mean performance of a student group, the GPA, the COG and the TRFAM methods focus on its quality performance by assigning weight coefficients to the higher scores achieved by students. This explains why, although D_1 demonstrated a better performance with respect to the calculation of the means and the measurement of the system's uncertainty, the performance of D_2 was found to be equal or better than the performance of D_1, when using the GPA the COG and the TRFAM methods

The coefficients attached to the y_i's in the last three methods -see equation (7) and the first of equations (2) and (4) respectively- are presented in the following Table 2:

Table 2. Weight coefficients of the y_i's

y_i	GPA	COG (x_c)	TRAFM (X_c)
y_1	0	½	7
y_2	1	3/2	14
y_3	2	5/2	21
y_4	3	7/2	28
y_5	4	9/2	35

From Table 2 it becomes evident that the two fuzzy assessment methods (GOC and TRFAM) assign greater coefficients to the higher scores than GPA. In other words these two methods are *more sensitive* than GPA to the higher scores. This explains why the quality performance of the two groups was found to be the same with respect to the GPA index, while D_2 demonstrated a slightly better performance with respect to the COG technique and the TRFAM.

Notice also that, since the COG technique and the TRFAM treat differently the ambiguous cases of the students' scores being at the boundaries between two successive assessment grades, the conclusions obtained by applying these two assessment methods could differ in certain (other than the present) cases. This is the reason for which we have included both COG and TRFAM, and not only one of them, in our collaborative fuzzy assessment framework presented in this paper.

In concluding, the above performed comparison of our fuzzy methods with the two traditional assessment methods provided a very strong indication for their efficiency. Also, from the previous discussion it becomes evident that, although the proposed fuzzy assessment methods can be applied independently, the combined use of them gives to the user a more comprehensive view of the system's performance. On the other hand, if someone has personal criteria of goals (e.g. he/she is interested to the mean system's performance only), it is suggested to choose the method (or methods) that fits better to these criteria.

7. Conclusion

Summarizing the discussion performed in this paper we reach to the following conclusion:

We proposed the combined use of three different fuzzy methods compensating for each other for assessing a system's performance: The measurement of its total possibilistic uncertainty focusing on its mean performance, as well as the COG technique and the TRFAM focusing on its quality performance by assigning greater coefficients to the higher scores. The TRFAM is a recently developed original variation of the COG technique treating better the ambiguous cases being at the boundaries between two successive assessment grades. An application (students' assessment) was also presented, through which the above three fuzzy assessment methods were compared to each other and with two traditional assessment methods based on principles of bivalent logic, the calculation of meansand of the GPA index.In this way we have obtained a very strong indication for the effectiveness of the proposed fuzzy assessment methods.

Concerning our plans for future research on the subject, we intimate that further experimental investigation is needed in order to obtain safer statistical conclusions about the effectiveness of the proposed fuzzy collaborative assessment framework. On the other hand, the recently developed TRFAM and the similar to it TFAM appear to have the potential of general assessment methods and therefore they could be used in practice in assessing several other human activities apart from the students' performance.

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