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A Study on the Difficulties of Understanding Geometric Proofs of Solids Among the First-grade Students in Senior High School

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Abstract

Solid geometry proof is one of the essential contents for students to learn solid geometry. It plays an important role in training and developing students' spatial imagination and logical reasoning. However, the learning effect of students in proof of solid geometry is not satisfactory at present. In order to help teachers understand what difficulties students have in learning the proof of solid geometry, especially, in the process of understanding, so as to take appropriate countermeasures, this study uses the questionnaire method to survey on the first-grade students in Senior High School. The results show: (1) students are difficult to understand the relationship between the positions of the figures, especially when the figures are complex. (2) Students have difficulties in understanding geometry problems, of which the most difficult is understanding the meaning of the problem. (3) Students have difficulties in understanding the way of knowledge expression, especially the understanding of words, which most students think is the most difficult. (4) It is difficult for students to understand the proving method of a solid geometry. Most of them do not understand the individual steps. (5) There are some difficulties in students' understanding of the three-dimensional geometry proving process explained by the teacher. Most of the students don't understand some steps. (6) There are some difficulties in students' understanding of the proof thinking of solid geometry. Most students say that the most difficult thing is why they choose such proof thinking. (7) It is difficult for students to understand how to add auxiliary lines, especially most students don't understand why to add such auxiliary lines. Therefore, it is suggested that teachers should explain the proof more carefully and with the knowledge which students gained before and give students more helps after class.

Keywords

First-grade Students, Senior High School, Solid Geometry, Proof, Understanding

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1. Introduction

Geometry is a mathematical subject that studies the relationship between the shape, size, and position of objects in the real world. As an important part of geometry, solid geometry is an important branch of high school mathematics and plays an irreplaceable role [1]. The teaching of solid geometry is helpful to cultivate and develop the students' ability of space imagination, the ability of communication, reasoning, and demonstration, and the ability of geometry intuition. Solid geometry plays an important role in the learning of the whole senior high school. It has always been a difficult point in senior high school learning, which is of great significance to the development of students' reasoning ability of geometric space thinking. Also, once the logical thinking ability to analyze problems is developed, it will also be of great help to the learning of other subjects [2]. Therefore, students must lay a good foundation when they start to study solid geometry. The first year of senior high school is the initial stage for students to learn solid geometry. However,

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many students feel difficult to learn solid geometry knowledge, especially proof of solid geometry. Currently, the proof of solid geometry has become a difficulty in current mathematics teaching. Therefore, it is of great significance for teachers and students to find out what difficulties they have when they study geometric proof, especially, in the process of understanding.

2. Literature Review

There are many pieces of research on the study of solid geometry for senior high school students. Ma points out that there are some difficulties in the study of solid geometry for senior high school students at present, and many students have some psychological obstacles. The lack of space imagination of students leads to difficulties in understanding, distinguishing, and drawing pictures. Due to the logic obstacles of students, the expression ability of students is poor, including the expression and mutual transformation of text language and graphic language; due to the lack of understanding of mathematical thinking methods, and students' own thinking obstacles in solving problems, students cannot flexibly use mathematical thinking methods to solve problems [3].

Zhu investigates the learning situation of high school students' solid geometry by questionnaire and test method, and found that there are three obstacles in the learning of high school students' solid geometry: first, cognitive obstacles: obstacles in the aspects of graphic perception, memory and understanding of definition theorems, thinking judgment and spatial graphic imagination; second, operational obstacles: obstacles in the application of knowledge, the choice of proof methods and ideas, theorems and methods of thinking, and drawing; third, emotional obstacles: emotion is the expression of positive or negative attitudes of people towards external things, including three aspects of obstacles such as attitude recognition, emotion and behavioral tendency [4].

Chi through the questionnaire survey and analysis of the results of the survey finds that students in solid geometry learning obstacles are as follows: junior high school plane geometry knowledge storage is insufficient; spatial imagination ability is poor, the ability to read pictures is insufficient; graphics language and symbol language conversion ability is not strong [5].

In addition to the analysis of students' obstacles in learning solid geometry, scholars also analyzed the causes of these obstacles. Wang combines with the current situation of students' solid geometry learning to find out six factors that affect students' solid geometry: students' interest in learning, confidence, and sense of achievement; students' spatial imagination ability; students' drawing ability; teachers' teaching methods; students' learning methods and learning habits; teachers' teaching evaluation [6].

Lu stresses that since students did not learn the knowledge of solid geometry before the first year of senior high school, they only learned some knowledge of plane geometry, so when students learn solid geometry, it's difficult to find the relevant knowledge from the existing cognitive structure to establish a connection with it, and they can only rely on a little life experience, which is far from enough for the abstract and logical three-dimensional geometry. Besides, plane geometry has a more or less negative impact on students' learning of solid geometry. Students are easy to transfer and analogy some knowledge learned in plane geometry at will. However, many concepts in plane geometry are not tenable in solid geometry [7].

Many scholars and educators have put forward their own opinions on students' learning difficulties in solid geometry. Yang believes that to learn solid geometry, the first thing is to help students develop their learning motivation and improve their interest in learning solid geometry, which requires teachers to create questions related to real-life or interested in students at the beginning of the new course to arouse students' curiosity, we should start teaching from the simple to the deep, from the "shape" to the "number", giving students more time to think and more encouragement; we cannot use a single teaching method, but can use courseware demonstration, physical display, student operation, group discussion, and other methods to let students participate in the classroom; while teaching book knowledge, teachers should also use alternative experience or verbal persuasion to make students establish correct learning objectives [8].

Liu points out that we should pay attention to the cultivation of students' ability to understand and draw pictures. At the beginning of learning, we should pay attention to the training of students' ability to observe and draw pictures, so that students can correctly understand and draw all kinds of spatial figures. When students can skillfully draw special geometry, they should change some of them, practice drawing general figures, and gradually get rid of dependence on physical models. Secondly, at the beginning of solid geometry learning, teachers should pay attention to students' understanding of basic concepts, axioms, and theorems. By citing counterexamples, students' cognitive conflicts are caused, students feel how the conditions play a decisive role and summarize the nature of the proposition. Finally, in the process of teaching, teachers should try to slow down the speed and clarify the relationship between knowledge [9].

To sum up, the previous researches on solid geometry mainly focus on the learning difficulties, learning obstacles, factors affecting students' learning of solid geometry, and what

measures teachers should take. Most of the proofs of solid geometry are mentioned a little, few of which are devoted to the study of students' difficulties in understanding solid geometry, while the proofs of solid geometry must be mastered by students, from this we can know that the problems of the research students in the proof of solid geometry are very valuable. Therefore, the author intends to use the questionnaire method to research the difficulties of students' understanding of solid geometry.

3. Theoretical Basis

3.1. Understanding

Understanding refers to the thinking process of understanding the internal relations, essence, and laws of things by rational cognition activities and logical or illogical ways of thinking such as analysis, comparison, generalization, association, and intuition with the help of concepts. It can be divided into direct understanding and indirect understanding. Direct understanding refers to the understanding realized by individuals through the current personal experience without the participation of intermediary thinking; indirect understanding refers to the understanding realized by intermediary thinking through a series of analyses, synthesis, abstraction, and generalization by borrowing previous experience and individual's previous experience [10].

3.2. Mathematical Understanding

On the meaning of mathematical understanding, many domestic experts and scholars have put forward their views. Shen believes that mathematical understanding is based on perception, through thinking processing, assimilating the new learning content into the existing cognitive structure, or reorganizing and expanding the original cognitive structure, including the new learning content, it is a kind of thinking activity to gradually realize the essence and law of mathematical objects [11].

According to Qin, mathematical understanding emphasizes two aspects: one is the essential cognition of mathematical concepts, theorems and laws; the other is the ability to establish a clear context with upper concepts and lower concepts [12].

According to Professor Yang, the process of mathematical understanding is essentially a process in which new knowledge is recognized by individuals, and then a vertical and horizontal connection is established between the two related processes of generic learning activities and mathematical deduction activities and the related knowledge in the original cognitive structure [13].

To sum up, the process of students' understanding of mathematical knowledge is the process of knowledge entering students' minds and becoming a part of the existing cognitive structure. In this process, new knowledge and existing knowledge are connected to make the original cognitive structure richer and more perfect, and students can skillfully apply new knowledge to solve problems.

3.3. Perceptual Difficulty

According to Gao, the difficulty of mathematical understanding means that students can't grasp the essence and connotation of mathematical knowledge and the internal relationship between them [14].

Bai points out that the difficulty of mathematical understanding refers to that the students have spent enough time and made enough efforts, but still can't understand the mathematical knowledge to the level required by the teaching [15].

Zhao believes that so-called mathematics understanding difficulty is that the learner has experienced subjective efforts. According to his existing cognitive structure, he tries to use his existing knowledge and experience to deal with the mathematics knowledge information. However, he cannot correctly and completely reinterpret and reconstruct its meaning, so that the new learning content cannot be correctly integrated into the original cognitive structure, and the cognitive state of errors or deviations occurs in the process of thinking [16].

To sum up, the so-called understanding difficulty is that students can't establish a reasonable connection between new knowledge and existing knowledge after a series of efforts, can't bring new knowledge into the cognitive structure, and become a part of it, and can't flexibly extract knowledge to solve problems.

4. Research Method

4.1. Research Sample

In this study, 99 students from class 6 and 7 in grade 1 of a senior high school in China are selected for a questionnaire survey, of which female students account for 52.5% and male students account for 47.5% (see Table 1 for details).

Table 1. The basic situation of the research sample.

	Boys	Girls	Total
Senior 1 (6) class	23	27	50
Senior 1 (7) class	24	25	49
Total	47	52	99

Source: Field Survey, 2019.

4.2. Instrument

This study intends to use the self-made questionnaire survey

method to study. The questionnaire includes 8 questions, which are divided into 7 multiple-choice questions and an open-ended question. The questions of the questionnaire are related to the proving method, the expression way, the position relation of the figure, the proving process, and the proving thought of the teacher's explanation, etc. The questionnaire is jointly considered by the mathematics teachers in the first-grade group of senior high school, and is finalized after many changes, and reaches a relatively high validity.

4.3. Data Collection

The questionnaire is distributed by the author to the students in the two classes. The answer time is limited to 20 minutes. A total of 99 questionnaires are distributed and 99 points are returned. Among them, the effective volume is 99 and the recovery rate is 100%.

4.4. Data Processing

Use Excel tables to count the answers to each question. For objective questions, the frequency of the survey results is statistics, that is, the number of students who choose this option as a percentage of the total number. For subjective questions, summarize the main points of the answer. (All percentages are approximate)

5. Research Results

5.1. Understanding of Proof Method

According to the survey of students' understanding of the solid geometry proving method explained by the teacher, it is found that only 32.3% of the students can fully understand, 55.6% of the students do not understand the individual steps, another 8.1% of the students say it is difficult to understand and 4.0% of the students do not understand at all (see Table 2 for details). Therefore, most of the students cannot fully understand the solid geometry proving steps.

Table 2. Answer results of question 1.

Question 1				
Option	**A. Understand**	**B. Hard to understand**	**C. Individual steps not understand**	**D. Entirely fail to understand**
Percentage	32.3	8.1	55.6	4.0

Source: Field Survey, 2019.

5.2. Understanding of Expression

By examining students' understanding of knowledge expressed in three different ways, it is found that a certain number of students express difficulty in understanding the knowledge expressed in the three ways, and 10.1% of students believe that the knowledge expressed in these three ways is relatively difficult to understand (see Table 3 for details), which means that the students are not very proficient in the conversion of these three languages, especially the translation of the text language into mathematical symbol language and mathematical graphics language.

Table 3. Answer results of question 2.

Question 2				
Option	**A. Text language**	**B. Mathematical symbols**	**C. Mathematical graphs**	**D. All three**
Percentage	47.4	25.3	17.2	10.1

Source: Field Survey, 2019.

5.3. The Judgment of the Position Relation of Graphs

Through the examination of the students' judgment on the position relationship of the graphs, it is found that 37.4% of the students have a little difficulty in recognizing the complex graphs, 35.4% of the students may make mistakes when making the judgment, 2.0% of the students say it is difficult to identify, and 25.2% of the students can make an accurate judgment (see Table 4 for details). These show that most of the students have some difficulties in judging the position relationship of graphs.

Table 4. Answer results of question 3.

Question 3				
Option	**A. The structure is simple, but it's OK. It's difficult to recognize the complexity.**	**B. I can judge but sometimes make mistakes.**	**C. Generally, it can be identified accurately.**	**D. Hard to recognize**
Percentage	37.4	35.4	25.2	2.0

Source: Field Survey, 2019.

5.4. The Choice and Effect of Reading Methods

Through the examination of students' choice of reading mode and the effect of the mode, it is found that 37.4% of the students can read carefully in combination with graphics and dig out the desired information from it. 35.4% of the students are also reading carefully, but there are still misunderstandings. 12.1% of the students read the questions very fast, but do not understand the meaning of the questions, resulting in errors. 6.1% of the students only understand the surface meaning (see Table 5 for details). These show that nearly two-thirds of the students have difficulties in reading comprehension, or they can't read the questions carefully enough or they can't make a correct understanding of the problems.

Table 5. Answer results of question 4.

Question 4				
Option	**A. Ten lines at a glance, only understand the surface meaning.**	**B. Careful reading, the combination of graphics, the combination of reading and memory, can excavate some deep meaning.**	**C. Read carefully, but sometimes you still can't understand the meaning of the topic, sometimes you even understand the error.**	**D. Read very fast, but can't understand the meaning of the topic. Sometimes you will understand the error.**
Percentage	6.1	37.4	35.4	12.1

Source: Field Survey, 2019.

5.5. The Understanding of the Proof Process

Through the examination of students' understanding of the demonstration process of solid geometry explained by the teacher, it is found that only 35.4% of the students can understand every step basically, 45.4% of the students don't understand some steps, 12.1% of the students don't understand most steps, and the rest of the students don't understand at all (see Table 6 for details). These show that most of the students have a different degree of understanding difficulties in the process of proving the teacher's explanation, most of which are lack of understanding of some of the steps.

Table 6. Answer results of question 5.

Question 5				
Option	**A. Every step can be understood.**	**B. Some steps don't understand.**	**C. Most steps don't understand.**	**D. Entirely fail to understand.**
Percentage	35.4	45.4	12.1	7.1

Source: Field Survey, 2019.

5.6. The Understanding of Proof Thinking

Through the examination of students' understanding of the solid geometry proof ideas explained by the teacher, it is found that 41.4% of the students can understand and understand why to do so, while 45.5% of the students can understand but cannot understand why to do so, and 13.1% of the students do not understand (see Table 7 for details). This shows that more than half of the students have some difficulties in understanding the teacher's proof thinking. Most of them say they can understand the teacher's solution thinking, but they don't understand why they choose such solution thinking. In fact, this still reflects the lack of understanding of the proof method.

Table 7. Answer results of question 6.

Question 6			
Option	**Can understand but don't know why to do so.**	**B. Can understand and understand why.**	**C. Can't understand the proof idea.**
Percentage	45.5	41.4	13.1

Source: Field Survey, 2019.

5.7. The Understanding of Adding Guides

Through the examination of students' understanding of adding auxiliary lines to the proof questions, it is found that only 38.4% of the students know the function of auxiliary lines and understand why to add such auxiliary lines, while 42.4% of the students know the function of auxiliary lines but do not understand why to do so, and another 20.2% of the students do not understand why to add auxiliary lines (see Table 8 for details). These show that most of the students have a different degree of difficulty in the understanding of adding auxiliary lines. The most important thing is to be able to understand what role auxiliary lines play in the topic, but do not

understand why.

Table 8. Answer results of question 7.

Question 7			
Option	**A. Know the role of auxiliary line in the topic, and understand why to do it.**	**B. Know the role of guides in the topic but don't understand why.**	**C. Don't understand why you want to add guides.**
Percentage	38.4	42.4	20.2

Source: Field Survey, 2019.

5.8. A Survey of the Open Question

Question 8: when the teacher explains the proof of solid geometry, what aspects are you difficult to understand?

To this question, the student's answer is also various but can summarize the following points basically: 1. How to add auxiliary lines when solving the problem? 2. Why can we make a direct transition from this step to the next step? 3. Why choose such proof thinking? 4. When the graph is too complex, it is difficult to extract the conditions I want and judge the relationship between points, lines, and surfaces.

6. Discuss

Reviewing the above statistics, we can find that:

(1) In the judgment of the position relationship of graphs, most students express that they can't accurately judge the position relationship between the points, lines, and surfaces, especially in the more complex solid geometry graphs, so it can be seen that students have some degree of understanding difficulties in judging the graphs relationship, especially when the graphs are more complex.

(2) In terms of reading comprehension, nearly two-thirds of the students either lack the patience to read the questions in detail or have a deviation in understanding the meaning of the questions. The latter situation is more serious, so it can be seen that the students have certain understanding difficulties in the reading comprehension of geometry questions, especially in the understanding of problem questions.

(3) In terms of the expression of knowledge, there are a certain number of students who do not understand the text, mathematical symbols, and mathematical graphics, of which the students who do not understand the text language account for the largest proportion. Therefore, students have certain understanding difficulties in the expression of knowledge, especially in the understanding of the text.

(4) In the proving method of solid geometry, most students can't fully understand the proving steps of solid geometry, and the students who don't understand the individual steps account for the largest proportion. Therefore, it can be seen that students have some difficulties in understanding the proving method of solid geometry, and most of them don't understand the individual steps.

(5) In the process of proving solid geometry, most of the students have different levels of difficulties in understanding the proving process explained by the teacher, most of which are not enough to understand some of the steps. Therefore, it can be seen that students have some difficulties in understanding the proving process of solid geometry explained by the teacher, and most of the students say they do not understand some of the steps.

(6) More than half of the students can't fully understand the proof thinking of solid geometry. Most of them can understand the teacher's solution thinking, but they don't understand why they should choose such solution thinking. Therefore, students have certain difficulties in the solution of the proof thinking of solid geometry, and most of them can listen clearly but don't understand.

(7) In the addition of auxiliary lines, most of the students do not understand why to add auxiliary lines or know the role but do not know why to do so. Most of them do not understand the latter. Therefore, students have certain difficulties in understanding the addition of auxiliary lines, especially do not understand why to add such auxiliary lines.

7. Conclusion

Through the above analysis, it is found that: (1) students have a certain degree of understanding difficulties in judging the position relationship of graphics, especially when the graphics are more complex. (2) Students have some difficulties in understanding geometry problems, of which the most difficult is understanding the meaning of the problem. (3) Students have some difficulties in understanding the way of knowledge expression, especially the understanding of the text, which most students think is the most difficult. (4) It is difficult for students to understand the proving method of a solid geometry. Most of them do not understand the individual steps. (5) There are some difficulties in students' understanding of the solid geometry proving process explained by the teacher. Most of the students don't understand some steps. (6) There are some

difficulties in students' understanding of the proof thinking of solid geometry. Most students say that the most difficult thing is why they choose such proof thinking. (7) It is difficult for students to understand how to add auxiliary lines, especially most students don't understand why to add such auxiliary lines.

8. Implications

It is suggested that: first of all, teachers should make use of physical objects and models, corresponding to geometric graphs, to help students clarify the relationship between points, lines, and surfaces. When the graphs are more complex, teachers can use multimedia teaching to analyze the relationship between graphs, and pay attention to the corresponding exercises from time to time, so that students gradually develop a solid sense; secondly, when explaining the problem, teachers should pay attention to guide students to carefully analyze the meaning of the question, and give students appropriate inspiration when necessary, to help students dig deeper meaning and develop the habit of reading the question seriously; thirdly, in the process of learning the basic knowledge of solid geometry, teachers should guide students to transform the text language, mathematical symbols and mathematical graphics into each other, and carry out certain training to deepen students' understanding of the text knowledge; in addition, when explaining the proving process and method of solid geometry, the teacher should make clear every step and the causal relationship between steps, what kind of theorem is used, and the most important thing is that the teacher should tell the students why they should choose such a solution idea and what its starting point is; finally, when the teacher explains the proof questions that need to add auxiliary lines, in addition to making the students clear about the role of auxiliary lines, the most important thing is to tell the students how to choose the appropriate auxiliary lines.

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