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# Research on Cultivating Students' Mathematical Abstract Literacy by Combining Textbooks

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## Abstract

High school mathematics textbook is an important textual basis for learning high school mathematics, an important resource for teachers' classroom teaching, an important carrier for realizing teaching objectives, and the task of cultivating students' core literacy is also inseparable from the textbook. With the deepening of the new curriculum reform, each subject has its own core literacy. Mathematical abstract literacy is the first of the six core literacy of mathematics and plays a unique and irreplaceable role in the formation of human rational thinking, scientific spirit and the promotion of personal intelligence development. Thus, mathematical abstract literacy is the basic accomplishment that every citizen should have in contemporary society. Under the background of the core literacy of the subject, it is a concern of mathematics teachers how to cultivate students' mathematical abstract literacy in the classroom teaching. This paper will follow the concept of the new curriculum and combine the knowledge of high school mathematics textbooks to conduct research from three aspects, including conceptual formation, formula and theoretic derivation and examples and exercises. At the same time, this paper will deeply analyze the problems existing in the teaching of concepts, theorems and formulas and examples and exercises. Finally, in view of these problems, this paper will also put forward specific suggestions to teachers from the point of view of fostering students' mathematical abstract literacy. It aims to improve students' mathematical abstract literacy, implement the fundamental tasks of taking high moral values establishment and people cultivation, provide assistance for the teaching of front-line teachers, and prepare for further research in the future.

## Keywords

High School Mathematics Teaching, High School Mathematics Textbook, Mathematical Abstract Literacy

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## 1. Introduction

"Mathematics Curriculum Standard in General Senior High School" focused on the implementation of the core literacy of mathematics and clearly pointed out that mathematical abstract literacy was the first of the six core literacy of mathematics. Mathematical abstraction is the basic idea of mathematics and an important basis for forming rational thinking. It reflects the essential characteristics of mathematics and runs through the process of the generation, development and application of mathematics [1]. Mathematical abstract literacy is one of the basic qualities that need to be cultivated and improved in senior high school students. Therefore, it is very necessary to cultivate mathematical abstract literacy in high school mathematics teaching. In order to help front-line teachers improve students' mathematical abstract literacy, provide reference for teachers' teaching, and better promote the cultivation of students' core literacy, this paper will conduct research from three aspects, including conceptual formation, formula and theoretic derivation, examples and exercises.

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## 2. The Importance of Mathematics Textbooks in Senior High School

High school mathematics textbooks are an important textual basis for studying high school mathematics, and the task of cultivating students' core literacy is inseparable from textbooks. "National Medium-and Long-Term Education Reform and Development Programs (2010-2020)" pointed out that the textbook was the concentrated reflection of the course objective. So the content of the textbooks should be optimized, the basic task of cultivating moral character should be implemented, and the all-round and life-long development of students should be promoted [2]. "Mathematics Curriculum Standard in General Senior High School" pointed out that mathematics textbooks provided learning themes, basic clues and specific contents for "teaching" and "learning" activities. They were important teaching resources for realizing the objectives of mathematics curriculum and developing students' mathematical abstract literacy [1]. Zheng Shanjun also pointed out that textbooks were the concrete embodiment of the "curriculum standard", the carrier of teaching content, the basis of teaching and learning, and the support of teaching process. Textbooks provided rich and colorful contents for teachers to carry out mathematics curriculum and classroom teaching reform, and also provided more clues for students' activities [3]. Ren Yun pointed out that textbooks were the source of teachers' teaching objectives and tasks, and the basic reference tools for students' learning and thinking [4]. Zeng Tianshan pointed out that teaching objectives had a restrictive and normative effect on textbooks, while textbooks were the concrete reflection and direct description of teaching objectives [5]. Li Xiaohua pointed out that textbooks, as the basis of imparting knowledge to students, were of great significance in cultivating students' core literacy and were also the key content of the reform of education system [6].

To sum up, it can be seen that mathematics textbooks, as the most important reliance and teaching resources of teachers and students, played an irreplaceable role in transferring mathematical knowledge, carrying out teaching activities, achieving teaching objectives and cultivating students' core literacy of mathematics. Therefore, it is necessary for us to reform mathematics education according to the concept of the current curriculum standard, combine the content of the textbooks, and improve the students' mathematical abstract literacy from the aspects of conceptual formation, formula and theoretic deduction, examples and exercises.

## 3. The Present Situation of Mathematics Teaching in Senior High School

### 3.1. Not Paying Enough Attention to the Quotations and the Process of Conceptual Formation

Conceptual teaching is generally divided into the introduction of concepts and the acquisition of concepts, which plays an important role in cultivating students' mathematical abstract literacy. However, many teachers do not attach importance to the role of quotations in textbooks in actual teaching [7]. In order to save time, the teacher directly gives the definition of the concept by skipping the cited examples in the textbooks, and the students lack the perceptual understanding of the concept, so they will naturally lack the correct understanding of the abstract concepts given directly, and their understanding of the problems will be biased when they do the exercises. Secondly, the process of conceptual acquisition is not neglected. Some teachers and students do not attach much importance to the formation process of mathematical concepts. They believe that as long as students learn the method of solving problems from the cases, they can solve problems when they are faced with them, and require students to carry out problem training after learning by rote [8]. Teachers can not truly understand the importance of the link from specific issues to defining concepts. It is unrealistic for teachers with this understanding to cultivate students' abstract literacy in place.

### 3.2. Formula and Theoretic Derivation Emphasizes Results but Neglects Processes

"Mathematics Curriculum Standard in General Senior High School" clearly pointed out that mathematics curriculum in senior high schools should return to its original state and try to reveal the development process and essence of mathematical concepts, rules and conclusions [1]. Mathematical formulas and theorems have the characteristics of abstractness and generality of formal symbolization, which are important carriers for the development of students' cognitive level and play important roles in the cultivation of students' mathematical abstract literacy [14]. However, in the teaching of mathematical formulas and theorems in middle schools, there is a teaching form of "formula plus examples". Teachers are concerned about the results of students' learning, while ignoring the derivation process of formulas and theorems. They also neglect the process of guiding students' abstraction [9, 13]. This form of teaching often leaves only the shell of formulas and theorems in students' minds, ignoring their origin and development, and students are not clear about the

conditions and scope of their application. For the formulas and theorems that need to be proved in the curriculum standard, due to the limitation of class time, many teachers will directly omit the proof of formulas and theorems, or the proof process is not strict. The proof that can be done by mathematical induction is proved by incomplete induction, which leads to the fact that students cannot experience the process of abstracting the essential attributes of mathematical objects from mathematical phenomena. And it also leads to the inadequacy of students' abstract ability. The concrete manifestations are that the students can memorize the formulas or conclusions by rote, and cannot clarify the successive relationship between the formulas and the theorems.

### 3.3. Attaching Importance to the "Sea-topic Tactics" and Lacking Generalization

Example teaching is an important part of mathematics teaching in senior high school. It can not only help students better grasp the basic knowledge and skills of mathematics, but also enable students to better understand the application of mathematics. Meanwhile, it can also improve students' thinking quality and mathematical literacy [10]. However, at present, although teachers and students are carrying out a large number of examples and training every day, students are still at a loss when they encounter some familiar questions in their daily exams, and the results of the exams are still not satisfactory. One of the reasons is that teachers are reluctant to leave time for students to think independently and use various methods to teach, which leads to confusion of students' thinking and lack of clarity of the nature of the problem. The second reason is that teachers emphasize problem-solving tactics and focus only on the the results of students' problem-solving, which leads to students' more operational memory, mechanical imitation of problem-solving, low level of thinking, and lack of optimization of examples and necessary reflection and induction and generalization of the same types of questions [10].

## 4. Training Measures

Based on the above analysis of the current situation of mathematics teaching in senior high school, the following teaching suggestions are put forward to effectively cultivate students' mathematical abstract literacy:

### 4.1. Paying Attention to the Process of Conceptual Formation and Improving the Mathematical Abstract Literacy

Concept is the most basic abstraction, concept is the cell of thinking. Therefore, the cultivation of abstract literacy should start with concepts. In the actual teaching, first of all, teachers should delve into the textbook and deeply understand the importance of citing examples in the textbook. These examples have been considered and written by experts for many times. They are typical, exemplary, and wide-ranging and can give students a rich perceptual experience. So teachers should make full use of these examples. Secondly, teachers should give students enough time to "think" and "say" the examples of common characteristics, timely comment on students, and attach great importance to the process of students' experience, so that students can observe and analyze in specific situations. When students have an intuitive embryonic form in their brains, they should be guided to think and discuss, discover the common characteristics of the object of study, then abstract the essential attributes of things, and finally form mathematical concepts. It is necessary to enable students to form a good habit of thinking and promote the improvement of students' core literacy. For example, when explaining "sine theorem", teachers should start with right triangle and guide students to discover the relationships between edges and corners by using the old knowledge. After that, teachers changed the original problem into an acute triangle. In this way, students can initially dig out the invariable mathematical essence from the changing problem forms. At the same time, teachers should further guide the study to explore and conclude conclusions. In this way, through the teaching of conceptual formation or abstraction of conclusions over and over again, students can accumulate some practical and abstract experience, learn to grasp the essence of mathematics, develop the habit of thinking in general in daily life and practice, and eventually improve mathematical abstract literacy.

### 4.2. Improving Mathematical Abstraction Literacy in Derivation of Formulas and Theorems

In view of the teaching of formulas and theorems that emphasize the conclusions but ignore the processes, teachers should clarify their role of demonstration and guidance in teaching, and teachers' beliefs directly affect students' beliefs. If even the teachers themselves feel that the teaching of mathematical formulas and theorems can emphasize the conclusion rather than the process, it is impossible to require students to master the deduction and proof of mathematical formulas and theorems. Therefore, in classroom teaching, teachers should effectively grasp the proof process of mathematical formulas and theorems, and students should give full play to their main role. The questions that students can deduce by themselves require students to complete by themselves. Meanwhile, teachers should inspire students to make connections between knowledge, set up a series of questions, pay attention to the deduction of formulas and theorems step by step, and pay attention to pointing out the

errors in students' deduction. For complicated formulas and theorems in the process of derivation, teachers should guide students to pay attention to analysis and clarify the origin and development of knowledge, instead of pursuing so-called "efficiency". Teachers should not allow students to memorize and recite instead of understanding, and problem-solving training instead of process experience. The teaching mode of pursuing "efficiency" violates the basic law of thinking development. Even if it is helpful for students to improve their short-term test scores, it is also very unfavorable for students to develop their long-term mathematical literacy [11]. Secondly, teachers should not only teach formulas and theorems at the application level, but also let students understand the abstractness of mathematics. Teachers should consciously reveal the relationships between formulas and theorems so as to help students form a knowledge network. For example, after explaining the theorem of "the nature of line-plane parallelism", teachers should inspire students to associate the theorem of "determination of line-plane parallelism" learned before. Then teachers should guide the students to find the differences and connections between the theorems. Ultimately, students will understand the role of each theorem, rather than memorize it by rote. Finally, teachers should properly introduce the history of mathematics and provide students with a variety of proof ideas, which will broaden students' horizons and promote the cultivation of students' abstract literacy. For example, to prove the "sinusoidal formula of the difference between two angles", after guiding students to prove it from an algebraic point of view, teachers can introduce knowledge of the history of mathematics and explain to students the methods of proving it from a geometric point of view in ancient times. Then let students analyze and discuss the differences and connections between modern and ancient proofs, analyze the advantages and disadvantages, and master the methods of proof and the mathematical ideas involved in comparison. Therefore, the derivation process of formulas and theorems is particularly important for the development of students' core literacy. Only when students experience the process of mathematical abstraction can their mathematical abstraction literacy be improved correspondingly. At the same time, students will feel at ease in the derivation and proof problems in the future.

### 4.3. Improving Mathematical Abstract Literacy in the Process of Summarizing Examples and Exercises

The ideas and methods contained in the examples and exercises are the essential understanding of the questions, and are of a general nature [12]. In view of the current situation of "sea-topic tactics", teachers should first study examples and exercises. Teachers should not only stay on the mathematical concepts and basic knowledge involved in examples and exercises, but also help students dig deeply into examples and exercises and extend them appropriately. At the same time, the mathematical ideas and methods contained in the examples and exercises should be refined, and students who encounter similar questions in the exam will be much easier. For students, students can not only solve problems on the surface, they must be good at summarizing, abstracting and inducing the ideas and methods contained in the questions after completing the questions, so as to cultivate students' abstract literacy. In this way, the essence of mathematics can be revealed to the greatest extent, which not only consolidates the knowledge acquired, but also expands students' mathematical vision and improves their ability of abstraction and generalization. Secondly, in the process of solving problems on the board, teachers should present conditions in stages, stimulate students' existing cognition and establish connections, find out students' thoughts by asking questions, and continuously guide students to supplement and evaluate, so as to ask students to express their own ideas. For example, what are the drawbacks of this idea? Can you improve it? Any other ideas? What is the best solution? Thirdly, teachers should add, delete and adjust examples and exercises reasonably according to students' actual cognitive level and learning conditions. Some knowledge can be learned ahead of time, which is more conducive to students' learning and understanding; some of the more knowledgeable and difficult comprehensive exercises can be put back. If the teacher forcibly puts them here, the students will not accept them. This will interfere with students' understanding and mastery of this part of the knowledge. Finally, the teacher changes the form of the problems by changing the conditions or conclusions of the topic, setting up gradient problems with a variety of ways to solve the problem, so that every student can participate in the process of example teaching, which will avoid the phenomenon of "students can't eat enough" and "not enough to eat". For example, teachers can guide students to think from the perspective of function or algebra about "the maximum value of the sum of arithmetic sequence". Only in this way can the students' mathematical abstract literacy be truly developed.

## 5. Conclusion

From the perspective of core literacy and on the basis of " Mathematics Curriculum Standard in General Senior High School", this paper firstly sorts out the importance of senior high school mathematics textbooks to cultivate students' mathematical abstract literacy and the present situation of senior high school mathematics teaching. For the former, scholars have fully affirmed that senior high school mathematics textbooks are crucial to the cultivation of students' mathematical abstract literacy, and the

development of students' mathematical abstract ability cannot be separated from textbooks. They are interdependent and mutually reinforcing; for the latter, scholars have studied extensively, pointing out that the teaching of concept formation is neglected, the teaching of formula and theorem stresses results over process and attaches importance to the teaching of sea-topic tactics. All of these studies are reasonable and have profound significance for the study of this paper. However, from the above analysis, it can be seen that the research on the way to cultivating students' mathematical abstract literacy with textbooks is still blank, which has profound enlightening significance for the study of this paper. On the basis of previous research results, aiming at the problems existing in the cultivation of students' mathematical abstract literacy in teaching, this paper puts forward some suggestions on how to apply textbooks to cultivate students' mathematical abstract literacy from three perspectives of concept, theorem and formula and examples and exercises teaching. Teachers should attach importance to the process of introducing examples and students' experience in the formation of concepts; pay attention to the relationships between formulas and theorems and the various deduction processes and methods of formulas and theorems in ancient and modern times; attach importance to the induction and generalization of examples and exercises, the extraction of mathematical thinking methods and the reorganization of examples and exercises. These methods have reference significance for teachers' practical teaching and play an important role in the development of students' abstract thinking and the cultivation of their abstract literacy. It is believed that through the implementation of the above ways, students' mathematical abstract literacy will be greatly cultivated. The above is only an analysis of how teachers combine textbooks to cultivate students' mathematical abstract literacy. However, there is no in-depth analysis on how students use textbooks reasonably to cultivate their mathematical abstract literacy. This will be the further improvement direction of the paper.

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