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Solitary Wave Solutions of Modified Telegraphist Equations Modeled in an Electrical Line

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Abstract

In this paper, we are using an ordinary electrical line to find the modified Telegraphist equations in terms of current and voltage. Then, we define the nonlinear analytical shape that must respect the conductance per unit length so that the new obtained lines accept the propagation of solitary waves. From the analytical definition of the conductance of the modified Telegraphist equations, we obtain new nonlinear partial differential equations that govern the dynamics of solitary waves in the new lines. Having constructed the exact solitary wave solutions of the new higher-order nonlinear partial differential equations, we have confirmed that these lines accept the simultaneous propagation of a set of two signals which are current and voltage of type (Kink; Kink) or type (Pulse; Pulse). The obtained results have advantages generally in the domain of physics and particularly in the domain of engineering of telecommunication because the new lines obtained accept the propagation of solitary waves of type (Kink; Kink) or type (Pulse; Pulse) on longer distances maintaining their shape, their velocity, without loss of energy contrary to sinusoidal waves that we have obtained with non-modified Telegraphist equations whose amplitude decreases exponentially and loss a lot of energy.

Keywords

Telegraphist Equations, Construction, Model, Soliton Solution, Solitary Wave, Nonlinear Partial Differential Equation, Kink, Pulse

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1. Introduction

During the last decades, Solitary waves have evolved from simple water waves to the propagation of Pulse solitons in optical fibers [1]. From the definition of solitary wave which is a wave capable to move on longer distances maintaining its shape and velocity; it has come to our mind that if such a signal is used in engineering of information through an electrical line described by modified Telegraphist equations, it will resist best on diverged dissipation factors. We have in this regard decided to come up with two definitions of nonlinear conductance per unit length of resistors constituting networks of an electrical line described by modified Telegraphist equations. Then, we have applied them to model new higher-order nonlinear partial differential equations which govern the dynamics of solitary wave in the given line. The construction of solitary wave solutions of each modeled nonlinear partial differential equation by mathematical methods presented in [2-15] and new Bogning-Djeumen Tchaho-Kofane method presented in [16-21] has enabled to obtain solitary wave solutions of type (Kink; Kink) and Solitary wave solution of type (Pulse; Pulse). The work we are presenting in this paper is partitioned as follows: in part 2, we present the general modeling of modified Telegraphist equations. In part 3, we find out the solution of Telegraphist equations, in part 4, we construct the solitary wave solution of type (Kink; Kink) of the modified Telegraphist equations, in part 5, we construct the

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solitary wave solution of type (Pulse; Pulse) of the modified Telegraphist equations. We finally present the conclusion in part 6.

2. Modeling of Modified Telegraphist Equations

Let us consider an ordinary electrical line constituting a good number of identical networks shown in Figure 1 where: $i(x,t)$ is the current flowing through the resistor with resistance per unit length R and inductor with inductance per unit length L; $u(x,t)$ is the voltage across the capacitor with capacitance per unit length C and across another resistor with conductance per unit length G.

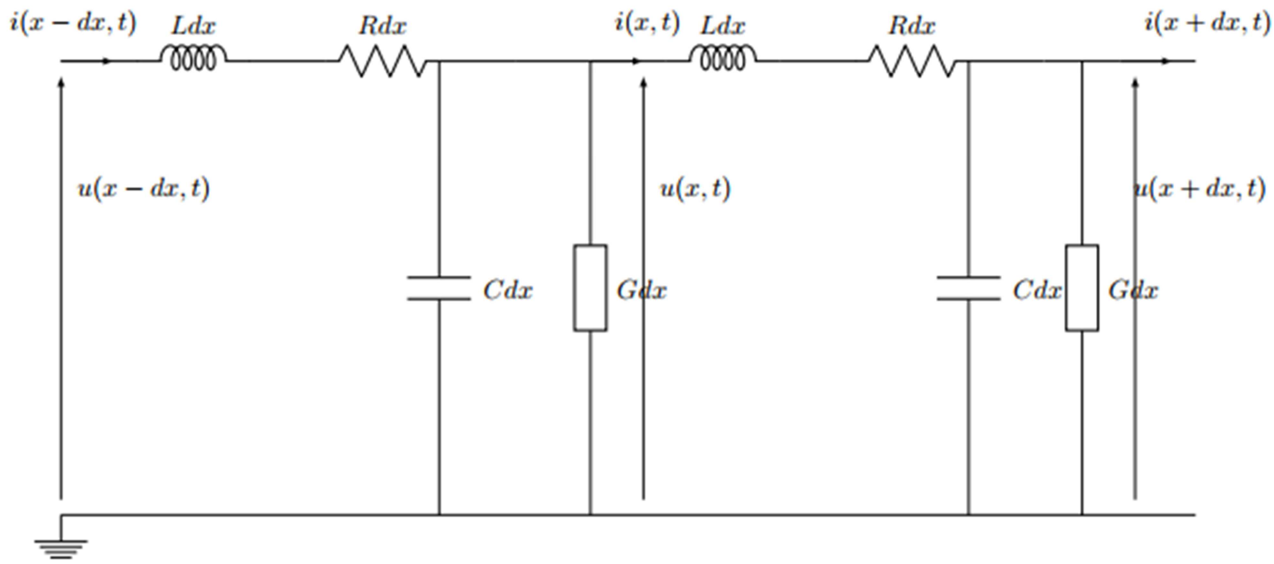

Figure 1. Presentation of an electrical line which describe modified Telegraphists equations.

Applying Kirchhoff laws to the circuit of Figure 1, we obtain the following equations:

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}, \quad (1)$$

$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}. \quad (2)$$

By using derivatives definitions $\frac{u(x+\partial x,t)-u(x,t)}{\partial x} = \frac{\partial u(x,t)}{\partial x}$ and $\frac{i(x,t)-i(x-\partial x,t)}{\partial x} = \frac{\partial i(x,t)}{\partial x}$ the equation (1) and (2) are rewritten as follows:

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}, \quad (3)$$

$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}. \quad (4)$$

By deriving each of equations (3) and (4) relative to the position x we obtain the following equations:

$$\frac{\partial^2 u(x,t)}{\partial x^2} = -R\frac{\partial i(x,t)}{\partial x} - L\frac{\partial^2 i(x,t)}{\partial x \partial t}, \quad (5)$$

$$\frac{\partial^2 i(x,t)}{\partial x^2} = -G\frac{\partial u(x,t)}{\partial x} - C\frac{\partial^2 u(x,t)}{\partial x \partial t}. \quad (6)$$

By deriving each of equations (3) and (4) relative to the time t we obtain the equations bellow:

$$\frac{\partial^2 u(x,t)}{\partial x \partial t} = -R\frac{\partial i(x,t)}{\partial t} - L\frac{\partial^2 i(x,t)}{\partial t^2}, \quad (7)$$

$$\frac{\partial^2 i(x,t)}{\partial x \partial t} = -G\frac{\partial u(x,t)}{\partial t} - C\frac{\partial^2 u(x,t)}{\partial t^2}. \quad (8)$$

Substituting in the one hand equations (4) and (8) in (5) and on the other hand equations (3) and (7) in (6) we obtain Telegraphists equations in voltage and current that we present in the following order:

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial x^2} - LC\dfrac{\partial^2 u(x,t)}{\partial t^2} - (LG+RC)\dfrac{\partial u(x,t)}{\partial t} - RGu(x,t) = 0 \\ \dfrac{\partial^2 i(x,t)}{\partial x^2} - LC\dfrac{\partial^2 i(x,t)}{\partial t^2} - (LG+RC)\dfrac{\partial i(x,t)}{\partial t} - RGi(x,t) = 0 \end{cases}. \quad (9)$$

We deduce from Telegraphist equations (9) the modified Telegraphist equations (10) by considering that the conductance per unit length G of resistor varies in a nonlinear manner relative to the voltage $u(x,t)$ across it.

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial x^2} - LC\dfrac{\partial^2 u(x,t)}{\partial t^2} - \left(LG\left(u(x,t)\right)+RC\right)\dfrac{\partial u(x,t)}{\partial t} - RG\left(u(x,t)\right)u(x,t) = 0 \\ \dfrac{\partial^2 i(x,t)}{\partial x^2} - LC\dfrac{\partial^2 i(x,t)}{\partial t^2} - \left(LG\left(u(x,t)\right)+RC\right)\dfrac{\partial i(x,t)}{\partial t} - RG\left(u(x,t)\right)i(x,t) = 0 \end{cases}. \quad (10)$$

3. Solution of Telegraphist Equations (9)

The two Telegraphist equations in voltage and current being identical, the solving one implies the solution of the other. Let us find out the solution of equation (9) under the shape:

$$u(x,t) = v(x)\exp(jwt). \quad (11)$$

Where $j^2 = -1$ and $v(x)$ a real function of x. By substituting $u(x,t)$ of (11) in the Voltage equation of (9), we obtain:

$$\frac{d^2 v(x)}{dx^2} - \left(\frac{1}{2}\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}} + j\frac{w(LG+RC)}{\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}}} \right)^2 v(x) = 0. \quad (12)$$

Since the propagation equation is that of losses phenomena, the solution of equation (12) is written as follows:

$$v(x) = U_0 \exp\left(-\left(\frac{1}{2}\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}} + j\frac{w(LG+RC)}{\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}}} \right) x \right). \quad (13)$$

Where U_0 is a non-nil real number. Substituting $v(x)$ of (13) in (11) we obtain a sinusoidal wave solution of Telegraphist equations (9) that we present in (14) as follows:

$$\begin{cases} u(x,t) = U_0 \exp\left(-\left(\frac{1}{2}\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}} \right) x \right) \times \\ \exp\left(jw\left(t - \dfrac{(LG+RC)}{\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}}} x \right) \right) \\ i(x,t) = I_0 \exp\left(-\left(\frac{1}{2}\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}} \right) x \right) \times \\ \exp\left(jw\left(t - \dfrac{(LG+RC)}{\sqrt{-2w^2LC+2RG+2\sqrt{w^4L^2C^2+R^2G^2+w^2L^2G^2+w^2R^2C^2}}} x \right) \right), \end{cases} \quad (14)$$

Where U_0 and I_0 is a non-nil real number. In the sinusoidal wave solution (14), the first exponential show that wave amplitude decreases exponentially meanwhile the second exponential show the propagation of wave in the x direction with velocity given by:

$$V = \frac{w\sqrt{-2w^2LC + 2RG + 2\sqrt{w^4L^2C^2 + R^2G^2 + w^2L^2G^2 + w^2R^2C^2}}}{LG + RC} \tag{15}$$

The sinusoidal wave solutions (14) of Telegraphist equations (9) proof that the signals that are displaced in the electrical line modeled by those equations are sinusoidal waves whose amplitude decreases exponentially and losses a lot of energy. To remedy this situation, we construct in the following parts of this work the solitary wave solutions of modified Telegraphist equations (10), since those solitary wave are displaced on longer distances by maintaining their shape, their velocity and resist best on dissipation factors.

4. Construction of Type Kink Solitary Wave Solutions of Modified Telegraphist Equations (10)

We define the nonlinear conductance per unit length under the analytical shape given as follows:

$$G\left(u(x,t)\right) = \frac{A_3u^3(x,t) + A_2u^2(x,t) + A_1u(x,t) + A_4}{A_5u^2(x,t) + A_6u(x,t) + A_7}. \tag{16}$$

With $u(x,t) \neq \frac{-A_6 \pm \sqrt{A_6^2 - 4A_5A_7}}{2A_5}$; $A_6^2 > 4A_5A_7$. A_1, A_2, A_3, A_4, A_5, A_6 and A_7 are non-nil real numbers whose conditions of choice will be established. Replacing nonlinear conductance per unit length $G(u(x,t))$ of (16) in each of the differential equations (10), we obtain higher-order nonlinear partial differential equations given below:

$$\begin{aligned}
&\left(-A_5u^2(x,t) - A_6u(x,t) - A_7\right)\frac{\partial^2 u(x,t)}{\partial x^2} + \left(LCA_5u^2(x,t) + LCA_6u(x,t) + LCA_7\right)\frac{\partial^2 u(x,t)}{\partial t^2} \\
&+\left(LA_3u^3(x,t) + \left(LA_2 + RCA_5\right)u^2(x,t) + \left(LA_1 + RCA_6\right)u(x,t) + LA_4 + RCA_7\right)\frac{\partial u(x,t)}{\partial t} \\
&+RA_3u^4(x,t) + RA_2u^3(x,t) + RA_1u^2(x,t) + RA_4u(x,t) = 0,
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\left(-A_5u^2(x,t) - A_6u(x,t) - A_7\right)\frac{\partial^2 i(x,t)}{\partial x^2} + \left(LCA_5u^2(x,t) + LCA_6u(x,t) + LCA_7\right)\frac{\partial^2 i(x,t)}{\partial t^2} \\
&+\left(LA_3u^3(x,t) + \left(LA_2 + RCA_5\right)u^2(x,t) + \left(LA_1 + RCA_6\right)u(x,t) + LA_4 + RCA_7\right)\frac{\partial i(x,t)}{\partial t} \\
&+RA_3u^4(x,t) + RA_2u^3(x,t) + RA_1u^2(x,t) + RA_4u(x,t) = 0.
\end{aligned} \tag{18}$$

Let us use Bogning-Djeumen Tchaho-Kofane method [16-21] to find out the exact solution of nonlinear partial differential equation (17) under the analytical shape written as follow:

$$u(x,t) = a\tanh(kx - vt). \tag{19}$$

Where a, k and v are non-nil real numbers that will be determined in terms of modeled electrical line parameters described by nonlinear partial differential equation (17). Replacing $u(x,t)$ of (19) in equation (17), we obtain the following equation:

$$\begin{aligned}&Ra^2A_1+Ra^4A_3+\begin{pmatrix}RCA_5a^3v+LA_2a^3v+Ra^4A_3\\+2LCA_6a^2v^2-2A_6a^2k^2\end{pmatrix}\frac{1}{\cosh^4(kx-vt)}\\&+\begin{pmatrix}-Ra^2A_1-2Ra^4A_3-RCA_7av+2A_6a^2k^2\\-RCA_5a^3v-LA_4av-LA_2a^3v-2LCA_6a^2v^2\end{pmatrix}\frac{1}{\cosh^2(kx-vt)}\\&+\begin{pmatrix}2A_7ak^2-Ra^3A_2-LA_3a^4v+2A_5a^3k^2\\-2LCA_7av^2-LA_1a^2v-RCA_6a^2v-2LCA_5a^3v^2\end{pmatrix}\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)}\\&+\left(LA_3a^4v+2LCA_5a^3v^2-2A_5a^3k^2\right)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)}+\left(Ra^3A_2+RaA_4\right)\frac{\sinh(kx-vt)}{\cosh(kx-vt)}=0.\end{aligned}\tag{20}$$

Equation (20) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permit us to obtain the set of six equations presented below:

$$\begin{cases}Ra^2A_1+Ra^4A_3=0,\\RCA_5a^3v+LA_2a^3v+Ra^4A_3+2LCA_6a^2v^2-2A_6a^2k^2=0,\\-Ra^2A_1-2Ra^4A_3-RCA_7av+2A_6a^2k^2-RCA_5a^3v-LA_4av-LA_2a^3v-2LCA_6a^2v^2=0,\\2A_7ak^2-Ra^3A_2-LA_3a^4v+2A_5a^3k^2-2LCA_7av^2-LA_1a^2v-RCA_6a^2v-2LCA_5a^3v^2=0,\\LA_3a^4v+2LCA_5a^3v^2-2A_5a^3k^2=0,\\Ra^3A_2+RaA_4=0.\end{cases}\tag{21}$$

Solving the set of six equations (21), we obtain the solution and the conditions given in (22) of higher-order nonlinear partial differential equation (17) which govern the dynamic of solitary wave of type Kink in the modeled corresponding electrical line:

$$v=\frac{A_5}{L},\ a=\frac{A_6}{R},\ k=\pm\frac{\sqrt{4A_6^2A_5^2LC-2A_1A_6L^2R}}{2A_6L},\ 2A_6^2A_5^2C>A_1A_6LR,\ A_7=-Lva^2,\ A_4=RCva^2,\ A_2=-Rcv,$$

$$A_3=\frac{2\left(k^2-LCv^2\right)}{a},\ u(x,t)=\frac{A_6}{R}\tanh\left(\pm\frac{\sqrt{4A_6^2A_5^2LC-2A_1A_6L^2R}}{2A_6L}x-\frac{A_5}{L}t\right).\tag{22}$$

In the same light it is equally verified that by respecting the given conditions in (22) the solution of the modified Telegraphist equation of current (18) is also written as follows:

$$i(x,t)=\frac{A_6}{R}\tanh\left(\pm\frac{\sqrt{4A_6^2A_5^2LC-2A_1A_6L^2R}}{2A_6L}x-\frac{A_5}{L}t\right)\tag{23}$$

5. Construction of Type Pulse Solitary Wave Solutions of Modified Telegraphist Equations (10)

We define the nonlinear conductance per unit length under the analytical shape given as follows:

$$G\left(u(x,t)\right)=\frac{A_1+A_2u^2(x,t)+A_3\sqrt{1-\left(\frac{u(x,t)}{A_0}\right)^2}}{A_4+A_5\sqrt{1-\left(\frac{u(x,t)}{A_0}\right)^2}}.\tag{24}$$

With $|A_0| > |u(x,t)|$ and $A_5\sqrt{1-\left(\frac{u(x,t)}{A_0}\right)^2} \neq -A_4$. A_1, A_2, A_3, A_4 and A_5 are non-nil real numbers whose conditions of choice will be established. Replacing nonlinear conductance per unit length $G(u(x,t))$ of (24) in each of the differential equations (10), we obtain higher-order nonlinear partial differential equations given below:

$$\begin{aligned}&\left(-A_4A_0 - A_5\sqrt{A_0^2-u^2(x,t)}\right)\frac{\partial^2 u(x,t)}{\partial x^2}+\left(LCA_4A_0+LCA_5\sqrt{A_0^2-u^2(x,t)}\right)\frac{\partial^2 u(x,t)}{\partial t^2}\\&+\left(LA_0A_2u^2(x,t)+(LA_3+RCA_5)\sqrt{A_0^2-u^2(x,t)}+LA_1A_0+RCA_4A_0\right)\frac{\partial u(x,t)}{\partial t}\\&+RA_0A_2u^3(x,t)+RA_0A_1u(x,t)+RA_3u(x,t)\sqrt{A_0^2-u^2(x,t)}=0,\end{aligned} \tag{25}$$

$$\begin{aligned}&\left(-A_4A_0 - A_5\sqrt{A_0^2-u^2(x,t)}\right)\frac{\partial^2 i(x,t)}{\partial x^2}+\left(LCA_4A_0+LCA_5\sqrt{A_0^2-u^2(x,t)}\right)\frac{\partial^2 i(x,t)}{\partial t^2}\\&+\left(LA_0A_2u^2(x,t)+(LA_3+RCA_5)\sqrt{A_0^2-u^2(x,t)}+LA_1A_0+RCA_4A_0\right)\frac{\partial i(x,t)}{\partial t}\\&+RA_0A_2u^3(x,t)+RA_0A_1u(x,t)+RA_3u(x,t)\sqrt{A_0^2-u^2(x,t)}=0.\end{aligned} \tag{26}$$

Let us use Bogning-Djeumen Tchaho-Kofane method [16-21] to find out the exact solution of nonlinear partial differential equation (25) under the analytical shape written as follow:

$$u(x,t) = a\,\mathrm{sech}(kx - vt). \tag{27}$$

Where a, k and v are non-nil real numbers that will be determined in terms of modeled electrical line parameters described by nonlinear partial differential equation (25). Replacing $u(x,t)$ of (27) in equation (25), we obtain the following simplified equation when $a = A_0$:

$$\begin{aligned}&\left(RA_1+vLA_3-A_4k^2+vRCA_5+LCA_4v^2\right)\frac{1}{\cosh(kx-vt)}\\&+\left(-A_5k^2+LCA_5v^2+vRCA_4+vLA_1+RA_3\right)\frac{\sinh(kx-vt)}{\cosh^2(kx-vt)}\\&+\left(-2LCA_4v^2-vRCA_5+RA_2-vLA_3+2A_4k^2\right)\frac{1}{\cosh^3(kx-vt)}\\&+\left(2A_5k^2+A_0^2vLA_2-2LCA_5v^2\right)\frac{\sinh(kx-vt)}{\cosh^4(kx-vt)}=0.\end{aligned} \tag{28}$$

Equation (28) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permit us to obtain the set of four equations presented below:

$$\begin{cases} RA_1+vLA_3-A_4k^2+vRCA_5+LCA_4v^2=0,\\ -A_5k^2+LCA_5v^2+vRCA_4+vLA_1+RA_3=0,\\ -2LCA_4v^2-vRCA_5+RA_2-vLA_3+2A_4k^2=0,\\ 2A_5k^2+A_0^2vLA_2-2LCA_5v^2=0.\end{cases} \tag{29}$$

Solving the set of four equations (29), we obtain the solution and the conditions given in (30) of higher order nonlinear partial differential equation (25) which govern the dynamic of solitary wave of type Pulse in the modeled corresponding electrical line:

$$A_1 = 0\,,\; A_2 = 0\,,\; A_3 = \frac{-RCA_5}{L}\,,\; A_4 = RA_0^2\,,\; a = A_0\,,\; v = \frac{A_5}{LA_0^2}\,,\; k = \pm\frac{\sqrt{LCA_5^2}}{LA_0^2}\,,\; u(x,t) = A_0\,\mathrm{sech}\left(\pm\frac{\sqrt{LCA_5^2}}{LA_0^2}x - \frac{A_5}{LA_0^2}t\right). \quad (30)$$

In the same light it is equally verified that by respecting the given conditions in (30) the solution of the modified Telegraphist equation of current (26) is also written as follows:

$$i(x,t) = A_0\,\mathrm{sech}\left(\pm\frac{\sqrt{LCA_5^2}}{LA_0^2}x - \frac{A_5}{LA_0^2}t\right). \quad (31)$$

6. Conclusion

We have used an electrical line which permitted us to model modified Telegraphist equations. We have also defined and exploited nonlinear conductance per unit length to find out two different higher-order nonlinear partial differential equations which has enabled us to construct for each one solitary wave solutions; it is therefore necessary to point out that the results obtained will first of all enable us in the domain of physics and engineering of telecommunication, the manufacturing of two new transmission lines notably those of electrical lines presented in figure 1 where a nonlinear conductance per unit length of resistors of its networks varies for one in nonlinear manner defined in (16) and the other in nonlinear manner defined in (24). In addition, these results will permit an amelioration of the quality of signals that will be propagated in the new lines. In fact, those signals are solitary waves of type Kink obtained in (12), (23) and type Pulse obtained in (30); (31) which by their definition are displaced on long distance without changing their shape, velocity and resisting best on different dissipation factors contrary to sinusoidal waves obtained in (14) with non-modified Telegraphist equations whose amplitude decreases exponentially and loss a lot of energy. Finally, in a typical mathematical domain, the results obtained has permitted us to define in (17) and in (25) two new higher-order nonlinear partial differential equations which have respectively exact solitary wave solutions (22) and (30); this by increasing the field of mathematical knowledge. In order to bring new ideas on the stability of the obtained solitary waves; it is necessary for us to study next their modulational instability before carrying out a practical study where we will experiment the applicability and the perfection of those two new lines.

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