

Analysis of Radiation from Photosphere of the Sun

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Abstract

In the present paper an analysis of the continuous radiation from photosphere of the Sun at equilibrium temperature of it is carried out. The radiation is assumed due to quantum jumps of the electron, in a high density ionized semi-gaseous type material of the photosphere of the Sun, from its amplitude states considering types of damped frequency oscillations and eigen frequency damped oscillations analogous to that exist in a low density plasma with electron-molecule collisions (Nandedkar 2016) [8]. Damped and eigen frequency damped oscillations of the electron result due to density fluctuations of the charge-carriers with opposite signs in the body of the photosphere due to nuclear processes going deep in the core of the Sun. The density fluctuations alternatively builds up and withdraws a d.c. electric field in the body of the photosphere. The necessary damping for the type of damped oscillations mentioned, is provided by electron-ion collision type interaction at equilibrium temperature of the photosphere. The minimum value of the amplitude the electron takes which can be quantized in the field of the neighbouring ion and in the absence of the d.c. electric field when eigen-frequency damped oscillations result, govern the minimum value of the wavelength of radiation from the continuous spectrum of the photosphere. In general the wavelength is shown to be a function of the average density of the charge-carriers. Thus lower limit of the electron density corresponding to an intermediate value of the chromosphere, brings an upper limit for the radiation-wavelength of the radiation spectrum of the photosphere. The continuous radiation from photosphere of the Sun corresponds to electron density N_e variations in the range $4.774(2) \times 10^{29} \text{ m}^{-3} \geq N_e \geq 1 \times 10^{16} \text{ m}^{-3}$ which corresponds to wavelength λ of radiation in the range $0.2476(8) \times 10^{-6} \text{ m} \leq \lambda \leq 8.984(9) \times 10^{-3} \text{ m}$. However recently the solar spectrum is photographed up to a wavelength of about $0.2099 \times 10^{-6} \text{ m}$ which can be explained due to a mixture of a doubly charged ion and a singly charged ion in right proportion with one of electrons in each cases be considered, then the average distance of the electron from the ion can tend to lower down the minimum value of radiation wavelength.

Keywords

Solar-Spectrum, Photosphere, Ultra-Violet, Cut-Off, Plasma-Model, Quantum-Jumps

Received: September 16, 2015 / Accepted: December 18, 2015 / Published online: January 17, 2016

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1. Introduction

The Sun's energy arises from nuclear processes deep in its interior and flows outward in a relentless stream. The temperature of the center is over 10 million degrees Kelvin; at the surface direct measurements indicate a temperature of about 7000 °K, (where °K is Degree Kelvin). Thus Sun consists of a central hot core terminating in a surface having

temperature of about 7000 °K. The surface of this central hot core is termed as the photospheres' surface.

At the surface of the Sun, however a series of remarkable phenomenon occurs. The temperature continues to drop in the first few hundred kilometers outwards, but then rises through a series of steps to a plateau at a temperature over 1 million degrees Kelvin. The entire change takes place in a few thousand kilometers. The million degree outer envelope,

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called the corona, is a tenuous region stretching irregularly outward all the way to the earth, 149 million kilometers away – refer to Shklovsky 1963 [1].

The transition region between corona and photosphere appears during eclipse as a bright red ring around the Sun, and is therefore termed as chromosphere. When the chromosphere is examined closely, it is seen not to be a homogeneous layer but a rapidly changing filamentary structure. The term ‘burning prairie’ has been used to describe it. Numerous short-lived jets called ‘spicules’ are continuously shooting up to heights of several kilometers. The red color of the chromosphere is due to dominance of radiation in the red H_α line of Hydrogen – refer to Thomas and Athay 1962 [2].

The solar corona and chromosphere contain terrestrial ionized gases, where electron and ion densities are equal. In the solar corona the electron and ion densities are equal. In the solar corona the electron density varies from about 10^{10} m^{-3} to 10^{14} m^{-3} , whereas in the chromosphere the electron density can go as high as 10^{18} m^{-3} (Ginzburg 1964 [3]; Zirin 1966 [4]). The transition region between the photosphere, the chromosphere and the corona are not distinctly sharp.

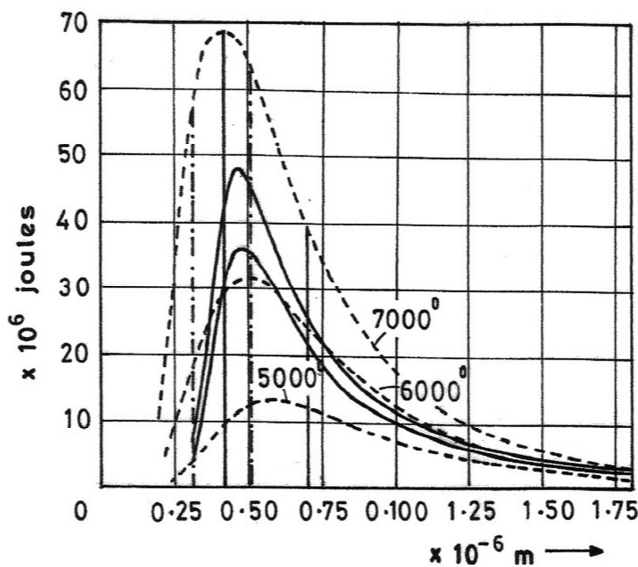


Fig. 1. Distribution of energy in the solar spectrum.

The photosphere may be defined as the outer surface of the Sun as seen in white light radiation. The photosphere represents a fairly uniform surface roughened by granulation. Its density is rather high, so that collisions help set-up equilibrium at a temperature of about 7000°K . Best of all, only the few hundred kilometers of the photosphere can be seen. Physically the photosphere includes the region of upper chromosphere.

Next question is that whether the photosphere of the Sun

radiates like a black Body – refer to De Jager 1965 [5]. For this the distribution of emittance in the spectrum of the Sun has to be studied. This has been, done by various workers like Plaskett and Wilsing etc. The results are discussed by Brill and Bernheimer exhaustively. For these results treatises in Astrophysics can be consulted. The results are illustrated in the books like those of Shah and Srivastava (Shah and Srivastava 1958 [6] & Kraus 1966 [7]) and are summarized in the following paragraph.

The full curves (Fig. 1) represent the observed values of emission for the different wavelengths, the upper curve for radiation from the central part of the disc, the lower for the integrated radiation from the disc. The broken curves represent blackbody radiation for the temperatures 7000°K , 6000°K and 5000°K . It is seen that the energy curve deviates considerably from that of a black body. Taking the curve for central radiation, it is found that from the long wavelength side to about the wavelength $\lambda = 0.78 \times 10^{-6} \text{ m}$, the curve can be made to coincide with the curve of 6000°K , but between this wavelength, and the wavelength of maximum emission ($\lambda = 0.4680 \times 10^{-6} \text{ m}$), the emission lies between 6000°K and 7000°K . From the wavelength of maximum emission, the fall is too steep, and the curve can under no circumstances be made to agree with that for any black body radiation at any temperature. This rapid fall on the violet side is certainly to be ascribed to greater diminution of ultraviolet light by scattering in the atmosphere of the sun, as well as by terrestrial atmosphere. Below the wavelength $\lambda = 0.295 \times 10^{-6} \text{ m}$ the solar spectrum entirely disappears though even for a temperature of 5000°K , it ought to be continued to the wavelength $\lambda = 0.2 \times 10^{-6} \text{ m}$. This total absorption has been traced to a layer of ozone which is formed in the upper layers of our atmosphere. Recently by using V_2 –rockets spectrographs have been carried up to heights of 110 kilometers, i.e. above the ozone layer, and the solar spectrum photographed to wavelength $\lambda = 0.2099 \times 10^{-6} \text{ m}$. It is therefore fairly certain that the radiation reaching us from the Sun differs widely in quality from black body. The temperature of the Sun, which is a star of G 0 – class is given values ranging from 5980°K to 6200°K by various workers. But the actual temperature of the photosphere is probably in the neighborhood of 7000°K . Plaskett investigated the intensity distribution of the light emitted from the center of the disc and finds that between wavelength $\lambda = 0.38 \times 10^{-6} \text{ m}$ to wavelength $\lambda = 0.76 \times 10^{-6} \text{ m}$, the radiation is identical to that given by a black body at 6700°K . Wilsing obtains a temperature of 6740°K . But the fact that a certain part of the emission curve can be made to coincide with the black body can hardly justify the

conclusion that the Sun emits like a black body. However, although the loss for each wavelength due to passage through the Sun's atmosphere is not definitely determined, even then the emission could not be black because the surface is open and contains large proportion of electrons.

In this analysis of photospheres' radiation from the Sun, it is assumed that the photosphere is made up of high density terrestrial ionized semi-gaseous type material where electron and ion densities are equal. The charged particles of the ionized semi-gaseous type material are in thermal equilibrium at temperature T ($=7000^\circ\text{K}$) of the photosphere. Electron-ion collisions in the photosphere helps establishing the thermal equilibrium.

The nuclear processes going deep inside the core of the Sun builds up a difference of charge densities of the charge carriers with opposite signs in the body of the photosphere. This in turn develops a junctional potential difference across which the difference of charge densities exists. And hence it gives rise to a d.c. potential difference across which the difference of charge exists. And hence, this gives rise to a d.c. electric field E_{dc} in the body of the photosphere. When the difference of the charge densities of the charge carriers with opposite signs vanishes in the course of time, then E_{dc} also vanishes. Thus fluctuations in the density of charge carriers with opposite signs, alternatively builds up and withdraws E_{dc} in the body of the photosphere.

Then the electron in the vicinity of an ion alternatively takes the positions R_{oc} and R_o in the presence and in the absence of E_{dc} (refer to Nandedkar 2016 [8]). Quantization of the electron in the vicinity of the ion gives rise to absorption and emission of radiation due to electronic jumps from state R_{oc} to state R_o and vice versa at equilibrium temperature T of the photosphere. The frequency of radiation thus obtained is a function of the electron density in the photosphere. In the distribution of emittance of radiation as a function of the frequency at temperature T , upper limit of the frequency can be obtained due to the minimum number of the quantum state of the electron in the vicinity of the ion available. Whereas the lower limit of the frequency is governed by the minimum value of the electron density available near photosphere which is arbitrarily chosen to be $1 \times 10^{16} \text{ m}^{-3}$ – a value intermediate to the chromosphere as the transition between photosphere and corona is not very sharp.

Further the radiation from the photosphere thus emitted has to pass through dielectric medium provided by the material of photosphere in general. Moreover, the radiation also passes through the solar atmosphere. This again introduces its own absorption. The total absorption can be assumed to produce the full line curve (Fig. 1), for the radiation curve for $T = 7000^\circ\text{K}$ with lower cut-off wavelength corresponding to

the state of minimum quantum number for the electron available in the vicinity of ion in the photosphere. Everywhere in this research-paper, wavelength of radiation emitted from photosphere of the Sun corresponds to the free space wavelength, considering absorption and emission of radiation due to electronic jumps from state R_{oc} to state R_o and vice versa at equilibrium temperature T of the photosphere.

2. Electron Charge Density Fluctuations in the Photosphere

In the presence of a d.c. electric field E_{dc} (which is randomly oriented) developed under favorable conditions due to charge density fluctuations in the photosphere as mentioned in Secn. 1, the differential equation of motion of an electron in the vicinity of an ion can be written down (for instance refer to Nandedkar 2016 [8], [9] and [10]) as follows:

$$m_e \frac{\partial^2 r}{\partial t^2} + f_{1ei} \frac{\partial r}{\partial t} + f_{2ei} r = e E_{dc}, \quad (1)$$

or,

$$\frac{\partial^2 r}{\partial t^2} + \frac{f_{1ei}}{m_e} \frac{\partial r}{\partial t} + \frac{f_{2ei}}{m_e} r = \frac{e}{m_e} E_{dc}, \quad (2)$$

where m_e is the mass of an electron of charge e . $f_{1ei} \frac{\partial r}{\partial t}$ is the damping force due to electron-ion scattering, $f_{2ei} r$ is an electric restoring force due to quasi-bound nature of the electron with respect to the ion. r gives the displacement of the electron measured with respect to the ion. $\frac{\partial r}{\partial t}$ and $\frac{\partial^2 r}{\partial t^2}$ are the velocity and acceleration of the electron at time t under consideration. Further,

$$f_{1ei}/m_e = v_{ei}, \quad (3)$$

and

$$f_{2ei}/m_e = \omega_o^2, \quad (4)$$

here v_{ei} is the (actual) collision frequency of electron-ion scattering at average temperature T of the Photosphere for quasi-free: quasi bound electron model of ionized semi-gaseous type material of photosphere in presence of E_{dc} , and ω_o is the angular frequency of electronic damped oscillations such that,

$$\omega_o^2 \gg (v_{ei}/2)^2, \quad (5)$$

is the assumption.

Using eqns. (3) and (4), eqn. (2) gives,

$$\frac{\partial^2 r}{\partial t^2} + v_{ei} \frac{\partial r}{\partial t} + \omega_o^2 r = \frac{e}{m_e} E_{dc} \quad (6)$$

Here E_{dc} is considered a randomly oriented one.

Steady state solution of eqn. (6), in the presence of electron-ion scattering phenomenon denoted by v_{ei} , gives,

$$r \rightarrow R_{oc} = \frac{e}{m_e} \frac{E_{dc}}{\omega_o^2}, \quad (7)$$

where R_{oc} is the final position of the electron from the ion in the presence of E_{dc} .

When E_{dc} ceases under favourable conditions due to charge density fluctuations in the photosphere as mentioned in Secn. 1, then the differential equation of motion of the electron in the vicinity of the ion can be written down (for instance refer to Nandedkar 2016) [9] as follows:

$$\frac{\partial^2 r}{\partial t^2} + v_{ei} \frac{\partial r}{\partial t} + \omega_j^2 r = \frac{e}{m_e} \langle E_i \rangle, \quad (8)$$

where $\langle E_i \rangle$ is the average value of electric field due to the ion at distance R_o given by,

$$R_o = \sqrt[3]{(3/4\pi) N_e^{-1/3}}, \quad (9)$$

where R_o denotes average separation of an electron from the neighbouring ion. Here $\langle E_i \rangle$ is reached in the limit when E_{dc} ceases to act. ω_j in eqn. (8), gives eigen angular frequency of the damped oscillations which are not sustained (for instance refer to Nandedkar 2016) [8]. The value of r considered in eqn. (8) is in the vicinity of R_o where $\langle E_i \rangle$ can be treated as constant.

Steady state solution of eqn. (8), in the presence of electron-ion interaction denoted by v_{ei} , is given by,

$$r \rightarrow R_o = \frac{e}{m_e} \frac{\langle E_i \rangle}{\omega_j^2}. \quad (10)$$

In eqn. (7) and (10), R_{oc} and R_o denote the position of the electron with respect to ion, where electronic damped oscillations asymptotically exit, provided damping is low. R_{oc} and R_o denote the steady state asymptotic positions of the electron with respect to the neighbouring ion in respective cases.

Relationship between R_o and R_{oc} in the presence of $\langle E_i \rangle$ and E_{dc} respectively can be obtained as follows:

In the vicinity of the ion, the value of electric field due to the ion varies inversely as the square of the distance in accordance with Coulomb's law and the ultimate value of average electric field $\langle E_i \rangle$ is reached when R_o is attained by the electron in presence of ω_j . Whereas in the case of d.c. electric field E_{dc} which remains everywhere the same in the vicinity of the ion, and R_{oc} is the value attained by the electron in the presence of ω_o .

If the conditions in the vicinity of the ion with $\langle E_i \rangle$ and with

E_{dc} are different, then the displacement of the electron in the presence of E_{dc} is constrained in comparison to that in the presence of $\langle E_i \rangle$. Let the constrained displacement of the electron with respect to the ion be denoted by r_c in the presence of d.c. electric field E_{dc} . When R_o and R_{oc} in the presence of $\langle E_i \rangle$ and E_{dc} respectively, are to be compared, then the non-constrained displacement of the electron in the presence of E_{dc} be denoted by r_{nc} with respect to the ion.

In the hypothetical case of E_{dc} and $\langle E_i \rangle$ both tending to zero, the value of r_{nc} and r_c would also tend to zero.

Any change of the value of $(R_{oc} - r_c)$ with respect to r_{nc} is considered to be proportional to $(R_{oc} - r_c)$ itself, i.e.,

$$\frac{\partial}{\partial r_{nc}} (R_{oc} - r_c) \propto - (R_{oc} - r_c). \quad (11)$$

The negative sign outside the bracket on the left hand side of eqn. (11) denotes that $(R_{oc} - r_c)$ decreases as r_{nc} is increased. Eqn. (11) can be rewritten as follows:

$$\frac{\partial}{\partial r_{nc}} (R_{oc} - r_c) = - \frac{(R_{oc} - r_c)}{R_o}, \quad (12)$$

where $1/R_o$ is the constant of proportionality. R_o is the value with which R_{oc} is to be compared. R_o is given by eqn. (9).

Integration of eqn. (12) gives that,

$$(R_{oc} - r_c) = A_c \exp \left(- \frac{r_{nc}}{R_o} \right), \quad (13)$$

where A_c is a constant of integration. As mentioned before in the limit when,

$$r_{nc} \rightarrow 0, \quad (14a)$$

then,

$$r_c \rightarrow 0. \quad (14b)$$

Thus, using eqns. (14a) and (14b), eqn. (13) gives:

$$A_c = R_{oc}. \quad (15)$$

Substituting the value of A_c from eqn. (15) in eqn. (13), eqn. (13) gives that,

$$r_c = R_{oc} [1 - \exp \left(- \frac{r_{nc}}{R_o} \right)]. \quad (16)$$

The value of R_{oc} in terms of R_o is obtained when, r_c and r_{nc} simultaneously tend to R_o , then eqn. (16) gives,

$$R_{oc} = \frac{R_o}{1 - \exp(-1)}. \quad (17)$$

From eqn. (17), the value of R_o / R_{oc} is given by the following relationship, viz.,

$$\frac{R_o}{R_{oc}} = 1 - \exp(-1) = 0.6321(2), \quad (18)$$

and eqn. (18) indicates that when R_o and R_{oc} in the presence of $\langle E_i \rangle$ and E_{dc} respectively are compared then R_o is 63.21(2)% smaller than R_{oc} .

The derivation assumes that the effects of the charges outside the sphere of radius R_o or R_{oc} under consideration, have no interference.

Here R_{oc} and R_o are the alternate positions with respect to the neighbouring ion, the electron takes when E_{dc} is developed and vanished successively during the process of the appearance of the gradient of the charge densities of the carriers with opposite signs and disappearance of that gradient respectively. Thus, due to the process of fluctuations in charge densities of charge-carriers with opposite signs in the body of the photosphere, the electron with respect to the neighbouring ion takes alternatively the positions of R_{oc} and R_o .

3. Quantization of the Electron in the Vicinity of the Ion

The ion is considered to be much heavier as compared to the electron. The electron in the vicinity of the ion is in the central force field of the ion. In the presence of the central force field, the electron acquires a potential energy $V_e(r)$, which depends on the radial distance r of the electron from the ion. If E_e be the total energy of the quasi-bound electron with respect to the ion, then $[E_e - V_e(r)]$ gives the kinetic energy of the electron under consideration. If p_e is the momentum of the electron corresponding to the kinetic energy $[E_e - V_e(r)]$, then,

$$\frac{p_e^2}{2m_e} = [E_e - V_e(r)], \quad (19)$$

The time independent (Schrödinger) wave equation of the electron of momentum p_e associated with a matter wave of (D' Broglie) wavelength λ_{me} of value,

$$\lambda_{me} = h/p_e, \quad (20)$$

where h is Planck constant, is given by (for instance for analogy refer to Nandedkar and Bhagavat, 1970 [11]) – (where “Corrigendum/Corrigenda” for [11] is given by Nandedkar and Bhagavat 1970 [12]),

$$\nabla^2 \Psi_{em} + \frac{4\pi^2}{h^2} p_e^2 \Psi_{em} = 0, \quad (21)$$

where ∇^2 is Laplacian operator in a system of a three dimensional co-ordinates with the stationary ion at the origin. Ψ_{em} gives the amplitude of the matter wave and is the wave function of the electron. Using eqn. (19), eqn. (21) gives,

$$\nabla^2 \Psi_{em} + (2m_e/\hbar^2) [E_e - V_e(r)] \Psi_{em} = 0, \quad (22)$$

where,

$$\hbar = h/2\pi. \quad (23)$$

In the case of spherical symmetry with respect to the ion assumed in this analysis, Ψ_{em} only depends on r . Thus expanding ∇^2 in eqn. (22) in terms of r , eqn. (22) gives,

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_e}{dr} \right) + (2m_e/\hbar^2) [E_e - V_e(r)] R_e = 0, \quad (24)$$

where R_e is the radial part of Ψ_{em} . R_e is the radial eigen wave function of the electron with eigen energy as E_e .

Coulomb force acting on the electron due to the ion of charge e at distance r is given by,

$$F_c = \left(\frac{e^2}{4\pi\epsilon_0 r^2} \right). \quad (25)$$

Thus the potential energy $V_e(r)$ of the electron is given by,

$$V_e(r) = \int_{\infty}^r F_c dr = - \frac{e^2}{4\pi\epsilon_0 r}. \quad (26)$$

Using eqn. (26), eqn. (24) gives,

$$\frac{d^2 R_e}{dr^2} + \frac{2}{r} \frac{dR_e}{dr} + [(2m_e/\hbar^2) E_e + (2m_e/\hbar^2) \left(\frac{e^2}{4\pi\epsilon_0 r} \right)] R_e = 0. \quad (27)$$

Now consider first discrete state where E_e is negative for the quasi-bound electron with respect to the ion in the ionized semi-gaseous type material of the photosphere. Further introduce a new eigen value parameter n by means of the following relationship, viz.,

$$E_e = - \frac{m_e e^4}{32 \pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2}. \quad (28)$$

Using eqn. (28), eqn. (27) gives,

$$\frac{d^2 R_e}{dr^2} + \frac{2}{r} \frac{dR_e}{dr} + \left[- \frac{m_e^2 e^4}{16 \pi^2 \epsilon_0^2 \hbar^4} \frac{1}{n^2} + \frac{m_e e^2}{2\pi\epsilon_0 \hbar^2} \frac{1}{r} \right] R_e = 0. \quad (29)$$

Let then introduce a new parameter x , by substituting,

$$x = \frac{2}{n A_0} r, \quad (30)$$

where,

$$A_0 = \frac{4\pi\hbar^2\epsilon_0}{m_e e^2}. \quad (31)$$

Thus,

$$r = n \left(\frac{2\pi\hbar^2\epsilon_0}{m_e e^2} \right) x. \quad (32)$$

Using eqn. (32), eqn. (29) gives,

$$\frac{d^2 X_e(x)}{dx^2} + \frac{2}{x} \frac{dX_e(x)}{dx} + \left(-\frac{1}{4} + \frac{n}{x} \right) X_e(x) = 0, \quad (33)$$

where,

$$R_e(r) = X_e(x). \quad (34)$$

Here the physically significant limits of x correspond to those

of r , viz., zero to infinity.

An indication of the solution of eqn. (33) can be obtained by taking x very large, so that the corresponding asymptotic equation is,

$$\frac{d^2 X_e(x)}{dx^2} - \frac{1}{4} X_e(x) = 0, \quad (35)$$

of which the solutions are,

$$X_e(x) = \exp(x/2), \quad (36a)$$

and,

$$X_e(x) = \exp(-x/2). \quad (36b)$$

Since x may vary from 0 to ∞ , the former of these solutions [(eqn. (36a))] will increase as x increases, this will lead to a non-acceptable wave function. On the other hand, the second solution [(eqn. (36b))] will be satisfactory, since it decreases to zero as x (– and hence r), the distance of the electron from the positive ion-increases to ∞ .

Bearing in mind the asymptotic solution, a possible solution to eqn. (33) is,

$$X_e(x) = \exp\left(-\frac{x}{2}\right) v_E(x), \quad (37)$$

where $v_E(x)$ is another function of the variable x . Thus,

$$R_e(r) = X_e(x) = \left[\exp\left(-\frac{x}{2}\right)\right] v_E(x). \quad (38)$$

Using eqn. (38), eqn. (33) gives,

$$x \frac{d^2 v_E(x)}{dx^2} + (2-x) \frac{dv_E(x)}{dx} + (n-1) v_E(x) = 0. \quad (39)$$

Now consider the following differential equation:

$$x \frac{d^2 u_E(x)}{dx^2} + (1-x) \frac{du_E(x)}{dx} + n u_E(x) = 0. \quad (40)$$

Differentiating eqn. (40) with respect to x gives,

$$x \frac{d^2 v_E(x)}{dx^2} + (2-x) \frac{dv_E(x)}{dx} + (n-1) v_E(x) = 0, \quad (41)$$

where,

$$v_E(x) = \frac{d}{dx} [u_E(x)]. \quad (42)$$

Eqn. (41) is the same as eqn. (39).

In order to solve eqn. (40), substitute the following power series expression for $u_E(x)$, viz.,

$$u_E(x) = \sum_{\gamma_k=0}^{\infty} a_{\gamma_k} x^{\alpha_k + \gamma_k}. \quad (43)$$

Then eqn. (40) gives,

$$\sum_{\gamma_k=0}^{\infty} a_{\gamma_k} (\alpha_k + \gamma_k)^2 x^{\alpha_k + \gamma_k - 1} - \sum_{\gamma_k=0}^{\infty} a_{\gamma_k} (\alpha_k +$$

$$\gamma_k - n) x^{\alpha_k + \gamma_k} = 0. \quad (44)$$

Equating the lowest power of x to zero, eqn. (44) gives,

$$a_0 \alpha_k^2 = 0. \quad (45)$$

Since $a_0 \neq 0$ for a non-trivial solution, eqn. (45) gives,

$$\alpha_k = 0. \quad (46)$$

Substituting $\alpha_k = 0$ in eqn. (44), eqn. (44) gives,

$$\sum_{\gamma_k=0}^{\infty} a_{\gamma_k} (\gamma_k)^2 x^{\gamma_k - 1} - \sum_{\gamma_k=0}^{\infty} a_{\gamma_k} (\gamma_k - n) x^{\gamma_k} = 0. \quad (47)$$

Equating to zero the coefficient of j -th term of x in eqn. (47), it is found that,

$$a_{j+1} (j+1)^2 = a_j (j-n).$$

So that, there results the following recurrence relation, viz.,

$$a_{j+1} = \frac{(j-n)}{(j+1)^2} a_j. \quad (48)$$

Using eqn. (48), eqn. (43) where $\alpha_k = 0$, gives,

$$y = a_0 \left[1 - nx + \frac{n(n-1)}{(2!)^2} x^2 - \dots + (-1)^q \frac{n(n-1)\dots(n-q+1)}{(q!)^2} x^q + \dots \right], \quad (49)$$

where,

$$y = u_E(x). \quad (50)$$

The expression given by eqn. (49) becomes a polynomial when n is a positive integer. Choosing,

$$a_0 = (-1)^n n! \quad (51)$$

eqn. (49) gives,

$$y = L_n(x) = (-1)^n \left[x^n - \frac{n^2}{1!} x^{n-1} + \frac{n^2(n-1)^2}{2!} x^{n-2} + \dots + (-1)^n n! \right]. \quad (52)$$

For the positive integer n , $L_n(x)$ of eqn. (52) denotes a finite polynomial of degree n . And hence eqn. (42) represents integrable solution of eqn. (41) or eqn. (39) to the wave function of electron, because n is a positive integer.

Coming to eqn. (42), v_E is given by,

$$v_E \equiv \frac{d}{dx} y = \frac{d}{dx} L_n(x) = L'_n(x), \quad (53)$$

using eqns. (50) and (52). Here L'_n is a polynomial of degree $(n-1)$. Returning to eqn. (52), eqn. (52) can be alternatively represented as follows:

Consider,

$$u_{E_n} = \frac{1}{2\pi i} \oint \frac{z^{-n-1}}{(1-z)} \exp\left(\frac{-xz}{1-z}\right) dz, \quad (54)$$

where counter is taken to include the origin. Differentiation with respect to x can be performed under the integral sign.

Hence,

$$\frac{du_{E_n}}{dx} = -\frac{1}{2\pi i} \oint \frac{z^{-n-1}}{(1-z)^2} \exp\left(\frac{-xz}{1-z}\right) dz,$$

and

$$\frac{d^2 u_{E_n}}{dx^2} = \frac{1}{2\pi i} \oint \frac{z^{-n+1}}{(1-z)^3} \exp\left(\frac{-xz}{1-z}\right) dz.$$

On substituting in the left hand side of eqn. (40), it is found that,

$$\frac{1}{2\pi i} \oint \left[\frac{xz^2}{(1-z)^2} - \frac{(1-x)z}{(1-z)} + n \right] \frac{z^{-n-1}}{(1-z)} \exp\left(\frac{-xz}{1-z}\right) dz.$$

But this reduces to,

$$-\frac{1}{2\pi i} \oint \frac{d}{dz} \left[\frac{z^{-n}}{1-z} \exp\left(\frac{-xz}{1-z}\right) \right] dz,$$

an expression which vanishes because the quantity in the brackets takes on the same value at the initial and final point of the contour. Hence eqn. (54) is a solution of eqn. (40). Moreover it is a polynomial as the analysis of the theorem of residues shows its relationship to $L_n(x)$ can be established by computing both $u_{E_n}(x)$ and $L_n(x)$ for a particular value of x , say zero. From eqn. (52),

$$L_n(0) = n!$$

From eqn. (54),

$$\begin{aligned} u_{E_n}(0) &= \frac{1}{2\pi i} \oint \frac{z^{-n-1}}{1-z} dz \\ &= \frac{1}{2\pi i} \oint z^{-n-1} (1+z+z^2+\dots) dz = 1. \end{aligned}$$

Therefore

$$L_n = n! u_{E_n}.$$

Using the theorem of residues, it is found, since

$$L_n = \frac{n!}{2\pi i} \oint \frac{z^{-n-1}}{1-z} \exp\left(-\frac{xz}{1-z}\right) dz, \quad (55)$$

then

$$(-z)^{-1} \exp\left(\frac{-xz}{1-z}\right) = \sum_{n=0}^{\infty} u_{E_n} z^n = \sum_{n=0}^{\infty} \frac{L_n(x)}{n!} z^n. \quad (56)$$

On differentiating eqn. (56) with respect to x is seen that,

$$(-1)(1-z)^{-1} \left(\frac{z}{1-z}\right) \exp\left(\frac{-xz}{1-z}\right) = \sum_{\lambda_p=1}^{\infty} \frac{L'_{\lambda_p}(x)}{\lambda_p!} z^{\lambda_p}. \quad (57)$$

Coming to eqn. (53) the polynomial $L'_n(x)$, or more correctly the polynomial multiplied by a constant G_e will be a solution of eqn. (39) or (40), so it is possible to write,

$$v_E(x) = G_e L'_n(x). \quad (58)$$

The complete expression for the function $R_e(r)$ is thus seen from eqn. (38) to be given by,

$$R_e(r) = X_e(x) = G_e \exp\left(-\frac{x}{2}\right) L'_n(x). \quad (59)$$

Eqn. (59) denotes an acceptable solution since as already seen, has a finite number of terms only. The constant G_e in eqn. (59) can be made equal to the normalization factor by the following procedure:

The normalizing condition for the physically significant interval of zero to infinity is,

$$\int_0^{\infty} R_e(r) R_e^*(r) r^2 dr = 1. \quad (60)$$

The factor r^2 being necessary to convert the length dr into an element of volume. If $R_e(r)$ and the complex conjugate of $R_e(r)$ i.e. $R_e^*(r)$ be the same, then eqn. (60) gives,

$$\int_0^{\infty} [R_e(r)]^2 r^2 dr = 1. \quad (61)$$

Using eqns. (30) and (59), eqn. (61) gives,

$$\left(\frac{nA_0}{2}\right)^3 G_e^2 \int_0^{\infty} \exp(-x) [L'_n(x)]^2 x^2 dx = 1. \quad (62)$$

Now consider the integral,

$$I_n = \int_0^{\infty} [\{\exp(-x)\} \{L'_n(x)\}]^2 x^2 dx. \quad (63)$$

From eqn. (57), there results the following identity:

$$\sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \frac{L'_n(x) L'_{n'}(x)}{n! n'!} z^n z^{n'} = \frac{\exp\left\{-\frac{xz}{1-z} - \frac{xZ}{1-Z}\right\}}{(1-z)^2 (1-Z)^2} zZ,$$

from which it follows that,

$$\begin{aligned} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \frac{z^n z^{n'}}{n! n'!} \int_0^{\infty} \{\exp(-x)\} x^2 L'_n(x) L'_{n'}(x) dx \\ = \frac{(zZ)}{(1-z)^2 (1-Z)^2} \int_0^{\infty} x^2 \exp\left\{-x - \frac{xZ}{1-Z}\right\} dx. \end{aligned}$$

Solving the integration, there results the following expression for Left Hand Side of above equation, viz.,

$$\frac{(2)! (zZ)(1-z)(1-Z)}{(1-zZ)^3},$$

and by means of binomial expression, this function can be expanded to,

$$(1-z-Z-zZ) \sum_{r'=0}^{\infty} \frac{(r'+2)!}{r'!} (zZ)^{r'+1}.$$

Now the coefficient of $(zZ)^n$ in this expansion is,

$$\frac{(2n)(n!)}{(n-1)!}.$$

But this is equal to $I_n/(n!)^2$. Hence,

$$I_n = \int_0^\infty \exp(-x) [L'_n(x)]^2 x^2 dx = \frac{(2n)(n!)^3}{(n-1)!} \quad (64)$$

Using eqn. (64), eqn. (62) gives,

$$G_e = \left[\left(\frac{2}{nA_0} \right)^3 \frac{(n-1)!}{(2n)(n!)^3} \right]^{1/2}. \quad (65)$$

Using eqns. (65) and (30), eqn. (59) gives,

$$R_{en}(r) = \left[\left(\frac{2}{nA_0} \right)^3 \frac{(n-1)!}{(2n)(n!)^3} \right]^{1/2} \exp\left(-\frac{r}{nA_0}\right) L'_n\left(\frac{2r}{nA_0}\right), \quad (66)$$

where $R_e(r)$ is replaced by $R_{en}(r)$ which is the acceptable eigen function of the electron in n -th quantum state in the vicinity of the ion in the ionized semi-gaseous type material of the photosphere.

The corresponding value of eigen energy E_{en} of the electron in n -th quantum state is given from eqn. (28), where E_e is replaced by E_{en} , as follows:

$$E_{en} = -\frac{m_e e^4}{32 \pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2} = -\frac{3e^2}{16\pi\epsilon_0} \frac{1}{\left(\frac{3}{2} A_0 n^2\right)}. \quad (67)$$

The present analysis shows that n has to be a positive integer if $L'_n\left(\frac{2r}{nA_0}\right)$ function is a finite polynomial of degree $(n-1)$, thus making as an acceptable wave function. The minimum value of n , this shows has to be unity. Thus, in eqn. (66) or (67),

$$n = 1, 2, 3, \dots, \quad (68)$$

which gives quantum number of the electron in the vicinity of the ion. The condition $n=1$ gives ground state of the electron in the system under consideration.

4. Absorption and Emission of Radiation by the Electron

In order to study absorption and emission of radiation due to quantum jumps of the electron from distance R_{oc} to R_o with respect to the ion, and vice versa, the knowledge of the average distance of the electron from the ion in terms of the quantum number n is required. This can be obtained as follows:

Coming to eqn. (62) the distribution function $P_e(x)$ to find electron at distance x can be given by,

$$P_e(x) = D_e x^2 \{\exp(-x)\} \{L'_n(x)\}^2, \quad (69)$$

where

$$D_e = \left(\frac{nA_0}{2}\right)^3 G_e^2, \quad (70)$$

here in eqn. (69), x is given by eqn. (30).

Mean value of r [- which is distance of the electron from the ion], for this distribution denoted by eqn. (69), is given by,

$$\bar{r} = \frac{\int_0^\infty P_e(x) r dr}{\int_0^\infty P_e(x) dx}. \quad (71)$$

Using eqn. (30), eqn. (71) gives,

$$\bar{r} = \frac{\left(\frac{nA_0}{2}\right) \int_0^\infty P_e(x) x dx}{\int_0^\infty P_e(x) dx}. \quad (72)$$

Substituting the value of $P_e(x)$ from eqn. (69) in eqn. (72), eqn. (72) gives,

$$\bar{r} = \left(\frac{nA_0}{2}\right) \frac{\int_0^\infty x^3 \{\exp(-x)\} \{L'_n(x)\}^2 dx}{\int_0^\infty x^2 \{\exp(-x)\} \{L'_n(x)\}^2 dx}. \quad (73)$$

In order to solve the integrals involved in eqn. (73), consider the following general integral, viz.,

$$I_{n,m} = \int_0^\infty \{\exp(-x)\} L'_n(x) L'_m(x) x^{q'} dx.$$

here q' is another integer which is either equal to Z or z . Furthermore the interest is mainly confined to $I_{n,n}$.

Multiply eqn. (57) in which z_1 is written for z , by a similar one in which z is replaced by z_2 , then there results the following expression, viz.,

$$\begin{aligned} & \sum_{\lambda_p, \mu_p=1}^\infty \frac{z_1^{\lambda_p} z_2^{\mu_p}}{\lambda_p! \mu_p!} L'_{\lambda_p}(x) L'_{\mu_p}(x) \\ &= (z_1 z_2) (1 - z_1)^{-2} (1 - z_2)^{-2} \exp\left(\frac{-x z_1}{1 - z_1} + \frac{-x z_2}{1 - z_2}\right). \end{aligned}$$

Multiply each side of this equation by $\{\exp(-x)\} x^{q'}$ and then integrate with respect to x . In view of the definition of $I_{n,n}$, the result can be written down as follows:

$$\sum_{\lambda_p, \mu_p=1}^\infty \frac{z_1^{\lambda_p} z_2^{\mu_p}}{\lambda_p! \mu_p!} I_{\lambda_p, \mu_p} = (z_1 z_2) [(1 - z_1)(1 - z_2)]^{-2} \int_0^\infty x^{q'} \exp\left[x\left(1 - \frac{1}{1 - z_1} - \frac{1}{1 - z_2}\right)\right] dx. \quad (74)$$

Now,

$$\int_0^\infty [\exp(-\beta_q x)] x^{r_q} dx = \beta_q^{-r_q-1} \int_0^\infty [\exp(-t_q)] t_q^{r_q} dt_q,$$

or,

$$\int_0^\infty [\exp(-\beta_q x)] x^{r_q} dx = \beta_q^{-r_q-1} \Gamma(r_q + 1) \\ = \beta_q^{-r_q-1} (r_q!),$$

due to Γ -integrals.

If,

$$\beta_q = (1 - z_1)^{-1} + (1 - z_2)^{-1} - 1 \\ = (1 - z_1 z_2)(1 - z_1)^{-1}(1 - z_2)^{-1},$$

be substituted, then there results the following expression, viz.,

$$\sum_{\lambda_p, \mu_p=1}^\infty \frac{z_1^{\lambda_p} z_2^{\mu_p}}{\lambda_p! \mu_p!} I_{\lambda_p, \mu_p} = \frac{(z_1 z_2)(1 - z_1)^{q'-1}(1 - z_2)^{q'-1}}{(1 - z_1 z_2)^{q'+1}} (q'!). \quad (75)$$

Expansion of the denominator on the right by the binomial theorem, gives,

$$(1 - z_1 z_2)^{-q'-1} = \sum_{\lambda_p} \frac{(q' + \lambda_p)!}{q'! \lambda_p!} (z_1 z_2)^{\lambda_p},$$

and using this, Right Hand Side of Eqn. (75), gives,

$$(1 - z_1)^{q'-1}(1 - z_2)^{q'-1} \sum_{\lambda_p} \frac{(q' + \lambda_p)!}{\lambda_p!} (z_1 z_2)^{\lambda_p+1}. \quad (76)$$

Thus in the view of eqn. (75), $I_{n,m}$ is simply $(n!)^2$ times the coefficient of $(z_1 z_2)^n$ of this expression.

Case a:

When $q' = 2$, Eqn. (76) becomes,

$$\sum_{\lambda_p} \left(\frac{(\lambda_p + 2)!}{\lambda_p!} \right) \left[(z_1 z_2)^{\lambda_p+1} - z_1^{\lambda_p+2} z_2^{\lambda_p+1} - z_1^{\lambda_p+1} z_2^{\lambda_p+2} \right. \\ \left. + (z_1 z_2)^{\lambda_p+2} \right].$$

The second and third terms in the bracket in which z_1 and z_2 appear with different exponents cannot contribute to $I_{n,m}$; the first term contributes when $\lambda_p = n - 1$, and the last one when $\lambda_p = n - 2$. Hence,

$$I_{n,n} = (n!)^2 \left[\frac{(n+1)!}{(n-1)!} + \frac{n!}{(n-2)!} \right],$$

and,

$$[I_{n,n}]_{q'=2} = \frac{(n!)^3}{(n-1)!} (2n) = \int_0^\infty [\exp(-x)] [L'_n(x)]^2 x^2 dx \quad (77)$$

Case b:

When $q' = 3$, the significant parts of eqn. (76) are,

$$\sum_{\lambda_p} \frac{(\lambda_p + 3)!}{\lambda_p!} [(z_1 z_2)^{\lambda_p+1} + 4(z_1 z_2)^{\lambda_p+2} + (z_1 z_2)^{\lambda_p+3}],$$

where terms with different exponents of z_1 and z_2 are omitted. Consequently,

$$[I_{n,n}] = (n!)^2 \left[\frac{(n+2)!}{(n-1)!} + \frac{4(n+1)!}{(n-2)!} + \frac{n!}{(n-3)!} \right],$$

and,

$$[I_{n,n}]_{q'=3} = \frac{(n!)^3}{(n-1)!} (6n^2) = \int_0^\infty [\exp(-x)] [L'_n(x)]^2 x^3 dx. \quad (78)$$

Using eqns. (78) and (77), the value of $[I_{n,n}]_{q'=3} / [I_{n,n}]_{q'=2}$ is given by,

$$\frac{[I_{n,n}]_{q'=3}}{[I_{n,n}]_{q'=2}} = \frac{\int_0^\infty [\exp(-x)] [L'_n(x)]^2 x^3 dx}{\int_0^\infty [\exp(-x)] [L'_n(x)]^2 x^2 dx} = 3n. \quad (79)$$

Using eqn. (79), eqn. (73) gives,

$$\bar{r} = \frac{3}{2} A_0 n^2. \quad (80)$$

Thus the average distance r of the electron from the ion is a function of n and is given by,

$$\overline{r(n)} = \frac{3}{2} A_0 n^2. \quad (81)$$

where A_0 is given by eqn. (31).

Using eqn. (81) in eqn. (67), the energy of the electron in n -th quantum state is given by,

$$E_{en} = -\frac{3e^2}{16\pi\epsilon_0} \frac{1}{\overline{r(n)}}. \quad (82)$$

When the electron alternatively takes quantum jumps from the positions of R_0 and R_{oc} and vice versa by successive removal and application of the d.c. electric field due to fluctuations in charge densities of the charge-carriers with opposite signs in the photosphere, then assume that the electron occupies the quantum number, n_0 at $R_0 = [\overline{r(n)}]_{n_0}$ and, n_{oc} at $R_{oc} = [\overline{r(n)}]_{n_{oc}}$. Under these conditions,

$$R_0 \rightarrow R_{en_0} = \frac{3}{2} A_0 n_0^2, \quad (83)$$

$$R_{oc} \rightarrow R_{en_{oc}} = \frac{3}{2} A_0 n_{oc}^2, \quad (84)$$

using eqn. (81).

Corresponding energies of the electron in these quantum states of n_0 and n_{oc} i.e. E_{en_0} and $E_{en_{oc}}$ are, using eqn. (82) given by,

$$E_{\text{en}_o} = -\frac{3e^2}{16\pi\epsilon_o} \left(\frac{1}{R_{\text{en}_o}} \right) = -\frac{3e^2}{16\pi\epsilon_o} \left(\frac{1}{R_o} \right) \quad (85)$$

and

$$E_{\text{en}_{oc}} = -\frac{3e^2}{16\pi\epsilon_o} \left(\frac{1}{R_{\text{en}_{oc}}} \right) = -\frac{3e^2}{16\pi\epsilon_o} \left(\frac{1}{R_{oc}} \right). \quad (86)$$

When the electron makes quantum jumps from state R_{oc} to state R_o (where $R_{oc} > R_o$) at equilibrium temperature T of the photosphere, then it absorbs a quantum of energy hf of radiation of frequency f , such that,

$$E_{\text{en}_{oc}} - E_{\text{en}_o} = +hf. \quad (87)$$

Here + sign on the Right Hand Side of eqn. (87), denotes absorption of radiation.

On the other hand, when the electron makes transitions from state R_o to state R_{oc} at equilibrium temperature T of the photosphere, then the quantum of energy hf gets emitted, such that,

$$E_{\text{en}_o} - E_{\text{en}_{oc}} = -hf, \quad (88)$$

where the negative sign on the Right Hand Side of eqn. (88) denotes emission of radiation.

The successive quantum jumps of the electron from state R_{oc} to state R_o and vice versa due to fluctuations in the charge densities of the charge-carriers with opposite signs give rise to absorption and emission of radiation respectively in general of the photosphere.

The frequency of radiation f can be obtained by eqn. (87), using eqns. (86) and (85). Thus,

$$f = \frac{3e^2}{16\pi\epsilon_o h} \left[\frac{1}{R_o} - \frac{1}{R_{oc}} \right]. \quad (89)$$

Substituting the value of R_{oc} in terms of R_o from eqn. (17), eqn. (89) gives that,

$$f = \frac{3e^2 \exp(-1)}{16\pi\epsilon_o h R_o}. \quad (90)$$

Eqn. (90) gives frequency f of radiation as a function of R_o [– which is the average separation of the electron from the neighbouring ion].

If c be the velocity of radiation in free space and λ be the corresponding wavelength, then,

$$f\lambda = c. \quad (91)$$

Using eqn. (90), eqn. (91) gives,

$$\lambda = \frac{16\pi\epsilon_o h c R_o}{3e^2 \exp(-1)} = \frac{R_o}{3.204(7) \times 10^{-4}}. \quad (92)$$

Substituting value of R_o using eqn. (9), eqn. (92) gives that,

$$\lambda = \frac{\sqrt[3]{(3/4\pi)} 16\pi\epsilon_o h c}{3e^2 \exp(-1)} N_e^{-1/3} = 1.935(8) \times 10^3 / N_e^{1/3} \quad (93)$$

The minimum value of λ i.e. λ_{\min} available from photosphere corresponds to the minimum value of R_o i.e. $(R_o)_{\min}$ given by,

$$\begin{aligned} (R_o)_{\min} &= [R_{\text{en}} = \frac{3}{2} A_o n^2]_{n=1} \\ &= \frac{3}{2} A_o = \frac{4\pi\hbar^2\epsilon_o}{m_e e^2} = \frac{3\hbar^2\epsilon_o}{2\pi m_e e^2}. \end{aligned} \quad (94)$$

using eqns. (81), and (31).

The maximum value of N_e i. e. $(N_e)_{\max}$ corresponding to $(R_o)_{\min}$ can be obtained using eqn. (9) and is given by,

$$(N_e)_{\max} = \left(\frac{3}{4\pi} \right) (R_o)_{\min}^{-3} = \frac{2\pi^2 m_e^3 e^6}{9\hbar^6 \epsilon_o^3}. \quad (95)$$

Since the transition between photosphere and solar corona in which electron density varies from 10^{10} m^{-3} to $1 \times 10^{14} \text{ m}^{-3}$ as already mentioned in Secn. 1 is quite arbitrary, an arbitrary value of the of electron density i.e. $(N_e)_{\min}$ near the photosphere, intermediate to the chromosphere, is chosen such that,

$$(N_e)_{\min} = 1 \times 10^{16} \text{ m}^{-3} \quad (96)$$

Maximum value of R_o i.e. $(R_o)_{\max}$ corresponding to minimum value of N_e i.e. $(N_e)_{\min}$ is given by,

$$(R_o)_{\max} = \left(\frac{3}{4\pi} \right)^{1/3} (N_e)_{\min}^{-1/3}, \quad (97)$$

using eqn. (9).

Values of range of λ [of eqn. (92)] are defined by,

$$\lambda_{\max} \geq \lambda \geq \lambda_{\min}, \quad (98)$$

for range of R_o given by,

$$(R_o)_{\max} \geq R_o \geq (R_o)_{\min}, \quad (99)$$

where $(R_o)_{\max}$ and $(R_o)_{\min}$ are given by eqns. (97) and (94) respectively.

The distribution of N_e corresponding to the distribution of R_o [eqn. (9)] with minimum and maximum values given by eqns. (94) and (97) respectively, give rise to the distribution of λ with minimum λ_{\min} and maximum λ_{\max} values determined, at equilibrium temperature T of the photosphere.

5. Distribution of Emittance with Wavelength in Photospheres' Radiation Spectrum

Monochromatic noise energy density $W_{R\lambda}$ at wavelength λ

generated by the electrons at equilibrium temperature T of 7000°K of the photosphere which is radiation temperature of the Sun can be shown to be given by the methods similar to those treated in an earlier paper (Nandedkar and Bhagavat 1970 [11]), as follows:

$$W_{R\lambda} d\lambda = \frac{8\pi c h}{\lambda^5} \left[\frac{d\lambda}{\{\exp(\frac{hc}{\lambda kT})\}-1} \right], \quad (100)$$

where $W_{R\lambda} d\lambda$ is the radiation energy density in the wavelength band $d\lambda$ at λ . Here λ is given by eqn. (92), such that [refer to eqn. (98)],

$$\lambda_{\min} \leq \lambda \leq \lambda_{\max}.$$

where λ_{\min} and λ_{\max} can be determined by eqns. (92).

Further monochromatic radiation energy density $W_{R\lambda}$ at wavelength λ , using eqn. (100), is given by,

$$W_{R\lambda} = \frac{8\pi c h}{\lambda^5} \left[\frac{1}{\{\exp(\frac{hc}{\lambda kT})\}-1} \right], \quad (101)$$

whereas $c W_{R\lambda}$ which is emittance of the radiation at wavelength λ is obtained from eqn. (101), as follows:

$$c W_{R\lambda} = \frac{8\pi c^2 h}{\lambda^5} \left[\frac{1}{\{\exp(\frac{hc}{\lambda kT})\}-1} \right]. \quad (102)$$

Here $c W_{R\lambda}$ gives amount of the radiation power crossing the unit area normal to the direction of propagation in the wavelength interval $d\lambda$ between λ and $\lambda + \lambda$.

Equation (101) indicates that $W_{R\lambda}$ varies with the wavelength λ of the radiation. The wavelength λ_{pk} , at which $W_{R\lambda}$ becomes maximum, i.e. $(W_{R\lambda})_{\max}$ is given by condition:

$$\frac{d}{d\lambda} [\lambda^5 \{\exp(hc/\lambda kT) - 1\}]_{\lambda=\lambda_{pk}} = 0, \quad (103)$$

at which the denominator in the expression of $W_{R\lambda}$ becomes minimum. Eqn. (103) can also be written as follows:

$$[-\lambda^5 \{\frac{hc}{\lambda^2 kT}\} \exp(hc/\lambda kT) + 5\lambda^4 \{\exp(hc/\lambda kT) - 1\}]_{\lambda=\lambda_{pk}} = 0. \quad (104)$$

If,

$$\frac{hc}{\lambda_{pk} kT} = \theta_{pk}, \quad (105)$$

then eqn. (104) gives,

$$\exp(-\theta_{pk}) + (\theta_{pk}/5) - 1 = 0. \quad (106)$$

The solution of eqn. (106) is,

$$\theta_{pk} = 4.965 \quad (107)$$

Using eqn. (107), eqn. (105) gives,

$$\lambda_{pk} = \frac{hc}{kT} \frac{1}{4.965} = [0.4138(6) \times 10^{-6} \text{ m}]_{T=7000^\circ\text{K}} \quad (108)$$

From eqns. (101), (105) and (107), the maximum value of $W_{R\lambda}$ i.e. $(W_{R\lambda})_{\max}$ is given by,

$$(W_{R\lambda})_{\max} = \frac{8\pi c h}{\lambda_{pk}^5} \left[\frac{1}{\{\exp 4.965\}-1} \right]. \quad (109)$$

Thus,

$$\frac{W_{R\lambda}}{(W_{R\lambda})_{\max}} = \frac{c W_{R\lambda}}{(c W_{R\lambda})_{\max}} = \frac{\{\exp 4.965\}-1}{(\lambda/\lambda_{pk})^5 [\exp\{4.965/(\lambda/\lambda_{pk})\}-1]}. \quad (110)$$

where $c W_{R\lambda}$ is the emittance and $(c W_{R\lambda})_{\max}$ is maximum value of the emittance of photospheres' radiation spectrum. Eqn. (102) gives emittance of the radiation as a function of wavelength of radiation- refer to Fig. 2.

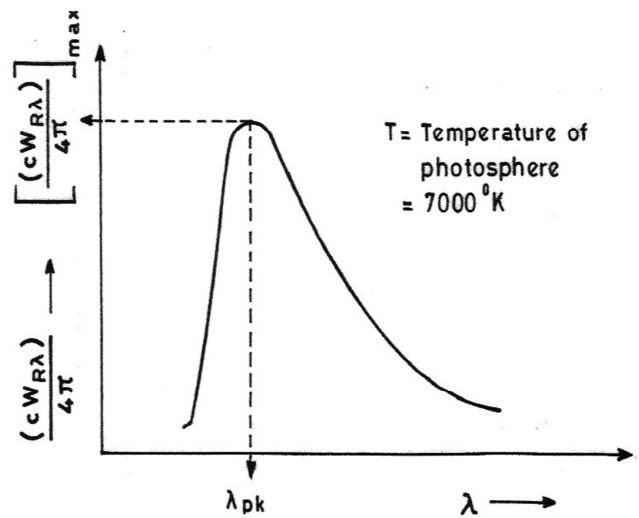


Fig. 2. $(c W_{R\lambda})/4\pi$ versus λ at $T = 7000^\circ\text{K}$, $[(c W_{R\lambda})/4\pi]_{\max} = 68.90(1) \times 10^{12} \text{ W-m}^{-2} \cdot \text{rad}^{-2} \cdot \text{m}^{-1}$. Here, $\lambda_{pk} = 0.4138(6) \times 10^{-6} \text{ m}$.

6. Conclusions

Equation (95) and (96) gives maximum and minimum values of average electron density N_e as a function of average distance of the electron from ion [eqn. (9)] which is quantized for ionized semi-gaseous type material of the photosphere as a function of free space wavelength λ of the radiation from the photosphere [eqn. (92)].

Limits for values of N_e are,

$$4.774(2) \times 10^{29} \text{ m}^{-3} \geq N_e \geq 1 \times 10^{16} \text{ m}^{-3}, \quad (111)$$

using eqns. (95) and (96), for $(N_e)_{\max}$ and $(N_e)_{\min}$ for upper and lower limits of the densities N_e respectively, where $(N_e)_{\min}$ corresponds to an arbitrary value of the of electron density i.e. $(N_e)_{\min}$ chosen near the photosphere, intermediate to the chromosphere.

Now with reference to above limits of N_e given by eqn. (111), limits of R_o are given by,

$$7.937(3) \times 10^{-11} \text{ m} \leq R_o \leq 2.879(4) \times 10^{-6} \text{ m} \quad (112)$$

using eqns. (9) & (95) and (9) & (96), for $(R_o)_{\min}$ & $(R_o)_{\max}$, respectively, where $(R_o)_{\min}$ corresponds to an average separation of an electron from ion in ionized semi-gaseous type material of photosphere of the Sun for ground state $n = 1$ given by eqn. (94).

Further with reference to above limits of R_o given by eqn. (112), limits of λ are given by

$$0.2476(8) \times 10^{-6} \text{ m} \leq \lambda \leq 8.984(9) \times 10^{-3} \text{ m}, \quad (113)$$

using eqn. (92) and values of $(R_o)_{\min}$ & $(R_o)_{\max}$ as given by eqn. (112), respectively.

From eqn. (93), it is clear that λ inversely varies as cube root of N_e with limits of λ and N_e as given by eqns. (113) and (111), respectively.

Further using eqn. (102), value of $(c W_{R\lambda})/4\pi$ which is radiation emittance per unit solid angle of 4π radians per unit wavelength interval of $d\lambda$ at wavelength λ , is a function of free space wavelength λ due to the photospheres' radiation from Sun and is given [from eqns. (105) and (107)] by,

$$\frac{(c W_{R\lambda})}{4\pi} = \frac{2 c^2 h}{\lambda^5} \left[\frac{1}{\{\exp(\frac{4.965\lambda_{pk}}{\lambda})\}-1} \right], \quad (114)$$

where [refer to eqn. (108)],

$$\lambda_{pk} = \frac{hc}{kT} \frac{1}{4.965} = [0.4138(6) \times 10^{-6} \text{ m}]_{T=7000^{\circ}\text{K}}$$

gives value of wavelength at which $\frac{(c W_{R\lambda})}{4\pi}$ or $c W_{R\lambda}$ is maximum at Photosphere's radiation temperature of 7000°K .

Variation of $[\frac{(c W_{R\lambda})}{4\pi}]$ with λ is shown in Fig. 2. In Fig. 2, the limits of λ are given by eqn. (113). Fig. 2 shows variation of $(c W_{R\lambda})/4\pi$ i.e. radiation emittance per unit solid angle of 4π radians per unit wavelength interval of $d\lambda$ at wavelength λ , is a function of free space wavelength λ due to the photospheres' radiation from Sun at radiation temperature 7000°K . Maximum value of $[\frac{(c W_{R\lambda})}{4\pi}]$ i.e. $[\frac{(c W_{R\lambda})}{4\pi}]_{\max}$ is $68.90(1) \times 10^{12} \text{ W-m}^{-2} - \text{rad}^{-2} - \text{m}^{-1}$ and this occurs for the free space wavelength $\lambda = \lambda_{pk}$ of $0.4138(6) \times 10^{-6} \text{ m}$ of eqn. (108).

Next eqn. (92) gives variation of R_o i.e. the average separation of an electron from an ion with free space wavelength λ of radiation. R_o Increases as λ increases. Limits of variations of R_o of eqn. (112) corresponding to the limits of λ mentioned in eqn. (113) are,

$$7.937(3) \times 10^{-11} \text{ m} \leq R_o \leq 2.879(4) \times 10^{-6} \text{ m}$$

Here lower limit of R_o corresponds to eqn. (94), which is ground state of $n=1$ for separation of an electron from ion as already mentioned.

In this analysis quantum jump of the electron from state R_{oc} to state R_o and vice versa is assumed to absorb and emit radiation. Further solar atmosphere introduces its own absorption for the radiation. Considering various absorption effects in the photosphere, a curve similar to that shown by full line in Fig. 1 can be obtained from the dotted curve at radiation temperature of 7000°K , with appropriate cut-off wavelength λ_{\min} of value $0.2476(8) \times 10^{-6} \text{ m}$ as discussed in this analysis.

As mentioned in Secn. 1 of the "Introduction", recently the solar spectrum is photographed up to $0.2099 \times 10^{-6} \text{ m}$.

In the present analysis, a singly charged ion with an electron model of the photosphere is discussed. If a mixture of a doubly charged ion and a singly charged ion in right proportion with one of electrons in each cases be considered, then the average distance of the electron from the ion can tend to lower down than that given by eqn. (81) for ground state. The net result of this quasi-free: quasi-bound electron-ion model of the ionised semi-gaseous type material of the photosphere is then to reduce λ_{\min} given by eqn. (92) by a fraction, i.e. in this approximate analysis new λ_{\min} gets lower than $0.2477 \times 10^{-6} \text{ m}$.

Again as mentioned before, when the radiation in the free space wavelength of interest proceeds, the radiation undergoes various types of absorption effects in the solar atmosphere. In general a lower limit is ultimately reached for the wavelength of the radiation. With present knowledge, the detected lower limit of solar radiation is $0.2099 \times 10^{-6} \text{ m}$ as already stated in Secn. 1.

In short in the present article, an analysis of the continuous radiation from photosphere of the Sun at equilibrium temperature of $T=7000^{\circ}\text{K}$ of it is carried out. The radiation is assumed due to quantum jumps of the electron, in a high density ionized semi-gaseous type material of the photosphere of the Sun, from its amplitude states considering types of damped frequency oscillations and eigen frequency damped oscillations analogous to that exit in a low density plasma with electron-molecule collisions (Nandedkar 2016) [8]. Damped and eigen frequency damped oscillations of the electron result due to density fluctuations of the charge-carriers with opposite signs in the body of the photosphere due to nuclear processes going deep in the core of the Sun. The density fluctuations alternatively builds up and withdraws a d.c. electric field in the body of the photosphere. The necessary damping for the type of damped oscillations mentioned, is provided by electron-ion collision type interaction at equilibrium

temperature of the photosphere. The minimum value of the amplitude the electron takes which can be quantized in the field of the neighbouring ion and in the absence of the d.c. electric field when eigen-frequency damped oscillations result, govern the minimum value of the wavelength of radiation from the continuous spectrum of the photosphere. In general the wavelength is shown to be a function of the average density of the charge-carriers. Thus lower limit of the electron density corresponding to an intermediate value of the chromosphere, brings an upper limit for the radiation-wavelength of the radiation spectrum of the photosphere. The continuous radiation from photosphere of the Sun corresponds to electron density N_e variations in the range $4.774(2) \times 10^{29} \text{ m}^{-3} \geq N_e \geq 1 \times 10^{16} \text{ m}^{-3}$ which corresponds to wavelength λ of radiation in the range $0.2476(8) \times 10^{-6} \text{ m} \leq \lambda \leq 8.984(9) \times 10^{-3} \text{ m}$. However recently the solar spectrum is photographed up to a wavelength of about $0.2099 \times 10^{-6} \text{ m}$ which can be explained due to a mixture of a doubly charged ion and a singly charged ion in right proportion with one of electrons in each cases be considered, then the average distance of the electron from the ion can tend to lower down the minimum value of radiation wavelength.

Acknowledgements

The author is thankful for the interesting discussions with reference to this research-paper given herewith he had, with his Guru (the Teacher) Dr. Ing. G. K. Bhagavat, [(now) Late] Professor of Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India.

Further he gives thanks also to Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India for giving permission to publish this research-work.

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