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Multi-Particle Quantum Fields

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Abstract

The goal of the current work is to propose a model which would allow to apply the methods of quantum field theory for description of the hadron scattering as bound systems of quarks and antiquarks. From one side by applying the quantization procedure to the multi-particle fields, hadronic creation and annihilation operators have been obtained. From other side, by considering the hadron quark structure allow one describe the hadron interactions with the help of interactions of the constituent quarks and antiquarks according to the gauge principle by requiring of the local $SU_c(3)$ symmetry. The gauge field which was obtained in this way revealed to be a multi-particle field. By analysing the dynamical equations of this multi-particle gauge field it was shown that one can impose the partialy solution of these equations. This solution allows one within same framework to describe the quark confinement as well as interactions between bound quarks by the gluon exchange. The important feature of the proposed model is that it describes the quark confinement as well as confinement of gluons.

Keywords

Multi-Particle, Spinor, Pseudo-Scalar, Confinement, Asymptotic Freedom, Quakrs, Gluons, Gauge Field, Hadrons

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1. Introduction

As was shown in references [1, 2, 3, 4, 5], in the framework of perturbative φ^3 theory, it was possible to reproduce the qualitative features of the hadron's total cross-section spectrum as a function of $\sqrt{s}$. A new mechanism has been proposed that may explain the origin of the asymptotic growth of the total cross-section of hadron-hadron scattering. In framework of those models also possible to recreate inclusive cross-section spectra within changes of energy, where spectrum has just one maximum point at low energies, which may be replaced by two symmetrical maximum with increase of energy [6]. However, results of such models allow to describe experimental data only qualitatively.

To move to the quantitative level one should manage to do the same computations in QCD, which is much more realistic theory of processes that occur in hadron scattering with respect to above mentioned φ^3 theory. On the one hand within QCD one may use the same computation methods and consider the same physical mechanisms as in [7, 8]. On the other hand, there is a significant problem, which makes impossible an automatic translation of the results, which were obtained using perturbation theory in φ^3, to perturbative QCD theory. This is a well-known problem, the interactions occur between quarks with help of gluons, while initial and final states always show up as their bound states

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(colorless), namely hadrons. From a general point of view of perturbation theory this observation manifested in the fact that Feynman diagrams use only quark and gluon lines, but there are no hadron lines. If one will consider such diagrams for construction of the scattering amplitude, the energy-momentum conservation law will be imposed on four-momenta of quarks, not on four-momenta of hadrons, as in experiments. This is a consequence of the fact that the Hamiltonian of a system do not reach asymptotically the Hamiltonian of free particles, if one will consider quarks and gluons as constituent particles of such a system. Therefore, we are not able to "turn on" and to "turn off" the interactions as in the regular $\hat{S}$ - matrix approach, since this interaction is the very essence of the mechanism that ensures the existence of hadrons as bound states of constituents quarks.

As it seems to us this problem is not associated exclusively with the application of the perturbation theory, rather it is related to the fact that all existing field theories are formulated in terms of single-particle occupation numbers of preceding states [9]. While quark states inside hadrons are not single-particle and principally cannot be expressed through single-particle states, since, due to relativistic invariance each such single-particle state should be characterized by a certain value of the energy-momentum. In the end, we return to the above mentioned problem of application of the energy-momentum conservation law, regardless of the choice of method for describing the scattering process.

Usually these difficulties are overcome by use of parton models [10, 11, 12]. However, these models are much more adapted to the calculation of the inclusive characteristics of scattering processes rather than to the complete description of those processes. In particular, the system of multi-parton distribution functions for hadrons in the initial state, when there is no interaction between partons, is unknown. Determination of these functions is the problem by itself which yet further complicates the description of the scattering process.

However, if we believe that a hadron in the initial and final state of the scattering process, i.e. before or after interaction with other particles, is composed of a certain number of specific constituent quarks [13, 14, 15, 16], then one may try to describe the internal state of such hadron using a nonrelativistic approximation. In other words, one may try to describe this internal state by the probability amplitude, which can be found as a solution of the Schrödinger equation with respect to the selected potential. It can be shown that if considering the nonrelativistic bound state, one turns to the Jacobi coordinates [17] and splits out the internal state from the system-center-of-mass with respect to the initial reference frame, then the transition to the new reference frame should not change the internal state, as was proved in [18, 19].

In this paper we propose to consider multi-particle field operators which will modify the occupation numbers of the multi-particle hadron states. The interaction of the colorless hadrons with gauge fields in this case can be described through the quark's internal degrees of freedom.

The constituent quarks will be described by bi-spinor fields $\Psi_x(x)$, $\bar{\Psi}_s(x)$, $s = 1,2,3,4,...$. The corresponding field functions of these fields are defined in Minkowski space, denoted by $M_x, x \in M_x$. The range of values of quark's field functions is a linear space where either the bi-spinor representation of the Lorentz group or its Dirac-conjugate representation is realized. These spaces are denoted by B and $\bar{B}$ respectively. Here we introduce the following notation:

$$\hat{\Psi} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix}, \hat{\bar{\Psi}} = \hat{\Psi}^\dagger \hat{\gamma}^0, \tag{1}$$

where $\hat{\gamma}^0$ is the Dirac matrix.

Let's consider the tensor product of the two $M \otimes M$, or three Minkowski spaces, $M \otimes M \otimes M$ (for meson and baryons respectively) and the tensor product $\bar{B} \otimes B$, or $B \otimes B \otimes B$. The representations of the Lorentz group are realized on the tensor products $\bar{B} \otimes B$ or $B \otimes B \otimes B$. By expanding these products into direct sums of the invariant subspaces with respect to these representations one may reduce these representations to a pseudo-scalar form in case of $\bar{B} \otimes B$, or bi-spinor form in case of $\bar{B} \otimes B$, or $B \otimes B \otimes B$:

$$\begin{aligned} &\bar{B} \otimes B = I \oplus \cdots, B \otimes B \otimes B = B \oplus \cdots, \\ &\bar{B} \otimes \bar{B} \otimes \bar{B} = \bar{B} \oplus \cdots. \end{aligned} \tag{2}$$

Here I is the invariant subspace on which the pseudo-scalar representation may be realized; the ellipses denote the rest of invariant subspaces which are not sufficient in this case.

Further, we can consider mapping $M \otimes M$ into I , which we denote by $\varphi(x_1, x_2)$ and $\varphi^*(x_1, x_2)$ $x_1, x_2 \in M$). The mapping of $M \otimes M \otimes M$ manifold into B and $M \otimes M \otimes M$ into $\bar{B}$ is denoted through $\Psi_s(x_1, x_2, x_3)$, $(s = 1,2,3,4, \text{and } x_1, x_2, x_3 \in M)$ and $\bar{\Psi}_s(x_1, x_2, x_3)$ respectively.

Afterwards, we want consider the nonrelativistic approximation for the internal dynamics of the constituent quarks. In this approximation the field functions $\varphi(x_1, x_2)$, $\Psi_s(x_1, x_2, x_3)$, $\bar{\Psi}_s(x_1, x_2, x_3)$ have to become the probability amplitudes, which have to be computed for the same values of time coordinates for all particles with respect to the reference frame, where we do our measurements. These issues were discussed in detail in Ref.[19, 20]. Consequently, later in the paper we will examine fields only on a submanifold of the entire tensor product $M \otimes M$, or $M \otimes M \otimes M$, which has the following boundaries $x_1^0 = x_2^0$, or $x_1^0 = x_2^0 = x_3^0$. It is convenient to make a transition to this submanifold by introducing the 4-space Jacobi coordinates on these tensor products:

$$\begin{aligned} X^a &= \frac{x_1^a + x_2^a}{2}, \qquad y_1^a = x_2^a - x_1^a, \\ X^a &= \frac{x_1^a + x_2^a + x_3^a}{3}, \; y_1^a = x_3^a - \frac{x_1^a + x_2^a}{2}, \; y_2 = x_2^a - x_1^a, \end{aligned} \tag{3}$$

where $a = 0,1,2,3$. Here the first line corresponds to two-quark systems, and the second line to three-quark systems.

Using variables from Eq.3 the fields on the subset which we will consider are:

$$y_1^0 = 0, \quad \text{or} \quad y_1^0 = y_2^0 = 0. \tag{4}$$

The variables

$$\mathbf{y}_i = \left(y_i^1, y_i^2, y_i^3\right), i = 1,2,3, \tag{5}$$

which we will denote as internal variables, and the fields $\varphi(X, \mathbf{y}_1)$, $\Psi_s(X, \mathbf{y}_1, \mathbf{y}_2)$, $\bar{\Psi}_s(X, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3)$, where we use the notation

$$X \equiv \begin{pmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \end{pmatrix}, \tag{6}$$

which we will call multi-particle fields.

Of course, the conditions in Eq.4 cannot be imposed in a Lorentz invariant way. That is, observers in different inertial systems, by imposing the conditions of Eq.4, will mark out different submanifolds on the corresponding tensor products of the Minkowski spaces. However, as was discussed in detail in [18, 19] in case of a general Lorentz transformation (i.e. one which has boost and cannot be reduced to pure rotations) internal changes in the different inertial reference frames cannot be connected between each other neither via Lorentz transformations nor by using any other methods. In case of rotations each of them behaves like a regular three-dimensional vector. Speaking further about Lorentz transformations we will merely bear in mind a transformation that cannot be reduced to rotations. Instead, the expression in Eq.6 is transformed as a contravariant four-vector under Lorentz transformations. Thus, as was shown in [18, 19], the dependence of multi-particle fields on internal variables is the same in different inertial reference frames. Therefore, considering, for instance, the quantity

$$A = \int \bar{\Psi}_s\left(X, \mathbf{y}_1, \mathbf{y}_2\right) \Psi_s\left(X, \mathbf{y}_1, \mathbf{y}_2\right) d\mathbf{y}_1 d\mathbf{y}_2, \tag{7}$$

we get in another reference frame (the repetition of the index s implies summation over this index):

$$\begin{aligned} A = \int & d\mathbf{y}_1' d\mathbf{y}_2' \bar{\Psi}_{s_1}\left(X = \Lambda^{-1} X', \mathbf{y}_1', \mathbf{y}_2'\right) D_{s_1 s}^{-1}(\Lambda) \\ & \times D_{s s_2}(\Lambda) \Psi_{s_2}\left(X = \Lambda^{-1} X', \mathbf{y}_1', \mathbf{y}_2'\right). \end{aligned} \tag{8}$$

Here Λ is a Lorentz transformation, $D_{ss_2}(\Lambda)$ and $D_{s_1 s}^{-1}(\Lambda)$ are the corresponding elements of the Λ matrix transformation in the bi-spinor representation of the Lorentz group and its inverse transformation, respectively. At the same time the $\mathbf{y}_1'$, $\mathbf{y}_2'$ do not couple to $\mathbf{y}_1$, $\mathbf{y}_2$. Given that the integrands in Eq.7 and Eq.8 are the same functions of the internal variables, namely the integrand in Eq.7 is the same function of variables $\mathbf{y}_1$, $\mathbf{y}_2$ as is the integrand in Eq.8 with respect to the variables $\mathbf{y}_1'$, $\mathbf{y}_2'$. One may note that the integrals differ only by notations of the integration variables. Therefore, A takes the same values in different inertial reference frames, similar to the $\bar{\Psi}_s(X)\Psi_s(X)$ expression which is the Lorentz invariant in case of ordinary single-particle field.

Integration over $\mathbf{y}_1$, $\mathbf{y}_2$ becomes the analog of summation of color and flavor indices, which justifies the use of the term "internal variables" for these variables, since they are not connected to Lorentz transformations.

The above reasoning leads to the conclusion that, even though one cannot decouple expression Eq.4 in a Lorentz-invariant way, the end result can still be harmonized with the principle of relativity.

For imposition of the multi-particle fields using the described method further in this work, we propose the dynamical equations, construct the corresponding Lagrangians and constants of motions. Moreover, the quantization procedure will be implemented. An important role in the interpretation of field operators as those that is modify the occupation numbers of the single-particle states played by their transformation with respect to spacetime shift, not the boost [20]. We would like to emphasize this statement because due to this shift multi-particle fields are manifested. As seen from the definition of Eq.3, the internal Jacobi coordinates stay unchanged, while the coordinates X^a are modified similarly to normal single-particle coordinates. As will be shown later, this leads to the fact that the energy-momentum of a system becomes the energy-momentum of two-particle or three-particle systems, via the multi-particle field operators, that appear as a result of quantization of the multi-particle fields. Thus they can be interpreted as the hadron's creation and annihilation operators. Therefore, by considering a theory with such operators one obtains the conservation law especially for four-momenta of hadrons as it should be.

So far, for brevity we did not write out the internal indices corresponding to the color, c , and flavor, f , for single-particle quark fields. Henceforth, we will assume that single-particle quark fields have appropriate degrees of freedom, and they transform with respect to the fundamental representation of the groups $SU_c(3)$ and $SU_f(3)$ respectively. As a result of the tensor multiplication of these fields, the obtained multi-particle fields will transform with respect to tensor products of this representation. These representations are implemented on a linear space of values of the tensor multi-particle field functions. Laying out this space into a direct sum of subspaces that are invariant with respect to all transformations of considered representation, we select the subspace on which a trivial representation of $SU_c(3)$ group has been implemented. Then we will consider a multi-particle field as an element of this subspace. In this way we take care of the colorlessness of hadrons. However, the presence of internal indices of the field, which is a consequence of the existence of internal quark degrees of freedom, allows one to introduce the interaction with the gauge field through the conventional derivative extension procedure. The requirement of simultaneity, Eq.4, leads to the fact that this field should also be considered as multi-particle field. Operators of this field, as will be shown further, correspond to the creation and annihilation operators of bound states of gluons.

Using the multi-particle hadron fields that can interact by multi-particle gauge fields, which in turn may generate secondary particles by interactions with hadron fields, one may obtain a method to describe the processes of elastic and inelastic scattering of hadrons with the "correct" law of energy-momentum conservation, which is imposed on the energy-momenta of hadrons rather than constituent quarks.

2. Pseudo-Scalar Meson Fields Constructed from Two Bi-spinor Fields

In the work [21] we have considered an example of the two-particle scalar field and analyze the physical sense of the operators of that field, which appear as result of the quantization procedure. The aim of this section is to construct the pseudo-scalar field with the help of two bi-spinor fields, keeping in mind that mesons are bound states of quark and antiquarks. Creation and annihilation operators of this field will correspond to creation and annihilation of mesons. Due to internal coordinates such a meson can interact with the gluon field, which will help to describe the production of such a meson in hadron-hadron scattering.

The tensor product of the two bi-spinor fields $\Psi_{s_2}(x_2)$ and $\bar{\Psi}_{s_1}(x_1)$, that correspond to quarks, can be represented as matrix:

$$\Psi_{s_1 s_2}(x_1, x_2) = \bar{\Psi}_{s_1}(x_1)\Psi_{s_2}(x_2) \qquad (9)$$

This matrix can be decomposed on the basis of 16 matrices $\hat{\Gamma}_a, a = 1, 2, .., 16$ [20], the basis of algebra generated by the Dirac matrices:

$$\Psi_{s_1 s_2}(x_1, x_2) = \sum_{a=1}^{16} \varphi_a(x_1, x_2)(\Gamma_a)_{s_1 s_2}. \qquad (10)$$

The set of matrices $\hat{\Gamma}_a, a = 1, 2, .., 16$ can be decomposed to a subset; each of these subsets create a basis of the invariant subspace with respect to a Lorentz transformation. Accordingly, the functions $\varphi_a(x_1, x_2)$ decomposed into sets of functions, which are transformed according to the irreducible representations of the Lorentz group. Especially, we are interested in the pseudo-scalar representation, which is realized in the one-dimensional subspace spanned by the matrix $\hat{\gamma}^5$:

$$\Psi_{s_1 s_2}(x_1, x_2) = \varphi(x_1, x_2)\gamma^5_{s_1 s_2} + \cdots. \qquad (11)$$

Here the ellipses denote the rest of the terms in the expression Eq.10. By taking into account the properties of the matrix $\hat{\Gamma}_a$, the term $\varphi_a(x_1, x_2)$ in this expression can be written as:

$$\varphi(x_1, x_2) = \frac{1}{4}\left(\Psi_{s_1 s_2}(x_1, x_2)\gamma^5_{s_1 s_2}\right), \qquad (12)$$

where there is a sum over repeated indices s_1 and s_2 .

In this section we would like to find the dynamical equation which defines the two-particle function $\varphi_a(x_1, x_2)$.

Thus we require that the dynamical operator that will be included in this equation should conserve the structure of the invariant subspaces with respect to the Lorentz group that is defined by Eq.10. Hence as a result of the action of this operator, the element of each of the invariant subspaces should be mapped to the elements of the same subspace. In other words, the subspaces that are invariant under Lorentz transformations, should remain invariant with respect to dynamical operator of the two-particle field $\Psi_{s_1 s_2}(x_1, x_2)$. As was shown in [21] if one will use the Dirac equations as dynamical equations for each of the psi-functions in the Eq.9, then expansion Eq.10 will not be invariant anymore with respect to the corresponding dynamical operators.

In order to overcome this issue one may use the fact that the components of the bi-spinor should also satisfy the Klein-Gordon-Fock equation in addition to Dirac equation:

$$\begin{cases} -g^{a_1 a_2} \dfrac{\partial \bar{\Psi}_{s_1}(x_1)}{\partial x_1^{a_1} \partial x_1^{a_2}} - m^2 \bar{\Psi}_{s_1}(x_1) = 0, \\ -g^{a_1 a_2} \dfrac{\partial \Psi_{s_2}(x_2)}{\partial x_2^{a_1} \partial x_2^{a_2}} - m^2 \Psi_{s_2}(x_2) = 0. \end{cases} \tag{13}$$

By considering a linear combination of Eqs.13 with coefficients $\Psi_{s_2}(x_2)$ and $\bar{\Psi}_{s_1}(x_1)$ in the same way as in the previous section, we get:

$$\begin{aligned} &-g^{a_1 a_2} \frac{\partial \Psi_{s_1 s_2}(x_1, x_2)}{\partial x_1^{a_1} \partial x_1^{a_2}} - g^{a_1 a_2} \frac{\partial \Psi_{s_1 s_2}(x_1, x_2)}{\partial x_2^{a_1} \partial x_2^{a_2}} \\ &-2m^2 \Psi_{s_1 s_2}(x_1, x_2) = 0. \end{aligned} \tag{14}$$

The dynamical operator on the left hand side now conserves the invariant subspaces in Eq.10. Therefore, by convoluting Eq.14 with the matrix $\hat{\gamma}^5$, one will obtain equations for two-particle pseudo-scalar field $\varphi(x_1, x_2)$ Eqs.11,12:

$$-g^{a_1 a_2} \frac{\partial \varphi(x_1, x_2)}{\partial x_1^{a_1} \partial x_1^{a_2}} - g^{a_1 a_2} \frac{\partial \varphi(x_1, x_2)}{\partial x_2^{a_1} \partial x_2^{a_2}} - 2m^2 \varphi(x_1, x_2) = 0. \tag{15}$$

Note, that from the definitions of Eqs.9-12 it follows that $\varphi(x_1, x_2)$ is complex. Hence, instead of solving Eq.15 for real and imaginary parts of this function let's add the complex conjugated terms to it. These operations give us the system of Euler-Lagrange equations for Lagrangian:

$$\begin{aligned} L(x_1, x_2) &= g^{a_1 a_2} \frac{\partial \varphi^*(x_1, x_2)}{\partial x_1^{a_1}} \frac{\partial \varphi(x_1, x_2)}{\partial x_1^{a_2}} \\ &+ g^{a_1 a_2} \frac{\partial \varphi^*(x_1, x_2)}{\partial x_2^{a_1}} \frac{\partial \varphi(x_1, x_2)}{\partial x_2^{a_2}} \\ &-2m^2 \varphi^*(x_1, x_2) \varphi(x_1, x_2). \end{aligned} \tag{16}$$

As bi-spinor fields we consider those fields that correspond to quarks and antiquarks. In addition to bi-spinor indices these fields have color indices, denoted as c_1 and c_2, and flavor indices, denoted as f_1 and f_2. Then the two-particle field will transform, not only with respect to the tensor product of the Lorentz representations, but also with respect to the tensor product of the $SU_c(3)$ and $SU_f(3)$ group representations. This two-particle field will be denoted as:

$$\Psi_{s_1, s_2; c_1, c_2; f_1, f_2}(x_1, x_2) = \bar{\Psi}_{s_1, c_1, f_1}(x_1) \Psi_{s_2, c_2, f_2}(x_2). \tag{17}$$

Instead of Eq.12 we write:

$$\varphi_{c_1, c_2}^{f_1, f_2}(x_1, x_2) = \frac{1}{4} \bar{\Psi}_{s_1, c_1, f_1}(x_1) \Psi_{s_2, c_2, f_2}(x_2) \gamma_{s_1 s_2}^5. \tag{18}$$

Here, the flavor indices are written as superscripts just in order to squeeze the notation. Respectively, instead of Lagrangian Eq.16 we get:

$$\begin{aligned} L &= g^{a_1 a_2} \frac{\partial \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^*}{\partial x_1^{a_1}} \frac{\partial \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2)}{\partial x_1^{a_2}} \\ &+ g^{a_1 a_2} \frac{\partial \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^*}{\partial x_2^{a_1}} \frac{\partial \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2)}{\partial x_2^{a_2}} \\ &-2m^2 \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^* \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2). \end{aligned} \tag{19}$$

Now one can define the interaction of the two-particle pseudo-scalar field $\varphi_{c_1, c_2}^{f_1, f_2}(x_1, x_2)$ with the gluon field by the extension of the derivatives. This will ensure that local $SU_c(3)$ invariance of the two-particle pseudo-scalar Lagrangian is satisfied, we get:

$$\begin{aligned} L &= g^{a_1 a_2} \left(\frac{\partial \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^*}{\partial x_1^{a_1}} + igA_{a_1}^{g_1}(x_1) \left(\varphi_{c_3 c_2}^{f_1 f_2}(x_1, x_2) \right)^* \left(\lambda_{c_3 c_1}^{g_1} \right)^* \right) \\ &\times \left(\frac{\partial \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2)}{\partial x_1^{a_2}} - igA_{a_2}^{g_2}(x_1) \lambda_{c_2 c_4}^{g_2} \varphi_{c_1 c_4}^{f_1 f_2}(x_1, x_2) \right) \\ &+ g^{a_1 a_2} \left(\frac{\partial \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^*}{\partial x_2^{a_1}} + igA_{a_1}^{g_1}(x_2) \left(\varphi_{c_3 c_2}^{f_1 f_2}(x_1, x_2) \right)^* \left(\lambda_{c_3 c_1}^{g_1} \right)^* \right) \\ &\times \left(\frac{\partial \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2)}{\partial x_2^{a_2}} - igA_{a_2}^{g_2}(x_2) \lambda_{c_2 c_4}^{g_2} \varphi_{c_1 c_4}^{f_1 f_2}(x_1, x_2) \right) \\ &-2m^2 \left(\varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2) \right)^* \varphi_{c_1 c_2}^{f_1 f_2}(x_1, x_2). \end{aligned} \tag{20}$$

Here g is the strong interaction coupling constant, g_1 and g_2 are the internal indices of the gluon field.

The law of the two-particle field $\varphi_{c_1, c_2}^{f_1, f_2}(x_1, x_2)$ transformations under $SU_c(3)$ coming from Eq.17 has the form:

$$\varphi'^{f_1f_2}_{c_1c_2}(x_1,x_2)=U_{c_2c_4}(x_1)\varphi^{f_1f_2}_{c_3c_4}(x_1,x_2)U^{\dagger}_{c_3c_1}(x_2). \quad (21)$$

Here $U_{c_2,c_4}(x_1)$ and $U_{c_3,c_1}(x_2)^{\dagger}$ are the elements for coordinate-dependent $SU_c(3)$ matrix and its Hermitian conjugated matrix, respectively.

By examining, in the special case, representations of the global transformation group of $SU_c(3)$ on the linear space of two index color tensors using the transformation law Eq.21, one may select the one-dimensional invariant subspace on trivial representation,

$$\begin{aligned}&\left(\varphi^{f_1f_2}_{c_1c_2}(x_1,x_2)\right)^{*}=\delta_{c_1c_2}\varphi^{*}_{f_1f_2}(x_1,x_2),\\&\varphi^{f_1f_2}_{c_1c_2}(x_1,x_2)=\delta_{c_1c_2}\varphi_{f_1f_2}(x_1,x_2).\end{aligned} \quad (22)$$

As seen from Eq.21 in the general case of the local transformation (colorless) of a meson could be defined through the manifold of configurations of type:

$$\varphi^{f_1f_2}_{c_1c_2}(x_1,x_2)=\left(U_{c_2c_3}(x_1)U^{\dagger}_{c_3c_1}(x_2)\right)\varphi_{f_1f_2}(x_1,x_2), \quad (23)$$

which is gauge equivalent to Eq.22.

However, as seen from Eq.23 the local transformation at $x_1\neq x_2$ does not conserve the structure of the invariant subspaces with respect to the global transformations. This in turn leads to violation of the local invariance if one will substitute Eq.22 into Lagrangian Eq.20. In order to overcome this issue, let's agree on the following rule. If one will do the local transformation, and then allocate the invariant subspace this will not violate local invariance. But not vice versa. Since these two operations are not permutable. In order to avoid the violation problem we adopt this rule for the multi-particle fields. Firstly we will apply the local transformation for the whole tensor $\varphi^{f_1f_2}_{c_1c_2}(x_1,x_2)$ then we divide it into tensors that are invariant with respect to the global transformations.

By using the above mentioned rule and substitute Eq.22 into Lagrangian Eq.20 we get:

$$\begin{aligned}L=g^{a_1a_2}&\left(\frac{\partial\varphi^{*}_{f_1f_2}(x_1,x_2)}{\partial x_1^{a_1}}\frac{\partial\varphi_{f_1f_2}(x_1,x_2)}{\partial x_1^{a_2}}\right.\\&\left.+\frac{\partial\varphi^{*}_{f_1f_2}(x_1,x_2)}{\partial x_2^{a_1}}\frac{\partial\varphi_{f_1f_2}(x_1,x_2)}{\partial x_2^{a_2}}\right)\\&-2m^2\varphi^{*}_{f_1f_2}(x_1,x_2)\varphi_{f_1f_2}(x_1,x_2)\\&+\frac{2}{3}g^2\varphi^{*}_{f_1f_2}(x_1,x_2)\varphi_{f_1f_2}(x_1,x_2)\\&\times g^{a_1a_2}\left(A^{g_1}_{a_1}(x_1)A^{g_1}_{a_2}(x_1)+A^{g_1}_{a_1}(x_2)A^{g_1}_{a_2}(x_2)\right).\end{aligned} \quad (24)$$

This Lagrangian is locally invariant in the sense discussed above. That is, if we want to make the local transformation we have to return to the previous Lagrangian Eq.20 and perform the local transformation on it, and then select the invariant subspace Eq.22 which is spanned by the unit tensor of color indices.

By turning to the four dimensional Jacobi coordinates in Eq.3 we again obtain Lagrangian up to a small constant factor on the subset in Eq.4:

$$\begin{aligned}&L=\left(L_0+L_{int}\right)\Big|_{y_1^0=0},\\&L_0=g^{a_1a_2}\frac{\partial\varphi^{*}_{f_1f_2}(X,y_1)}{\partial X^{a_1}}\frac{\partial\varphi_{f_1f_2}(x_1,x_2)}{\partial X^{a_2}}\\&\quad-\left((2m)^2\varphi^{*}_{f_1f_2}(X,y_1)\varphi_{f_1f_2}(X,y_1)\right.\\&\quad\left.+2(2m)\frac{1}{m}\sum_{b=1}^{3}\frac{\partial\varphi^{*}_{f_1f_2}(X,y_1)}{\partial y_1^{b}}\frac{\partial\varphi_{f_1f_2}(X,y_1)}{\partial y_1^{b}}\right),\\&L_{\text{int}}=\frac{4}{3}g^2\varphi^{*}_{f_1f_2}(X,y_1)\varphi_{f_1f_2}(X,y_1)\\&\quad\times g^{a_1a_2}\left(A^{g_1}_{a_1}\left(X-\frac{1}{2}y_1\right)A^{g_1}_{a_2}\left(X-\frac{1}{2}y_1\right)\right.\\&\quad\left.+A^{g_1}_{a_1}\left(X+\frac{1}{2}y_1\right)A^{g_1}_{a_2}\left(X+\frac{1}{2}y_1\right)\right).\end{aligned} \quad (25)$$

The Lagrangian L_0 is similar to the Lagrangian in the previous section and can be considered as the Lagrangian of a free meson field. Which after the quantization procedure correspond to the meson's creation and annihilation operators. Respectively, L_{int} is the Lagrangian of interaction for the meson fields with the gauge field. This Lagrangian can be used to describe multi-meson production in scattering processes. Taking into account that in experiments these processes are mostly observed in proton-proton(antiproton) scattering we need to construct the three-particle field that will correspond to protons.

In addition, the criteria of simultaneity in Eq.25 leads to the fact that gauge field which interact with the meson field as well as the proton field (this will be shown later in the paper) should be considered as multi-particle fields.

3. Three-Particle Bi-spinor Field

The three-particle field that will correspond to baryons is obtained by considering all possible products of three bi-spinor field components:

$$\begin{aligned}&\Psi^{f_1,f_2,f_3}_{s_1s_2s_3,c_1,c_2,c_3}\left(x_1,x_2,x_3\right)=\\&\Psi_{s_1,c_1,f_1}\left(x_1\right)\Psi_{s_2,c_2,f_2}\left(x_2\right)\Psi_{s_3,c_3,f_3}\left(x_3\right).\end{aligned}\tag{26}$$

For brevity, we will temporarily omit color and flavor indices and consider $\Psi_{s_1s_2s_3}(x_1,x_2,x_3)$, which under Lorentz transformations transform with respect to the tensor product of the three bi-spinor representations:

$$\begin{aligned}&\Psi'_{s_1s_2s_3}\left(x'_1,x'_2,x'_3\right)=\\&D_{s_1s_4}\left(\Lambda\right)D_{s_2s_5}\left(\Lambda\right)D_{s_3s_6}\left(\Lambda\right)\\&\times\Psi_{s_4s_5s_6}\left(x_a=\Lambda^{-1}x'_a\right),\ a=1,2,3.\end{aligned}\tag{27}$$

Here Λ is the Lorentz transformation, $D_{s_1s_4}(\Lambda)$ are the matrix elements of the bi-spinor representation of the Lorentz group.

From the linear space of the triple index tensors $\Psi_{s_1s_2s_3}(x_1,x_2,x_3)$ we have to pick out the invariant subspace that transforms with respect to the bi-spinor representation of the Lorentz group. This can be done in different ways. One of these options has been considered in [21]. For us this example does not matter, because we are interested in the dynamical equation for the three-particle bi-spinor field, which we denote as $\Psi_s(x_1,x_2,x_3)$ before introducing the Jacobi coordinates and before transition to the subset of simultaneity.

In order to obtain this equation we will proceed with the same reasons as in the previous section. Thus, we start from the Klein-Gordon-Fock equation for the components of each bi-spinor multipliers in Eq.27. With the reasoning that led us to Eq.14 by use of the Jacobi coordinates of Eq.3, we can write on the subset of simultaneity.

$$\begin{aligned}&-g^{a_1a_2}\frac{\partial^2\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)}{\partial X^{a_2}\partial X^{a_1}}-\left(\left(3m\right)^2\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)\right.\\&+2\left(3m\right)\left(-\frac{3}{4m}\Delta_{\mathbf{y}_1}\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)\right.\\&\left.\left.-\frac{1}{m}\Delta_{\mathbf{y}_2}\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)\right)\right)=0.\end{aligned}\tag{28}$$

As in [21] let's consider the internal Hamiltonian of the three particle system:

$$\hat{H}^{\text{internal}}=\left(3m\right)\hat{E}-\frac{3}{4m}\Delta_{\vec{y}_1}-\frac{1}{m}\Delta_{\vec{y}_2}.\tag{29}$$

Further as in [21] with an accuracy of the order of the square of the ratio of the characteristic internal energy of the three-particle system to its rest energy, we can write:

$$-g^{a_1a_2}\frac{\partial^2\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)}{\partial X^{a_2}\partial X^{a_1}}-\left(\hat{H}^{\text{internal}}\right)^2\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)=0.\tag{30}$$

It is obvious that the operators $\hat{H}^{\text{internal}}$ and $i\partial/\partial X^a$ for all values of the index a commute. This allows us to apply Eq.30 to the same factorization procedure as for the "ordinary" Klein-Gordon-Fock equation which leads to the "ordinary" Dirac equation. In our case this factorization gives us the three particle analog of it:

$$\begin{aligned}&i\gamma^a_{s_1s_2}\frac{\partial\Psi_{s_2}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)}{\partial X^a}\\&-\left(3m-\frac{3}{4m}\Delta_{\mathbf{y}_1}-\frac{1}{m}\Delta_{\mathbf{y}_2}\right)\Psi_{s_1}\left(X,\mathbf{y}_1,\mathbf{y}_2\right)=0.\end{aligned}\tag{31}$$

Furthermore, we will consider Eq.31 as a dynamical equation for the three bi-spinor field, which we are looking for.

The procedure, which describes interactions of the quark fields with the gauge field in the three-particle case is similar to the problem that was considered in the previous section, and has been described in details in [21]. It leads to the interaction Lagrangian of the three-particle bi-spinor field and the gluon field, which can be written as

$$\begin{aligned}L_{\text{int}}=&\frac{1}{9m}g^2\bar{\Psi}_{s_1}\left(x_1,x_2,x_3\right)\Psi_{s_1}\left(x_1,x_2,x_3\right)\\&\times\Big(\left(\varphi\left(x_1,x_2\right)+\chi\left(x_1,x_2\right)\right)\\&+\left(\varphi\left(x_1,x_3\right)+\chi\left(x_1,x_3\right)\right)\\&+\left(\varphi\left(x_2,x_3\right)+\chi\left(x_2,x_3\right)\right)\Big),\end{aligned}\tag{32}$$

where we use the notation:

$$\begin{aligned}&\varphi\left(x_{b_1},x_{b_2}\right)=g^{a_1a_2}A_{a_1,g_1}\left(x_{b_1}\right)A_{a_2,g_1}\left(x_{b_2}\right),\\&\chi\left(x_{b_1},x_{b_2}\right)=g^{a_1a_2}\Big(A_{a_1,g_1}\left(x_{b_1}\right)A_{a_2,g_1}\left(x_{b_1}\right)\\&+A_{a_1,g_1}\left(x_{b_2}\right)A_{a_2,g_1}\left(x_{b_2}\right)\Big).\end{aligned}\tag{33}$$

In this L_{int}, we can again turn to Jacobi coordinates and examine it on the subset of Eq.4.

Therefore, we can describe proton interactions with the gluon field, which can interact with meson fields generating secondary mesons. This allows us to describe processes of inelastic and elastic proton scattering withing the framework of multi-particle fields. However as was mentioned before the products of the gluon field functions, which were included into L_{int} Eq.25 and Eq.32, are also examined on the subset of Eq.4. That is why for the construction of these models we have to consider also a two-particle gluon field.

4. Two-Particle Gluon Field

As seen from L_{int} in Eqs.25 and 32 the gluon field $A_{a_1,g_1}(x_1)$ is included in the scalar combinations in Eq.33 with respect to the Lorentz transformation, as well as with respect to the adjoint representation of the global $SU_c(3)$ group. We will consider Eq.33 as two-particle fields. However, examination of the multi-particle gauge field from a physical point of view requires answers to several questions.

The consideration of the multi-particle fields in the previous sections was based on the fact that hadrons contain a certain amount of constituent quarks. In addition, those quarks have such mass that the energy of their interaction is not large enough for creation of new quarks. Therefore, the number of constituent quarks is fixed. This allows one to examine the internal hadron state using a nonrelativistic approximation. But there is a question with respect to the gauge field: what it represents from a physical point of view? If one considers the two-particle gauge field, what particles will represent the quanta of this field after the quantization procedure? How are they composed? How will the fact that this field is a two-partial field be manifest? We will try to give answers to these questions in this section, since these answers will be based on properties of solutions of the dynamical equations for two-particle gauge field. Let's consider these equations.

By multiplying each of the Euler-Lagrange equations for non-Abilian gauge field $A_{a_1,g_1}(x_1)$ with each component for the field $A_{a_2,g_2}(x_2)$, we have:

$$A_{a_4,g_4}(x_2)\,g^{a_3a_2}\hat{D}_{a_3}\left(A_{a_3,g_3}(x_1)\right)F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)=0. \quad (34)$$

Here we introduce the following notation for the tensor of the gauge field:

$$F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)= \frac{\partial A_{a_1,g_1}(x_1)}{\partial x_1^{a_2}}-\frac{\partial A_{a_2,g_1}(x_1)}{\partial x_1^{a_1}}-gc_{g_1g_2g_3}A_{a_1,g_2}(x_1)A_{a_2,g_3}(x_1), \quad (35)$$

where $c_{g_1g_2g_3}$ are the structure constants of the gauge group. Using the following notation $F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)$ we would like to emphasize that here we are considering the equation with respect to the field $A_{a_1,g_1}(x_1)$ and the field tensor just used as denotation. Moreover, in Eq.35 we use the notation for the extended derivative in the adjoint representation of the gauge group:

$$\hat{D}_{a_3}\left(A_{a_3,g_3}(x_1)\right)F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)= \frac{\partial F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)}{\partial x_1^{a_3}} -gA_{a_3,g_3}(x_1)c_{g_3g_1g_2}F_{a_1a_2,g_2}\left(A_{a_k,g_k}(x_1)\right). \quad (36)$$

Note, that the system of Eqs.34 is invariant with respect to local gauge redirection. Indeed, expressing $A_{a_1,g_1}(x_1)$ and $A_{a_2,g_2}(x_2)$ fields by gauge equivalent field configurations, we have:

$$\begin{aligned}&\left(\exp\left(\hat{I}_{g_5}\theta_{g_5}(x_2)\right)\right)_{g_4g_6}\left(\exp\left(\hat{I}_{g_7}\theta_{g_7}(x_1)\right)\right)_{g_1g_8}\\ &\times A'_{a_4,g_6}(x_2)\,g^{a_3a_2}\hat{D}_{a_3}\left(A'_{a_3,g_3}(x_1)\right)F_{a_1a_2,g_8}\left(A'_{a_k,g_k}(x_1)\right)\\ &+\frac{\partial\theta_{g_4}(x_2)}{\partial x^a}\left(\exp\left(\hat{I}_{g_7}\theta_{g_7}(x_1)\right)\right)_{g_1g_8}g^{a_3a_2}\\ &\times\hat{D}_{a_3}\left(A'_{a_3,g_3}(x_1)\right)F_{a_1a_2,g_8}\left(A'_{a_k,g_k}(x_1)\right)=0.\end{aligned} \quad (37)$$

where we denote this configurations with primes. In Eq.37 the parameters of a local gauge transformation denoted by $\theta_{g_k}(x_1)$ and $\theta_{g_k}(x_2)$ respectively, $\hat{I}_{g_k}$ are generators of the adjoint representation of the gauge group. As seen from this expression, the inhomogeneous term that includes derivatives of the transformation parameter entering the equality as a product on the expression which is equal to zero as a result of the dynamical equations for the single-particle field $A_{a_1,g_1}(x_1)$. Therefore, only the first term remains in Eq.37. Convoluting this term by indices g_1 and g_4 with matrices which are inverse of the matrices of the adjoint representation, we get the same system of equations for the field functions with primes as was written in Eq.34. Hereby one may say that in local gauge transformations, the left hand side of the system, Eq.34, transforms with respect to the tensor product of the two adjoint representations of the gauge group. Hence this left hand side of Eq.34 is a tensor of the Lorentz group with respect to indices a_1, a_4, and is also a tensor of local gauge transformations with respect to indices g_1, g_4. Furthermore we denote this tensor as $L_{a_1a_4;g_1g_4}(x_1,x_2)$. For each these pairs of indices one may identically represent this tensor as a sum of the unit tensor, the antisymmetric tensor and the symmetric tensor with zero trace:

$$L_{a_1a_4;g_1,g_4}(x_1,x_2)=l(x_1,x_2)\,g_{a_1a_4}\delta_{g_1g_4}+\cdots. \quad (38)$$

Here we just pick out the term that is composed of unit tensors, the rest of the terms which are irrelevant for us denoted as "...". Take into consideration that all such terms are linearly independent tensors Eq.34, we obtain:

$$l(x_1,x_2)\,g_{a_1a_4}\delta_{g_1g_4}=0, \tag{39}$$

or

$$\begin{aligned} &g^{a_1a_4}\delta_{g_1g_4}A_{a_4,g_4}(x_2)\,g^{a_3a_2} \\ &\times\hat{D}_{a_3}\left(A_{a_3,g_3}(x_1)\right)F_{a_1a_2,g_1}\left(A_{a_k,g_k}(x_1)\right)=0. \end{aligned} \tag{40}$$

Given that $x_1\neq x_2$, the tensor $\delta_{g_1g_2}$ is not transformed into itself under the local transformation of Eq.37 as was shown above. But because the local invariance of Eq.34 is achieved by a combination of the transformation of Eq.34 and a convolution with the components of matrices, that are inverse with respect to the matrices of the adjoint representation, then after such combination, the tensor $\delta_{g_1g_2}$ will already be transformed into itself. Moreover the Eqs.39-40 will preserve their form with respect to a local gauge transformation.

We symmetrize Eq.40 with respect to the variables x_1 and x_2. The obtained equation will include two tensors:

$$\begin{aligned} &A_{a_1,g_1}(x_1)A_{a_2,g_2}(x_2)=\varphi_{a_1,a_2;g_1g_2}(x_1,x_2), \\ &A_{a_1,g_1}(x_1)A_{a_2,g_2}(x_1) \\ &+A_{a_1,g_1}(x_2)A_{a_2,g_2}(x_2)=\chi_{a_1,a_2;g_1g_2}(x_1,x_2). \end{aligned} \tag{41}$$

For these tensors we have the relations:

$$\chi_{a_1,a_2;g_1g_2}(x_1,x_1)=2\varphi_{a_1,a_2;g_1g_2}(x_1,x_1). \tag{42}$$

Each of the tensors Eq.41 will be decomposed into invariant tensors with respect to Lorentz transformations and to global internal transformations marking out the scalar part:

$$\begin{aligned} &\varphi_{a_1,a_2;g_1g_2}(x_1,x_2)=\varphi(x_1,x_2)\,g_{a_1a_2}\delta_{g_1g_2}+\cdots, \\ &\chi_{a_1,a_2;g_1g_2}(x_1,x_2)=\chi(x_1,x_2)\,g_{a_1a_2}\delta_{g_1g_2}+\cdots. \end{aligned} \tag{43}$$

Note, that in this expression, we pick just these terms $\varphi(x_1,x_2)$ and $\chi(x_1,x_2)$ because they are defined by Eq.33 and entering into the interaction Lagrangian Eq.25 and 32.

Now we will try to impose symmetrical with respect to x_1 and x_2 Eq.40 the solution that contains only terms selected in Eq.43. The rest of the terms in this solution are equal to zero. Note, that if, for the construction of the tensors of Eq.41 one uses solutions of the single-particle equations, then the antisymmetric and the symmetric tensors with zero trace part, which are denoted in Eq.43 as "...", in general are not equal to zero. Making them vanish means that from this moment we start considering a new pure two-particle field. Furthermore, for the two-particle gauge field we adopt the same sequence of the local transformations as in the previous sections. Namely, firstly we transform the whole tensors from Eq.41, then we select from them a part that is proportional to the unit tensor. And by putting all other tensor parts equal to zero, except those that are selected in Eq.43, we obtain:

$$\begin{aligned} &\frac{\partial^2\varphi(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}g^{a_1a_2}+\frac{\partial^2\varphi(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}}g^{a_1a_2} \\ &-\frac{1}{2}g^2\varphi(x_1,x_2)\chi(x_1,x_2)=0. \end{aligned} \tag{44}$$

Now we introduce new two-particle fields $a(x_1,x_2)$ and $b(x_1,x_2)$ instead of $\varphi(x_1,x_2)$ and $\chi(x_1,x_2)$ using the following relation:

$$\begin{aligned} &\varphi(x_1,x_2)=a(x_1,x_2)-b(x_1,x_2), \\ &\chi(x_1,x_2)=a(x_1,x_2)+b(x_1,x_2). \end{aligned} \tag{45}$$

Taking into account Eq.45 instead of Eq.44, we get:

$$\begin{aligned} &g^{a_1a_2}\frac{\partial^2a(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}+g^{a_1a_2}\frac{\partial^2a(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}} \\ &-\frac{1}{2}g^2a^2(x_1,x_2)-\left(g^{a_1a_2}\frac{\partial^2b(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}\right. \\ &\left.+g^{a_1a_2}\frac{\partial^2b(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}}-\frac{1}{2}g^2b^2(x_1,x_2)\right)=0. \end{aligned} \tag{46}$$

If we denote the left hand side of Eq.46 that contains field $a(x_1,x_2)$ and its derivatives as $k(x_1,x_2)$ then:

$$\begin{aligned} &g^{a_1a_2}\frac{\partial^2a(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}+g^{a_1a_2}\frac{\partial^2a(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}} \\ &-\frac{1}{2}g^2a^2(x_1,x_2)=k(x_1,x_2), \end{aligned} \tag{47}$$

as seen from Eq.46 the part that includes the $b(x_1,x_2)$ field should be equal to the same function $k(x_1,x_2)$. We get the most simple problem by considering the case where $k(x_1,x_2)$ is equal to some constant. Let's consider the physical consequences which are driven by this particular case.

Therefore, we impose a partial solution for Eq.46 that is defined by the following relations:

$$g^{a_1 a_2}\frac{\partial^2 a(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}+g^{a_1 a_2}\frac{\partial^2 a(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}}-\frac{1}{2}g^2a^2(x_1,x_2)=k,$$
$$g^{a_1 a_2}\frac{\partial^2 b(x_1,x_2)}{\partial x_1^{a_1}\partial x_1^{a_2}}+g^{a_1 a_2}\frac{\partial^2 b(x_1,x_2)}{\partial x_2^{a_1}\partial x_2^{a_2}}-\frac{1}{2}g^2b^2(x_1,x_2)=k, \quad (48)$$

where k is some arbitrary constant.

Taking into account that the interaction Lagrangian Eq.32 depends only on the $a(x_1,x_2)$, we will examine only the first equation of Eq.48. Assuming that due to similarity of these fields, all obtained results, will also be relevant for the field $b(x_1,x_2)$. Turning to the Jacobi coordinates Eq.3, we get

$$\frac{1}{2}g^{a_1 a_2}\frac{\partial^2 a(X,y_1)}{\partial X^{a_1}\partial X^{a_2}}+2g^{a_1 a_2}\frac{\partial^2 a(X,y_1)}{\partial y_1^{a_1}\partial y_1^{a_2}}-\frac{1}{2}g^2a^2(X,y_1)=k. \quad (49)$$

Before we start examining this equation let's make it dimensionless. Considering it being dimensionless, we come straight forward to the conclusion that the gauge field tensor should have dimensions of $(1/l^2)$, where l is the unit of length. Then the gauge field has dimensions of $(1/l)$. Moreover, based on the form of the extended derivative, one may conclude that the coupling constant g is dimensionless. Based on this, in Eq.49 further we assume that the two-particle field $a(X,y_1)$ was made dimensionless by $(1/l^2)$, the constant k - by $(1/l^4)$, and the coordinates as well as the internal coordinates were made dimensionless by l. There is no special notation for dimensionless quantities, but we will assume that the following dimensionless procedure has already been done in Eq.49. Thus we will further consider that all quantities are already dimensionless.

The nonhomogeneous equation, Eq.49, has different features in cases $k\ge 0$ or $k<0$. In case of $k<0$, the equation allows for partial solutions $a_0=-\left(2|k|/g^2\right)^{1/2}$, or $a_0=\left(2|k|/g^2\right)^{1/2}$ which may be reduced to constants. Then by introducing a new field $a_1(X,y_1)$ with the help of the relation $a(X,y_1)=a_0+a_1(X,y_1)$ one may obtain a homogeneous equation for this field.

In case of $k\ge 0$ in order to come to the homogeneous equation, we need at least one partial solution of Eq.49. This solution should be a function of internal variables $a_0(y_1)$. On the subset of Eq.4 the equation for this function has the form:

$$-\Delta_{\mathbf{y}_1}a_0(\mathbf{y}_1)-\frac{1}{4}g^2a_0^2(\mathbf{y}_1)=k. \quad (50)$$

Before we start analyzing features of this expression, let's examine this expression from a physical point of view. Representing the field $a(X,y_1)$ as:

$$a(X,\mathbf{y}_1)=a_0(\mathbf{y}_1)+a_1(X,\mathbf{y}_1), \quad (51)$$

the substitution we should make in the Lagrangian in Eq.32. But then in the full baryon Lagrangian $L=L_0+L_{\text{int}}$, where L_0 is defined in Ref.[21] and L_{int} by Eq.32, the terms that include $a_0(\mathbf{y}_1)$ may be grouped with terms that contain $3m$. In this way, one may include terms with $a_0(\mathbf{y}_1)$ into L_0, at the same time terms with $a(X,\mathbf{y}_1)$ can be considered as the interaction Lagrangian. For this new L_0 Lagrangian one may find an approximated Euler-Lagrange equation and instead of Eq.31 obtain:

$$i\gamma^a_{s_1s_2}\frac{\partial\Psi_{s_2}(X,\mathbf{y}_1,\mathbf{y}_2)}{\partial X^a}-\left(3m-\frac{3}{4m}\Delta_{\mathbf{y}_1}-\frac{1}{m}\Delta_{\mathbf{y}_2}-\frac{g^2}{9m}a_0(\vec{y}_2)-\frac{g^2}{9m}a_0\left(\mathbf{y}_1-\frac{1}{2}\mathbf{y}_2\right)-\frac{g^2}{9m}a_0\left(\mathbf{y}_1+\frac{1}{2}\mathbf{y}_2\right)\right)\Psi_{s_1}(X,\mathbf{y}_1,\mathbf{y}_2)=0. \quad (52)$$

Thus the function $(-a_0(\mathbf{y}_1))$ included into the internal Lagrangian as potential energy of quark interactions inside the hadron in nonrelativistic approximation with accuracy up to terms with $g^2/9m$. Based on this we want to analyze features of the function $(a_0(\mathbf{y}_1))$ as a solution of Eq.50.

We turn Eq.50 into spherical variables and examine the simplest spherically symmetric solution of this equation. Denote $|\mathbf{y}_1|\equiv r$ and introduce a new unknown function $(a_2(\mathbf{y}_1))$ instead of $(a_0(\mathbf{y}_1))$, using the following relation:

$$a_0(r)=\frac{a_2(r)}{r}. \quad (53)$$

Then for the function $(-a_2(r))$ we have:

$$\frac{d^2(-a_2(r))}{dr^2}=kr+\frac{1}{4}g^2\frac{(-a_2(r))^2}{r}. \quad (54)$$

As was noted from Eq.53 we get the function $(-a_0(r))$ with finite value at $r = 0$, also with finite derivative at this point if we apply the boundary conditions of Eq.54 as:

$$a_2(r)\big|_{r=0} = 0, \frac{da_2(r)}{dr}\bigg|_{r=0} = C. \quad (55)$$

The value of C is not defined from the analyticity of the function $(-a_0(r))$ and therefore it can be arbitrary. Thus let's analyze the features of the solution of Eq.54 with respect to the different values of C. The features of the function $(-a_2(r))$ can be analyzed with the help of an expansion into a Taylor series taking into account that we are looking for the function $(-a_2(r))$ at the manifold of functions that are analytic at the vicinity of the point $r = 0$. Due to the boundary conditions Eq.55, the expansion of the function $(-a_2(r))$ into a Taylor series can be started from the second order term. But if we substitute the expression:

$$(-a_2(r)) = \sum_{l=2}^{\infty} q_l r^l, \quad (56)$$

into Eq.54, we get:

$$\begin{aligned} &q_2 = 0, q_4 = 0, q_5 = 0, q_6 = 0, \\ &q_3 = \frac{k}{6}, q_7 = \frac{g^2k^2}{2688}, \dots \end{aligned} \quad (57)$$

It is noticeable from these relations that Eq.54 has a nontrivial solution that satisfies the boundary conditions Eq.55. From these relations one may also note that in the case $k = 0, C = 0$ this equation will just have a trivial solution with the same boundary conditions. The case $k = 0$ we will be discussed in detail later in the paper.

By integrating Eq.54 twice with the conditions of Eq.55 we get:

$$(-a_2(r)) = \frac{k}{6}r^3 + \frac{1}{4}g^2 \int_0^r dr_1 \int_0^{r_1} dr_2 \frac{(a_2(r_2))^2}{r_2}, \quad (58)$$

whence, using the inalienability of the integrand in expression Eq.58 and relation Eq.53 we obtain:

$$(-a_0(r)) \geq \frac{k}{6}r^2. \quad (59)$$

Thus, obtained from two-particle equations, the QCD potential of quark interactions, $(-a_0(r))$, provides not only the possibility of existence of hadrons as bound particles, but also quark confinement. Note, that the case, $k > 0, C = 0$, which has just been discussed in addition to quark confinement also describes the features of asymptotic freedom.

Now we examine the case $k > 0, C > 0$. By integrating Eq.54 twice we get:

$$(-a_2(r)) = Cr + \frac{k}{6}r^3 + \frac{1}{4}g^2 \int_0^r dr_1 \int_0^{r_1} dr_2 \frac{(-a_2(r_2))^2}{r_2}. \quad (60)$$

From this relation we can make an estimation of:

$$(-a_2(r)) > Cr \geq 0, \quad (61)$$

and as a consequence we get the inequality

$$(-a_2(r)) > Cr + \frac{1}{6}\left(k + \frac{g^2C^2}{4}\right)r^3. \quad (62)$$

Thus, as seen from this relation, in case of $C > 0$ the potential $(-a_0(r))$ will ensure the existence of a bound state of quarks and confinement not only in case of $k > 0$, but also at $k = 0$. By considering the Taylor series at the vicinity of $r = 0$, by analogy with Eq.56 and Eq.57 we get:

$$(-a_2(r)) \xrightarrow{r \to 0} Cr + \frac{1}{6}\left(k + \frac{1}{4}g^2C^2\right)r^3. \quad (63)$$

Therefore, the conclusion of the description of the asymptotic freedom remains valid in the case of $k \;\; 0, C \neq 0$, as well as in the case of $k = 0, C \neq 0$.

We now analyze the existence of the bound states and confinement in the case $k > 0, C < 0$. We are interested in the asymptotic behavior of the function $(-a_0(r))$, hence the behavior of $(-a_2(r))$ at high r. In order to make this analysis, we rewrite Eq.61 as:

$$(-a_2(r)) = r\left(C + \frac{k}{6}r^2\right) + \frac{1}{4}g^2 \int_0^r dr_1 \int_0^{r_1} dr_2 \frac{(-a_2(r_2))^2}{r_2}. \quad (64)$$

The expression $\left(C + \frac{k}{6}r^2\right)$ at relatively high values of r becomes positive. If we denote by r_0 the value of r at which this expression is equal to some positive value C_0, then for $r > r_0$ we get:

$$(-a_2(r)) > C_0 r + \frac{1}{4}g^2 \int_{r_0}^r dr_1 \int_{r_0}^{r_1} dr_2 \frac{(-a_2(r_2))^2}{r_2} > C_0 r > 0. \quad (65)$$

As a result of this inequality we have the estimate:

$$\left(-a_2\left(r\right)\right) > C_0 r + \frac{1}{24} g^2 \left(C_0\right)^2 r^3. \tag{66}$$

Thus, as seen, the conclusion about the description of the quark confinement and asymptotic freedom for the case $k>0$ took place on arbitrary value of the constant C.

For the case $k<0$, features of the solutions are much more diverse, for details see [21].

Considering the equations and the features of their solutions allows us to reply one question: What is the two-particle gauge field from a physical point of view? To reply it, we return to Eq.49 and examine its solution on the subset of simultaneity, Eq.4. This equation is nonhomogeneous. If we want to quantize the fields $a\left(X,\mathbf{y}_1\right)$ and $b\left(X,\mathbf{y}_1\right)$, we have to start from homogenues equations. This can be done by substituting Eq.49 into Eq.51, where $a_0\left(\mathbf{y}_1\right)$ is one of the solutions of Eq.50 and $a_1\left(X,\mathbf{y}_1\right)$ is a new unknown function. Therefore, we get the following equation for this unknown function:

$$\begin{aligned} &g^{a_1 a_2}\frac{\partial^2 a_1\left(X,\mathbf{y}_1\right)}{\partial X^{a_1}\partial X^{a_2}} + \left(-4\Delta_{\mathbf{y}_1} a_1\left(X,\mathbf{y}_1\right)\right. \\ &\left.+2g^2\left(-a_0\left(\mathbf{y}_1\right)\right)a_1\left(X,\mathbf{y}_1\right)\right) - g^2\left(a_1\left(X,\mathbf{y}_1\right)\right)^2 = 0. \end{aligned} \tag{67}$$

To describe the field $a_1\left(X,\mathbf{y}_1\right)$, we will apply methods of perturbation theory. Thus we will drop the nonlinear terms from Eq.67 and afterwards we get the equation that is generated by the action:

$$\begin{aligned} S = \int d^4 X d\mathbf{y}_1 &\left(\frac{1}{2} g^{a_1 a_2} \frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial X^{a_1}} \frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial X^{a_2}}\right. \\ &\left.-2\frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial y_1^b}\frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial y_1^b} + \frac{g^2}{2} a_0\left(\mathbf{y}_1\right)\left(a_1\left(X,\mathbf{y}_1\right)\right)^2\right). \end{aligned} \tag{68}$$

Noether's theorem for the field with such action leads to the following expression for the energy of the field $a_1\left(X,\mathbf{y}_1\right)$ at zeroth order in perturbation theory:

$$\begin{aligned} P_0 &= \frac{1}{2}\int dX^1 dX^2 dX^3 d\mathbf{y}_1 \\ &\times\left(\sum_{b=0}^{3}\left(\frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial X^b}\right)^2 + 4\sum_{b=1}^{3}\left(\frac{\partial a_1\left(X,\mathbf{y}_1\right)}{\partial y_1^b}\right)^2\right. \\ &\left.+g^2\left(-a_0\left(\mathbf{y}_1\right)\right)\left(a_1\left(X,\mathbf{y}_1\right)\right)^2\right). \end{aligned} \tag{69}$$

As can be seen from this expression, the requirement of finite energy leads to a quite fast approach to zero of all possible realizations of the field $a_1\left(X,\mathbf{y}_1\right)$ when $|\mathbf{y}_1|$ is approaching infinity in the case when $\left(-a_0(\mathbf{y}_1)\right)$ preserves quark confinement. Thus, the same function $\left(-a_0(\mathbf{y}_1)\right)$ that in Eq.52 provides quark confinement in equation for the zeroth order approximation for the field $a_1\left(X,\mathbf{y}_1\right)$ is providing gluon confinement:

$$\begin{aligned} &g^{a_1 a_2}\frac{\partial^2 a_1\left(X,\mathbf{y}_1\right)}{\partial X^{a_1}\partial X^{a_2}} \\ &+\left(-4\Delta_{\mathbf{y}_1} a_1\left(X,\mathbf{y}_1\right) + 2g^2\left(-a_0\left(\mathbf{y}_1\right)\right)a_1\left(X,\mathbf{y}_1\right)\right) = 0, \end{aligned} \tag{70}$$

Eq.70 is similar to 30 with internal variables but with the minor difference that the operator

$$\begin{aligned} &\left(\hat{H}^{\text{internal}}\right)^2\left(a_1\left(X,\mathbf{y}_1\right)\right) = \\ &-4\Delta_{\mathbf{y}_1} a_1\left(X,\mathbf{y}_1\right) + g^2\left(-a_0\left(\mathbf{y}_1\right)\right)a_1\left(X,\mathbf{y}_1\right), \end{aligned} \tag{71}$$

which is similar to the Hamiltonian of the two-particle system here is the square of the Hamiltonian of the internal system. Indeed if one will turn from the field $a_1\left(X,\mathbf{y}_1\right)$ to its Fourier representation using the coordinates $X^a, a = 0,1,2,3$:

$$a_1\left(X,\mathbf{y}_1\right) = \frac{1}{\left(2\pi\right)^{3/2}}\int d^4 X a_1\left(p,\mathbf{y}_1\right) \mathrm{e}^{ip_a X^a}, \tag{72}$$

then we get the equation:

$$\left(\hat{H}^{\text{internal}}\right)^2\left(a_1\left(p,\mathbf{y}_1\right)\right) = \left(g^{a_1 a_2} p_{a_1} p_{a_2}\right)a_1\left(p,\mathbf{y}_1\right). \tag{73}$$

Thus, as seen from this equation,the eigenvalues of the operator, Eq.71, are equal to the square of the internal energy of the two-gluon particle for different internal states of this particle. Indeed, in the case of the potential $\left(-a_0(\mathbf{y}_1)\right)$ that provides confinement, one may set boundary conditions for Eq.54 in a way that all eigenvalues of the operator, Eq.71, will be positive. However, as follow from the previous discussion, this is true for cases where $k>0, C\ge 0$ or $k=0, C>0$. Given Eq.73, this means that the two-gluon particle has nonzero mass and there exists of a rest reference frame for it. As follow from Eq.73, the eigenvalues of the operator in Eq.71 is equal to the square of the energy of the two-gluon particle in this reference frame.

From a formal point of view this situation looks similar to the one that was considered in [21] for the two-particle field that composed of two massive fields, but with one significant difference. In [21] we have contribution from terms with $(2m)^2$, that comes from the mass of bound particles. Thus one may assume that the eigenvalues of a sum of the kinetic energy and potential energy operators, that are in a bound state, are small with respect to the total rest energy of these particles. In this way one may just keep linear terms with respect to these operators in the square of the internal Hamiltonian. But in the current case we are considering the bound state of two massless gluons, which can be noticed from Eq.71 due to absence of a term similar to $(2m^2)$. Thus we can no longer assume that the eigenvalues of the operator in Eq.71 are small additive terms to something, so in order to examine the internal Hamiltonian of the two-gluon particle we have to take the square root of the operator in Eq.71.

A formal definition of this square root does not pose any problems, since in case of the confinement operator, Eq.71, it has just a discrete spectrum of eigenvalues. By choosing the system of eigenvalues that corresponds to this operator as a basis, on may represent it as a matrix. This matrix in this basis will be diagonal. On the main diagonal it will contain the eigenvalues of this operator. As was noticed above by choosing appropriate boundary conditions, Eq.55, they could be positive. If we will change these eigenvalues by the square root of them, then we get a matrix that would be a self-conjugate operator due to the realness of values of the square root. Therefore this operator can be used to define the operator $\hat{H}^{\text{internal}} = \sqrt{\left(\hat{H}^{\text{internal}}\right)^2}$. After the quantization procedure the sign in front this square root could be interpreted in a natural way by considering corresponding coefficients as creation and annihilation operators of the two-gluon particles.

Nevertheless, there is no need for an arbitrary definition of the operator $\hat{H}^{\text{internal}}$, since as seen from the previous discussions all relations for the free field $a_1\left(X, \mathbf{y}_1\right)$, as well as for the field that interacts with others fields include especially the operator of Eq.71. Thus one may say that dynamics of the two-particle gauge field is defined not by the internal Hamiltonian but by the square of it. Analysis of such kind of "weird" situation allows one to make a conclusion about the nature of the two-particle gluon field. Since the operator $\left(-4\Delta_{\mathbf{y}_1}\right)$ that is included in Eq.71 may be written as:

$$-4\Delta_{\mathbf{y}_1} = -2\left(\Delta_{\mathbf{x}_1} + \Delta_{\mathbf{x}_2}\right) + \Delta_{\mathbf{X}}. \tag{74}$$

We will consider the two-gluon particle in their center mass reference frame. Then the eigenvalue of $\Delta_{\mathbf{X}}$ is equal to zero, the rest of the operators could be considered as operators of the square of the energy each of the gluons:

$$\begin{aligned} \hat{E}_1^2 &= -2\Delta_{\mathbf{x}_1}, \\ \hat{E}_2^2 &= -2\Delta_{\mathbf{x}_2}, \end{aligned} \tag{75}$$

The operator $\left(-g^2 a_0(\mathbf{y}_1)\right)$ describes the interaction between gluons. Due to this interaction the internal state of the two-gluon particle is not an eigenstate anymore, neither of energy nor of momentum of these gluons. But we can discuss the mean values of those quantities. Then from the definition of Eq.75 we have:

$$\left\langle E_a^2 \right\rangle = 2\left\langle \mathbf{p}_a^2 \right\rangle, a = 1, 2. \tag{76}$$

If we consider the "initial gluon", namely those that appear in QCD at the zero order of pertubation theory, then one will leave just linear terms in the equation. When QCD equations coincide with well-known QED equations, then due to masslessness of this gluon the relation between eigenvalues of the energy, E, and momentum, $\mathbf{p}$, in a state that is an eigenstate for both of these quantities, has the form $E = |\mathbf{p}|$. Then for the square of the average values in the noneigenstate for these quantities, we get exactly Eq.76. The factor of two appears in this relation due to two polarization states of the "initial gluon". Therefore, masslessness of the "initial gluon" leads to the fact that the square of the average momentum determines the square of the average energy of this particle, but not the average energy. This was shown previously for nonrelativistic massive particles.

One may conclude from the above discussion, that two interacting gluons which form the two-gluon particle stay massless, even if their properties are significanly different from the properties of the "initial gluon" from pertubation theory. Each of these gluons interact with one another and are in the state that is located in space, hence not in the eigenstate with respect to momentum. Due to this issue the average energy of the massless gluon in the examined state is not zero. Thus if we consider the possibility of creating a new gluon (or several gluons) in this reference frame, then it does not matter if it is massless because this creation will require nonzero energy (similar to the case when the gluon is massive). So, if the interaction of the two-gluon particle with another field cannot provide the required energy, then a new gluon will be not created, and the system will stay a two-gluon system. This makes sense for the consideration of the two-gluon field that describes the bound states of the two interactions between each massless gluons. Candidates for such two-gluon bound states are being searched for in the experiments [22, 23] as well as in theory [24].

By using a representation of the interaction, we can define all multi-particle operators through eigenfunctions of the internal Hamiltonians or squares of internal Hamiltonias that correspond to the lowest eigenvalues. By substituting these representations into the interaction Lagrangian, we can make integration over internal variables and obtain a model. In this model, the operators that correspond to creation and annihilation of hadrons are coupled to operators that correspond to creation and annihilation of two-gluon states.

5. Discussion and Conclusions

The obtained results for the multi-particle fields allow one to compute quantities that are observed in experiments, for instance, in the inelastic and elastic processes of proton scattering. Indeed, we have a proton three-particle bi-spinor field that may interact with the gluon field, and this interaction is described by the Lagrangian in Eq.32. In addition, as seen from dynamical equation, see Eq.67, for the two-gluon field this gluon field is self interacting. For this dynamical equation one can write a Lagrangian that turns it into an Euler-Lagrange equation. Given that Euler-Lagrange equation should include derivatives from the Lagrangian with respect to its fields, then this Lagrangian will include a Lagrangian of self interaction of cubic power, in order to provide terms of second power, see the last term in Eq.67. If one will transform these multi-particle fields with respect to the representation of the interaction, then one has to substitute solutions of the equations for the free field into the Lagrangian of the interaction and self interaction. Since the dependence of the field functions of the free fields from internal variables was found by us as a solution of the eigenvalue problem for internal Hamiltonians, there is a possibility to calculate integrals over internal variables in the interaction Lagrangian. Thus, we come to the problem, of formally matching the problem of ordinary single-particle field theory. For such a model one may use the usual methods of diagram techniques, and in this way compute differential and total cross sections that can be compared with available experimental data [25, 26]. The framework shown in this paper allows one to compute the experimental properties of the inelastic scattering by adding to this model the interaction of the two-gluon field with a pseudo-scalar meson field, see the Lagrangian in Eq.25. For both elastic and inelastic scattering we obtain the "correct" law of the energy-momentum conservation in the sense as was discussed in the introduction. The model obtained in this way will look similar to the famous φ^3 model. Thus the results obtained in the framework of this model [2] allow us to hope for a successful description of the experimental data.

Note, that for the reasons mentioned in the introduction, one can make the assumption that in principle the multi-particle field operators cannot be expressed through the single-particle field operators. It seems that the dynamics of multi-particle fields in principle may not be obtained from single-particle field theories. Indeed, if we consider the single-particle fields, then we come to the momentum representation starting from any kind of representations. But as a result of the requirements of a relativistic theory, the presence of the "self" momenta in the single-particle state requires the presence of the "self" energy. While the two-particle system only has the energy of the whole system, and not the energies of individual particles. It look like the multi-particle fields are "independent" objects that should be considered regardless of single-particle fields.

Using this fact one can explain the difference in the potentials that provide confinement and asymptotic freedom that we found in this work with respect to those that are used in lattice calculations [27]. Really, the lattice calculations are aimed to calculate the continiuos integral with respect to single-particle field configurations. But, as discussed in the introduction, the problem occurred before we choose the method, for instance, lattice calculations, for computing the time evolution matrix element or the operator of scattering. The problem is manifested in the choice of states for which we would like to compute the matrix element. In lattice calculations these states are constructed as result of acting with the single-particle creation operator on the vacuum. Even if one will consider the matrix element between vacuum states, then these states are defined in the way that the single-particle annihilation operator set them to zero. And integration is done over single-particle fields. Thus, if this assumption is true, then no matter how precise the calculations would be that use the single-particle fields, they are not able to take into account the multi-particle effects.

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