Physics Journal
Vol. 2, No. 2, 2016, pp. 104-113
http://www.aiscience.org/journal/pj

Further Studies on D.C. Noise in Plasma Due to Electron-Ion Collisions

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Abstract

D.C. noise in the plasma is generated due to the charge-oscillations set in the colliding ions by the electrons of the electron-ion pairs in equilibrium with the ambient temperature of the plasma. These electron-ion pairs are formed on otherwise free space. This d.c. noise radiation is a coupled one having a maximum value of the frequency denoted by a cut-off frequency which is determined by the density of ions in the plasma. The d.c. noise radiation temperature for this coupled noise is 851^0K at ambient temperature of 273^0K for singly charged particles of ionized hydrogen type gas molecules with low charge-carrier average density of electrons or ions i.e $N_e \sim 10^{15} m^{-3}$ in a typical star dust of interstellar space(s) in the galaxy/galaxies with a characteristic temperature of 1021^0K having a cut off frequency of 21.29 x 10^{12} Hz, where it is assumed that there exit a randomly oriented average d.c. electric field of 10 V/m due to accumulation of charge-carriers of oppositely charged particles in some distant places in the galaxy/galaxies.

Keywords

Electron, Ion, Collisions, Charge-Oscillations, Coupled-Noise, D.C.-Noise, Radiation, Radiation-Temperature

Received: September 16, 2015 / Accepted: October 21, 2015 / Published online: January 17, 2016

1. Introduction

This analysis holds good for a plasma in interstellar space of galaxy/galaxies having star dust of singly charged particles of ionized hydrogen type gas molecules with low charge-carrier average density of electrons or ions i.e $N_e \sim 10^{15}\ m^{-3}$ in thermal equilibrium at temperature T (~273 ^{0}K, where ^{0}K is Degree Kelvin) of the ambient - (for General Reference, refer to Dyson 1997 [1], Kennicutt and Evans 2012 [2] and Weedman etc. 1981 [3]). Moreover there exists a randomly oriented average d.c. electric field over this type of plasma due to accumulation of oppositely charge-carries in some distant regions in the space of the galaxy/galaxies giving an average d.c electric field $E_{dc}$of order ~10 V/m, where the charge-carriers are moving about constituting local currents in the star dust. Here E_{dc} is oriented randomly in space.

General references for “Noise in Plasmas” may be found with Monnig and Hieftje1989 [4], Kojima, Takayama and Shimauchi 1954 [5], Kojima and Takayama 1950 [6], Epstein 1988 [7], Peperone 1962 [8].

In a previous plasma model described (Nandedkar and Bhagavat 1970) [9], the d.c. noise spectrum in the plasma due to electron-ion collisions is analyzed, where the electrons and the ions in the plasma are considered to be in thermal equilibrium at ambient temperature T of the plasma. Objective of this paper is to illustrate D.C. noise phenomenon of the previous plasma model (Nandedkar and Bhagavat 1970 [9]) in light of the plasma work done in presence of plasma damped oscillations (Nandedkar and Bhagavat 1970 [10] & Nandedkar 2016 [11]) with reference to specific numerical analysis which was not done then and to provide more correct solution to the problem of electron-ion collision frequency in the plasma which is considered in appendix of this paper.

In this paper further aspects of the phenomenon of coupled noise (Nandedkar and Bhagavat 1970 [9]) are analyzed with

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reference to interstellar plasma of star dust of the galaxy/galaxies mentioned above. At the time of electron-ion collisions, charge-oscillations are considered to set in the colliding ions which generate the d.c. noise. The absorption of the d.c. energy by the electron when the electron moves through a distance of mean free path parallel to a d.c. electric field E_{dc} in the plasma is considered to generate the d.c. (coupled) noise by setting in the charge-oscillations in the colliding ions. The charge-oscillations of the ions generate coupled noise. These ions in the plasma are separated on an average by a distance of $2R_o$ where R_o is the average separation of an electron from an ion in the plasma given by $R_o = \left(3/_{4\pi}\right)^{1/3} N_e^{-1/3}$, where N_e is the electron or the ion density in the plasma. Since the mass of an ion is extremely large as compared to that of an electron, so that the ions are considered to be stationary ones as compared to the moving electrons in the plasma. In other words, the ions in the plasma are considered as fixed ones (separated on an average by a distance of $2R_o$) with reference to the lighter electrons. These ions are electrostatically coupled with respect to each other and at electron-ion collisions they are assumed to generate and support coupled noises by setting in coupled charge-oscillations of the ions. Or, in other words the coupled noise is associated with coupled charge-oscillatory modes of ions in the plasma at electron-ion collisions in presence of the d.c. electric field E_{dc} in the interstellar space mentioned above. The maximum frequency of the coupled noise depends on the density of ions in the plasma. Whereas the minimum frequency of the noise is considered to be zero for a finite volume of plasma of dimensions much larger than dimensions of the electron-ion pair, where the finite volume of the plasma is considered to have a large number of the charge-carriers so as to use the methods of gas-kinetics and quantum statistics. This coupled noise is the (electromagnetic) radiation in nature.

Here the electron and the ion in the plasma form an electron-ion pair. The electron of the pair absorbs energy from the d.c. electric field (in moving through a distance of electron-ion mean free path parallel to the d.c. electric field) and this energy is transferred to the ion of the pair at the collision, and the ion in turn executes a coupled charge-oscillatory mode described in terms of the quantum energy states. Absorption of the d.c. energy by the electron of the pair and generation of the coupled noise by the ion of the pair are both inter related phenomenon and in this case neither the electron nor the ion of the pair in the plasma interacts back with the generated noise. And as such the noise radiation would find itself in otherwise free space on which electron-ion pairs of the plasma are formed. So that, the noise radiation in the plasma has a velocity which corresponds to the velocity of an electromagnetic wave in free space. In short, absorption and emission of energy by electron-ion pairs in the plasma is analyzed in this paper, which accounts for the r.f. radiation noise in sub-millimeter and less range of r.f. radiation spectrum with a cut-off frequency f_N determined by ion density in the plasma which is ~ 10^{13} Hz, which is near boundary of far infra-red and sub-millimetre range of spectrum of electromagnetic radiation, for the plasma under consideration.

2. Mean Free Path of an Electron during Collision

Electron-ion collision frequency in the plasma is given by following expression [refer to eqns. (A.45) and (A.46) of Appendix of this research-paper]:

$$\nu_{ei} = 8\pi N_e \left(\frac{e^2}{4\pi\epsilon_o m_e}\right)^2 \left(\frac{\pi m_e}{8kT}\right)^{3/2} \alpha\,, \ldots \qquad (1)$$

where ν_{ei} is (actual) collision frequency for *quasi-bound: quasi-free* "electron model" of this analysis which is different than the case of *bound* "electron model" considered by Lorentz in his book on "The Theory of Electrons" (Lorentz 1915 [12]). Here in this analysis motion of ion being much heavier than electron, is neglected. (For this analysis refer also to Nandedkar 2016) [13] and for the electron-ion collision frequency called earlier g_{ei} (Nandedkar 1983) [14]) and here α is a positive quantity given by,

$$\alpha = \left[\left\{\ln(1+\beta^2)^{1/2}\right\} - \frac{1}{2}\left\{\frac{\beta^2}{1+\beta^2}\right\}\right]. \qquad (2)$$

In eqn. (2) the value of β is given [refer to eqns. (A.44) of Appendix of this research-paper] by

$$\beta = \frac{R_o}{p_o} = \left(\frac{32\epsilon_o kT}{e^2}\right)\left(\frac{3}{4\pi N_e}\right)^{1/3}, \qquad (3)$$

here R_o is the average separation of an electron from an ion in the plasma or it is the average separation of an electron-ion pair in the plasma. p_o is the collision parameter of the electron in the hyperbolic orbit with the ion at its internal focus when the electronic deflection is $\pi/2$ (refer to Fig. 1 of the Appendix). In eqn.(1) or (3), e gives the magnitude of the charge of an electron or of an ion, N_e is the ion or the electron density in the plasma, ϵ_o is permittivity of free space and T is equilibrium (ambient) temperature of the plasma. The d.c. electric field in the plasma is randomly oriented, in this case.

When

$$\beta^2 >> 1, \qquad (4)$$

then eqn.(1), using eqns. (2) and (3) gives that,

$$\nu_{ei} = 8\pi N_e \left(\frac{e^2}{4\pi\epsilon_o m_e}\right)^2 \left(\frac{\pi m_e}{8KT}\right)^{3/2} \left[\left\{\ln\left(\frac{32\varepsilon_0 kT}{e^2}\right)\left(\frac{3}{4\pi N_e}\right)^{1/3}\right\} - \frac{1}{2}\right]. \quad (5)$$

Now if λ_{ei} is the mean free path of an electron with respect to an ion in the plasma, then as the electron moves through a distance of λ_{ei} parallel to E_{dc} , then it derives energy $e\lambda_{ei}E_{dcc}$ from the field. Hence the d.c. energy density W_{ei} derived by N_e electrons per unit volume in the plasma under these circumstances is given by,

$$W_{ei} = N_e(e\lambda_{ei}E_{dc}). \quad (6)$$

Eqn. (6) gives the energy density W_{ei} absorbed by the electros in the plasma at electron-ion collisions in the plasma.

In eqn. (6), the value of λ_{ei} using gas-kinetics is given by,

$$\lambda_{ei} = \frac{v_e}{\nu_{ei}}, \quad (7)$$

where v_e is the average thermal velocity of the electron at equilibrium (ambient) temperature T of the plasma given by,

$$v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}, \quad (8)$$

here k is Boltzmann constant and m_e is mass of an electron.

Using eqns. (5) and (8) in eqn. (7), eqn. (7) gives for the value of λ_{ei} that,

$$\lambda_{ei} = \frac{\frac{\left(\frac{4\pi m_e\epsilon_0}{e^2}\right)^2}{8\pi N_e}\left(\frac{8kT}{\pi m_e}\right)^2}{\left[\left\{\ln\left(\frac{32\epsilon_0 kT}{e^2}\right)\left(\frac{3}{4\pi N_e}\right)^{\frac{1}{3}}\right\}-\frac{1}{2}\right]}. \quad (9)$$

3. Phenomenon of Coupled Noise Generation in the Plasma

In the present analysis, the d.c. energy density W_{ei} derived by the electrons of Eqn. (6) in the presence of E_{dc}, is absorbed by the electrons and is scattered randomly at the collisions.. This scattered energy density is assumed to generate charge-oscillations in the colliding ions, where the heavier ions are considered to be stationary and fixed ones relative to lighter (moving) electrons and here the ions are separated with an average separation of $2R_0$ [where $R_o = (3/4\pi)^{1/3}N_e^{-1/3}$]. Here R_o gives the average separation of an ion from an electron. The colliding ions can oscillate without affecting the laws of conservation of energy and angular momentum in the process of the electron-ion collisions as already considered in the treatment of electron-ion collision frequency (refer to Appendix of this research paper, and also Nandedkar 1983 [14]), where the electron-ion collision frequency ν_{ei} is to be considered. The ionic oscillator undergoing charge-oscillations would generate random radiation noise in general. Then the monochromatic noise energy density $(W_{nf})_{dc}$ per unit noise frequency interval ∂f between frequencies f and $f+\partial f$, at the noise frequency f can be considered here as follows:

3.1. Monochromatic Noise Energy Density in the Plasma

The above referred fixed ions in the plasma represent (charge) oscillator generating electromagnetic (noise) radiation (quanta) of various energies and hence momenta. Now consider a small volume element ∂V of the plasma in which the magnitude of momentum vector of the quanta lies between p and $p+\partial p$. The momentum volume associated with ∂V is the volume of a spherical shell of radii p and $p+\partial p$ which is $4\pi p^2\partial p$. And the phase volume ∂V_p between p and $p+\partial p$ at ∂V is given by,

$$\partial V_p = (4\pi p^2\,\partial p)\,\partial V. \quad (10)$$

An elementary unit cell of the phase space has a volume σ_u and is given by,

$$\sigma_u = h^3, \quad (11)$$

where h is Planck constant. This follows from Heisenberg's uncertainty principle.

Hence the number of unit cells ∂G_c contained in the phase volume ∂V_p is given by,

$$\partial G_c = \frac{\partial V_p}{\sigma_u} = \frac{(4\pi p^2\,\partial p)\,\partial V}{h^3}. \quad (12)$$

The number of unit cells per unit volume of the plasma viz., ∂G is further given by,

$$\partial G = \frac{\partial G_c}{\partial V} = \frac{4\pi p^2\,\partial p}{h^3}. \quad (13)$$

Now each unit cell can accommodate a radiation which can have two types of polarizations, so that the density of cells $\partial g'$ is given by,

$$\partial g' = 2\,\partial G = \frac{8\pi p^2\,\partial p}{h^3}. \quad (14)$$

The momentum p of the quantum of radiation is further given by,

$$p=hf/c, \quad (15)$$

using Einstein's energy relationship, where c is the velocity of electromagnetic radiation in free space. Eqn. (15) further gives that,

$$\partial p = \frac{h}{c}\partial f. \quad (16)$$

Equation (14), using eqns. (15) and (16) gives that

$$\partial g' = \frac{8\pi f^2}{c^3}\,\partial f = Z_{ic}(f)\,\partial f, \quad (17)$$

where,

$$Z_{ic} = \frac{8\pi f^2}{c^3}. \quad (18)$$

Here $Z_{ic}(f)\partial f$ gives the number of modes of ionic oscillators or ionic (charge) oscillations or electromagnetic (noise) radiations (waves) generated by the ionic (charge) oscillators per unit volume of the plasma in the noise frequency interval ∂f between frequencies f and f+∂f, whereas $Z_{ic}(f)$ represents the weight factor at frequency f for the electromagnetic (noise) waves or the ionic oscillators where f either denotes the noise frequency or the frequency of the ionic charge-oscillations associated with the ionic oscillators. Here each of the ionic oscillators undergoes the (charge) oscillations with the same frequency.

Equation (17) with reference to the present analysis assumes that the finite volume element of the plasma is large enough as compared to the dimensions of the electron-ion pair but small enough as compared to the overall dimension of the plasma, where the volume element of the plasma under consideration contains a large number of charge-carriers so as to apply the results of quantum (statistical) mechanics and then in this case the minimum value of the (noise) frequency is taken as zero, for all practical purposes.

Further if W is the average energy of the ionic oscillator, then the energy of the electromagnetic (noise) radiations per unit volume of the plasma in the noise frequency interval ∂f between frequencies f and f+∂f is given by,

$$(W_{nf})_{dc}\,\partial f = Z_{ic}(f) W\,\partial f = \frac{8\pi f^2}{c^3} W\,\partial f. \quad (19)$$

Here it is assumed that the frequency of the ionic oscillator(s) is the same as that of the noise radiation. With reference to eqn. (19),

$$(W_{nf})_{dc} = Z_{ic}(f) W = \frac{8\pi f^2}{c^3} W, \quad (20)$$

gives the monochromatic noise energy density i.e. it is the noise energy per unit volume of the plasma per unit noise frequency interval of ∂f between frequencies f and f+ ∂f at the noise frequency f.

Further the value of W in eqn. (20) is obtained by the following analysis:

The oscillating fixed ion in the plasma already mentioned is considered not to have any value of energy from 0 to ∞, but can take only discrete values equal to nW_o, where n=0, 1, 2… and here W_o is a finite discrete amount given by

$$W_o = hf, \quad (21)$$

where h is Planck constant.

In equilibrium condition, at the temperature T_{dc} of the d.c. noise radiation, the value of nW_0 for any energy of the oscillating ion occurs with relative probability,

$$\exp\left(-\frac{nW_o}{kT_{dc}}\right), \quad (22)$$

known as the Boltzmann factor, Here T_{dc} also represents the d.c. noise radiation temperature of ions generating the noise radiation at T_{dc} in equilibrium conditions.

The energy W is obtained by averaging over all values of nW_o with the above probability i.e.

$$W = \frac{\sum_{n=0}^{\infty} nW_o \exp\left(-\frac{nW_o}{kT_{dc}}\right)}{\sum_{n=0}^{\infty} \exp\left(-\frac{nW_o}{kT_{dc}}\right)},$$

or,

$$W = -\frac{\partial}{\partial\left(\frac{1}{kT_{dc}}\right)} \ln \sum_{n=0}^{\infty} \exp\left(-\frac{nW_o}{kT_{dc}}\right). \quad (23)$$

If $\exp(-W_o/kT_{dc}) < 1$, then it follows from eqn. (23) that,

$$W = -\frac{\partial}{\partial\left(\frac{1}{kT_{dc}}\right)} \ln \frac{1}{1-\exp\left(-\frac{W_o}{kT_{dc}}\right)},$$

or,

$$W = \frac{W_o}{\{\exp(W_o/kT_{dc})\}-1}. \quad (24)$$

Thus in eqn. (20), the average energy W is given by eqns. (21) and (24), viz,

$$W = \frac{hf}{\{\exp(hf/kT_{dc})\}-1}. \quad (25)$$

As this noise radiation is generated due to the d.c. energy W_{ei} of eqn.(6) derived and hence absorbed by the electrons in moving through a distance λ_{ei} of mean free path of the electron(s), and the conversion of this absorbed d.c. energy by the electrons into noise radiation via ions by setting in charge-oscillations in the ions at electron-ion collisions, so that this noise radiation does not interact back either with the ions generating the noise radiation directly or with the electrons acting as an indirect source of noise radiation via ions. Or in other words the noise radiation does not interact back with the electron-ion pairs in the plasma [where the average separation of an electron-ion is given by $R_o = (3/4\pi)^{1/3} N_e^{-1/3}$]. And hence this noise radiation finds itself in free space over which the electron-ion pairs in the plasma are formed. Thus the noise radiation in the plasma has velocity c which corresponds to the velocity of an electromagnetic wave in free space.

3.2. The Coupled Noise in the Plasma

Now come to eqn. (17) viz.,

$$Z_{ic}(f)\,\partial f = \frac{8\pi f^2}{c^3}\partial f.$$

Here $Z_{ic}(f)\,\partial f$ gives the number of modes of ionic oscillators or ionic (charge) oscillations or electromagnetic (noise) radiations (waves) generated by the ionic oscillators per unit volume of the plasma in noise frequency interval between frequencies f and f+∂f. The minimum value of the frequency of the ionic oscillations is taken as zero when (linear) dimensions of the plasma are extremely large as compared to the dimensions of the electron-ion pair. The ionic oscillators are here considered not to oscillate mutually independent of each other but they are coupled one supported by the fixed ions in the plasma those are electrostatically coupled. This gives the maximum number of modes of ionic oscillations as $3N_e$ corresponding to three degrees of freedom for each of N_e ions, per unit volume of the plasma, undergoing the charge-oscillations along three mutually perpendicular axes. This limits the maximum frequency (cut-off frequency) of the ionic oscillations denoted by f_N. If the minimum frequency of the ionic oscillations is taken as zero as mentioned above, then integration of eqn. (17) gives that,

$$\int_0^{f_N} Z_{ic}(f)\,\partial f = \int_0^{f_N}\frac{8\pi f^2}{c^3}\partial f = 3N_e\,. \tag{26}$$

Equation (26) means that in presence of electrostatically coupled fixed ions of plasma, coupled charge-oscillatory modes of ions take place associated with the generation of coupled electromagnetic waves. And further each of the ions has a band of frequency ranging 0 to f_N, when (linear) dimensions of the plasma are extremely large as compared to that of an electron-ion pair. f_N of the plasma (under consideration) can exist in (far) infra-red region of the spectrum of electromagnetic radiations.

Further the value of f_N on solving the integral of eqn. (26) is given by,

$$f_N = \left(\frac{9N_e c^3}{8\pi}\right)^{1/3}. \tag{27}$$

Coming to eqn. (19) and using eqn. (25), the value of noise energy density at frequency f in frequency interval ∂f is given by,

$$(W_{nf})_{dc}\,\partial f = \frac{8\pi h f^3}{c^3}\frac{1}{\left\{\exp\left(\frac{hf}{kT_{dc}}\right)\right\}-1}\partial f. \tag{28}$$

Further, the total noise energy U_{dc} per unit volume of the plasma or the total noise energy density over the values of frequency ranging 0 to f_N of the coupled noise, is given by,

$$U_{dc} = \int_0^{f_N}(W_{nf})_{dc}\,\partial f. \tag{29}$$

Using eqns. (27) and (28), eqn. (29) gives that,

$$U_{dc} = \frac{9N_e}{f_N^3}\int_0^{f_N}\frac{hf^3\,\partial f}{\{\exp(hf/kT_{dc})\}-1}. \tag{30}$$

Now substitute,

$$y = \frac{hf}{kT_{dc}}, \tag{31}$$

and

$$x_N = \frac{hf_N}{kT_{dc}}, \tag{32}$$

in eqn. (30). Then eqn. (30) gives that,

$$U_{dc} = 9N_e\left(\frac{kT_{dc}}{hf_N}\right)^3 (kT_{dc})\int_0^{x_N}\frac{y^3\,\partial y}{\{\exp(y)\}-1}. \tag{33}$$

Further define a characteristic temperature T_N of the coupled noise by the following relationship, viz.,

$$T_N = \frac{hf_N}{k} = \frac{h}{k}\left(\frac{9N_e c^3}{8\pi}\right)^{1/3}, \tag{34}$$

where the value of f_N is substituted from eqn.(27).

Equation (33) on using eqn. (34) gives that,

$$U_{dc} = 9N_e\left(\frac{T_{dc}}{T_N}\right)^3 (kT_{dc})\int_0^{T_N/T_{dc}}\frac{y^3\,\partial y}{\{\exp(y)\}-1}. \tag{35}$$

Equation (35) gives the energy density of the coupled noise radiation in the plasma. This is due to the absorption of the energy density W_{ei} from eqn. (6) by the electrons from the d.c. electric field E_{dc} in the plasma in the presence of electron-ion collisions. Then equating the expressions for W_{ei} and U_{dc} as given by eqns. (6) and (35), gives that,

$$\frac{e\lambda_{ei}E_{dc}}{3kT_{dc}} = \frac{3}{(T_N/T_{dc})^3}\int_0^{T_N/T_{dc}}\frac{y^3\,\partial y}{\{\exp(y)\}-1}. \tag{36}$$

Transcendental expression of eqn. (36) determines T_{dc} when λ_{ei}, E_{dc} and T_N have specific values.

The coupled noise radiation considered here is in equilibrium with oscillating ions at temperature T_{dc} of the d.c. noise radiation. However, this coupled noise radiation is not in equilibrium with the electrons, as the electrons are indirect source of radiation, viz., they absorb the d.c. energy and transfer it to the ions during collisions. And the ions in turn give rise to coupled noise radiation via ionic charge-oscillations. This is a process of quasi-equilibrium phenomenon of absorption of the d.c. energy by electrons and giving rise to the coupled noise radiation by the ions of electron-ion pairs in the plasma.

Here indirectly or directly the electrons or the ions of the electron-ion pairs in the plasma play part in the generation of the coupled noise radiation as mentioned above and this coupled noise radiation does not interact back with the electrons and the ions or the electron-ion pairs in the plasma. And thus the coupled noise radiation finds itself in free space over which the electron-ion pairs in the plasma are formed. So that the coupled noise radiation in the plasma has a velocity c that corresponds to the velocity of an electromagnetic wave in free space.

Thus the finite value of electron-ion collision frequency/electron mean free path with reference to ion in the plasma given by eqn. (5)/eqn. (9), gives rise to the characteristics of the coupled noise radiation from the plasma.

4. Numerical Analysis and Conclusions

D.C. (coupled) noise radiation phenomenon due to electron-ion collisions in the plasma of cold star dust in interstellar spaces of the galaxy/galaxies is illustrated here, assuming average electron or ion density i.e. N_e is $10^{15}\ m^{-3}$ and ambient temperature T is 273 ^{0}K at which electrons and ions in the plasma are in thermal equilibrium. The d.c. electric field is chosen to be 10 V/m.

In Table 1, chosen value of N_e of $10^{15}\ m^{-3}$ is given in first column of Table 1 at the chosen ambient temperature T of the plasma of 273 ^{0}K. Value of β is given in second column using eqn. (3). Here value of β is 2.581(4) x 10^2. It is clear that condition of eqn. (4) viz., $\beta^2 >> 1$ is satisfied here.

v_e (the electron thermal velocity) is obtained using eqn. (8). Here value of v_e is 1.026(6) x 10^5 m/sc

ν_{ei} (the electron-ion collision frequency) is obtained using eqn. (5). Here value of ν_{ei} is 7.529(9) x 10^6 sc^{-1}.

λ_{ei} (the electron-ion mean free path) is obtained using eqn. (7)/eqn. (9). Here value of λ_{ei} is 1.363(3) x 10^{-2} m.

f_N (the cut-off frequency of coupled noise mode) is obtained using eqn.(27). Here value of f_N is 2.128(7) x 10^{13} Hz.

T_N (the characteristic temperature of the noise mode) is obtained using eqn. (34). Here value of T_N is 1.021(4) x 10^3 ^{0}K

T_{dc} (the d.c. noise radiation temperature of the coupled noise mode) is obtained using eqn. (36) {- the value of T_{dc} here is calculated from Table 4e-7 on pp 4-112 of 'American Institute of Physics

Table 1. Various Parameters in Connection with Coupled Noise Radiation in the plasma of star dust at T = 273 ^{0}K having ~~of~~ E_{dc} = 10 V/m.

N_e x 10^{-15}	β x 10^{-2}	ν_{ei} x 10^{-6}	λ_{ei} x 10^2	f_N X 10^{-13}	T_N X 10^{-3}	T_{dc} x 10^{-2}	ω_px 10^{-9}
(m^{-3})		(sc^{-1})	(m)	(Hz)	(^{0}K)	(^{0}K)	(rad/sc)
1.00(0)	2.581(4)	7.529(9)	1.363(3)	2.128(7)	1.021(4)	8.511(7)	1.783(9)

Handbook Coordinated by Gray, 1972 [15], where the values of the integrals of the from: $\left({}^{3}/_{Y^3}\right) \int_0^Y \left[\frac{y^3\ \partial y}{\{(\exp y)-1\}}\right]$, for various values of Y are tabulated}. Here value of T_{dc} is obtained as 8.511(7) x 10^2 ^{0}K.

The last column of the Table 1 gives angular frequency of plasma oscillations given (Tonks and Langmuir 1929) [16] by,

$$\omega_p = \sqrt{\frac{N_e\, e^2}{m_e\, \epsilon_o}}. \tag{37}$$

Here the value of ω_p is 1.783(9) x 10^9 rad/sc. This value of $\omega_p >> \nu_{ei}$. And then in this case, ω_p also denotes eigen angular frequency of damped oscillations which are not sustained in an analogues way to the case of electron-molecule collisions treated earlier (Nandedkar and Bhagavat 1970 [10]. Nandedkar 2016 [11]). Whence it is concluded that since E_{dc} is extremely large as compared to the average ionic field $\langle E_i \rangle$ which is the average electric field at a distance of R_o from center of the screening sphere where the ion is situated and here $\langle E_i \rangle$ is given by,

$$\langle E_i \rangle = \left(\frac{e}{\epsilon_o}\right)\frac{R_o}{(4\pi/3)R_o^3} = N_e\left(\frac{e}{\epsilon_o}\right)R_o\ , \tag{38}$$

in an analogues way to the case of electron-molecule collisions treated earlier (Nandedkar and Bhagavat 1970 [10]. Nandedkar 2016 [11]), where R_o/ R_{oc} is separation/constrained separation of an electron-ion pair in the plasma, where

$$R_{oc} = \frac{R_o}{[1-\exp(-1)]} \tag{39}$$

and

$$R_0 = \left(\frac{3}{4\pi N_e}\right)^{1/3}, \tag{40}$$

here R_0 is to be considered for $\langle E_i \rangle$ and R_{oc} is to be considered for E_{dc}, so that sustained electronic damped oscillations are also there in the screening sphere in presence of hyperbolic orbit in which the electron is also orbiting around the ion situated at internal focus of the hyperbola and generating quantum coupled radiation noise via electron-ion pairs described in this research-paper. The angular frequency of sustained damped oscillations in presence of E_{dc} is also very very large as compared to ν_{ei} which is electron-ion collision frequency in the plasma, this is so as $E_{dc} \gg \langle E_i \rangle$.

Ojective of this paper is to illustrate D.C. noise phenomenon of the previous plasma model (Nandedkar and Bhagavat 1970 [9]) in light of the plasma work done in presence of plasma damped oscillations (Nandedkar and Bhagavat 1970 [10] & Nandedkar 2016 [11]) with reference to specific numerical analysis which was not done then and to provide more correct solution to the problem of electron-ion collision frequency in the plasma which is considered in appendix of this paper.

Appendix

A-1. Introduction

In the plasma, electrons and ions are considered under thermal equilibrium at ambient temperature T. First of all here the orbit of the electron with reference to the ion in the plasma is considered. Then the scattering angle of the electron in the orbit is treated. Ultimately, a complete solution for the electron-ion collision frequency for the electron in the plasma is given, considering that the cut-off collision parameter of the electron in the orbit corresponds to the average separation of an electron from an ion in the plasma.

A-2. The Orbit of the Electron

The ion and the electron are treated as point charges, i.e. mere centers of Coulomb force, thus the dimensions of the interacting particles are not taken into account. Further, the ion is treated to be much heavier as compared to the electron, so that the ion is considered to be stationary (and fixed) one as compared to the moving electron in the plasma.

The positive ion of charge e is at the origin S of a system of polar co-ordinate r, θ; p (which is the collision parameter) is shortest distance between the initial path of the electron and ion. The position of electron is in general given by r, θ (Fig. 1).

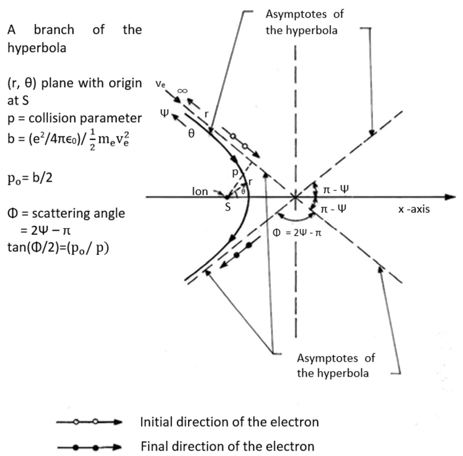

Fig. 1. The {Hyperbolic) path of the electron with reference to the ion in the plasma (indicated by the arrow heads).

The angular momentum L of the electron about the ion is given by,

$$L = m_e r^2 \frac{\partial \theta}{\partial t}, \tag{A.1}$$

where m_e is the mass of the electron, and t is the time under consideration.

Conservation of the angular momentum in this case, gives that L is constant. Its value when r = p is given by,

$$L = m_e p\, v_e , \tag{A.2}$$

where v_e is the initial velocity of the electron, and for the plasma at thermal equilibrium at temperature T, it is given by,

$$v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}, \tag{A.3}$$

which is the average thermal velocity of the electron. Here k is the Boltzmann constant.

The Coulomb attractive force between the electron and the ion is $F = -e^2/4\pi\epsilon_o r^2$, where e is the magnitude of charge of an electron or of an ion and ϵ_o is permittivity of free space. Here the charge of an ion is equal and opposite in sign to that of an electron. The force F gives rise to a radial acceleration,

$$a = \frac{\partial^2 r}{\partial t^2} - r\left(\frac{\partial \theta}{\partial t}\right)^2 = \frac{F}{m_e} = \frac{-e^2}{(4\pi\epsilon_o r^2) m_e} \cdot \ldots \tag{A.4}$$

Eqns. (A.1) and (A.4), are two simultaneous equations in r, θ and t; if t is eliminated between them, a single differential equation in r and θ is obtained which gives the orbit of the electron. For convenience, put

$$r = 1/u , \tag{A.5}$$

and get from eqn. (A.1),

$$\frac{\partial \theta}{\partial t} = \frac{L}{m_e r^2} = \frac{L u^2}{m_e}. \tag{A.6}$$

It follows that,

$$\frac{\partial r}{\partial t} = \frac{\partial r}{\partial \theta}\frac{\partial \theta}{\partial t} = -\frac{1}{u^2}\frac{\partial u}{\partial \theta}\frac{L u^2}{m_e} = -\frac{L}{m_e}\frac{\partial u}{\partial \theta}, \tag{A.7}$$

and that

$$\frac{\partial^2 r}{\partial t^2} = \frac{\partial \theta}{\partial t}\frac{\partial}{\partial \theta}\left(\frac{\partial r}{\partial t}\right) = \frac{L u^2}{m_e}\left(-\frac{L}{m_e}\frac{\partial^2 u}{\partial \theta^2}\right) = -\frac{L^2 u^2}{m_e^2}\frac{\partial^2 u}{\partial \theta^2}. \tag{A.8}$$

Substituting $\partial^2 r/\partial t^2$ and $\partial\theta/\partial t$ in eqn. (A.4), eqn. (A.4) gives that

$$\frac{\partial^2 u}{\partial \theta^2} + u = \frac{b}{2p^2}, \tag{A.9}$$

using eqn. (A.2). Here

$$b = \frac{e^2/4\pi\epsilon_o}{\frac{1}{2} m_e v_e^2}. \tag{A.10}$$

Equation (A.9) is the differential equation of the orbit of the electron; it has a solution.

$$u = \frac{1}{r} = A\cos(\theta - \theta_0) + \frac{b}{2p^2}, \qquad (A.11)$$

where A and θ_0 are arbitrary constants. The constant θ_0 just determines the orientation of the orbit in the (r, θ) plane. If x- axis be chosen such that $\theta_0 = 0$, then the equation of the orbit becomes

$$u = \frac{1}{r} = \frac{b}{2p^2} + A\cos\theta\,. \qquad (A.12)$$

The equation,

$$u = \frac{1}{r} = B + A\cos\theta\,. \qquad (A.13)$$

represents a conic section in general with eccentricity given by,

$$\epsilon_a = \frac{A}{B}. \qquad (A.14)$$

Since B is known and is equal to $b/2p^2$, it remains to determine A or ϵ_a. On setting,

$$A = \epsilon_a B = \epsilon_a\left(\frac{b}{2p^2}\right). \qquad (A.15)$$

Now eqn. (A.12) can be rewritten as follows:

$$u = \frac{1}{r} = \frac{b}{2p^2}(\epsilon_a\cos\theta + 1)\,, \qquad (A.16)$$

using eqns.(A.14) and (A.15).

Further the equation for conservation of energy of the electron in the path near the ion is given by

$$\frac{1}{2}m_e v_e^2 = \frac{1}{2}m_e v_{er}^2 - \frac{e^2}{4\pi\epsilon_o r}, \qquad (A.17)$$

where v_{er} is the velocity of the electron at a distance r from the ion when the potential energy of the electron with respect to the ion is $-e^2/4\pi\epsilon_o r$, and $m_e v_{er}^2/2$ gives the appropriate kinetic energy of the electron there. Here $m_e v_e^2/2$ gives the initial energy of the electron, where v_e is given by eqn. (A.3).

The constant ϵ_a of eqn. (A.16) can be determined by making use of eqn. (A.17) as follows:

Using eqn. (A.10), eqn. (A.17) gives that,

$$\frac{v_{er}^2}{v_e^2} = 1 + \left(\frac{b}{r}\right). \qquad (A.18)$$

Here the quantity v_{er}^2 is given by,

$$v_{er}^2 = \left(\frac{\partial r}{\partial t}\right)^2 + \left(r\,\frac{\partial\theta}{\partial t}\right)^2 = \left[\left(\frac{\partial r}{\partial\theta}\right)^2 + r^2\right]\left(\frac{\partial\theta}{\partial t}\right)^2. \qquad (A.19)$$

From eqn. (A.16), it follows that,

$$\left(\frac{\partial r}{\partial\theta}\right) = \frac{\epsilon_a b}{2p^2} r^2\sin\theta\,, \qquad (A.20)$$

and eqns. (A.1) and (A.2) gives that,

$$\left(\frac{\partial\theta}{\partial t}\right) = \frac{v_e p}{r^2}, \qquad (A.21)$$

so that eqn. (A.18), using eqns. (A.20) and (A.21) gives that,

$$\frac{v_{er}^2}{v_e^2} = \left(\epsilon_a^2\,\frac{b^2}{4p^2}\right)\sin^2\theta + \frac{p^2}{r^2} = 1 + \left(\frac{b}{r}\right). \qquad (A.22)$$

Eliminating r from eqns. (A.16) and (A.22), eqn. (A.22) gives that,

$$\epsilon_a^2 = 1 + (4p^2/b^2)\,,$$

or,

$$\epsilon_a = \left(1 + \frac{4p^2}{b^2}\right)^{1/2}. \qquad (A.23)$$

Further the constant A of eqn. (A.15) is given by

$$A = \epsilon_a\frac{b}{2p^2} = \left(1 + \frac{4p^2}{b^2}\right)^{1/2}\left(\frac{b}{2p^2}\right) = \frac{1}{p}\left(1 + \frac{b^2}{4p^2}\right)^{1/2}, \qquad (A.24)$$

using eqn. (A.23).

Substituting the value of A from eqn. (A.24) in eqn. (A.12), eqn. (A.12) for the central orbit of the electron gives that,

$$\frac{1}{r} = \frac{b}{2p^2} + \frac{1}{p}\left(1 + \frac{b^2}{4p^2}\right)^{1/2}\cos\theta\,. \qquad (A.25)$$

here the eccentricity ϵ_a of the central orbit of the electron is given by eqn. (A.23), and since b and p are both positive so that $\epsilon_a > 1$, and hence the orbit of the electron of eqn. (A.25) with reference to the ion is a hyperbola where the ion is situated at (internal) focus S of the hyperbola (Fig. 1).

A-3. The Scattering Angle

When $r\to\infty$, then $\theta\to\psi$, where $(\pi - \psi)$ is the angle between x- axis and asymptote(s) of the hyperbola (Fig. 1). Using these boundary conditions eqn. (A.25) gives that,

$$\cos(\pi - \psi) = \frac{(b/2p)}{[1+(b/2p)^2]^{1/2}}. \qquad (A.26)$$

On solving for $\tan(\pi - \psi)$, eqn.(A.26) gives that,

$$\tan(\pi - \psi) = 2p / b\,. \qquad (A.27)$$

The scattering angle ϕ of the electron is now given by

$$\phi = 2\,\psi - \pi, \qquad (A.28)$$

where ϕ is the angle between the final and initial directions of the electron which are along the asymptotes of the hyperbola (Fig. 1).

Substituting the value of ψ from eqn. (A.28) in eqn. (A.27), eqn. (A.27) gives that,

$$\tan\frac{\phi}{2} = \frac{b}{2p}. \qquad (A.29)$$

If the collision parameter has a value p_o, when the deflection is $\pi/2$ (i.e. $\phi=\pi/2$), then eqn. (A.29) gives,

$$p_o = \frac{b}{2} = \frac{(e^2/4\pi\epsilon_o)}{m_e v_e^2}, \quad (A.30)$$

using eqn. (A.10).

From eqns. (A.29) and (A.30), the scattering angle ϕ is given by,

$$\tan\frac{\phi}{2} = \frac{p_o}{p}. \quad (A.31)$$

A-4. The Electron-Ion Collision Frequency in Plasma

In the present analysis v_e is considered as the initial speed of the electron. If δv_e be the change in electron's speed perpendicular to its original direction of motion after a collision with the collision parameter p, then

$$(\delta v_e)^2 = (v_e \sin\phi)^2 = 4\, v_e^2 \sin^2\frac{\phi}{2}\cos^2\frac{\phi}{2}. \quad (A.32)$$

By making use of eqn. (A.31), eqn. (A.32) gives,

$$(\delta v_e)^2 = \frac{4 v_e^2 (p/p_o)^2}{[1+(p/p_o)^2]^2}. \quad (A.33)$$

On an average such collisions would be $N_e(2\pi p\ \partial p)\, v_e$ per unit time, where N_e is the number density of ions in the plasma which is also equal to the number density of electrons. $(2\pi p\ \partial p)$ gives the differential collision cross section and $N_e(2\pi p\ \partial p)\, v_e$ gives the differential electron-ion collision frequency in the present case.

The average value of $(\delta v_e)^2$ over various collision parameters, is given by

$$\langle(\delta v_e)^2\rangle = \int N_e (2\pi p\ \partial p)\, v_e (\delta v_e)^2 ,$$

or,

$$\langle(\delta v_e)^2\rangle = 8\pi N_e v_e^3 p_0^2 \int_o^{R_o/p_o} \frac{(p/p_o)^3\, \partial(p/p_o)}{[1+(p/p_o)^2]^2}. \quad (A.34)$$

The integral in eqn. (A.34) diverges at infinity and hence, for the plasma under consideration, a cut off collision parameter is introduced which corresponds to R_o that gives the average separation of an electron from an ion in the plasma. The value of R_o is given by

$$R_o = \left(\frac{3}{4\pi N_e}\right)^{1/3}, \quad (A.35)$$

considering spherical symmetry of the electron with respect to the ion. Here N_e gives density of electrons or ions is the plasma.

Equation (A.34) contains an integral I defined by

$$I = \int_0^{R_o/p_o} \frac{(p/p_o)^3\, \partial(p/p_o)}{[1+(p/p_o)^2]^2}. \quad (A.36)$$

This can be solved by substituting,

$$\frac{p}{p_o} = \tan y\,, \quad (A.37)$$

and

$$\frac{R_o}{p_o} = \tan y_0\,, \quad (A.38)$$

whence eqn. (A.36) gives that,

$$I = \int_0^{\tan y_0} (1-\cos^2 y)\tan y\ \partial y$$

or,

$$I = [\ln(\sec y)]_0^{\tan y_0} + \frac{1}{4}[\cos 2y]_0^{\tan y_0}. \quad (A.39)$$

Using eqn. (A.38), eqn. (A.39) gives that,

$$I = \left[\ln\left\{1+\left(\frac{R_o}{p_o}\right)^2\right\}^{1/2}\right] - \frac{1}{2}\left[\frac{\left(\frac{R_o}{p_o}\right)^2}{1+\left(\frac{R_o}{p_o}\right)^2}\right]. \quad (A.40)$$

Now the following relationship holds good for the deflection time of $1/\nu_{ei}$, for the case of electrons moving through the ions in the plasma, viz.,

$$\langle(\delta v_e)^2\rangle \frac{1}{\nu_{ei}} = v_e^2\,, \quad (A.41)$$

where ν_{ei} is the electron ion-collision frequency in the plasma.

Using eqns. (A.34), (A.36), (A.40) and (A.41), ν_{ei} is given by,

$$\nu_{ei} = 8\pi N_e v_e p_0^2 \left[\left\{\ln\sqrt{1+\left(\frac{R_o}{p_o}\right)^2}\right\} - \frac{1}{2}\left\{\frac{(R_o/p_o)^2}{1+(R_o/p_o)^2}\right\}\right]. \quad (A.42)$$

The value of p_0 using eqns. (A.30) and (A.3), is given by

$$p_o = \left(\frac{e^2}{32\epsilon_0 kT}\right). \quad (A.43)$$

The value of R_0 is given by eqn. (A.35), viz.,

$$R_o = \left(\frac{3}{4\pi N_e}\right)^{1/3}.$$

Thus the value of factor $\beta = R_o/p_0$, using eqns. (A.35) and (A.43), is given by,

$$\beta = \left(\frac{R_o}{p_o}\right) = \left(\frac{32\varepsilon_0 kT}{e^2}\right)\left(\frac{3}{4\pi N_e}\right)^{1/3}. \quad (A.44)$$

Substituting the values of v_e, p_0 and R_o/p_0 as given by eqns. (A.3), (A.30) and (A.44), eqns. (A.42) gives (Nandedkar 1983) [14] that,

$$\nu_{ei} = 8\pi N_e \left(\frac{e^2}{4\pi\epsilon_o m_e}\right)^2 \left(\frac{\pi m_e}{8kT}\right)^{3/2} \alpha\,, \qquad \text{(A.45)}$$

where α is a positive quantity given by,

$$\alpha = \left[\{\text{In}(1+\beta^2)^{1/2}\} - \frac{1}{2}\left\{\frac{\beta^2}{1+\beta^2}\right\}\right], \qquad \text{(A.46)}$$

for ν_{ei} is to have a positive value. In eqn. (A.46), the value of β is given by eqn. (A.44).

Acknowledgements

The author is thankful for the interesting discussions with reference to this research-paper given herewith he had, with his Guru (the Teacher) Dr. Ing. G. K. Bhagavat, [(now) Late] Professor of Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India.

Further he gives thanks also to Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India for giving permission to publish this research-work.

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