Physics Journal
Vol. 2, No. 3, 2016, pp. 196-219
http://www.aiscience.org/journal/pj

Quantum Statistical Plasma Model Biased by a D.C. Electric Field and Perturbed by Low Power R.F. Waves

D. P. Nandedkar*

Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India

Abstract

In previous papers by Bhagavat and/ Nandedkar [1] to [11] and [29], an analysis of a plasma model biased by a d.c. electric field and perturbed by low power r.f. waves has been carried out, where damped oscillations in plasma were discovered [2]. The aim of the present paper is to give a review of all this work and to complete some portions which have remained uncompleted previously. First of all the choice of a plasma model chosen in absence of a d.c. electric field is considered where eigen frequency damped oscillations due to electrons and ions exit which are not sustained in the screening sphere. Then d.c. resistivity of the plasma due to electron-molecule and ion-molecule collisions is described in presence of a d.c. electric field. Afterwards electronic and ionic frequencies of damped oscillations are illustrated in presence of a d.c. electric field. Electronic and ionic frequencies of damped oscillations also exit inside screening sphere and they are sustained there. In presence of electronic and ionic frequencies of damped oscillations, electrons and ions absorb d.c. energies because of finite d.c. resistivity due to collisions of charge-carriers with neutral molecules in plasma which reappears as noise spectrum which cancels out due to electrons and ions as charge of an electron is equal and opposite to that of a considering singly charged ion in plasma, which is described in turn of quantum statistical theory of finite d.c. resistivity of the plasma. After explaining the quantum statistical theory of finite d.c. resistivity of the plasma, perturbation of the plasma model (biased by a d.c. electric field) by low power r.f. waves is considered in which anomalous dispersion of r.f. waves occurs where permittivity of plasma, energy transport, phase & group, and wave front & signal velocities of the r.f. wave in the region near resonance in the plasma and quantum theory of noise radiation due to finite r.f. resistivity of the plasma, are illustrated one by one. In the end, conclusions of the ‘quantum statistical plasma model biased by a d.c. electric field and perturbed by low power r.f. waves’ are discussed. The absorption of r.f. energy is maximum near the resonance and falls on either side of it and the absorption of r.f. energy in plasma gives rise to a continuous spectrum (black-body type) of all noise frequencies of waves with a peak of extremely low power at sub-millimeter range of wavelengths for low power interacting electromagnetic U.H.F. wave(s) with plasma.

Keywords

Quantum, Statistics, Plasma, Low-Power, Radio-Frequency, Wave-Velocities, Perturbation, Noise, Spectrum

Received: September 16, 2015 / Accepted: December 17, 2015 / Published online: February 17, 2016

1. Introduction

The term plasma refers to an Ionized gas in some kind of temperature equilibrium, that is to say a medium consisting of electrons, ions and neutral particles (viz. molecules or atoms), which is on the whole neutral.

Study of propagation of electromagnetic (r.f.) waves through the plasma started from the wave propagation in the ionosphere.

Theory of propagation of r.f. waves through the ionosphere in the absence of earth’s magnetic field can be understood in

* Corresponding author
E-mail address: dpn@ee.iitb.ac.in

terms of the theory of complex dielectric constant of an ionized gas basically developed by Appeton and Chapman (Appleton and Chapman 1932 [12]; further refer to Ramo, Whinnery and Van Duzer 1970 [13] and also to Ratcliffe 1959 [14]). According to them, the complex dielectric constant of an ionized gas, can be shown to be given by

$$\epsilon_p = \epsilon_p' - i\epsilon_p'' = \epsilon_p'\left(1 + \frac{\sigma_p}{i\omega\epsilon_p'}\right), \quad (1)$$

where

$$\frac{\epsilon_p'}{\epsilon_o} = 1 - \frac{(N_e e^2/m_e\epsilon_o)}{\omega^2 + \nu^2}, \quad (2)$$

and

$$\sigma_p = \frac{N_e e^2 \nu}{m_e(\omega^2+\nu^2)}. \quad (3)$$

Here ϵ_p = complex dielectric constant of the ionized gas or plasma, ϵ_p' = real part of ϵ_p i.e. permittivity of the plasma, ϵ_p'' = imaginary part of ϵ_p i.e. loss factor of the plasma, ϵ_o = permittivity of free space, $i = \sqrt{-1}$, N_e = electron or ion density in the plasma, e = charge of an electron, m_e = mass of an electron and ν = electron collision frequency with neutral molecules in the plasma. This assumes that the plasma is weakly ionized. And further, the gas pressure is large enough to consider mainly electron-molecule (or atom) collisions in the plasma. These collisions are elastic in nature. Further, ω = angular frequency of the wave and σ_p = r.f. conductivity of the plasma.

In this case, no account of ion-temperature or ionic motion relative to electronic one is considered while deriving eqns. (2) and (3), simply because mass of an ion is much large as compared to that of an electron.

In eqn. (2), the quantity $(N_e e^2/m_e\epsilon_o)^{1/2}$ is referred to as ω_p, i.e. here

$$\omega_p = \left(\frac{N_e e^2}{m_e\epsilon_o}\right)^{1/2}, \quad (4)$$

where ω_p is electronic plasma angular frequency. It has been shown by Tonks and Langmuir (Tonks and Langmuir 1929) in 1929 [15] that ω_p / 2π is the frequency at which the electrons oscillate about their rest positions. That is, if the electrons in a group, are displaced from their rest positions by an external force, a restoring force will be established due to the electric field between the electrons and ions which have been left behind. In order for the plasma to remain electrically neutral, the electrons will tend to return to their original positions and will oscillate about their original positions due to inertial effects. The frequency of these oscillations is the electronic plasma frequency given by ω_p / 2π Here no account of ion-temperature or ionic motion relative to electronic one is considered while obtaining eqn. (4), as ionic mass is much large as compared to electronic one.

Equations (2) and (3) show that, for a given value of ν and N_e, $\frac{\varepsilon_p'}{\varepsilon_0}$ increases as ω is increased such that $\frac{\epsilon_p'}{\epsilon_o}$ approaches the value of unity in the limit and σ_p decreases with increase of ω.

The above expressions denoted by eqns. (2) and (3) are approximate only, and Margenau (Margenau 1946) [16]; (further also refer to Marton 1955 [17]) in 1946, has tried to make them more precise by considering the detailed statistics of electron's collisions. In this theory, elastic collisions of electrons with neutral gas molecules (or atoms) are of basic importance. For a given electron temperature and mean free path, this theory shows that, the relative permittivity of the ionized gas increases as angular frequency of the interacting wave increases and in the limit, relative permittivity approaches the value of unity; whereas the (r.f.) conductivity of the plasma falls with rise of angular frequency of the wave.

Adler (Adler 1949 [18]) in 1949, has published a paper in which he has described the results on the determination of complex conductivity of plasma at microwave range of frequencies. In this experiment, the positive column of a mercury glow discharge is placed along the axis of a cylindrical cavity excited in the TM_{010} mode. The transmission of 3 cm waves through the cavity and the shift in the resonant frequency have been calculated as a function of pressure as well as of discharge current. From these measurements, the complex conductivity of the ionized gas has been calculated as a function of pressure as well as of current. Using a theoretical formula for the conductivity (Margenau 1946 [16]) electron density in turn, has been calculated from both real and imaginary parts of the measured complex conductivity. Langmuir's probe studies have been carried out to check the results obtained for the electron densities and an adequate agreement is found at low pressures. Further, the electron temperature coming into the picture of conductivity, practically corresponds to that given by Langmuir's probe method. This experiment is carried in the region where electron collision frequency lies in the vicinity of angular frequency of the interacting wave.

Some investigations on the complex dielectric constant of ionized gases (devoid of humidity) are experimentally considered by Bhagavat and Nandedkar (Bhagavat and Nandedkar 1968 [19]) in 1968.

Impedance offered to a slab line in V.H.F. range (150 Mc/s - 200 Mc/s) by a plasma-slab is measured here. In one part of the experiment, the plasma-slab is filled by gaseous discharge of negative glow and Faraday dark space, of Argon. The discharge is obtained in the pressure-range 0.8 x 10^{-1} torr to 4 x 10^{-1} torr and discharge current is varied from 15mA to 40mA, where anode and cathode of the discharge tube which are made of nickel have area of 76 x 10^{2} mm^{2} each. Here variation of the discharge current from 15mA to 40 mA is obtained, by keeping the pressure fixed at a particular value of the pressure in the pressure-range mentioned already, using an external power supply and a high resistance in series with the tube. From impedance measurements of the plasma-slab by VSWR technique, relative permittivity $\frac{\epsilon_p}{\epsilon_o}$ and conductivity σ_p of the plasma are obtained as function of angular frequency ω of the interacting waves. $\frac{\epsilon_p'}{\epsilon_o}$ increases as ω increases. $\frac{\epsilon_p'}{\epsilon_o}$ is not found to exceed the value of unity. σ_p decreases with increase of ω. Here the experiment is performed at a fixed pressure as mentioned already [Electron-neutral particle (atom) collision frequency ν can be obtained from gas pressure in the discharge tube and value of classical radius for gas atom due to gas-kinetics. Order of the classical radius for any type of the gas atom or molecule is 10^{-10} m by kinetic theory of gases. For the investigations of the present type, it is sufficient to consider the value of classical radius of the neutral gas particle as 10^{-10} m while calculating the classical cross-section of electron-neutral particle collision where electron temperature is given by Langmuir's probe method. It is found that $\nu \sim 10^{8}$ sc^{-1}]. In the present investigations $\omega \sim 10^{9}$ rad/sc. Thus, Here $\omega^2 \gg \nu^2$. Here the experimental variations of $\frac{\epsilon_p'}{\epsilon_o}$ vs ω and σ_p vs ω, qualitatively show the behaviour as expected from eqns. (2) and (3), provided N_e and ν are regarded as constants.

Coming to eqns. (2) and (3), when $\omega^2 \gg \nu^2$, then it is seen that the refraction of the wave through the plasma is possible without too much absorption. This is the theory of Larmor (Larmor 1924 [20], [21]) in which he has shown the importance of considering the dielectric rather than the conducting properties of the ionosphere.

In 1902, it has been suggested by Kennelly and Heaviside independently that radio waves can be propagated from England to America 'round the protuberance of the earth', by reflection from an electrified layer in the upper atmosphere.

The first attempt to formulate mathematically the suggestions of Kennelly and Heaviside has been done by Eccles (Eccles 1912 [22]) in 1912. He has derived expressions for velocity and absorption of waves passing through a medium containing free charges now considered to be electrons. His expressions apply only when the velocity of the waves is not much different from that in free space.

In general when $\omega^2 \gg \nu^2$, then eqns. (1) to (3) can be considered to represent mathematically Eccles-Larmor theory of propagation of electromagnetic waves through ionosphere (in the absence of earth's magnetic field).

In the E-region of the ionosphere it is thought that the (electron) temperature is about 300 ^{0}K (where ^{0}K is Degree Kelvin), and number density of air molecules is of order 10^{19} m^{-3}. Magnitude of classical radius of an air molecule is 10^{-10} m according to gas-kinetics. This gives the value of ν ~ 10^{5} sc^{-1}. In this region electron (or ion) density i.e. N_e is of order 10^{11} m^{-3} (-refer to Ratcliffe 1959 [14]).

Thus in the E-region of the ionosphere, where electron-molecule collisions are of basic importance eqns. (1) to (3) of Eccles-Larmor theory can be applied while considering the r.f. wave interaction (in the absence of earth's magnetic field) when the wave frequency is large enough as compared to ν.

In recent years, a large number of experiments have been carried out on the dielectric constant of ionized air with the object of verifying Eccles-Larmor theory. This theory demands that the relative permittivity with respect to free space of an ionized gas (i.e. $\frac{\epsilon_p'}{\epsilon_o}$) should be less than unity. Study of the literature on the subject reveals, however, that most of the investigations are of qualitative nature, and further that the experimental results obtained are of abnormal type, the recorded relative permittivity being frequently equal to, or greater than, instead of less than, unity.

The earliest experiments, to verify in the laboratory the theory of ionospheric wave propagation, have been carried out by Barton and Kilby (Barton and Kilby 1913 [23]) in 1913. Using gaseous ionization produced at atmospheric pressure by X-ray or radium bromide, they have been able to detect by high frequency methods, the effect of conductivity of the gas. Their experimental apparatus includes an oscillatory circuit in which the condenser consisting of two plates has been filled by ionized air. Variations of apparent capacity are found, which have been attributed to the effects of the gas acting as a leak in parallel with the condenser.

Gutton (Gutton 1930 [24]) in 1930, has found a diminution of the relative permittivity for low ionic concentration, but at higher concentrations he has found an increase in the relative permittivity. Gutton and Clement (Gutton and Clement 1927 [25]) in 1927, have attempted to explain the increase in the relative permittivity above unity as due to the existence of quasi-elastically bound electric carriers in the ionized gas. According to them, the electric carriers in the ionized gas are not free and they possess a natural frequency. The natural frequency of the electrons has been considered to be proportional to $N_e^{3/8}$ where N_e gives the electron density in the plasma (Gutton, 1930 [24]).

Appleton and Childs (Appleton and Childs 1930 [26]) in 1930 have conducted some experiments and have found that, in the absence of an imposed magnetic field, the relative permittivity with respect to free space of ionized air, decreases with the increase of the ionic concentration up to a certain value of the latter. Beyond this value the relative permittivity gradually attains the value of unity and subsequently becomes greater than unity for large ionic concentrations. They have attributed this abnormal behaviour to the formation of ion sheath on the condenser plates when they are placed inside the discharge tube.

Rahman and Khastigir (Rahman and Khastigir 1940 [27]) in 1940, have carried out investigation experiments with external plates round a discharge tube on the relative permittivity of ionized air inside such a tube, having different pressures and different currents. At higher discharge tube currents, relative permittivity of the ionized air has shown abnormal behaviour, viz., the relative permittivity has exceeded the value of unity. The abnormal increase of the permittivity of the ionized air has been explained as due to the effect of the positive ionic sheath formed on the inner surface of the discharge tube.

Some investigations on the complex dielectric constant of ionized air (devoid of humidity) are experimentally considered by Bhagavat and Nandedkar (Bhagavat and Nandedkar 1968 [19]), in 1968.

Here studies on the variation with frequency of permittivity and conductivity of ionized air in the V.H.F. range (150 Mc/s-200 Mc/s) have been carried out. Impedance offered to a slab line by the plasma column (of the weakly ionized air in the form of a plasma-slab) has been determined from VSWR measurements. In the case of ionized air here, observations are taken in the abnormal region of electron molecule collision frequency ~ angular frequency of the wave.

In this case the plasma-slab is made of two nickel electrodes of size 76 x 10 mm^2 connected in a pyrex glass tube of 90 mm long. The central conductor of the slab line is placed along the axis of the glass tube. The glass tube is filled by air. The pressure in the tube is varied in the range 0.8 x 10^{-1} torr to 4 x 10^{-1} torr and discharge current varied from 15 mA to 40 mA, by keeping the pressure fixed at a particular value in the range mentioned, using an external power supply and a high resistance in series with the tube. Here the plasma-slab is filled by negative glow and Faraday dark space, of air.

In this case of ionized air, abnormal results are obtained at high discharge currents. Here relative permittivity first falls with increase of (angular) frequency of the wave and then after attaining a minimum value rises again with the frequency. The angular frequency ω at which relative permittivity of ionized air attains a minimum value is regarded to correspond to the order of electron-molecule collision frequency ν The order of ω at which minimum value of the permittivity occurs, is found to be 10^9 rad/sc. Here the experiment is performed at a fixed pressure as mentioned already. [Electron-molecule collision frequency ν can be obtained if gas pressure inside the tube is known. Further major constituent of air is nitrogen gas. Here collision cross-section for electron-molecule collisions is considered to be that due to Ramsauer-Townsend effect for the electrons in nitrogen gas having energy equivalent to one election volt. Here this cross-section chosen lies away from the peak in the cross-section versus energy curve for the electrons in the case of nitrogen gas (Von Engle 1965 [28]). Energy involved here is considered to be equivalent to one electron-volt in one collision. This energy is insufficient to produce excitation or ionization in the gas atoms or molecules. Thus the electron-molecule collisions treated here are considered to be elastic ones. Further defining an equivalent temperature corresponding to the energy of one electron-volt of the electrons chosen, the value of electron-molecule collision frequency ν can be obtained. Order of ν thus obtained is 10^9 sc^{-1}]. When $\omega^2 \gg \nu^2$, then the effect of the conductivity of the ionized air is small, such that the assumption ${\epsilon_p''}^2$ (where ϵ_p'' refers to the loss factor of the plasma) is small enough as compared to ${\epsilon_p'}^2$ (where ϵ_p' gives permittivity of the plasma) is valid and consequently the assumption which is made to obtain the value of ϵ_p' from the position of the first minimum on the slab line from plasma load, while considering VSWR holds good. Here recorded relative permittivity shows an increase as angular frequency ω of the wave is increased. In this case the relative permittivity is not found to exceed the value of unity. The behaviour of $\frac{\epsilon_p'}{\epsilon_o}$ vs ω, in this region when $\omega^2 \gg \nu^2$, qualitatively shows the nature as expected from the theory of Margenau (Margenau, 1946 [16]), provided electron temperature and electron mean free path or electron-molecule collision frequency and electron density are regarded as constants. But when ω is in the vicinity of ν and ω approaches ν, then the effect of conductivity of the ionized air becomes appreciable such that ${\epsilon_p''}^2$ is not small enough as compared to ${\epsilon_p'}^2$. Hence apparently a larger value of the position of the first minimum is being recorded in this process. The result is that, this effect gives apparently a smaller value for the relative permittivity than what it should be actually. In these two opposite situations, ϵ_p' decreases with ω till a minimum is obtained at ω ~ ν. And after this just reverse situation occurs, and eventually when $\omega^2 \gg \nu^2$, then ϵ_p' shows an increase with ω - as mentioned already. Thus the conductivity acquired by the ionized air is the main cause for the abnormal behaviour of the relative permittivity near ω equal to ν region. Further, in this case of the ionized air, conductivity decreases as ω increases. This qualitatively shows the behaviour as expected from the theory of Margenau (Margenau 1946 [16]), provided electron density and electron-molecule collision frequency (which means, in turn, the electron temperature and the electronic mean free path with respect to molecules) are regarded as constants. This means that the assumptions made in obtaining the value of conductivity, in this case, from VSWR measurements are valid.

Negative glow in the case of a gaseous discharge of air is, further, found to provide quite interesting results (Bhagavat and Nandedkar 1968 [29]).

Experimental investigation on the negative glow in the case of a gaseous discharge of air, under the conditions of low currents, have given to the results, which prove the existence of region of anomalous dispersion. Here an analysis of the phase propagation constant of a T.E.M. wave in the plasma is made by introducing a new factor referred to as the angular frequency of electronic damped oscillations, now denoted by ω_{oe} - that is proportional to the electric restoring force on an electron in the space charge region under the influence of the electric field. From the experimental determination of the phase propagation constant of a r.f. wave in the plasma terminated on a slab line in the form of a plasma-slab (which consists of the ionized air), in the region of anomalous dispersion it is concluded that in this region, relative permittivity $\frac{\epsilon_p'}{\epsilon_o}$ of the plasma decreases with increase of angular frequency ω of the wave passing through the value of unity at resonance defined by $\omega = \omega_{oe}$ whereas the absorption of rf energy is maximum near the resonance and falls on either side of it.

In this case, the plasma-slab is made of two nickel electrodes of size 30 x 18 mm^2, connected in a pyrex glass-tube of 50 mm length. The central conductor of the slab line is placed along the axis of the glass tube. The glass tube is filled with air. The pressure in the tube is fixed at 2 x 10^{-1} torr and the discharge current is varied from 0.5mA to 2.0 mA using an external power supply and a high resistance in series with the tube. Here the plasma-slab is filled by negative glow of a gaseous discharge of air. From the impedance measurements of the plasma-slab terminated on the main slab line in frequency range 310 Mc/s to 330 Mc/s, the value of phase constant of the wave in the plasma is determined at various frequencies with discharge tube current as a parameter.

From the geometry of the experimentally obtained curve of phase propagation constant of the wave versus its angular frequency in the region of anomalous dispersion, it is possible to determine electron density N_e and electron collision frequency ν. Experimentally determined values of the collision frequency agree well with those obtained from kinetic theory data, assuming that the electrons make elastic collisions with molecules in the plasma at equilibrium temperature of the ambient – which is room temperature, within experimental limitations.

Further investigations on the damped oscillations in plasma are carried out in the positive column of a gaseous discharge of air (Bhagavat and Nandedkar 1968 [1]). Here experimental investigations of the variation of electronic angular frequency of damped oscillations i.e. ω_{oe} with the electron density N_e and a d.c. electric field E_{dc} in the plasma column are described below: →

Experimental investigations on the positive column, in the case of a gaseous discharge of air, are carried out here. In this case the plasma-slab is made of two nickel electrodes 34 mm in diameter sealed in a pyrex glass tube of 120 mm length. Near the anode two stainless steel meshes of sleeve size 1.0 x 1.0 mm^2 are fitted parallel to the electrodes. The central conductor is fixed with the help of polystyrene spacers. The slab is connected to main slab line by appropriate adapters. The glass tube is filled with air. The pressure in the tube is fixed at 2 x 10^{-1} torr and the discharge current is varied from 5 mA to 20 mA, using a power supply and a high resistance in series with the tube. Here the plasma-slab is filled by positive column of a gaseous discharge of air. Potential difference across the meshes in the positive column of the glow discharge of air is varied from 50 V to 80 V from an external power supply unit. From the impedance measurements of the plasma-slab terminated on the main slab line, in frequency range 312 Mc/s to 396 Mc/s, the value of phase propagation constant of the wave in the plasma in the region of anomalous dispersion is determined with electron density N_e and a d.c. electric field E_{dc} in the positive column as parameters, respectively.

From the geometry of the experimentally obtained curve of phase propagation constant of the wave versus its angular frequency in the region of anomalous dispersion of the plasma, it is possible to determine electron collision frequency and angular frequency of electronic damped oscillations.

The values of electron collision frequency thus obtained experimentally, agree well with those given by gas-kinetics, where it is assumed that the electrons in the plasma make elastic collisions with gas molecules at equilibrium temperature, which is the room temperature, within experimental limitations.

Further, for a given d.c. electric field E_{dc} , ω_{oe} is experimentally found to be directly proportional to $N_e^{1/6}$. And for a given value of the electron density N_e , ω_{oe} is experimentally found to be directly proportional to $E_{dc}^{1/2}$. ω_{oe} is proportional to $N_e^{1/6}$ is a result which is entirely different than that considered by Gutton and his followers (Gutton and Clement 1927 [25]; Gutton 1930 [24]), where quasi-elastically bound electrons have angular frequency proportional to $N_e^{3/8}$. Thus, now, it is concluded that, the electrons undergoing damped oscillations are not elastically bound, but on the other hand the electron is electrically quasi-bound with respect to an ion in the space charge region provided by a screening sphere at the center of which the ion lies. And the necessary damping to the electron is provided by electron-molecule collisions in the plasma (Nandedkar and Bhagavat 1970 [10]). Further the electrons in this analysis, whose density is N_e and temperature is T which is ambient or room temperature are referred to as 'slow-electrons'. These 'slow-electrons' from about 1% part of the electrons in the other group in the plasma, which are characterized by density N_{eL} and temperature T_{eL} which can be determined by Langmuir's probe method. The electrons having density N_{eL} and temperature T_{eL} can be referred to as relatively, fast-electrons' as against the 'slow-electrons' which are characterized by density N_e and temperature T. Condition of overall charge neutrality in the plasma is experimentally verified here, where electrons, ions and neutral molecules are in thermal equilibrium at temperature T of the ambient.

From the observations those have obtained for the phase constant of the wave as a function of its frequency in the region of anomalous dispersion provided by the positive column of glow discharge of air (Bhagavat and Nandedkar 1968 [1]) which determine the values of electron density N_e and angular frequency of electronic damped oscillations ω_{oe}, it is possible to determine the value of ratio of charge to mass of an electron i.e. e/m_e by using the following theoretical formula given by (Nandedkar 2016 [11]):

$$\omega_{oe} = \left[\frac{1-\exp(-1)}{\left(\frac{3}{4\pi}\right)^{1/3}}\left(\frac{e}{m_e}\right)\right]^{1/2} E_{dc}^{1/2} N_e^{1/6}, \qquad (5)$$

where E_{dc} gives the d.c. electric field in the plasma column. Mean value of e/m_e thus obtained is 1.7352 x 10^{11} C/Kgm. The presently accepted value of e/m_e is 1.7591 x 10^{11} C/Kgm. The difference between the presently accepted value of e/m_e and that obtained experimentally is 1.3586 % (experimentally determined value of e/m_e is less than actual one) which is due to the experimental limitations. This confirms one part of the theory of electronic damped oscillations in the plasma, where the electrons and ions are in thermal equilibrium with the ambient at room temperature T.

Further from the analysis of the experimental curve of the phase constant of the wave versus its frequency in the region of anomalous dispersion of the plasma provided by the positive column of glow discharge of air (Bhagavat and Nandedkar 1968 [1]), it is possible to determine electron-molecule collision frequency g_E which is given by (Nandedkar 2016 [11]) the following expression, viz.,

$$g_E = N_m(\pi R_m^2)\left(\frac{2kT}{\pi m_e}\right)^{1/2}, \qquad (6)$$

using gas kinetics here N_m = number density of gas molecules in the plasma = p/kT, where p is the gas pressure T is the ambient temperature and k is the Boltzmann's constant. $\pi R_m^2 =$ classical cross-section for electron-molecule collisions. R_m = classical radius of the gas molecule whose order is the same for any type of a gas molecule which is 10^{-10} m according to gas kinetics. $(2kT/\pi m_e)^{1/2}$ = average thermal velocity of an electron corresponding to one dimensional electron-gas model at ambient temperature T. m_e = mass of an electron. Eqn. (6) assumes electron-molecule elastic collisions at ambient temperature T. Thus knowing g_E experimentally (Bhagavat and Nandedkar 1968 [1] and Nandedkar 2016 [11]), it is possible to determine the mean value of R_m. Mean value of R_m thus obtained is 1.0799 x 10^{-10}m for the case of an air molecule. This value of the classical radius R_m of the air molecule is quite reasonable, in view of gas kinetics which gives the order of classical value of R_m as 10^{-10} m for any type of gas molecule, for the type of experimental investigation carried out here in the case of the plasma provided by positive column of glow discharge of air. Now the major constitution of air is nitrogen gas. Hence the value of R_m = 1.0799 x 10^{-10} m obtained for the air molecule as mentioned already can also be assumed to represent the classical radius of the nitrogen gas molecule for all practical purposes for the type of experimental investigation illustrated here. This confirms other part of the theory of electronic damped oscillations in the plasma, in which electrons make elastic collisions with neutral gas molecules at ambient temperature T. Present theory holds good when $\omega_{oe}^2 >> g_E^2$.

Since g_E is practically the same for any type of a gas molecule (as the order of classical radius R_m is the same for any type of gas molecule which is 10^{-10} m according to gas kinetics) for the purpose of investigations of the present type of experimentation where $g_E^2 << \omega_{oe}^2$, and ω_{oe} just depends on N_e and E_{dc}, hence the present theory is applicable to any gaseous discharge, irrespective of the gas used. Thus, the theory is quite general in its application, provided the condition of thermal equilibrium of ions, electrons and neutral molecules at ambient temperature T of the room holds good.

In the light of the theory of electronic damped oscillations illustrated here, it is clear the increase of relative permittivity of the plasma above unity in the region of anomalous dispersion is a phenomenon which is entirely different than that of the phenomenon of a positive ion sheath formation at the walls in the discharge tube as considered by previous investigators (Appleton and Childs 1930 [26], Rahman and Khastigir 1940 [27]). Now Gutton- effect (Gutton and Clement 1927 [25]; Gutton 1930 [24]) is a phenomenon which is entirely different than the phenomenon of electronic frequency of damped oscillations considered here, as mentioned already. Hence the increase in relative permittivity of the ionized gas beyond unity in the analysis considered herewith is not due to the Gutton effect or ionic sheath effect, but it is due to the existence of electronic frequency of damped oscillations as mentioned before. This Gutton effect and sheath effect are entirely different effects, than the effect of electronic frequency of damped oscillations considered in this analysis.

Moreover, the present theory naturally leads to the similar expressions for, the electronic plasma angular frequency due to Tonks, and Langmuir i.e. eqn. (4), and the theory of complex dielectric constant of ionized gases basically due to Appleton-Chapman or Eccles-Larmor i.e. eqns. (1) to (3) when $\omega^2 >> \nu^2$ for the case of low electronic damping where electron-molecule collisions are of main importance - similar to that occurring in the case of 'E' region of the ionosphere where electrons make elastic collisions with neutral molecules of air at ambient temperature of 300 ^{0}K, as the limiting cases. Here it is assumed that the power of r.f. wave is very small. So that it does not practically disturb the plasma.

The region of anomalous dispersion in the case of the plasma described in this work (Bhagavat and Nandedkar 1968 [1]) is further, analyzed from energy transport, phase and group and wave front & signal velocities point of view. Energy absorbed by the oscillating electrons in the plasma undergoing damped oscillations when the plasma is biased by a d.c. electric field illustrated here and perturbed by low power r.f. waves, (- which measure the plasma parameters as electron density and electron collision frequency assuming them to be uniform meaning thereby the average values are concerned), give rise to various emission spectra of electromagnetic waves, which are analyzed using quantum statistical methods. All this work is done in a series of previous papers (Bhagavat and Nandedkar 1968 [1], [2], [3], [4], [29]; Nandedkar and Bhagavat 1969 [5], [6], [7]; 1970 [[8], [9], [10] and Nandedkar 2016 [11]). The aim of the present paper is to give a review of all the work under the title 'Quantum statistical plasma model biased by a d.c. electric field and perturbed by low power r.f. waves' and to complete some portion(s) which remained incomplete, previously.

First of all the choice of plasma model chosen in the absence of a d.c. electric field is considered. Then d.c. resistivity of the plasma due to electron-molecule and ion-molecule collisions is described. Afterwards electronic and ionic frequencies of damped oscillations are illustrated. After explaining the quantum theory of finite d.c. resistivity of the plasma, perturbation of the plasma-model (biased by a d.c. electric field) by low power r.f. waves is considered in which anomalous dispersion of r.f. waves in the plasma, energy transport, phase & group and wave front & signal velocities of the wave near resonance in the plasma and quantum theory of noise radiation due to finite r.f. resistivity of the plasma are illustrated one by one. In the end conclusions of the quantum statistical plasma model biased by a d.c. electric field and perturbed by low power r.f. waves are discussed.

Further here,

Corrigendum to paper (Bhagavat and Nandedkar 1968) [29], is given in [30].

Corrigenda to papers (Bhagavat and Nandedkar 1968) [29], [1] and [3] is given in [31].

Corrigenda to papers (Nandedkar and Bhagavat 1969 & 1970) [5] and [8] is given in [32].

2. The Plasma

The term plasma introduced here, refers to an ionized gas which contains electrons, ions and neutral molecules which are in thermal equilibrium at ambient temperature which is the room temperature. The plasma consists of equal densities of electrons and ions in order for it to be macroscopically neutral. Here the ions are considered to be singly charged with the charge numerically equal to that of an electron.

The plasma considered here is a weekly ionized gas (nearly one percent). In such a plasma thermal equilibrium is mainly established by electron-molecule and ion-molecule collisions at the ambient temperature T, which is the room temperature. By term thermal equilibrium of the plasma at temperature T, it is meant that the mean kinetic energies of an electron, an ion and a neutral molecule in the plasma, are equal to each other, i.e.,

$$\frac{1}{2}m_e v_{er}^2 = \frac{1}{2}m_i v_{ir}^2 = \frac{1}{2}M v_{mr}^2 = \frac{3}{2}kT, \quad (7)$$

considering three degrees of freedom of the respective particle, along three axes of coordinates. Here m_e, m_i and M denote mass of an electron, an ion and a molecule respectively. v_{er}, v_{ir} and v_{mr} denote rms velocity of an electron, an ion and a molecule respectively, at equilibrium temperature T of the plasma. k is Boltzmann constant. Eqn.(7) is a result due to gas kinetics, where it is assumed that the electrons, the ions and the molecules all are in thermal equilibrium characterized by Maxwellian distribution at temperature T, for each type of the particles.

Electron-molecule frequency ν_{em} at equilibrium temperature T of the plasma is given by (Nandedkar 2016) [11],

$$\nu_{em} = \frac{v_e}{\lambda_e} = N_m(\pi R_m^2)\left(\frac{8kT}{\pi m_e}\right)^{\frac{1}{2}}, \quad (8)$$

where,

$$\lambda_e = \frac{1}{N_m \pi R_m^2}, \quad (9)$$

and,

$$v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}. \quad (10)$$

Here λ_e is the mean free path of the electron in the plasma with respect to molecules and v_e gives average thermal velocity of the electron corresponding to a three dimensional electron-gas model. R_m is the classical radius of the gas molecule due to gas kinetics, whose order is the same for any type of a gas molecule which is 10^{-10} m. πR_m^2 gives classical cross-section for electron-molecule collisions. N_m gives the number density of the gas molecules in the plasma at temperature T, such that,

$$p = N_m\, k\, T, \quad (11)$$

where p gives the gas pressure, ν_{em} gives the collision frequency for elastic collision between electron and molecules.

Ion-molecule collision frequency ν_{im} at equilibrium temperature T of the plasma is further given by,

$$\nu_{im} = \frac{v_i}{\lambda_i} = N_m[\pi(2R_m)^2]\left(\frac{8kT}{\pi m_i}\right)^{1/2}, \quad (12)$$

using gas kinetics where,

$$\lambda_i = \frac{1}{N_m[\pi(2R_m)^2]}, \quad (13)$$

and,

$$v_i = \left(\frac{8kT}{\pi m_i}\right)^{1/2} \quad (14)$$

Here λ_i is the mean free path of the ion with respect to molecules in the plasma. v_i is average thermal velocity of the ion corresponding to a three dimensional ion-gas model. $2R_m$ is the classical diameter of the gas molecule and $\pi(2R_m)^2$ gives classical cross-section of ion- molecule collisions. ν_{im} gives the collision frequency for elastic collisions between ion and molecules.

Comparing eqns. (8) and (12), it is clear that the electron-molecule collision frequency predominates the ion-molecule collision frequency in the plasma at equilibrium temperature T, because mass of an ion is very much large as compared to that of an electron.

The ionized gas considered here can be treated as plasma, provided its volume is much large as compared to that of the screening (Debye) sphere. The radius $\sqrt{2}/k_{D'}$ of the screening sphere in the plasma at equilibrium temperature T of the ambient, is given by (Nandedkar and Bhagavat 1970) [10]:

$$\frac{\sqrt{2}}{k_{D'}} = \left(\frac{\epsilon_o kT}{N_e e^2}\right)^{1/2} \quad (15)$$

Here ϵ_o is permittivity of free space, N_e is electron or ion density in the plasma and e is the magnitude of the charge of an electron or ion. Here the plasma is considered to have ions which are singly charged with the magnitude of electronic charge as mentioned earlier.

Further the potential ϕ_i at distance r due to an ion of charge e at the center of the screening sphere is expressed by (Nandedkar and Bhagavat 1970) [10]:

$$\phi_i = \phi_c \exp(-k_{D'}\, r) \quad (16)$$

where ϕ_c is the Coulomb's potential at distance r from the ion such that

$$\phi_c = \frac{e}{4\pi\epsilon_o r}. \quad (17)$$

From eqn. (16) it is clear that when $r << 1/k_{D'}$ then $\phi_i \rightarrow \phi_c$

If N_e gives number density of electron-ion pairs in the plasma, then the volume $(4\pi/3)R_o^3$ occupied by an electron-ion pair in the plasma is given by

$$\frac{4\pi}{3}R_o^3 = N_e^{-1}, \quad (18)$$

considering spherical symmetry with respect to the ion in the plasma. Here R_o gives average separation of an electron from an ion in the plasma. The potential due to this ion at distance R_o can be expressed by eqn. (16), (Nandedkar and Bhagavat 1970) [10] provided

$$\frac{\sqrt{2}/k_{D'}}{N_{D'}} \leq R_o \leq \frac{\sqrt{2}}{k_{D'}}, \quad (19)$$

where $N_{D'}$ gives the number of electrons in the screening sphere.

In a previous paper (Nandedkar 2016) [11], it is shown that the (equivalent) differential equation of motion of the electron in the plasma in the screening sphere with the ion at the center, in the vicinity of distance R_o given by eqn.(18), can be expressed by,

$$\frac{d^2r}{dt^2} + \nu_{em}\frac{dr}{dt} + \omega_{je}^2 r = \frac{e}{m_e}\langle E_i\rangle, \tag{20a}$$

or,

$$\frac{d^2r}{dt^2} + \nu_{em}\frac{dr}{dt} + \omega_{je}^2 r = \frac{e}{m_e}\left(\frac{N_e er}{\epsilon_o}\right), \tag{20b}$$

where,

$$\langle E_i\rangle = N_e er \tag{21a}$$

gives polarization of electron-ion pairs in the plasma, and

$$\omega_{je} = \left(\frac{N_e e^2}{m_e\epsilon_o}\right)^{1/2}, \tag{21b}$$

gives angular eigen frequency of electronic damped oscillations in the plasma, when

$$\omega_{je}^2 >> \left(\frac{\nu_{em}}{2}\right)^2, \tag{22}$$

i.e. under the case of low damping. In eqn. (20a) or (20b), the distance r is measured with respect to the center of the screening sphere. $\frac{d^2r}{dt^2}$, $\frac{dr}{dt}$ gives acceleration and velocity of the electron at time t whereas r gives displacement of the electron. ν_{em} gives electron-molecule collision frequency given by eqn. (8). $m_e\nu_{em}\frac{dr}{dt}$ gives damping force acting on the electron due to collisions whereas $m_e\,\omega_{je}^2 r$ gives restoring force acting on the electron due to fluctuation in the space charge density near r in the screening sphere. Here $\langle E_i\rangle$ gives average value of the electric field due to the ion at the center of the screening sphere at distance R_o. $\langle E_i\rangle$ can also be considered as the electric field created due to the effect of relative polarization of electron-ion pairs in the plasma with respect to free space. The average separation of an electron-ion pair is r. Thus $\langle E_i\rangle = N_e er / \epsilon_o$. ω_{je} can also be expressed by,

$$\omega_{je}^2 = \frac{e}{m_e}\frac{\langle E_i\rangle}{R_o}, \tag{23}$$

in terms of distance R_o, which physically denotes the amplitude of the damped oscillations of the electron with angular frequency ω_{je}. R_o exists inside the screening sphere. Here R_o is obtained as a steady state solution of eqn. (20a) Using eqn. (21b), eqn. (20b) gives,

$$\frac{d^2r}{dt^2} + \nu_{em}\frac{dr}{dt} = 0, \tag{24}$$

which means eigen frequency damped oscillations denoted by eqn. (20a) are not sustained as such in the steady state. This is because the effect of the relative polarization of electron-ion pairs nullifies the effect of the restoring tendencies on the electron in the screening sphere. Under these circumstances, the electrons in the plasma behave as if they are ‘free’.

Angular eigen frequency of electronic damped oscillations given by eqn. (21b) and denoted by ω_{je} is similar to the electronic angular plasma frequency due to Tonks and Langmuir (Tonks and Langmuir 1929) [15].

Similarly in analysis for the screening sphere, in steady state, with an electron at its center can be carried out. And an expression of ω_{jp} i.e. the angular eigen frequency of ionic damped oscillations in the plasma can be obtained whose value is.

$$\omega_{jp} = \left[\frac{e}{m_i}\frac{\langle E_e\rangle}{R_o}\right]^{\frac{1}{2}} = \left[\frac{N_e e^2}{m_i\epsilon_o}\right]^{\frac{1}{2}}, \tag{25}$$

where R_o gives distance from the electron at center of the screening sphere where ionic damped oscillation of eigen angular frequency ω_{jp} exist. $\langle E_e\rangle$ is average electric field acting on the ion due to the electron at the center of the screening sphere. Like in eqn. (20a) or (20b), damping in the present case for the ionic-oscillations is provided by ν_{im} which is ion-molecule collision frequency in the plasma. Here also the case of low dapping is concerned. These eigen frequency ionic damped oscillations in the plasma at distance R_o measured with respect to the electron at the center of the screening sphere, exit inside the screening sphere. These eigen frequency ionic damped oscillations are also not sustained as such when steady state approaches, as in the case of eigen frequency electronic damped oscillations denoted by eqn. (24). And the ions also behave as if they are ‘free’.

The electron and ion having angular eigen frequency of electronic and ionic damped oscillations as given by eqns. (21b) and (25) respectively have displacement that tend to zero with respect to the respective distance of R_o in the steady state. However, in the steady state both eigen frequency damped oscillations are not sustained as such.

3. D.C. Conductivity of the Plasma

On an application of a practically uniform d.c. electric field E_{dc} to the plasma described in Secn. 1., where it is assumed that E_{dc} is large as compared to the internal electric fields between charges, the electrons and the ions start moving in the opposite direction parallel to the direction of E_{dc} and constitute drift currents. The charges are accelerated during their mean free paths with respect to heavy particles which are molecules in the present case. The charges scatter at their collisions with the molecules and this results into the d.c. resistivity of the plasma, because motion of the charges gets opposed at the collision, in the presence of E_{dc}. Energy acquired by the electrons and ions at the collisions with the molecules, from the d.c. electric field due to the finite value of the d.c. resistivity of the plasma is further considered. In general the d.c. conductivity of the plasma is the sum of d.c. conductivities due to the electrons and ions individually, because total drift current is the sum of the magnitudes of the drift currents due to the electrons and ions moving in opposite direction parallel to $E_{dc}$where charge of an electron is equal and opposite to that of an ion.

3.1. D.C. Conductivity of the Plasma Due to the Electrons

In between successive collisions in the direction parallel to E_{dc} the electron is subjected to a uniform acceleration a_{em} due to E_{dc} given by,

$$a_{em} = \frac{e}{m_e} E_{dc} \tag{26}$$

where e is charge and m_e is mass of the electron. The average distance s_e travelled by the electron between two collisions mentioned above, is given by,

$$s_e = \frac{1}{2} a_{em} t_E^2, \tag{27}$$

where,

$$t_E = \frac{1}{g_E}, \tag{28}$$

and here t_E is the mean free time between two collisions, with g_E as electron-molecule collision frequency given by (Nandedkar 2016) [11],

$$g_E = \frac{v'_e}{\lambda_e} = N_m(\pi R_m^2)\left(\frac{2kT}{\pi m_e}\right)^{1/2}, \tag{29}$$

where,

$$\lambda_e = \frac{1}{N_m(\pi R_m^2)},$$

which is eqn. (9), gives electronic mean free path with respect to gas molecules in the plasma having gas density as N_m. R_m is the classical radius of the gas molecule due to gas kinetics whose order is the same for any type of a gas molecule which is 10^{-10} m. In eqn. (29), πR_m^2 gives classical collision cross-section of the electron with respect to a molecule.

Further in eqn. (29), v'_e is given by,

$$v'_e = \left(\frac{2kT}{\pi m_e}\right)^{1/2}, \tag{30}$$

where v'_e gives the average thermal velocity of the electron corresponding to a one dimensional electron gas model. v'_e acts along the direction in which the electron moves, in the presence of the unidirectional E_{dc} mentioned here.

In writing eqn. (27) it is assumed that initial drift velocity of the electron in the presence of E_{dc} is zero. This means that all the drift velocity acquired by the electron in the presence of E_{dc} during previous mean free path is completely lost at the end of the path where electron-molecule collision takes place. And the electron starts afresh with zero drift velocity at the start of new mean free path. From eqn. (27) average drift velocity of the electron v_{ed} of the electron in the presence of E_{dc} is given by,

$$v_{ed} = \frac{s_e}{t_E} = \frac{1}{2}\frac{e}{m_e}\frac{E_{dc}}{g_E}, \tag{31}$$

using eqns. (26) and (28).

The drift current density due to N_e electrons per unit volume in the plasma having average drift velocity v_{ed} given by eqn. (31) is given by,

$$(j_{ed})_{dc} = N_e e v_{ed} = \frac{N_e e^2}{m_e(2g_E)} E_{dc}, \tag{32}$$

using eqns. (31). Hence d.c. conductivity of the plasma due to electrons viz., $(\sigma_e)_{dc}$ is given by,

$$(\sigma_e)_{dc} = \frac{(j_{ed})_{dc}}{E_{dc}} = \frac{N_e e^2}{m_e(2g_E)}, \tag{33}$$

using eqn. (32). When E_{dc} acts in one direction only, then in deriving eqn. (33) the distribution of electronic mean free paths is neglected (Nandedkar 2016) [11].

Coming back to eqn. (27), s_e gives the average distance travelled by the electron between two collisions. So s_e represents the mean free path given by eqn. (9). Thus,

$$s_e = \lambda_e \tag{34}$$

Using eqns. (28) to (30) and (34), eqn. (31) gives,

$$v_{ed} = v'_e = \left(\frac{2kT}{\pi m_e}\right)^{1/2}. \tag{35}$$

Thus the average drift velocity of the electron in the presence of E_{dc} is the average thermal velocity of the electron corresponding to a one dimensional electron gas model at ambient temperature T.

Hence if a unit cube in the plasma is considered across which a voltage of E_{dc} volts is applied then one electron in travelling a unit distance in direction of E_{dc} is accelerated through a potential difference of eE_{dc} electron volts. This is so, as the average thermal velocity of the electron viz., v'_e

always remains constant in the presence of E_{dc}. Thus the energy density acquired by N_e electrons in the presence of voltage E_{dc} is given by,

$$(W_{dc})_e = N_e e E_{dc}, \quad (36)$$

due to finite value of the electronic conductivity given by eqn. (33) in the presence of electron- molecule collisions.

3.2. D.C. Conductivity of the Plasma Due to the Ions

Previous analysis of the d.c. conductivity of the plasma due to electrons can be extended to the case of d.c. conductivity due to ions. Similar treatment gives,

$$(\sigma_i)_{dc} = \frac{N_e e^2}{m_i(2g_I)}, \quad (37)$$

where $(\sigma_i)_{dc}$ = d.c. conductivity of the plasma due to ions, N_e = density of ions in the plasma, e = magnitude of charge of an ion m_i = mass of an ion, g_I = ion-molecule collision frequency with average thermal velocity of the ion as,

$$v_i' = \left(\frac{2kT}{\pi m_i}\right)^{1/2}, \quad (38)$$

which refers to a one dimensional ion-gas model at equilibrium temperature T of the plasma. v_i' is considered in the direction parallel to E_{dc}. Using gas kinetics g_I is given by

$$g_I = \frac{v_i'}{\lambda_i} = N_m[\pi(2R_m)^2]\left(\frac{2kT}{\pi m_i}\right)^{1/2}, \quad (39)$$

with,

$$\lambda_i = \frac{1}{N_m[\pi(2R_m)^2]},$$

which is eqn. (13), gives mean free path of an ion with respect to molecules in the plasma whose density is N_m. R_m gives classical radius of the gas molecule due to gas kinetics whose order is the same for any type of a gas molecule which is 10^{-10}. $[\pi(2R_m)^2]$ gives classical cross section for ion-molecule collisions.

The energy density acquired by N_e ions in the presence of a voltage E_{dc} (applied to a unit cube) can further shown to be given by (as in the case of electrons):

$$(W_{dc})_i = N_e e E_{dc}, \quad (40)$$

due to finite value of ionic conductivity given by eqn. (37) in the presence of ion-molecule collisions.

3.3. Total DC Conductivity of the Plasma

Since charge of an electron is equal to opposite to that of an ion, the drift currents due to the electrons and ions, which are in opposite direction in practice, add together. Thus total d.c. drift current density J_{dc} due to the electrons and ions in the plasma is given by,

$$J_{dc} = [(\sigma_e)_{dc} + (\sigma_i)_{dc}]E_{dc} = \sigma_{dc}E_{dc}, \quad (41)$$

where,

$$\sigma_{dc} = (\sigma_e)_{dc} + (\sigma_i)_{dc}, \quad (42)$$

gives total d.c. conductivity of the plasma. Using eqns. (33) and (37), eqn. (42) gives,

$$\sigma_{dc} = \frac{N_e e^2}{2}\left[\frac{1}{m_e g_E} + \frac{1}{m_i g_I}\right] \quad (43)$$

Since,

$$m_e g_E << m_i g_I, \quad (44)$$

[refer to eqns.(29) and (39)], eqn.(43) gives,

$$\sigma_{dc} \sim \frac{N_e e^2}{m_e(2g_E)} \sim (\sigma_e)_{dc}. \quad (45)$$

This is obvious, as the ions are massive as compared to the electrons, so major contribution to the conductivity of the plasma is due to the electrons.

Derivation of $(\sigma_e)_{dc}$ or $(\sigma_i)_{dc}$ here assumes that λ_e or λ_i is at least ten times smaller than the length of the plasma column across which E_{dc} acts.

Here the case of a relatively high pressure plasma is considered which is having a low electron (or ion) density. So resistivity of the plasma is mainly due to electron-molecule collisions and to a second approximation due to ion-molecule collisions.

In this paper, the effect of electron-ion collisions is neglected as compared to electron-molecule collisions, on d.c. resistivity of the plasma in presence of the d.c. electric field.

4. Electronic and Ionic Frequencies of Damped Oscillations

Consider the uniform d.c. electric field E_{dc} applied to the plasma treated in Secn. 3. E_{dc} is larger than the local fields between the charge particles of $\langle E_i \rangle$ type. $\langle E_i \rangle$ is the average value of ionic field acting on the electron at distance R_o measured with respect to the center of the screening sphere where the ion is situated.

Now consider the (equivalent) differential equation of motion of the electron of the ion-electron pair in the plasma in the presence of E_{dc}, instead of $\langle E_i \rangle$ treated previously in Secn. 2. The electron is in the screening sphere and the ion lies at the center of the screening sphere. Following forces act on the electron, viz., (i) a damping force (i.e. due to the collisions of the electron with molecules) against the motion of the electron which is proportional to its velocity given by $-m_e(2g_E)dr/dt$ where g_E is electron-molecule collision frequency given by eqn. (29) and (ii) a resorting force which is proportional to its displacement given by $-m_e\omega_{oe}^2 r$ where ω_{0e} is the angular frequency of electronic damped oscillations given by eqn. (5). The displacement of the electron in the screening sphere deviates the plasma from the condition of charge neutrality. To maintain the charge neutrality of the plasma $m_e\omega_{oe}^2$ term comes into the picture.

The (equivalent) differential equation of motion of the electron is given by (Nandedkar 2016) [11]:

$$m_e\frac{d^2r}{dt^2} + m_e(2g_E)\frac{dr}{dt} + m_e(\omega_{oe}^2)r = eE_{dc}, \quad (46a)$$

which gives,

$$\frac{d^2r}{dt^2} + 2g_E\frac{dr}{dt} + \omega_{oe}^2 r = \frac{e}{m_e}E_{dc} \quad (46b)$$

Here $\frac{d^2r}{dt^2}$ and $\frac{dr}{dt}$ give acceleration and velocity of the electron at time t. r gives displacement of the electron measured with respect to the ion at the center of the screening sphere. Here it is assumed that the damping force acting on the electron per unit mass per unit velocity is $2g_E$. Whereas in the case of eqn. (20a) the damping force per unit mass per unit velocity acting on the electron is ν_{em}. Factor $2g_E$ in the case of eqn.(46a or 46b) indicates that distribution of free paths of the electron with respect to molecules are to be neglected in the plasma in the presence of an externally applied d.c. electric field, which acts in one preferred direction which is the direction of application (Nandedkar 2016) [11]. Factor ν_{em} in the case of eqn. (20a or 20b) indicates that the distribution of electronic free paths with respect to molecules are to be considered when the field $\langle E_i\rangle$ is randomly oriented (Nandedkar 2016) [11]. Here the ions are in equilibrium with the ambient at temperature T. In eqn. (46a or 46b) it is assumed that,

$$\omega_{oe}^2 >> g_E^2, \quad (47)$$

i.e. the case of very low damping is considered.

Total solution of eqn. (46 b) is given by

$$r = \exp(-g_E t)[A'_{do}\exp(i\omega_{oe}t) + B_{do}'\exp(-i\omega_{oe}t)] + \frac{e}{m_e}\frac{E_{dc}}{\omega_{oe}^2}. \quad (48)$$

A'_{do} and B_{do}' are constants of electronic motion. The steady state in this case is reached when,

$$t >> 1/g_E, \quad (49a)$$

and then,

$$r \to R_{oc} \quad (49b)$$

Applying the boundary condition given by eqns. (49a) and (49 b), eqn. (48) gives:

$$R_{oc} = \frac{e}{m_e}\frac{E_{dc}}{\omega_{oe}^2} \quad (50)$$

Here R_{oc} is obtained as a steady state solution of eqn. (48). Physically R_{oc} gives the amplitude of electronic damped oscillations of angular frequency ω_{oe}. R_{oc} is measured with respect to the ion situated at the center of the screening sphere.

Relationship between R_o [- which gives amplitude of eigen angular frequency damped oscillations denoted by ω_{je} (refer to eqn. (21b), which are not sustained -] and R_{oc} is given by (Nandedkar 2016) [11]:

$$\frac{R_{oc}}{R_o} = \left[\frac{1}{1-\exp(-1)}\right], \quad (51)$$

i.e. R_o is 63.21% smaller than R_{oc}. R_{oc} is inside the screening sphere. Using eqns. (18) and (51), eqn. (50) gives:

$$f_{oe} = \frac{\omega_{oe}}{2\pi} = \frac{1}{2\pi}\left[\frac{e}{m_e}\left(\frac{4\pi}{3}\right)^{1/3}\{1-\exp(-1)\}N_e^{1/3}E_{dc}\right]^{1/2}, \quad (52)$$

where f_{oe} gives frequency of electronic damped oscillations. Eqn. (52) is experimentally verified and used to determine the value of $\frac{e}{m_e}$. The value of $\frac{e}{m_e}$ thus obtained is correct within 1.3586% which is due to the experimental limitations (Nandedkar 2016) [11]. The experimentally determined value of $\frac{e}{m_e}$ is less than actual one.

Comparing eqns. (46b) and (20a), it is clear that when E_{dc} vanishes, then in the limit $\langle E_i\rangle$ is approached and in this limit $2g_E \to \nu_{em}$. And further in this limit $\omega_{oe} \to \omega_{je}$ [refer to eqns. (50) and (23)] since in this case $R_{oc} \to R_o$

Thus in the limit when E_{dc} vanishes, then the present theory of ω_{oe} naturally leads to the expression similar to the plasma electronic angular frequency [refer to eqn. (4)] which is due to Tonks and Langmuir (Tonks and Langmuir 1929) [15].

Equation (46b) denotes that the electron is bound with respect to the ion in the plasma with angular frequency ω_{oe}. ω_{oe} gives the angular frequency of electronic damped oscillations in the plasma. But analysis given in Secn. 3 indicates that the electrons move freely in the presence of a d.c. electric field and constitute the d.c. conductivity of value $N_e e^2/m_e(2g_E)$ - eqn. (33). Hence the present model of the plasma is described by quasi-bound (or quasi-free) electron model of the plasma.

Multiplying eqn. (46b) by $N_e e$ it can be written down as follows:

$$\frac{m_e}{N_e e^2}\frac{dj_e}{dt} + \frac{m_e(2g_E)}{N_e e^2}j_e + \frac{\int j_e dt}{(N_e e^2/m_e\omega_{oe}^2)} = E_{dc}, \quad (53)$$

where,

$$j_e = N_e e \frac{dr}{dt}. \tag{54}$$

Here j_e is the electron current density in the plasma and N_e is the electron density.

Now consider a unit cube of series L_{pe} C_{pe} R_{pe} circuit to which a potential difference of V_{dc} is applied across the parallel faces when j_e is the current flowing. The differential equation of the current in this case is:

$$L_{pe}\frac{dJ_e}{dt} + R_{pe}J_e + \frac{1}{C_{pe}}\int J_e dt = V_{dc}. \tag{55}$$

Comparing eqns. (53) and (55) for the unit cube of the plasma, it is found that:

Equivalent inductance of the plasma due to electrons:

$$L_{pe} = \frac{m_e}{N_e e^2}. \tag{56}$$

Equivalent capacitance of the plasma due to electrons:

$$C_{pe} = \frac{N_e e^2}{m_e \omega_{oe}^2}. \tag{57}$$

Equivalent resistance of the plasma due to electrons:

$$R_{pe} = \frac{m_e(2g_E)}{N_e e^2}. \tag{58}$$

It is interesting to note that R_{pe} is the same as $1/(\sigma_e)_{dc}$, i.e.,

$$R_{pe} = \frac{1}{(\sigma_e)_{dc}} = \frac{m_e(2g_E)}{N_e e^2}, \tag{59}$$

comparing eqns. (33) and (58). Thus eqn. (58) gives the d.c. resistivity of the plasma due to the electrons considered in Secn. 3.

Similarly an analysis can be carried out for ionic frequency of damped oscillations in the plasma. Here the screening sphere can be considered with an electron at its center. And an expression for ω_{op} i.e. the angular frequency of ionic damped oscillations in the plasma can be obtained whose value is:

$$\omega_{op} = \left[\frac{e}{m_i}\frac{E_{dc}}{R_{oc}}\right]^{1/2}, \tag{60}$$

where R_{oc} is the amplitude of ionic damped oscillations. R_{oc} is measured with respect to the center of the screening sphere. R_{oc} lies inside the screening sphere. Magnitude of R_{oc} is given by eqn. (60). Like the value of g_E in eqn. (46a or 46b) damping in the present case for the ionic damped oscillations is provided by g_I [eqn. (39)], which gives ion-molecule collision frequency corresponding to a one dimensional ion-gas model. The value of ionic frequency of damped oscillations is further given by,

$$f_{op} = \frac{\omega_{op}}{2\pi} = \left[\frac{e}{m_i}\left(\frac{4\pi}{3}\right)^{1/3}\{1-\exp(-1)\}N_e^{1/3}E_{dc}\right]^{1/2} \tag{61}$$

As in the case of eqn.(52), here also ω_{op} defined by eqn. (61) tends to ω_{jp} [refer to eqn. (25)] when E_{dc} tends to vanish, then in the limit $\langle E_e\rangle$ is approached [– where $\langle E_e\rangle$ is the average electric field acting on the ion due to the electron at distance R_o -] and in this limit $2g_I \to \nu_{im}$ so also $R_{oc} \to R_o$

Present model of the plasma is described by 'quasi-bound' (or' quasi-free') ion-model of the plasma - similar to the 'quasi-bound' (or 'quasi-free') electron-model of the plasma mentioned in this Secn.

The electron and ion having frequency of electronic and ionic damped oscillations as given by eqns. (52) and (61) respectively, have displacements that tends to zero with respect to the respective distance of R_{oc} in the steady state. In the steady state both of these damped oscillations due to the electron and ion are sustained as such.

For this 'quasi-bound' ion-model of the plasma, for a unit cube of the plasma (as in the case of the 'quasi-bound' electron model of the plasma mentioned in this Secn.), it is found that:

Equivalent inductance of the plasma due to ions:

$$L_{pi} = \frac{m_i}{N_e e^2}. \tag{62}$$

Equivalent capacitance of the plasma due to ions:

$$C_{pi} = \frac{N_e e^2}{m_i \omega_{op}^2} \tag{63}$$

Equivalent resistance of the plasma due to ions:

$$R_{pi} = \frac{m_i(2g_I)}{N_e e^2}. \tag{64}$$

It is interesting to note that R_{pi} is the same as $1/(\sigma_i)_{dc}$, i.e.

$$R_{pi} = \frac{1}{(\sigma_i)_{dc}} = \frac{m_i(2g_I)}{N_e e^2}, \tag{65}$$

comparing eqns. (37) and (64). Thus eqn. (64) gives the d.c. resistivity of the plasma due to the ions considered in Secn. 3.

5. Quantum Theory of Finite D.C. Resistivity of the Plasma

The quasi-bound electrons, each having the frequency of damped oscillations given by f_{oe} [eqn. (52)], absorb d.c. energy density $(W_{dc})_e$ given by eqn. (36) in the presence of finite d.c. conductivity $(\sigma_e)_{dc}$ [refer to eqns. (33) and (58)] of the plasma due to the electrons at electron-molecule collisions. During the process of electronic collisions with molecules in the plasma when the d.c. energy is absorbed, d.c. energy reappears in the form of a d.c. noise spectrum whose frequency distribution is governed by the quantum law of radiation of energy density $(W_R)_e$, proportional to the fourth power of the noise temperature T_e of the electrons. Here $(W_R)_e$, using methods of quantum statistical mechanics is given by (Nandedkar and Bhagavat 1970) [8]:

$$(W_R)_e = \int_0^\infty (W_{Rf})_e df = \frac{8\pi h}{c^3}\int_0^\infty \frac{f^3 df}{\exp(hf/kT_e)-1} = lT_e^4, \quad (66)$$

where,

$$l = \frac{8}{15}\frac{\pi^5 k^4}{c^3 h^3}. \quad (67)$$

Here $(W_{Rf})_e$ = monochromatic noise energy density at frequency f due to the electrons, h = Planck's constant and c = velocity of electromagnetic radiation in the free space.

In this analysis it is assumed that the noise frequency f varies between the following limits:

$$0 \le f \le \infty, \quad (68)$$

for all practical purposes.

Since $(W_R)_e$ is generated due to $(W_{dc})_e$, so equating the two expressions given by eqns. (66) and (36), it is found that:

$$(W_R)_e \equiv (W_{dc})_e$$

which gives,

$$T_e = \left[\frac{N_e e E_{dc}(15/8)}{\pi^5 k^4/c^3 h^3}\right]^{1/4}. \quad (69)$$

Using eqn. (66) the monochromatic noise energy density viz., $(W_{Rf})_e$ and emittance of noise viz., $(cW_{Rf})_e$ at frequency f is given by:

$$(W_{Rf})_e = \frac{8\pi h}{c^3}\frac{f^3}{\exp(hf/kT_e)-1}, \quad (70)$$

and,

$$(cW_{Rf})_e = \frac{8\pi h}{c^2}\frac{f^3}{\exp(hf/kT_e)-1}. \quad (71)$$

The noise radiation do not interact back on electrons in the plasma generating the noise radiation.

Using eqns. (70) and (71) it can be shown (Nandedkar and Bhagavat 1970) [8] that, $(W_{Rf})_e$ or $(cW_{Rf})_e$ becomes maximum at wavelength $(\lambda_{\widetilde{m}})_e$ given by:

$$(\lambda_{\widetilde{m}})_e T_e = \frac{ch}{k}\frac{1}{4.965}. \quad (72)$$

Using eqn. (69), eqn. (72) gives,

$$(\lambda_{\widetilde{m}})_e = \frac{(ch/k)(1/4.965)}{[e N_e E_{dc}/(8/15)(\pi^5 k^4/c^3 h^3)]^{1/4}} \quad (73)$$

Similarly, the quasi-bound ions, each having the frequency of damped oscillations given by f_{op} [eqn. (61)], absorb d.c. energy density $(W_{dc})_i$ given by eqn. (40) in the presence of finite dc conductivity $(\sigma_i)_{dc}$ [refer to eqns. (37) and (64)] of the plasma due to the ions at ion-molecule collisions. During the process of ionic collisions with molecules in the plasma when the d.c. energy is absorbed, the distribution of the ions is altered. The absorbed d.c. energy reappears in the form of a d.c. noise spectrum whose frequency distribution is governed by the quantum law of radiation of energy density $(W_R)_i$ proportional to the fourth power of the noise temperature T_i of the ions. Here $(W_R)_i$ using methods of quantum statistical mechanics, is given by (Nandedkar and Bhagavat 1970) [9]:

$$(W_R)_i = \int_0^\infty (W_{Rf})_i df = \frac{8\pi h}{c^3}\int_0^\infty \frac{f^3 df}{\exp(hf/kT_i)-1} = lT_i^4, \quad (74)$$

where,

$$l = \frac{8}{15}\frac{\pi^5 k^4}{c^3 h^3} \quad (75)$$

Here $(W_{Rf})_i$ = monochromatic noise energy density at frequency f, due to the ions. The noise frequency f varies between the limits,

$$0 \le f \le \infty, \quad (76)$$

for all practical purposes.

Using eqn. (74), the monochromatic noise energy density $(W_{Rf})_i$ and the noise emittance $(cW_{Rf})_i$ at frequency f, are given by:

$$(W_{Rf})_i = \frac{8\pi h}{c^3}\frac{f^3}{\exp(hf/kT_i)-1}, \quad (77)$$

and,

$$(cW_{Rf})_i = \frac{8\pi h}{c^2}\frac{f^3}{\exp(hf/kT_i)-1} \quad (78)$$

The noise radiation do not interact back on ions in the plasma generating the noise radiation.

Using eqn. (77) or (78), it can be shown that $(W_{Rf})_i$ or $(cW_{Rf})_i$ becomes maximum at noise wavelength (given by for instance refer to Nandedkar and Bhagavat 1970) [9]:

$$(\lambda_{\widetilde{m}})_i T_i = \frac{ch}{k}\frac{1}{4.965}. \quad (79)$$

Since $(W_{dc})_i$ which is the d.c. energy density absorbed by the ions given by eqn. (40) in the presence of finite d.c. resistivity of the plasma due to the ions given by eqn. (65), is responsible to generate the radiation noise energy density $(W_R)_i$ given by eqn. (74), hence equating $(W_{dc})_i$ and $(W_R)_i$, it is found that:

$$(W_{dc})_i \equiv (W_R)_i,$$

which gives,

$$T_i = \left[\frac{N_e e\, E_{dc}(15/8)}{\pi^5 k^4 /c^3 h^3}\right]^{1/4} \quad (80)$$

Using eqn. (80), eqn. (79) gives,

$$(\lambda_{\widetilde{m}})_i = \frac{(ch/k)(1/4.965)}{[e\, N_e E_{dc}/(8/15)(\pi^5 k^4/c^3 h^3)]^{1/4}} \quad (81)$$

Comparing eqn. (69) with eqn. (80), it is clear that the electron temperature T_e responsible for the noise generation is the same as the ion temperature T_i governing its noise and further from eqns. (73) and (81), it is clear that the wavelength $(\lambda_{\widetilde{m}})_e$ at which maximum noise emittance is obtained due to the electrons is also equal to $(\lambda_{\widetilde{m}})_i$ at which the maximum noise is emittance due to the ions is generated. Thus the noise emittance spectra due to the electrons [eqn. (71)] and the ions [eqn. (78)] are identical in magnitude and the magnitude of the noise emittance versus noise frequency plot is shown in Fig. 1 for $T_e = T_i = T_c$ (say).

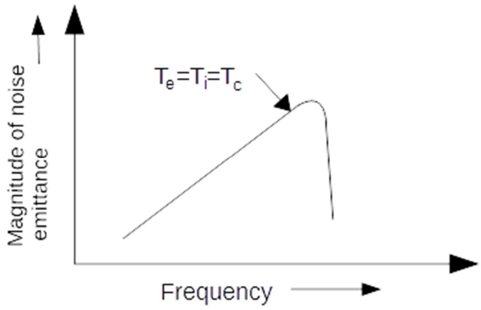

Fig. 1. Magnitude of noise emittance as a function of noise frequency.

If an ion has a charge which is equal and opposite to that of an electron, then between each wavelength of the noise emittance spectrum due to the electrons and the ions having the same magnitudes a phase difference of π radians exists.

Since the noise emittance spectra due to the electrons and the ions are exactly identical, so far as the magnitudes are concerned and further a phase difference of π radians exists between each wavelength of the noise due to the electrons and the ions, the resultant noise spectrum due to them cancels out everywhere, considering power flow of total noise radiation compromising various frequencies in the entire region of the spectrum of electromagnetic radiations (Nandedkar and Bhagavat 1970) [9].

Thus in the presence of the collisions, of electrons with molecules and of ions with molecules, electronic and ionic damped oscillations result in the plasma under the effect of a uniform d.c. electric field. The damped oscillators, viz., electronic and ionic, absorb energy (in the presence of respective collisions with molecules) from the d.c. electric field because of the finite d.c. conductivity. The absorbed energy density by electronic and ionic damped oscillators, respectively, reappears in the form of an electromagnetic spectrum which is in equilibrium state (when electronic and ionic damped oscillators are in thermal equilibrium with the ambient), thereby getting cancelled the entire electromagnetic spectrum of power having (noise) frequencies practically ranging 0 to ∞. This effect is analyzed using quantum statistical methods (Nandedkar and Bhagavat 1970) [8], [9], as mentioned briefly here. This is regarded as the quantum theory of finite dc resistivity of the plasma.

The plasma model biased by a d.c. electric field, is practically achieved by obtaining the positive column of a glow discharge of, say, air and putting two meshes in the column, across which the floating potential difference, is varied with the help of an external power supply and, is measured from which the value of d.c. electric fields viz., E_{dc} is obtained in the direction of axis of the column (Nandedkar 2016) [11].

6. Perturbation of the Plasma Model Biased by a D.C. Electric Field, by Low Power R.F. Waves

The plasma model biased by a d.c. electric field which is considered here is further perturbed by a low power r.f. wave, whose frequency lies in the vicinity of the electronic frequency of damped oscillations viz., f_{oe} Whence various interesting results are obtained which are described step by step herewith :-

6.1. Anomalous Dispersion of R.F. Waves in the Plasma

Now suppose a uniform r.f. wave (T.E.M. mode) of low power be made to interact with the quantum statistical plasma model biased by a d.c. electric field, described in various previous sections of this paper. Let $E = E_o \exp(i\omega t)$ be the r.f. electric field, of peak value E_o and angular frequency ω, which interacts with the electron satisfying differential equation of its motion given by eqn. (46b). The resultant (fictitious) differential equation of motion of the electron, in the present case, then becomes:

$$\frac{d^2r}{dt^2} + 2g_E\frac{dr}{dt} + \omega_{oe}^2 r = \frac{e}{m_e}[E_{dc} + E_o \exp(i\omega t)], \quad (82)$$

here $i = \sqrt{-1}$ and t is the instantaneous time.

Since mass of an electron is much less than the mass of an ion, the motion of the ion in the presence of the electric field of the r.f. wave is neglected in comparison to the motion of the electron.

In the steady state, the velocity of the electron can be shown to be given by (for instance refer to Bhagavat and Nandedkar 1968 [1], Nandedkar and Bhagavat 1969 [5] and Nandedkar 2016) [11]):

$$\frac{dr}{dt} = \frac{i\omega(e/m_e)E_o \exp(i\,\omega\, t)}{(\omega_{oe}^2-\omega^2)+i(2g_E)\omega}, \quad (83)$$

as a solution of eqn. (82).

Equation (82) physically leads to the following expression for the complex dielectric constant ϵ_p of the plasma (for instance refer to Nandedkar and Bhagavat 1969 [5]; Nandedkar 2016 [11]):

$$\frac{\epsilon_p'}{\epsilon_o} = 1 + \frac{\omega_{je}^2(\omega_{oe}^2-\omega^2)}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}, \quad (84)$$

and,

$$\frac{\epsilon_p''}{\epsilon_o} = \frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}, \quad (85)$$

where,

$$\epsilon_p = \epsilon_p' - i\epsilon_p'' \quad (86)$$

Here ϵ_p' gives the real part of the complex dielectric constant. ϵ_p' also, gives the permittivity of the plasma. ϵ_p'' gives the imaginary part of the complex dielectric constant of the plasma. ϵ_p'' also, gives the loss factor of the plasma. ϵ_p' and ϵ_p'' in eqns. (84) and (85) are normalized with respect to the permittivity ϵ_o of the free space.

Further, it can be shown that the r.f. energy density U_{ab} which is absorbed by the electrons in the plasma biased by a d.c. electric field, with respect to free space energy density U_f of the wave is given by (for instance refer to Nandedkar and Bhagavat, 1969 [5]: Nandedkar 2016 [11]):

$$\frac{U_{ab}}{U_f} = 2\pi\frac{\epsilon_p''}{\epsilon_o} = 2\pi\left[\frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}\right] \quad (87)$$

Here the case of low damping of the electron is considered, which is given by,

$$\omega_{oe}^2 >> g_E^2,$$

which is eqn. (47).

From this analysis, it is clear that $\frac{\epsilon_p'}{\epsilon_o}$ is unity at $\omega = \omega_{oe}$ and decreases/increases as ω increases/decreases in the neighbourhood of $\omega = \omega_{oe}$ The absorption of r.f. energy U_{ab}/U_f is maximum at $\omega = \omega_{oe}$ and U_{ab}/U_f falls on either side of it. This region in the neighborhood of the angular frequency ω_{oe} is referred to as the region of 'anomalous dispersion' for the r.f. waves. This is indicated by dotted lines in Fig. 2. In the region of anomalous dispersion, the absorption of r.f. power also reaches maximum at $\omega = \omega_{oe}$ and this power absorption falls on either side of the angular frequency ω_{oe} (for instance refer to Nandedkar and Bhagavat 1969 [5]).

Further it is interesting to note that when $E_{dc} \to 0$, $\omega_{oe} \to 0$ [eqn. (52)] and in the limit $\langle E_i \rangle$ is approached such that $2g_E \to \nu_{em}$ (Secns. 2 and 4), and then eqns. (84) and (85) give:

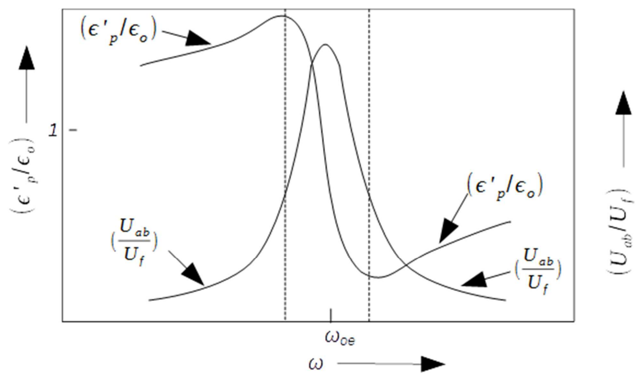

Fig. 2. Variation of ϵ'_p / ϵ_o and U_{ab}/U_f versus angular frequency ω of the interacting wave near resonance ($\omega=\omega_{oe}$), in plasma in the region of anomalous dispersion.

$$\frac{\epsilon_p'}{\epsilon_o} = 1 - \frac{(N_e e^2/m_e\varepsilon_o)}{\omega^2+\nu_{em}^2}, \quad (88)$$

and,

$$\frac{\epsilon_p''}{\epsilon_o} = \frac{(N_e e^2/m_e\varepsilon_o)\,\nu_{em}/\omega}{(\omega^2+\nu_{em}^2)} \quad (89)$$

R.f. conductivity σ_p of the plasma in this case is given by,

$$\sigma_p = \omega\epsilon_p'' = \frac{N_e e^2\nu_{em}}{m_e(\omega^2+\nu_{em}^2)} \quad (90)$$

Equations (88) and (90) are similar to eqns. (2) and (3). Thus the present theory naturally leads to the similar expression for the theory of complex dielectric constant of ionized gases basically due to Appleton-Chapman or Eccles-Larmor, when $\omega^2 \gg \nu_{em}^2$ for the case of low electronic damping where electron-molecule collisions are of main importance (similar to that occurring in the case of E-region of ionosphere where electrons make elastic collisions with neutral molecules of air at 300 ^{0}K, as the limiting case). In this case it is assumed that the power of r.f. wave is very small. So that, it does not practically disturb the plasma.

Experimentally variation of phase constant of a r.f. wave with wave-frequency is measured in the region of anomalous dispersion in the plasma. In practice this is achieved by terminating the plasma-slab biased by a d.c. electric field mentioned in Secn. 5, as a load to the slab line - through which r.f. wave of variable frequency can be sent. Standing wave ratio measurements give the value of phase constant of the wave in the plasma (Bhagavat and Nandedkar, 1968 [1]; Nandedkar and Bhagavat, 1969 [5]; Nandedkar 2016 [11]). From the geometry of the experimentally determined curve of the phase constant of the interacting wave vs. wave-frequency, electron density N_e and collision frequency g_E of the electron with molecules in the plasma, are determined (Bhagavat and Nandedkar, 1968 [1]; Nandedkar and Bhagavat, 1969 [5]; Nandedkar 2016 [11]). With these values of g_E and N_e, curves like those given in Fig. 2, are plotted, knowing experimentally the resonance angular frequency, ω_{oe}.

6.2. Energy Transport, Phase, Group, Wave Front and Signal Velocities in the Region of Anomalous Dispersion of the Plasma

Next an analysis of energy transport, phase, group, wave front and signal velocities of the r.f. wave in the plasma in the vicinity of resonance angular frequency ω_{oe} is considered.

(i) Velocity of energy transport (u_p):

The velocity with which energy of the r.f. wave propagates is referred to as the velocity of energy transport and is denoted by u_p in the case of wave propagation through the plasma.

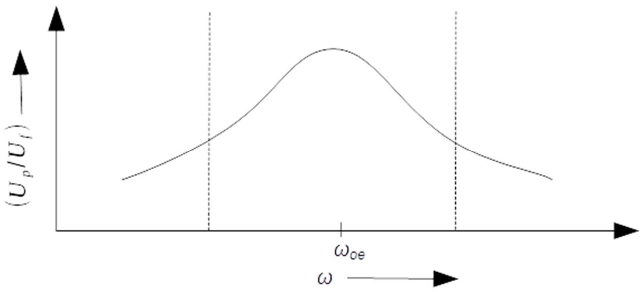

Fig. 3. Normalized energy density – U_p/U_f – in plasma of the interacting wave as a function of wave angular frequency ω near resonance ($\omega=\omega_{oe}$).

When the wave propagates through the plasma, energy is stored in the medium. This energy considering unit volume of the medium is stored in free space capacitance ϵ_o, free space inductance μ_o and equivalent inductance L_{pe} [eqn.(56)] of the plasma, as well as its equivalent capacitance C_{pe} [eqn. (57)]-refer to Bhagavat and Nandedkar 1968 [2]. This stored energy density is denoted by U_p. This stored energy density U_p normalized with respect to free space energy density U_f of the wave i.e. U_p/U_f becomes maximum at the resonance ($\omega = \omega_{oe}$), and falls on either side of it, as shown in Fig. 3 (for instance refer to Bhagavat and Nandedkar 1968 [2], [3]).

If P gives the magnitude of Poynting vector in the plasma, then velocity of energy transport u_p is given $u_p = P/U_p$ numerically.

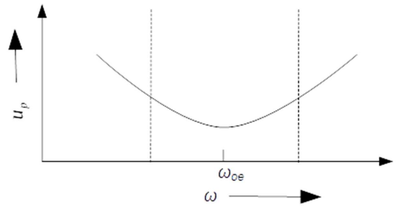

Fig. 4. Variation of u_p versus angular frequency ω of the interacting wave in plasma near resonance ($\omega=\omega_{oe}$).

Fig. 4 shows, variation of u_p with ω in the region of anomalous dispersion in the plasma near resonance. u_p has a pronounced minimum at $\omega = \omega_{oe}$. u_p does not exceed the value of velocity of a T.E.M. wave in free space i.e. c here (Bhagavat and Nandedkar 1968 [2], [3]).

(ii) Phase velocity (v_{pp}):

The velocity with which surface of constant phase of the wave moves is regarded as the phase velocity and is denoted by v_{pp} in the case of wave propagation through the plasma.

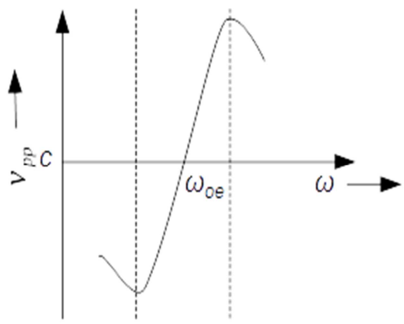

Fig. 5. Variation of v_{pp} versus angular frequency ω of the interacting wave in the plasma near resonance ($\omega=\omega_{oe}$).

Fig. 5 indicates that the phase velocity of the r.f. wave at first decreases with increase of angular frequency of the wave. Then it reaches a minimum and afterwards increases with increase of the angular frequency until it reaches a maximum value. After this v_{pp} is found to decrease with the angular frequency. The maximum and minimum values of v_{pp} are located at the two boundaries of the region of anomalous dispersion marked by dotted lines. At $\omega = \omega_{oe}$ the phase velocity is approximately equal to that of a T.E.M. wave in free space i.e. c. For $\omega > \omega_{oe}$, v_{pp} is found to be greater than c (Bhagavat and Nandedkar 1968 [4]).

(iii) Group velocity (v_{gp}):

If β_p be the phase propagation constant of the wave in the plasma [where $\beta_p = (\omega/c)(\epsilon_p'/\epsilon_0)^{1/2}$ - here square of attenuation constant of the wave in the plasma is much smaller than β_p^2 and further $\epsilon_p''^2 \ll \epsilon_p'^2$ throughout this analysis], then $d\omega/d\beta_p$ gives the group velocity of the wave and is denoted by v_{gp}.

Fig. 6 shows that group velocity at first finitely increases with increase of frequency until it approaches the velocity of a T.E.M. wave in free space i.e. c. Then it reaches an infinite value and afterwards increases finitely unit it reaches a finite maximum value. After this, it again decreases finitely until it shoots up to infinity and then it is found to decrease finitely with angular frequency, passing through the value of c. Near the two boundaries of the region of anomalous dispersion marked by dotted lines, v_{gp} approaches the value of c and then shoos up to $\pm\infty$. While within the boundaries the group velocity is negative. At $\omega = \omega_{oe}$ the group velocity is maximum so far as the negative group velocity zone is concerned.

The above results indicate that the group velocity in the vicinity of $\omega = \omega_{oe}$ can be greater than the velocity of a T.E.M. wave in free space, can be infinite and even negative.

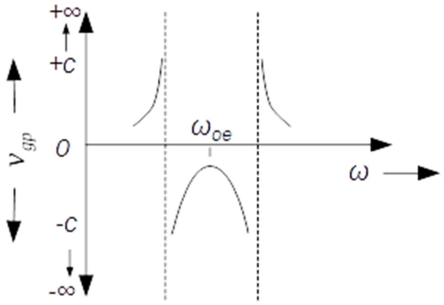

Fig. 6. Variation of v_{gp} versus angular frequency ω of the interacting wave in the plasma near resonance ($\omega=\omega_{oe}$).

Comparing the results of the velocity of energy transport in the neighbourhood of $\omega = \omega_{oe}$ (fig. 4), it is clear that the group velocity is entirely different from the velocity of energy transport in this region and hence the group velocity in the neighborhood of $\omega = \omega_{oe}$ does not give the velocity with which energy is propagated. Actually, here the velocity of energy transport is always less than the velocity of a T.E.M. wave in free space.

The region in the vicinity of $\omega = \omega_{oe}$ gives the zone of backward waves, because phase and group velocities are oppositely directed, the phase velocity is positive and the group velocity negative (Bhagavat and Nandedkar 1968 [4]).

(iv) Wave front velocity (v_w):

Whatever may be the frequency of a r.f. wave, the velocity of wave front in the plasma is always equal to the velocity of a T.E.M. wave in free space. Thus the wave front velocity in the region of backward waves is equal to the wave velocity in free space. When the wave front of the signal makes its way through the plasma, it finds the electrons which are capable of oscillating originally at rest. Originally, therefore plasma seems to be free space for electromagnetic field propagation; only after the electrons are set into motion, can influence the phase and form of the electromagnetic wave. The propagation of the wave front thus proceeds undisturbed with the velocity of a T.E.M. wave in free space, independently of character of oscillating electrons (Nandedkar and Bhagavat 1969 [7]).

(v) Signal velocity (v_{sp}):

The signal velocity for $\omega < \omega_{oe}$, increases with the increase of angular frequency of the r.f. wave in the region of backward waves. In this portion of the region v_{sp} is less than c. At ω_{oe}, the signal velocity becomes equal to the velocity of a T.E.M. wave in free space. For $\omega > \omega_{oe}$, the signal velocity decreases with increase of angular frequency of the r.f. wave, in this portion of the region, v_{sp} is again smaller than c. The left hand portion of the region of backward waves with respect to ω_{oe} is approximately a mirror image of the right half portion. One thing is certain, the signal velocity in the region of the backward waves is always smaller than the velocity of a T.E.M. wave in free space or at the most equal to c – refer to Fig. 7. (refer to Nandedkar and Bhagavat 1969) [7].

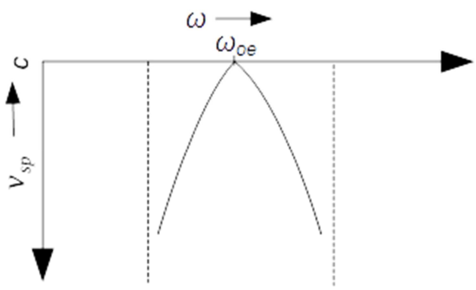

Fig. 7. Signal velocity v_{sp} versus angular frequency ω of the interacting wave in the plasma near resonance ($\omega=\omega_{oe}$).

Plots of u_p, v_{pp}, v_{gp} and v_{sp} near resonance i.e. $\omega = \omega_{oe}$ (figs. 4 to 7) in the plasma, versus angular frequency of the wave, are obtained, when electron density N_e, electron-molecule collision frequency g_E and resonance angular frequency ω_{oe} of electronic damped oscillations in the plasma are experimentally known. These quantities can be obtained by perturbation of the plasma model biased by a d.c. electric field, using low power r.f. waves (Bhagavat and Nandedkar 1968 [1], Nandedkar and Bhagavat 1969 [5] and Nandedkar 2016 [11]).

6.3. Quantum Theory of noise Radiation Due to Finite r.f. Resistivity of the Plasma

When the r.f. wave interacts with the plasma model biased by a d.c. electric field described in Secn. 5, then energy from the r.f. wave is lost in the medium. Since mass of an electron is much less than the mass of an ion, the motion of the ion in the presence of the electric field of the r.f. wave is neglected in comparison to the motion of the electron. And hence while considering the (fictitious) differential equation of motion of the electron in the presence of the field of the r.f. wave [eqn. (82)], the effect of the ion motion is neglected. So that, absorption of the r.f. energy from the wave, mainly, takes place due to quasi-bound electrons with frequency of damped oscillations denoted by f_{oe} [eqn. (52)], in the presence of electron-molecule collisions. The r.f. energy density U_{ab} which is absorbed by the quasi-bound electrons in the plasma in the presence of electron-molecule collisions normalized with respect to free space energy density U_f of the wave, is given by eqn. (87):

$$\frac{U_{ab}}{U_f} = 2\pi\left(\frac{\epsilon_p''}{\epsilon_o}\right) = 2\pi\left[\frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2 - \omega^2)^2 + (2g_E)^2\omega^2}\right],$$

where,

$$\frac{\epsilon_p''}{\epsilon_o} = \frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2},$$

which is eqn. (85), gives the loss factor ϵ_p'' of the plasma for the wave, normalized with respect to free space permittivity of ϵ_o as mentioned already. Further r.f. conductivity of the plasma viz., $(\sigma_{pe})_{r.f.}$ due to the electrons, in terms of ϵ_p'' can be expressed by the following relationship (for instance refer to Nandedkar and Bhagavat 1969 [5] and Nandedkar 2016. [11]):

$$(\sigma_{pe})_{r.f.} = \omega\epsilon_p'' = \frac{(\omega\epsilon_0)\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}, \quad (91)$$

and so the r.f. resistivity $(R_{pe})_{r.f.}$ of the plasma due to the 'quasi-bound' electrons undergoing electron-molecule collisions, is given by,

$$\frac{1}{(\sigma_{pe})_{r.f.}} = (R_{pe})_{r.f.} = \frac{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}{(\omega\epsilon_0)\omega_{je}^2(2g_E)\omega} \quad (92)$$

Thus because of the finite r.f. resistivity of the plasma due to the electrons given by eqn. (92), there occurs absorption of r.f. energy density U_{ab} as given by eqn. (87).

Thus, the 'quasi-bound- electrons, each having the frequency of damped oscillations as given by f_{oe} [eqn. (52)] in the presence of finite r.f. resistivity [refer to eqn. (92)] of the plasma due to electronic collisions with molecules, absorb r.f. energy density U_{ab} given by eqn. (87). During the process of electronic collisions with molecules when U_{ab} is absorbed, distribution of the electrons in the plasma is altered. The absorbed r.f. energy density reappears in the form of a noise spectrum whose frequency distribution is governed by the quantum law of radiation of energy density W_n proportional to the fourth power of the noise temperature T_n of the electrons in the plasma.

At electron-molecule collisions, when r.f. energy density U_{ab} is absorbed by the 'quasi bound' electrons as mentioned already, a pulse of noise radiation is generated which compromises practically all frequencies ranging 0 to ∞. Then all the electrons in the plasma, which constitute the noise radiation pulse, oscillate with common frequency of noise radiation f, such that,

$$0 \le f \le \infty, \quad (93)$$

for all practical purpose. The altered distribution state of the electrons having noise frequencies given by eqn. (93) is represented in terms of noise temperature T_n, which is the electron temperature that generates the noise. Quantization of the linear oscillating electron with noise frequency given by eqn. (93) at temperature T_n gives the following expression for quantum noise radiation energy density W_n (Nandedkar and Bhagavat 1969 [6]), viz.,

$$W_n = \int_0^\infty W_{nf}df = \frac{8\pi h}{c^3}\int_0^\infty \frac{f^3 df}{\exp(hf/kT_n)-1} = lT_n^4, \quad (94)$$

where,

$$l = \frac{8}{15}\frac{\pi^5 k^4}{c^3h^3}. \quad (95)$$

Here W_{nf} gives the monochromatic noise energy density at noise frequency f due to the electrons. The limits of variation of f, in this case, is given by eqn. (93). The value of constant as given by eqn. (95) is same as that given in eqn. (67) or (75). Since W_n is generated due to U_{ab}, so equating the two expressions given by eqn. (94) and (87) it is found that,

$$W_n \equiv U_{ab},$$

which gives,

$$T_n = \left[2\pi\frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}\left(\frac{15}{8}\frac{c^3h^3}{\pi^5k^4}\right)\right]^{1/4}(U_f)^{1/4} \quad (96)$$

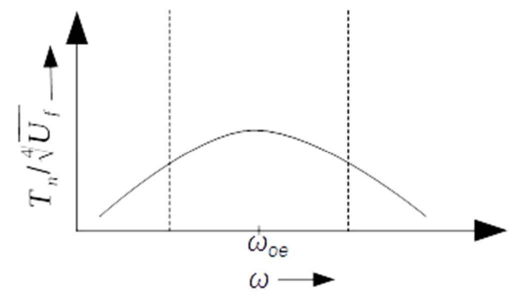

Fig. 8. Variation of $T_n/(U_f)^{1/4}$ versus angular frequency ω of the interacting wave in the plasma near resonance ($\omega=\omega_{oe}$).

Variation of T_n with angular frequency ω of the interacting wave, near resonance $\omega = \omega_{oe}$ is shown in Fig. 8. T_n reaches maximum at resonance and falls on either side of it (for instance refer to Nandedkar and Bhagavat 1969 [6]).

Using eqn. (94), the monochromatic noise energy density viz., (W_{nf}) and emittance of noise viz., (cW_{nf}), at noise frequency f, are given by,

$$(W_{nf}) = \frac{8\pi h}{c^3}\frac{f^3}{\exp(hf/kT_n)-1}, \tag{97}$$

and,

$$(cW_{nf}) = \frac{8\pi h}{c^2}\frac{f^3}{\exp(hf/kT_n)-1} \tag{98}$$

Here the effect of dispersing charge particles in the plasma on the emittance of noise radiation, is neglected. Using equation (97) or (98) it can be shown (Nandedkar and Bhagavat 1969) [6] that (W_{nf}) or (cW_{nf}) becomes maximum at wavelength λ_m given by,

$$\lambda_m T_n = \frac{ch}{k}\frac{1}{4.965} \ . \tag{99}$$

Using eqn. (96), eqn. (99) gives,

$$\lambda_m = \frac{(ch/k)(1/4.965)}{\left[2\pi\frac{\omega_{je}^2(2g_E)\omega}{(\omega_{oe}^2-\omega^2)^2+(2g_E)^2\omega^2}\left(\frac{15c^3h^3}{8\pi^5k^4}\right)\right]^{1/4}(U_f)^{1/4}} \tag{100}$$

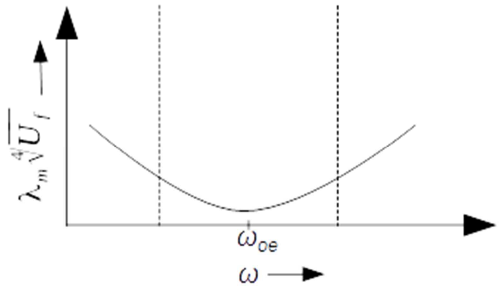

Fig. 9. Variation of $\lambda_m (U_f)^{1/4}$versus angular frequency ω of the interacting wave in the plasma near resonance ($\omega=\omega_{oe}$).

Variation of λ_m with angular frequency ω of the interacting wave, near resonance ($\omega = \omega_{oe}$) is shown in Fig. 9. λ_m reaches minimum at the resonance and increases on either side of it (for instance refer to Nandedkar and Bhagavat 1969 [6]).

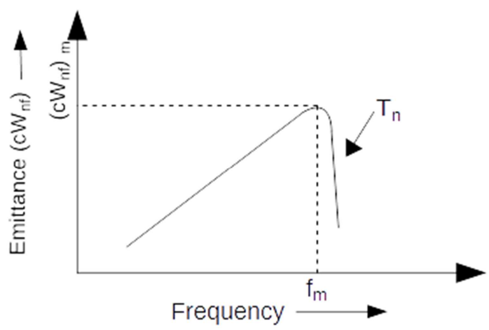

Fig. 10. Variation of (cW_{nf}) with noise frequency at a given noise temperature T_n. $(cW_{nf})_m$ is maximum value of (cW_{nf}) at frequency f_m.

Fig. 10 gives variation of (cW_{nf}) with noise frequency f at a given noise temperature T_n. In fig. 10, $(cW_{nf})_m$ gives the maximum value of (cW_{nf}) at noise frequency f_m, such that

$$f_m = c/\lambda_m, \tag{101}$$

where $\lambda_m$is given by eqn (100).

In Secn. 5, it is described that ion or electron noise radiation spectra in the d.c. biased plasma are in equilibrium state such that no net radiation power comes out. When such a plasma model is perturbed by a low power r.f. wave, a differential radiation spectrum (Fig.10) characterized by T_n [eqn. (96)] results where the noise power can come out, by creating a small disturbance in the d.c. biased plasma, due to the r.f. wave.

When $E_{dc} \to 0$ then $\omega_{oe} \to 0$, then the above quantum theory of (black body type) noise radiation due to perturbation of the plasma model on interactions with low power r.f. waves does not hold good.

7. Discussions and Conclusions

Present analysis considers the plasma in which electrons, ions and neutral molecules are in thermal equilibrium at ambient temperature T which is the room temperature. The plasma consists of equal densities of electrons and ions, thus making it macroscopically neutral. Here the case of a relatively high pressure plasma is considered which is having a relatively low density of charge particles. Thus thermal equilibrium of the plasma with the ambient is mainly obtained due to electron molecule and ion molecule collisions. These collisions considered are elastic in nature.

In the presence of internal ionic field of the plasma, electronic eigen frequency damped oscillations having frequency $\omega_{je}/2\pi$ [refer to eqn. (21b)] result in the presence of electron-molecule collisions, at the average distance of an electron from an ion in the screening sphere viz., R_o, where the ion is at the center of the screening sphere. The eigen frequency of electronic damped oscillations, is similar to the plasma frequency in the case of electrons due to Tonks and Langmuir (Tonks and Langmuir 1929) [15]. Similarly ionic eigen frequency damped oscillations having frequency $\omega_{jp}/2\pi$ [refer to eqn. (25)], taking into account the effect of ion-molecule collisions, is considered. These eigen frequency damped oscillations due to the electrons or ions are not sustained as such in practice when steady state approaches, considering the effect of relative polarization of electron-ion pairs in the plasma, thereby making the electrons or ions of the plasma as free particles.

On the application of a d.c. electric field E_{dc} to the above plasma model, the electrons and ions start moving in the opposite directions. The motion of the electrons and ions, in the presence of E_{dc} is opposed by the collisions of the electrons and ions with neutral molecules, in the plasma. This constitutes the d.c. conductivity of the plasma, which is the sum of the conductivities due to the electrons and ions [eqn. (43)]. Maintaining thermal equilibrium of the charge particles at temperature T, the electrons and ions absorb d.c. energy densities $(W_{dc})_e$ and $(W_{dc})_i$ from the d.c. electric field in the plasma [eqns. (36) and (40) respectively].

In the presence of low damping provided by electron-molecule collisions, electronic damped oscillations of frequency f_{oe} [eqn. (52)] result, whose amplitude R_{oc} is inside the screening sphere having an ion at its center. R_{oc} is measured with respect to the center of the screening sphere. The relation between R_o [which is the amplitude of eigen frequency electronic damped oscillations viz., $\omega_{je}/2\pi$ {refer to eqn. (21b)}] and R_{oc} [which is the amplitude of electronic damped oscillations of frequency $\omega_{oe}/2\pi$ {refer to eqn. (52)}] is given by $R_o/R_{oc} = 1 - \exp(-1)$, refer to eqn. (51). Here R_{oc} exists inside the screening sphere. The electronic damped oscillations of frequency f_{oe} [eqn. (52)] exist in the plasma in the presence of d.c. electric field, E_{dc} mentioned already. In the limit when E_{dc} vanishes, then the present theory of f_{oe} naturally leads to the expression, similar to the plasma electronic frequency, which is due to Tonks and Langmuir (Tonks and Langmuir 1929) [15]). Similarly ionic damped oscillations of frequency f_{op} [eqn. (61)], is obtained in the presence of E_{dc}, when low damping is provided by ion-molecule collisions. Ionic damped oscillations of frequency f_{op} exist inside the screening sphere in steady state, with an electron at the center of the screening sphere. When E_{dc} vanishes in the limit, then $\omega_{jp}/2\pi$ [refer to eqn. (25)] is obtained.

Further the electrons in this analysis whose density is N_e and temperature T (which is ambient room temperature) are referred to as 'slow electrons'. These 'slow electrons' form about 1% part of the electrons in the other group in the plasma, which are characterized by density N_{eL} and temperature T_{eL} which can be determined by Langmuir's probe method. The electrons having density N_{eL} and temperature T_{eL} can be referred to as relatively 'fast electrons' as against the 'slow electrons' which are characterized by density N_e and temperature T. Condition of overall charge neutrality in the plasma is experimentally verified here, where electrons, ions, and neutral molecules are in thermal equilibrium at temperature T of the ambient.

The 'quasi bound' electrons and ions having frequency of damped oscillations denoted by f_{oe} and f_{op} respectively, absorb d.c. energy density $(W_{dc})_e$ and $(W_{dc})_i$ at electron-molecule and ion-molecule collisions in the presence of a d.c. electric field E_{dc}, due to finite d.c. conductivity of the plasma. The absorbed energy density of electronic and ionic damped oscillators respectively, reappears in the form of an electromagnetic noise spectrum. Magnitude of the noise emittance spectrum versus noise frequency, due to either electrons or ions having noise temperature of $T_e = T_i = T_c$ (say), is shown in Fig. 1. Since the noise emittance spectrum due to either electrons or ions is exactly identical, so for the magnitude is concerned, and further a phase difference of π radians exists between each wavelength of noise due to electrons and ions, the resultant noise emittance spectrum due to them cancels out everywhere, considering power flow of the total noise radiations compromising various frequencies practically ranging 0 to ∞ in entire spectrum of electromagnetic radiations.

When such a plasma model biased by a d.c. electric field, is perturbed by low power r.f. waves whose angular frequencies is in the vicinity of the angular frequency of electronic damped oscillations namely $\omega_{oe}(= 2\pi\, f_{oe})$, then a region of anomalous dispersion exists as shown in Fig. 2, where permittivity of the plasma decreases with increase of angular frequency of the wave and absorption of energy of the r.f. wave is maximum at the resonance and falls on either side of it (Nandedkar and Bhagavat 1969) [5]. The r.f. wave does not practically disturb ions in the plasma as compared to that of electrons. Moreover, the present theory naturally leads to the similar expression for the theory of complex dielectric constant of ionized gases basically due to Appleton-Chapman (Appleton and Chapman 1932) [12] or Eccles-Larmor (Eccles 1912 [22], Larmor 1924 [20], [21]), when $\omega^2 >> \nu_{em}^2$ or $\omega^2 >> \nu^2$ for the case of low electronic damping where electron-molecule collisions are of basic importance [eqns.(88) and (90) or (2) and (3)] similar to that occurring in the case of E-region of ionosphere where electrons make elastic collisions with neutral air molecules at ambient temperature of $300\,^0$K (Ratcliffe 1959) [14] – as the limiting case.

In actual practice the d.c. biased plasma model is realized by obtaining the positive column of glow discharge of air and putting two meshes in the positive column, across which the floating potential difference is varied (using an external power supply unit) and measured from which the value of d.c. electric field, viz., E_{dc} is obtained in the direction of axis of the positive column. Experimentally variation of the phase constant of a r.f. wave with wave frequency is measured in the region of anomalous dispersion in the plasma. In practice this is achieved by terminating the plasma-slab biased by the d.c. electric field (and carrying a given d.c. current), as a load to a slab line through which the r.f. wave of variable frequency is sent. Measurements of the reflection coefficient give the value of phase constant of the wave (Nandedkar and Bhagavat 1969 [5], Nandedkar 2016 [11]). From the geometry of the experimental graph of phase constant of the wave versus its frequency in the region of anomalous dispersion, values of electron density N_e, electron-molecule collision frequency g_E and electronic frequency of damped oscillations f_{oe} are determined. Knowing the value of f_{oe}, N_e and E_{dc} experimentally (Nandedkar and Bhagavat 1969 [5], Nandedkar 2016 [11]), the value of the ratio of charge to mass of an electron i.e. e/m_e, is determined using theoretical formula given by eqn. (52). The value of e/m_e thus obtained is 1.7352 x 10^{11} C/kgm. The difference between presently accepted value of e/m_e and that obtained according to present analysis is 1.3586%, (actual value of e/m_e is higher than experimentally obtained value) which is due to the experimental limitations. Further knowing the gas pressure in the plasma-slab, ambient temperature T and g_E experimentally (Nandedkar and Bhagavat 1969, Nandedkar 2016) [5] and [11], the value of R_m i.e. classical radius for the gas molecule viz., that of air due to gas kinetics is determined using eqn. (29). The mean value of the classical radius of an air molecule obtained by the present analysis given here is 1.0799 x 10^{-10}m. This value of the classical radius R_m for the air molecule is quite reasonable in the view of gas-kinetics (which gives the order of classical radius R_m as 10^{-10} m for any type of a gas molecule) for the experimental investigations carried out here in the case of plasma provided by positive column of glow discharge of air. Now the major constituent of air is nitrogen gas. Hence the value of R_m= 1.0799 x 10^{-10} m obtained for the air molecule as mentioned earlier, can also be a assumed to represent the classical radius of the nitrogen gas molecule for all practical purposes, for the type of experimental investigations illustrated here. Since g_E [eqn.(29)], is practically the same for any type of a gas molecule [as order of classical radius R_m is the same for any type of a gas molecule which is 10^{-10} m according to gas-kinetics, for the purpose of investigations of the present type of experimentation where $g_E^2 << \omega_{oe}^2$, where ω_{oe} is obtained from eqn. (52)] and ω_{oe} depends on N_e and E_{dc} , hence the present theory is applicable to any gaseous discharge, irrespective of the gas used, provided the condition of thermal equilibrium of electrons, ions and molecules at ambient temperature holds good, Thus the theory is quite general in its application.

Now consider following expressions, of eqn. (8),

$$\nu_{em} = \frac{v_e}{\lambda_e} = N_m(\pi R_m^2)\left(\frac{8kT}{\pi m_e}\right)^{\frac{1}{2}},$$

and,

$$g_E = \frac{v_e'}{\lambda_e} = N_m(\pi R_m^2)\left(\frac{2kT}{\pi m_e}\right)^{1/2},$$

of eqn. (29). From the above two equations, it is clear that,

$$\nu_{em} = 2\, g_E \qquad (102)$$

This value of electron-molecule collision frequency ν_{em} is previous value of electron-molecule collision frequency considered in earlier papers (Bhagavat and Nandedkar 1968, Nandedkar 2016) [1], [11]. Since value of radius of air molecule obtained from the value of ν from a previous paper (Bhagavat and Nandedkar 1968) [1], is the same as that as obtained from the value of $g_E = g$ of a previous paper {(Nandedkar and Bhagavat, 1969) [5]} where eqn. (102) holds good, where ν_{em} or ν is to be considered as a real collision frequency and g_E or g is to be considered as a virtual collision frequency. The virtual collision frequency is introduced now to explain various quantum effects described herewith with this model of "quasi-bound: quasi-free charge-carriers".

Further analysis of phase, group and energy transport velocities of the r.f. waves in the plasma in the vicinity of resonance angular frequency ω_{0e} is carried out (for instance, refer to Fig. 5, 6 and 4).

Here phase velocity v_{pp} increases with increase of angular frequency of the wave tending to the value of velocity of a TEM wave in free space i.e. c near resonance (Fig. 5).

Group velocity v_{gp} near resonance can be greater than velocity of a TEM wave in free space and can be even negative (Fig. 6). In this region phase and group velocities are opposite directed. So this region, provides an example of the region of backward waves with respect to angular frequency of the wave relative to resonance. The backward wave region is inside dotted lines region shown (Fig. 6).

Group velocity v_{gp}, near resonance is entirely different from velocity of energy transport u_p ; u_p has a minimum near resonance and increases on either side of it with respect to angular frequency ω of the wave (Fig. 4). u_p does not exceed the value of c here.

Wave front velocity v_w is always equal to velocity of a T.E.M. wave in free space viz., c near resonance with reference to angular frequency ω of the r.f. wave, and otherwise also.

Signal velocity v_{sp} at resonance is equal to c and reduces on either side of the resonance with reference to angular frequency ω of the r.f. wave in approximately symmetrical way (Fig. 7).

It has been mentioned earlier that electron and ion noise radiation spectra in the plasma biased by a d.c. electric field, due to electron-molecule and ion-molecule collisions, are in equilibrium state such that no net radiation power come out. When the low power r.f. wave interacts with the plasma biased by a d.c. electric field, then r.f. energy is lost [refer to eqn. (87)] and this brings into picture the r.f. resistivity of the plasma [refer to eqn. (92)], in this case. In fact, the low power r.f. wave perturbs the plasma biased by a d.c. electric field, and as such a differential radiation spectrum (Fig.10) characterized by the noise temperature T_n [eqn. (96)] results, where the noise power can come out, by creating a small disturbance in the plasma biased by a d.c. electric field, and a continuous spectrum (black-body type) of all noise frequencies of waves can exit with a peak of extremely low power at sub-millimeter range of wavelengths for low power interacting electromagnetic U.H.F. wave(s) with plasma-refer to Nandedkar and Bhagavat 1969 [6].

Acknowledgements

The author is thankful for the interesting discussions with reference to this research-paper given herewith he had, with his Guru (the Teacher) Dr. Ing. G. K. Bhagavat, [(now) Late] Professor of Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India.

Further he gives thanks to Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, India for giving permission to publish this research-work.

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