Physics Journal
Vol. 1, No. 3, 2015, pp. 343-348
http://www.aiscience.org/journal/pj

Mathematical Modeling of Self - Oscillations in the Combustion Chamber of Liquid Rocket Engine with Variable Latency Combustion

B. I. Basok, V. V. Gotsulenko*

Department of Thermophysical Fundamentals of Energy-Saving Technologies, Institute of Engineering Thermal Physics of NAS of Ukraine, Kiev, Ukraine

Abstract

Self-oscillations and certain of their regularities determined by solution of a system of differential equations with variable delay argument equations that is used in considering combustion instability in combustion chambers of liquid-propellant rocket engines are modeled mathematically. Periodic solutions of the system of equations of nonstationary motion of a medium in a liquid-propellant rocket engine were obtained, with the aid of which the possibility of lowering the amplitude of the longitudinal self-oscillations of vibration combustion or their complete removal has been substantiated. Analytically determined critical time delay combustion, above which a stationary combustion becomes unstable and self-excited oscillations.

Keywords

Vibration Combustion, Instability, Self - Oscillation, Limiting Cycle, the Time Delay of Combustion

Received: September 16, 2015 / Accepted: December 12, 2015 / Published online: December 29, 2015

1. Introduction

The nonstationarity of the process of burning of a fuel in different devices is generated by the chemical-kinetic, diffusion-thermal, and convective phenomena [1]. Furthermore, according to the first low of thermo- dynamics, the heat released in combustion of the fuel in the flow is converted to the internal energy and components of the total head whose value is dependent on the flow rate. The necessary condition of excitation of self-oscillations in the case in question is the presence of the ascending branch on the head characteristic [2, 3], which is formed by the heat-to-head conversion.

In [4], it has been established that, in burning of the fuel in solid-fuel engines, the pressure dependence of the rate of formation of the gas can be such that the system will lose stability [5]. In liquid-propellant rocket engines (LPRE), the condition of intrachamber instability lies in the formation of the ascending branch of the $p = F(G)$ dependence of the pressure in the combustion chamber on the flow rate. Growth in the head or the pressure p in the combustion chamber with flow rate features prominently among the causes of excitation of vibrational-combustion self-oscillations; it has not been considered in theoretical description of the phenomenon of instability of combustion in liquid-propellant rocket engines. The reason is that the conditions of formation of the ascending branch on the $p = F(G)$ dependence of the head characteristic of the combustion chamber on the flow rate remained unknown.

Thus, e.g., in [5, 6], the above dependence of the head characteristic has been represented as being monotonically decreasing. In [5], the stability of the stationary regime with such a dependence has been substantiated. Therefore, combustion instability in liquid-propellant rocket engines

* Corresponding author
E-mail address: basok@ittf.kiev.ua (B. I. Basok), gosul@ukr.net (V. V. Gotsulenko)

was mainly determined by the mechanism of phenomenological time delay of the combustion of the fuel [5–7] and by the manifestation of different feedbacks which are internal and act universally.

2. Formulation of the problem

In this paper, we consider the problem of determining the critical time delay of combustion and the construction of the boundaries of the stability region when the delay is variable, depending on the pressure inside the combustion chamber. In Fig. 1 shows a diagram of the combustion chamber rocket engine.

3. System of Equations of Intrachamber Combustion Instability

The equation of motion for the sections of the flow of a fuel 0 – 0 and of combustion products 3 – 3 (Fig. 1) in the form of $d(mw) = (p_{\rm f} - h_{\rm T} - h_{\rm fr} - p_{\rm n})Sdt$, taking into account that the mass of the combustion is $m = \rho l S$, will be written as [8, 9]

$$L_{\rm a,c.ch}\frac{dG}{dt} = F(G) - p_{\rm n}, \qquad (1)$$

where $L_{\rm a,c.ch} = l/S$ – acoustic mass of the combustion chamber; G – mass flow rate in the combustion chamber; $p_{\rm f} = H(G)$ – pressure characteristic of parallel connection centrifugal pumps oxidant and fuel; $F(G) = p_{\rm f} - h_{\rm T}(G) - h_{\rm fr}(G)$ – pressure head characteristic of fuel flow in a combustion chamber [8-9]; $h_{\rm T}(G)$ – pressure losses because of heat supply (thermal resistance); $h_{\rm fr}(G)$ – viscous losses along the length of the combustion chamber.

The mass equation in the combustion chamber is represented, according to [8], in the form

$$dm = \left(G\left[t - \tau(p_{\rm n})\right] - G_{\rm n}\right)dt,$$

where $G = G_{\rm f} + C_{\rm ox}$, $G_{\rm f}$ – flow rate of a fuel, $C_{\rm ox}$ – flow rate of oxidizer, $\tau(p_{\rm n})$ – the time delay of combustion of fuel, $G_{\rm n}$ – mass flow rate through the nozzle. Since $dp_{\rm n}/d\rho$ is equal to c^2, where c – velocity of sound in the flow, it may be reduced to the form

$$C_{\rm a,c.ch}\frac{dp_{\rm n}}{dt} = G\left[t - \tau(p_{\rm n})\right] - \phi(p_{\rm n}), \qquad (2)$$

where $G_{\rm n} = \phi(p_{\rm n})$ – characteristic of the nozzle of a liquid-propellant rocket engine, $C_{\rm a,c.ch} = V/c^2$ – acoustic flexibility of the combustion chamber, V – the volume of the combustion chamber. Experimental studies [7] showed that the dependence of $\tau(p_{\rm n})$ is monotonically decreasing. Figure 2 shows the experimental points and their approximation using the following formula $\tau(p_{\rm n}) = b\exp(-ap_{\rm n})/\sqrt{p_{\rm n}}$, where $a = 0.12$, $b = 0.006$.

4. Calculation of Hydraulic Characteristics in the LPRE Combustion Chamber

The viscous losses occurring over the combustion chamber length in the smoke gas motion segment were calculated using the Darcy - Weisbach formula

$$h_{\rm fr}(G) = \lambda\frac{l}{d}\rho\frac{w^2}{2}, \qquad (3)$$

where λ is the coefficient of hydraulic losses.

Heat input to the fuel flow with a mass rate of flow $G = \rho w S$ in the channel of the LPRE combustion chamber with a constant normal cross-section area (Fig. 1) leads to a decrease in the density ρ of the flow of combustion products and an increase in its velocity w. Therefore, the heat resistance to such flow [10] is determined by the local hydraulic resistance in the region of heat input. In [10], we obtained a function for determining the energy loss in the combustion chamber $h_{\rm T}(G)$ at polytropic heat input to the fuel flow.

Let us define the function $h_{\rm T}(G)$ at isobaric heat input to the fuel flow in the LPRE combustion chamber. We write the energy equation for sections 1–1 and 2–2 (Fig. 1), and from this equation we determine the thermal resistance, which is the local one, appearing in the region of heat supply [10]

$$q + \frac{p_1}{\rho_1} + u_1 + \frac{w_1^2}{2} = \frac{p_2}{\rho_2} + u_1 + \frac{w_2^2}{2} + \Delta h_{\rm T}, \qquad (4)$$

where $\Delta h_{\rm T}$ is the energy loss in the flow due to the heat input. Since at isobaric combustion heat input to the ideal gas flow $q = c_{\rm p}(T_2 - T_1)$, the change in the internal energy of the gas $u_2 - u_1 = c_{\rm v}(T_2 - T_1)$, and $c_{\rm p} - c_{\rm v} = R$, then $q - \Delta u = R(T_2 - T_1)$. Further, in view of the relation $q - \Delta u = p_2/\rho_2 - p_1/\rho_1$ from Eq. (4) it follows that

$\Delta h_{\text{T}} = w_1^2/2 - w_2^2/2$, or in pressure units, assuming $h_{\text{T}} = \rho_1 \Delta h_{\text{T}}$, we have

$$h_{\text{T}} = \rho_1 \frac{w_1^2}{2}\left[1 - \left(\frac{w_2}{w_1}\right)^2\right].$$

Making use of the continuity equation $\rho_1 w_1 = \rho_2 w_2$ for the gas flow in the combustion chamber with a constant normal cross-section area and of the relation between the isobaric process parameters $T_2/T_1 = \rho_1/\rho_2$, we obtain [11]

$$h_{\text{T}}(G) = \frac{G^2}{2\rho_1 S^2}\left[1 - \left(\frac{T_2}{T_1}\right)^2\right]. \quad (5)$$

Determine the formula for the hydraulic characteristics of the jet nozzle. The maximum mass flow rate of gas G_{n} in the critical section of nozzle

$$G_{\text{n}} = S_{\min}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\frac{p_{\text{n}}}{w_{\text{n}}}},$$

or

$$G_{\text{n}} = \phi(p_{\text{n}}), \quad (6)$$

where $\phi(p_{\text{n}}) = S_{\min} k \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \frac{p_{\text{n}}}{\sqrt{kRT_{\text{n}}}}$; R – gas constant of combustion products; k – adiabatic exponent; T_{n} – absolute temperature of the flowing gas.

Found above dependencies (3), (5), (6) makes the system of equations (1) – (2) fully defined. Integrating this system of equations allows to determine the limit cycles and self-oscillations in the combustion chamber rocket engine. In Fig. 3 shows the limit cycles and their corresponding forms of self-oscillations in a saddle-node discharge characteristics $F(G)$ of the combustion chamber. Respectively in Fig. 4 shows the graphics data, when the pressure characteristic $F(G)$ of the combustion chamber is monotonically decreasing.

5. Calculating the Boundaries of the Stable Combustion

The parameters of stationary combustion mode are determined from the system of equations (1) – (2) believing in it

$$\left.\frac{dG}{dt}\right|_{G=G^*} = 0,\ \left.\frac{dp_{\text{n}}}{dt}\right|_{p_{\text{n}}=p_{\text{n}}^*} = 0. \quad (7)$$

Using conditions (7), we obtain $p_{\text{n}}^* = F(G^*)$ and $G^* = \phi(p_{\text{n}}^*)$. Also Using the Taylor decomposition

$$G(t-\tau) = G(t) - \tau\frac{dG}{dt} + O(\tau^2),$$

the system of equations (1) - (2), accurate to quantities of order $O(\tau^2)$, can be written in the following form

$$L_{\text{a,c.ch}}\frac{dG}{dt} = F(G) - p_{\text{n}},$$

$$C_{\text{a,c.ch}}\frac{dp_{\text{n}}}{dt} = G - \frac{\tau(p_{\text{n}})}{L_{\text{a}}}\left(F(G) - p_{\text{n}}\right) - \phi(p_{\text{n}}), \quad (8)$$

Further, it is more convenient to switch to dimensionless variables:

$$x = \frac{G - G^*}{G^*},\ y = \frac{p_{\text{n}} - p_{\text{n}}^*}{p_{\text{n}}^*}. \quad (9)$$

In the new variables (9) the system of equations (8) can be written in the following form:

$$\begin{cases} G^* L_{\text{a,c.ch}}\dfrac{dx}{dt} = F(G^* + G^* x) - p_{\text{n}}^*(y+1), \\ p_{\text{n}}^* C_{\text{a,c.ch}}\dfrac{dy}{dt} = (x+1)G^* - \tau\left(p_{\text{n}}^*(y+1)\right)\left(\dfrac{F(G^* + G^* x) - p_{\text{n}}^*(y+1)}{L_{\text{a,c.ch}}}\right) - \phi\left(p_{\text{n}}^*(y+1)\right). \end{cases} \quad (10)$$

Thus, the nature of stability of stationary combustion mode is reduced to the study of stability of zero equilibrium of the dynamic system (10). Using Lyapunov's first method of stability analysis let us consider the Jacobian matrix of the system (10) is calculated in the zero position of equilibrium

$$\mathbf{J} = \begin{bmatrix} \dfrac{F'(G^*)}{L_{\text{a}}} & -\dfrac{p_{\text{n}}^*}{G^* L_{\text{a,c.ch}}} \\ \dfrac{G^*}{p_{\text{n}}^* C_{\text{a,c.ch}}}\left(1 - \dfrac{\tau(p_{\text{n}}^*)}{L_{\text{a,c.ch}}}F'(G^*)\right) & \dfrac{1}{C_{\text{a,c.ch}}}\left(\dfrac{\tau(p_{\text{n}}^*)}{L_{\text{a,c.ch}}} - \phi'(p_{\text{n}}^*)\right) \end{bmatrix}. \quad (11)$$

where $F'(G^*) = \left.\frac{dF(G^* + G^* x)}{dx}\right|_{x=0}$, $\phi'(p_n^*) = \left.\frac{d\phi(p_n^* + p_n^* y)}{dy}\right|_{y=0}$.

To determine the critical time delay of combustion is necessary to pre-compute the roots of the characteristic equation

$$\det(\mathbf{J} - \lambda \mathbf{E}) = 0. \quad (12)$$

Calculating the determinant of (12), we obtain

$$\lambda_{1,2} = \frac{\mathrm{tr}(\mathbf{J}) \pm i\sqrt{4\det(\mathbf{J}) - \mathrm{tr}^2(\mathbf{J})}}{2},$$

where $\mathrm{tr}(\mathbf{J})$ trace, and $\det(\mathbf{J})$ the determinant of the Jacobian matrix $\mathbf{J}$. Moreover, according to (12), obtain the following representations for these characteristics:

$$\mathrm{tr}(\mathbf{J}) = \frac{F'(G^*)}{L_{\mathrm{a,c.ch}}} + \frac{1}{C_{\mathrm{a,c.ch}}}\left(\frac{\tau(p_n^*)}{L_{\mathrm{a,c.ch}}} - \phi'(p_n^*)\right),$$

$$\det(\mathbf{J}) = \frac{1}{L_{\mathrm{a,c.ch}} C_{\mathrm{a,c.ch}}}\left(1 - F'(G^*)\phi'(p_n^*)\right).$$

Thus, the critical delay time of combustion is determined from the condition:

$$\mathrm{Re}\{\lambda_{1,2}\} = 0 \;\Leftrightarrow\; \begin{cases} \mathrm{tr}(\mathbf{J}) = 0, \\ \det(\mathbf{J}) > 0. \end{cases} \;\Leftrightarrow$$

$$\begin{cases} \frac{F'(G^*)}{L_{\mathrm{a,c.ch}}} + \frac{1}{C_{\mathrm{a,c.ch}}}\left(\frac{\tau_{\mathrm{kr}}}{L_{\mathrm{a,c.ch}}} - \phi'(p_n^*)\right) = 0, \\ F'(G^*)\phi'(p_n^*) < 1, \end{cases}$$

where finally, we obtain that

$$\tau_{\mathrm{kr}} = L_{\mathrm{a,c.ch}}\phi'(p_n^*) - C_{\mathrm{a,c.ch}} F'(G^*). \quad (13)$$

Thus, the equilibrium of the dynamic system (10) that determines the parameters of stationary combustion mode becomes unstable if true the following inequality

$$\tau(p_n^*) > \tau_{\mathrm{kr}}.$$

Note also that the condition $F'(G^*)\phi'(p_n^*) < 1$ is executed automatically when pressure characteristic $F(G)$ of the combustion chamber is a monotonically decreasing function. For further analysis of the obtained formula (13) we well approximate the pressure characteristic of the combustion chamber by a polynomial of third degree, equating $F(G) \approx F_0 - k_F G^3$. Then from (13) we get the following dependence for the critical time delay from combustion-chamber pressure p_n^*

$$\tau_{\mathrm{kr}}(p_n^*) = \frac{L_{\mathrm{a,c.ch}}}{p_n^*}\left(\frac{F_0 - p_n^*}{k_F}\right)^{\frac{1}{3}} - 3k_F C_{\mathrm{a,c.ch}}\left(\frac{F_0 - p_n^*}{k_F}\right)^{\frac{2}{3}}. \quad (14)$$

In Fig. 5 shows a plot of (14) to the combustion chamber length $l = 0.15$ m and diameter $d = 0.1$ m, when as the oxidizing agent used liquid oxygen and fuel – hydrogen.

6. Conclusions

By numerically integrating the system of equations of the nonstationary motion of a medium in an LPRE, in transition of it to a degenerate form, for the first time the possibility of controlling the amplitude of oscillations of vibration combustion at different pressure head characteristic of a combustion chamber has been shown. The possibility of complete suppression of self-oscillations and attainment of a stationary regime of combustion in an LPRE has been established.

Analytically derived the ratio that determines the critical time delay of combustion of gaseous fuel that must be exceeded in the combustion chamber of rocket engines stationary combustion mode becomes unstable and excited vibration combustion. Also for the combustion chamber with a monotonically decreasing pressure characteristic was obtained dependence of the critical delay time of combustion.

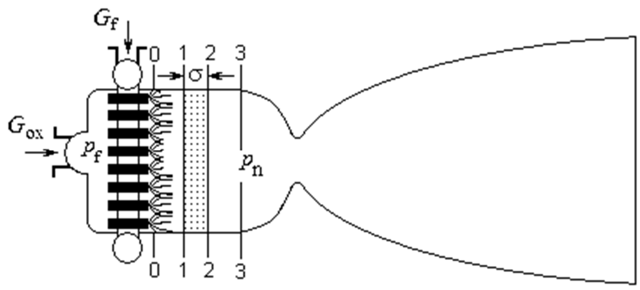

Fig. 1. Position of the sections in the LPRE combustion chamber that are used for construction of the equations of motion of combustion products.

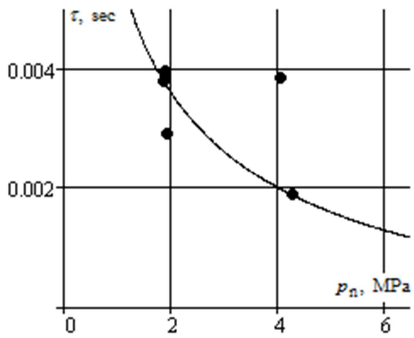

Fig. 2. The dependence of the time delay from combustion chamber pressure.

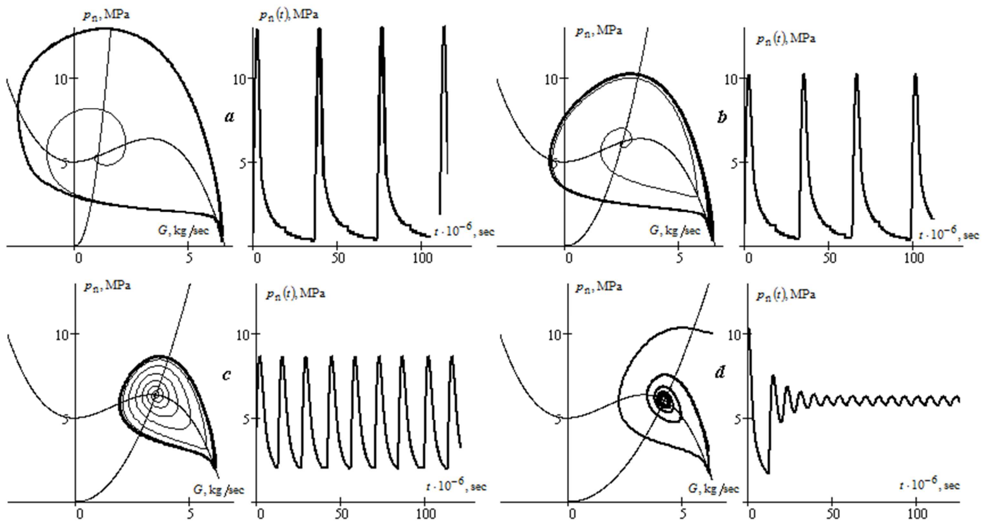

Fig. 3. Limit cycles and self - oscillations of vibration combustion when the characteristic $F(G)$ is a saddle-node: a) $G^{*}=1$ kg/sec ; b) $G^{*}=2.5$ kg/sec ; c) $G^{*}=3.5$ kg/sec ; d) $G^{*}=4.2$ kg/sec

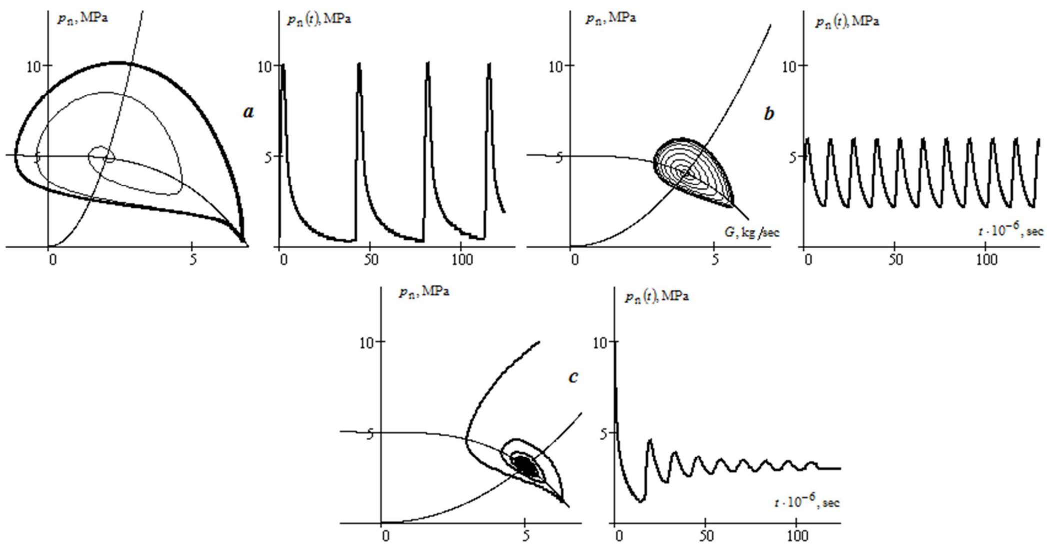

Fig. 4. Limit cycles and self - oscillations of vibration combustion when the characteristic $F(G)$ is monotonically decreasing: a) $G^{*}=2$ kg/sec ; b) $G^{*}=4$ kg/sec ; c) $G^{*}=5$ kg/sec .

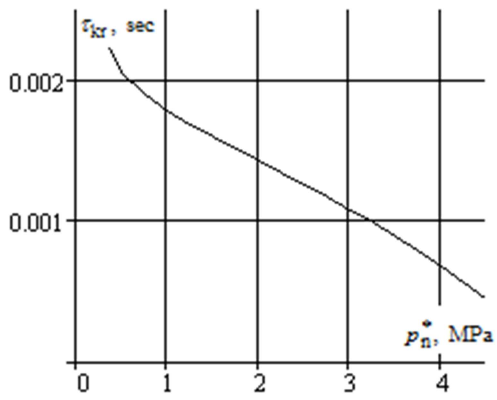

Fig. 5. The dependence of the critical time delay of combustion $\tau_{kr}\left(p_n^{*}\right)$.

References

[1] Ya. B. Zel'dovich, G. I. Barenblatt, V. B. Librovich, and G. M. Makhviladze, Mathematical Theory of Combustion and Explosion [in Russian], Nauka, Moscow (1980).

[2] V. V. Gotsulenko, Special modes of the Rijke phenomenon, Inzh.-Fiz. Zh., 16, No. 9, 160–164 (2005).

[3] V. V. Gotsulenko, Mathematical modeling of the decrease in the amplitudes of vibrations of vibrational combustion in large industrial units, Mat. Modelir., 17, No. 11, 16–24 (2005).

[4] Ya. B. Zel'dovich, O. I. Leipunskii, and V. B. Librovich, Theory of Nonstationary Combustion of Gunpowder [in Russian], Nauka, Moscow (1975).

[5] M. S. Natanzon, Instability of Combustion [in Russian], Mashinostroenie, Moscow (1986).

[6] K. I. Artamonov, Thermoacoustic Stability [in Russian], Mashinostroenie, Moscow (1982).

[7] L. Crocco and Sin-I Cheng, Theory of Combustion Instability in Liquid-Propellant Rocket Engines [Russian translation], IL, Moscow (1958).

[8] V. V. Gotsulenko and V. N. Gotsulenko, Mathematical modeling of the self-oscillations of vibrational combustion in liquid-propellant rocket engines due to combustion heat release, Mat. Modelyuv. (Dneprodzerzhinsk Gos. Tekh. Univ.), No. 1, 2 (15), 16–24 (2006).

[9] Gotsulenko V. V. and Gotsulenko V. N. On the Problem of Control of the Amplitude of Self-Oscillations of a Singing Flame // Journal of Engineering Physics and Thermophysics, Volume 87, Issue 2, 319–324 (2014).

[10] B. I. Basok and V.V. Gotsulenko, Negative thermal resistance in one-dimensional steady inviscid flow of a perfect gas, Journal of the Moscow physical-technical Institute, Vol 6, No. 4(24), 153–157 (2014).

[11] Basok B. I. and Gotsulenko V.V. Calculating the Parameters of Self - Oscillations in the Vertical Combustion Chamber of the Blast - Furnace Air Heater during Unstable Combustion // Thermal Engineering, Vol. 62, No. 1, 58–63 (2015).