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Determination of the Charge to Mass Ratio of an Electron and Classical Radius of a Gas Molecule Using the Knowledge of Electronic Damped Oscillations in Plasma

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Abstract

In a previous paper (Nandedkar and Bhagavat 1970) [4] an analysis of damped oscillations in the plasma has been carried out. In the present paper, it is shown that the steady state amplitude of sustained electronic damped oscillations in the plasma in presence of an external d.c. electric field is greater than that in the case of the eigen-frequency damped oscillations when the applied d.c. electric field is removed. In both the cases, the steady state amplitude exists well inside the screening sphere. The amplitude being measured with respect to an ion at the center of the screening sphere. Ultimately an expression for the frequency of sustained electronic damped oscillations, in the weakly ionized plasma in presence of a low damping is developed. Further electron collision frequency term, in the low density plasma, is considered to be different in the presence and in the absence of the applied d.c. electric field. The collision frequency being smaller in the previous case, than in the later case. Moreover the distribution of electronic free paths is not neglected while determining the damping force constant part in the equation of motion of the electron in the absence of the applied d.c. electric field unlike in the case when sustained damped oscillations exist. Knowing the electron density, collision frequency and frequency of damped oscillations in the plasma in the presence of the external d.c. electric field experimentally, the values of charge to mass ratio of an electron and classical radius of a gas molecule viz., that of air are determined. In the end it is illustrated that, how the present model of weakly ionized plasma leads to the similar expression for the plasma frequency due to Tonks and Langmuir (Tonks and Langmuir 1929) [5] and to the similar expression for the complex dielectric constant of plasma basically due to Appleton and Chapman (Appleton and Chapman 1932) [6] as the limiting cases.

Keywords

Damped-Oscillations, Screening-Sphere, Molecular-Radius, Electronic-Charge, Electronic-Mass

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1. Introduction

For General Reference to plasmas refer to Loeb 1955 [1], Von Engle 1965 [2] and Delcroix 1965 [3].

In a previous paper (Nandedkar and Bhagavat 1970) [4] an analysis of frequency of damped oscillations in a plasma is carried out. In the presence of an average local electric field $\langle E_i \rangle$ due to an ion at a neighbour electron in the plasma, there results the electronic damped oscillations of eigen-frequency f_j in the steady state; whereas with an external d.c. electric field E_{dc} electronic damped oscillations of frequency f_o are detected (Bhagavat and Nandedkar 1968) [7]. Both f_j and f_o type of electronic damped oscillations are shown to exist inside the screening sphere surrounding the given ion. In either case, the damping force is provided by electron collisions in the plasma. The eigen-frequency damped

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oscillations are not sustained as such in the practice, when the effect of relative polarization of electron-ion pairs in the plasma, is considered. The frequency f_o of the sustained electronic damped oscillations is a function of electron density N_e in the plasma and the applied d.c. electric field E_{dc} In this analysis, it has been shown that how f_o varies with electron density N_e and E_{dc}, in accordance with the experimental results.

In the present paper a more detailed study of f_o is carried out. It is considered that the steady state amplitude R_o (Nandedkar and Bhagavat 1970) [4] of the eigen-frequency damped oscillations in the presence of $\langle E_i \rangle$ is not the same as the steady state amplitude R_{oc} of the damped oscillations of frequency f_o. But the amplitudes R_o and R_{oc} are shifted with respect to each other in the screening sphere, R_{oc} being greater than R_o.

Then electron-molecule collision frequency in the weakly ionized plasma at ambient temperature T is further considered.

While treating the electron collision frequency in the plasma in presence of $\langle E_i \rangle$ when sustained damped oscillations do not exist, it is assumed that as $\langle E_i \rangle$ has not an preferred direction, since electron-ion pairs are randomly oriented in the plasma, so that the average thermal velocity of an electron at ambient temperature T which is thermal equilibrium temperature of the plasma, is to be considered in general with a three dimensional electron gas model.

Whereas in the case of the electron collision frequency in the plasma in the presence of the d.c. electric field E_{dc}, when sustained damped oscillations exist, it is assumed that as E_{dc} has a preferred direction (which is the direction of application of E_{dc} in the plasma, so that the average thermal velocity of the electron at thermal equilibrium temperature T, is to be considered with a one dimensional electron gas model.

Further an expression for f_o is derived in terms E_{dc} and R_{oc}. Then f_o and f_j are compared.

Afterwards the role electron collision frequency in determining the value of damping constant in the differential equation of motion of the electron leading to f_j and f_o respectively is considered. In the case of f_j when $\langle E_i \rangle$ is randomly oriented, then the distribution of free paths of the electrons is to be considered, whereas in the case of f_o, when E_{dc} is present in a given direction, then the distribution of free paths of the electrons is to be neglected, while considering the electronic motion in the presence of respective fields.

Then using the expression for f_o derived in terms of E_{dc} and R_{oc}, the value of e/m_e, i.e. the ratio of charge to mass of an electron is obtained from experimentally obtained values of f_o and N_e with the known value of E_{dc}. Further experimentally obtained value of electron collision frequency in the presence of E_{dc} is used to find the value of the classical radius of a gas molecule, viz., that of air in the plasma.

Ultimately in Section of the 'Discussions and Conclusions' it is shown that how the present model of weakly ionized plasma leads to the similar expression of plasma frequency due to Tonks and Langmuir (Tonks and Langmuir 1929) [5] and, to the similar expression for complex dielectric constant of the plasma basically due to the Appleton and Chapman (Appleton and Chapman 1932) [6] - as the limiting cases.

The Method of Analysis of this research-paper consists of following sections for this article:

1. Introduction
2. Determination of R_{oc} in terms of R_o
3. The electron collision frequency in the plasma
4. Comparison of f_j and f_o
5. Behaviour of the electron collision frequency in the determination of the values of f_1' / m_e and f_1''/m_e
6. Charge to mass ratio of an electron and classical radius of a gas molecule
7. Measurement of the phase constant β_p
8. Experimental work

followed by,

9. Discussions and Conclusions

The above sections of the article are illustrated one after the other in this article.

2. Determination of R_{oc} in Terms of R_o

Consider a quasi-stationary plasma with equal densities of electrons and ions. The plasma is in thermal equilibrium at ambient temperature T and it is weakly ionized. Density of electron-ion pairs in the plasma is N_e. Here N_e is the density of electrons or ions in the plasma.

If R_o be the average separation of an electron from an ion in the plasma, then the volume $(4\pi/3)\,R_o^3$ occupied by an electron-ion pair, is given by:

$$\frac{4\pi}{3} R_o^3 = N_e^{-1}, \quad (1)$$

where the case of spherical symmetry with respect to the ion in the plasma is considered. From eqn. (1), R_o is given by

$$R_o = \left(\frac{3}{4\pi}\right)^{1/3} N_e^{-1/3}\,, \quad (2)$$

here R_o is measured with respect to the center of the screening sphere of radius, R_s, given by (Nandedkar and Bhagavat 1970) [4]:

$$R_s = \frac{\sqrt{2}}{k_{D'}} = \left(\frac{\epsilon_0 kT}{N_e\, e^2}\right)^{1/2}\,, \quad (3)$$

where,

$$k_{D'} = \left(\frac{2N_e e^2}{\epsilon_0 kT}\right)^{1/2}, \quad (4)$$

here $k_{D'}$ is a factor inversely proportional to R_s. ϵ_0 is the permittivity of free space. e is the charge of an electron or ion. k Boltzmann constant. R_o lies well inside the screening sphere of radius R_s, i.e.

$$R_o << \sqrt{2}/k_{D'}\,. \quad (5)$$

Here R_o is interpreted as the steady state separation of an electron of the plasma from the neighbouring ion reached when an average electric field due to the ion acts on the electron in the presence of (i) a damping force (i.e. due to electron collisions) against the motion of the electron proportional to its velocity and (ii) a restoring force proportional to the displacement of the electron, in the vicinity of R_o. The displacement of the electron in presence of this average electric field, say $\langle E_i \rangle$, inside the screening sphere, deviates the plasma from the condition of neutrality, so the second force mentioned comes into the picture. In this case a low damping is considered. The ion lies at the center of the screening sphere mentioned.

Electric potential ϕ_i at distance r due to the ion at the center of the screening sphere is given by (Nandedkar and Bhagavat 1970) [4],

$$\phi_i = \phi_c \exp(-k_{D'}\, r), \quad (6)$$

where $k_{D'}$ is given by eqn. (4) and $\emptyset_c$ is the Coulomb's potential at distance r given by

$$\emptyset_c = \frac{e}{4\pi\varepsilon_0 r}, \quad (7)$$

where e is the charge of the ion.

When,

$$r << \frac{\sqrt{2}}{k_{D'}}, \quad (8)$$

then eqn. (6) gives:

$$\emptyset_i = \emptyset_c = \frac{e}{4\pi\epsilon_0 r}\,, \quad (9)$$

i.e. well inside the screening sphere, the electric potential due to the ion is merely Columbian.

Electric field E_I due to $\emptyset_i$ at distance r, using eqn. (9) is given by,

$$E_I = -\left(\frac{\partial \emptyset_i}{\partial r}\right) = \frac{e}{4\pi\epsilon_0 r^2}\,. \quad (10)$$

For the plasma under consideration, the average value of E_I i.e. $\langle E_I \rangle$ over a sphere of radius r is given by:

$$\langle E_I \rangle = \frac{\int_0^r E_I 4\pi r^2\, \partial r}{\int_0^r 4\pi r^2\, \partial r}\,,$$

i.e.

$$\langle E_I \rangle = \left(\frac{e}{\epsilon_0}\right)\left[\frac{r}{(4\pi/3)r^3}\right] = \left(\frac{e}{\epsilon_0}\right)\frac{1}{(4\pi/3)r^2}, \quad (11)$$

where it is assumed that (for instance, refer Nandedkar and Bhagavat 1970) [4],

$$\frac{\sqrt{2}/k_{D'}}{N_{D'}} << r, \quad (12)$$

where $N_{D'}$ is the number of electrons in the screening sphere. When $r \rightarrow R_o$, such that,

$$\frac{\sqrt{2}/k_{D'}}{N_{D'}} << R_o << \frac{\sqrt{2}}{k_{D'}}, \quad (13)$$

then, $\langle E_I \rangle \rightarrow \langle E_i \rangle$.

Using eqn. (11) and (1), the value of $\langle E_i \rangle$ is given by

$$\langle E_i \rangle = \left(\frac{e}{\epsilon_0}\right)\frac{R_o}{(4\pi/3)R_o^3} = N_e\left(\frac{e}{\epsilon_0}\right)R_o\,. \quad (14)$$

Thus within the range of distance of interest, inside the screening sphere, the value of electric field due to the ion at the center, varies inversely as the square of the distance and the ultimate value of the electric field $\langle E_i \rangle$ - given by eqn. (14), is reached when R_o is attained.

Now suppose a uniform d.c. electric field E_{dc} be applied to the plasma. Ion being much heavier as compared to the electron, the ionic motion in comparison to the electronic motion is neglected in the presence of E_{dc}. Unlike the ionic field inside the screening sphere, which decreases as the square of the distance from the center of the sphere is increased, E_{dc} remains everywhere the same inside the screening sphere. Hence the steady state separation R_{oc} measured with respect to the ion at the center of the screening sphere in the presence of E_{dc} is different than which is in the case of $\langle E_i \rangle$. In this present case of E_{dc}, the damping force acts on the motion of the electron due to electron collisions and a restoring forces acts on the electron due to its displacement in the screening sphere. However, the damping force in the present case is different than the one considered in the presence of $\langle E_i \rangle$. But here also, as before, the case of low damping is considered. In the presence of E_{dc} and the above mentioned damping and restoring forces, the value of R_{oc} is attained by the electron.

If the conditions inside the screening sphere with $\langle E_i \rangle$ and with E_{dc} are different, then the displacement of the electron in the presence of E_{dc} in the sphere is constrained in comparison to that in the presence of $\langle E_i \rangle$. Let the constrained displacement of the electron with respect to the center of the screening sphere where the ion lies, be denoted by r_c in the presence of the applied d.c. electric field E_{dc}. When R_o and R_{oc} in the presence of $\langle E_i \rangle$ and E_{dc} respectively are to be compared, then let the non-constrained displacement of the electron in the presence of E_{dc} be denoted by r_{nc} with respect to the ion at the center of the screening sphere.

In the case when r_{nc} tends to zero, which means that E_{dc} and $\langle E_i \rangle$ both tend to zero, i.e., neutral gas is formed in the limit, then r_c also tends to zero.

Any Change of the value (R_{oc} - r_c) with respect to r_{nc} is considered to be proportional to (R_{oc} - r_c) itself, I.e.,

$$\frac{\partial(R_{oc}-r_c)}{\partial r_{nc}}\ \alpha\ -\ (R_{oc}-r_c)\,, \qquad (15)$$

the negative sign outside the bracket on left hand side of eqn.(15) denotes that (R_{oc} - r_c) decreases as r_{nc} is increased. Eqn. (15) can be rewritten as follows:

$$\frac{\partial(R_{oc}-r_c)}{\partial r_{nc}} = -\frac{(R_{oc}-r_c)}{R_o}, \qquad (16)$$

where $(1/R_o)$ is the constant of proportionality. R_o is the value with which R_{oc} is to be compared. R_o is given by eqn. (2).

Integration of eqn. (16) gives,

$$(R_{oc}-r_c) = A_c \exp\left(-\frac{r_{nc}}{R_o}\right), \qquad (17)$$

where A_c is a constant of integration as mentioned before.

When

$$r_{nc} \to 0, \qquad (18a)$$

then

$$r_c \to 0. \qquad (18b)$$

Thus using eqns. (18a) and (18b), eqn. (17) gives,

$$A_c = R_{oc}\,. \qquad (19)$$

Substituting the value of A_c from eqn. (19) in eqn. (17), eqn. (17) gives:

$$r_c = R_{oc}\left[1-\exp\left(-\frac{r_{nc}}{R_o}\right)\right]. \qquad (20)$$

The value of R_{oc} in terms of R_o is obtained when r_c and r_{nc} simultaneously tend to R_o and then eqn. (20) gives,

$$R_{oc} = \frac{R_o}{[1-\exp(-1)]}\,. \qquad (21)$$

Using eqn. (2), eqn. (21) gives,

$$R_{oc} = \left[\frac{(3/4\pi)^{1/3}}{1-\exp(-1)}\right] N_e^{-1/3}. \qquad (22)$$

Coming to eqn. (21), the value of R_o/R_{oc} is given by the following expression, viz.,

$$\frac{R_o}{R_{oc}} = [1-\exp(-1)] = 0.6321. \qquad (23)$$

Eqn. (23) indicates that when R_o and R_{oc} in the presence of $\langle E_i \rangle$ and E_{dc} respectively, are compared then R_0 is 63.21% smaller than R_{oc}. Here R_0 and R_{oc} both exist inside the screening sphere.

3. The Electron Collision Frequency in the Plasma

In the present plasma model, it is assumed that electrons, ions and neutral molecules are in thermal equilibrium at ambient temperature T of the plasma. In the presence of an electric field in the plasma, when the electron undergoes a drift motion, then it encounters collisions with neutral molecules on its way. In the weakly ionized plasma at ambient temperature T, electron-molecule collisions are much larger than electron-ion collisions. So only electron-molecule collisions are considered here.

To study the electron collision frequency, the knowledge of average thermal velocity of the electron at equilibrium temperature T of the plasma is required.

The average thermal velocity of the electron is treated in two cases. In the first case a three dimensional electron gas model is considered whereas in the second case a one dimensional electron gas model is chosen at the equilibrium temperature T of the plasma.

Case 1:

Taking a three dimensional electron gas in the plasma at equilibrium temperature T, the number of electrons having velocity components between w_e and $w_e + \partial w_e$, $\theta_e + \partial\theta_e$, and $\emptyset_e + \partial\emptyset_e$ where w_e varies from 0 to ∞, θ_e varies from 0 to π and $\emptyset_e$ varies from 0 to 2π in a system of spherical polar coordinates, can be given by using methods of gas-kinetics (for instance refer Max Born 1963) [16]:

$$\psi_e(w_e, \theta_e, \emptyset_e)\,\partial w_e\,\partial\theta_e\,\partial\emptyset_e = A_e w_e^2 \exp\left(-\frac{m_e w_e^2}{2kT}\right)\partial w_e\,\sin\theta_e\,\partial\theta_e\,\partial\emptyset_e, \qquad (24)$$

where A_e is a constant for the given system of the electron gas. Here m_e is the mass of an electron.

The average value of the thermal velocity of the electron, i.e., v_e in the electron-gas defined by eqn. (24) is given by:

$$v_e = \frac{\int_{w_e=0}^{\infty}\int_{\theta_e=0}^{\pi}\int_{\emptyset_e=0}^{2\pi} w_e \psi_e(w_e,\theta_e,\emptyset_e)\,\partial w_e\,\partial\theta_e\,\partial\emptyset_e}{\int_{w_e=0}^{\infty}\int_{\theta_e=0}^{\pi}\int_{\emptyset_e=0}^{2\pi} \psi_e(w_e,\theta_e,\emptyset_e)\,\partial w_e\,\partial\theta_e\,\partial\emptyset_e}. \quad (25)$$

Using eqn. (24), eqn. (25) gives:

$$v_e = \frac{\int_{w_e=0}^{\infty}\int_{\theta_e=0}^{\pi}\int_{\emptyset_e=0}^{2\pi} w_e^3 \exp\left(-\frac{m_e w_e^2}{2kT}\right)\partial w_e \sin\theta_e\,\partial\theta_e\,\partial\emptyset_e}{\int_{w_e=0}^{\infty}\int_{\theta_e=0}^{\pi}\int_{\emptyset_e=0}^{2\pi} w_e^2 \exp\left(-\frac{m_e w_e^2}{2kT}\right)\partial w_e \sin\theta_e\,\partial\theta_e\,\partial\emptyset_e}$$

or,

$$v_e = \frac{\int_{w_e=0}^{\infty} w_e^3 \exp\left(-\frac{m_e w_e^2}{2kT}\right)\partial w_e}{\int_{w_e=0}^{\infty} w_e^2 \exp\left(-\frac{m_e w_e^2}{2kT}\right)\partial w_e}. \quad (26)$$

Solving the integrals involved in eqn. (26), (for instance refer Max Born 1963) [16], eqn. (26) gives,

$$v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}. \quad (27)$$

Equation (27) gives, the average thermal velocity of an electron in the plasma at equilibrium temperature T, considering a three dimensional electron-gas model.

Case 2:

Now consider a one dimensional electron-gas in the plasma at equilibrium temperature T. The number of electrons in such an electron-gas, having velocity components, say between w_{ex} and $w_{ex} + \partial w_{ex}$, where w_{ex} varies from 0 to ∞ along x axis of a system of rectangular co-ordinates, can be given by using methods of gas-kinetics (for instance refer Max Born 1963) [16]:

$$\psi_{ex}(w_{ex})\,\partial w_{ex} = A_{ex}\exp\left(-\frac{m_e w_{ex}^2}{2kT}\right)\partial w_{ex}, \quad (28)$$

where A_{ex} is a constant for the given system of electron-gas under consideration.

The average value of thermal velocity of an electron, i.e. v'_e in the electron gas defined by eqn. (28), is given by,

$$v'_e = \frac{\int_{w_{ex}=0}^{\infty} w_{ex}\psi_{ex}(w_{ex})\,\partial w_{ex}}{\int_{w_{ex}=0}^{\infty} \psi_{ex}(w_{ex})\,\partial w_{ex}}. \quad (29)$$

Using eqn. (28), eqn. (29) gives:

$$v'_e = \frac{\int_{w_{ex}=0}^{\infty} w_{ex}\exp\left(-\frac{m_e w_{ex}^2}{2kT}\right)\partial w_{ex}}{\int_{w_{ex}=0}^{\infty} \exp\left(-\frac{m_e w_{ex}^2}{2kT}\right)\partial w_{ex}}. \quad (30)$$

Solving the integrals involved in eqn. (30), (for instance refer Max Born 1963) [16], eqn. (30) gives,

$$v'_e = \left(\frac{2kT}{\pi m_e}\right)^{1/2}. \quad (31)$$

Equation (31) gives, the average thermal velocity of an electron in the plasma at equilibrium temperature T, considering a one dimensional electron-gas model.

3.1. The Electron Collision Frequency in the Presence of the Average Ionic Electric Field $\langle E_i \rangle$

The average ionic electric field $\langle E_i \rangle$ acting on the electron in the plasma is not in a specific direction, but on the other hand it is randomly oriented. Thus while considering the collisions of the electron in the presence of the drift due to $\langle E_i \rangle$ the average thermal velocity corresponding to a three dimensional electron-gas model, i.e. v_e given by eqn. (27) is used.

If size of the electron is much smaller than the size of a gas molecule, then the closest distance the electron can approach the molecule is the classical radius R_m of the molecule. The order of the classical radius R_m as given by gas-kinetics, for any type of a gas molecule is 10^{-10} m.

Now imagine a cylinder in the plasma of length l_c and radius of cross-section of the cylinder as R_m. Volume occupied by the cylinder is $(\pi R_m^2 l_c)$. If N_m be the number density of the gas molecules in the plasma, then the cylinder under consideration contains $(N_m \pi R_m^2 l_c)$ number of gas molecules.

In the simple picture of electron-molecule collisions, it is assumed that, a single electron while travelling through the cylinder, mentioned already, along length l_c with the average thermal velocity v_e would make collisions with all the gas molecules present therein. As such the number of electron collisions would be equal to the number of gas molecules present therein i.e. $(N_m \pi R_m^2 l_c)$. These many collisions, the electron, would make in time (l_c / v_e). Hence the number of collisions the electron makes with neutral molecules in one second, which is the electron-molecule collision frequency ν_E, is given by,

$$\nu_E = \frac{(N_m \pi R_m^2 l_c)}{(l_c / v_e)}. \quad (32)$$

Here it is assumed that the number density N_e of the electrons in the plasma is very much smaller than the number density N_m of the gas molecules. Equation (32) can be rewritten as follows, viz.,

$$\nu_E = N_m \pi R_m^2 v_e \quad (33)$$

Substituting the value of v_e from eqn. (27) in eqn. (33), eqn. (33) gives,

$$\nu_E = N_m \pi R_m^2 \left(\frac{8kT}{\pi m_e}\right)^{1/2}. \quad (34)$$

Equation (34) gives electron-molecule collision frequency in the plasma in the presence of $\langle E_i \rangle$.

3.2. The Electron Collision Frequency in the Presence of an Externally Applied d.c. Electric Field E_{dc}

The externally applied d.c. electric field E_{dc} acting on the electron in the plasma, has a fixed one direction, say along x-axis, which is the direction of application of the field. So, in the determination of the electron collision frequency with gas molecules, in the presence of the drift of the electron under E_{dc}, the average thermal velocity corresponding to a one dimensional electron gas model, i.e. v'_e given by eqn. (31) is used.

Hence replacing v_e by v'_e in eqn. (33), electron-molecule collision frequency g_E in the present case, is given by:

$$g_E = N_m \pi R_m^2 v'_e. \tag{35}$$

Using eqn. (31) for the value of v'_e, eqn. (35) gives,

$$g_E = N_m \pi R_m^2 \left(\frac{2kT}{\pi m_e}\right)^{1/2}. \tag{36}$$

Equation (36) gives electron-molecule collision frequency in the plasma in the presence of E_{dc} .

Comparing eqns. (34) and (36), it is seen that,

$$g_E < \nu_E. \tag{37}$$

Thus, in the presence of the drift of the electron under directed E_{dc}, the electron suffers less number of collisions with gas molecules as compared to its drift under randomly oriented field $\langle E_i \rangle$.

4. Comparison of f_j and f_o

To study f_j, consider the differential equation of motion of the electron in the plasma, in the screening sphere with the ion at its center, under the action of the average ionic electric field $\langle E_i \rangle$ given by eqn. (14). The following forces act on the electron, viz.,

(i) a damping force against the motion of the election which is proportional to its velocity given by $-f'_1\ (\partial r/\partial t)$ and

(ii) a restoring force which is proportional to its displacement r given by $-f'_2$ r. The displacement of the electron in the screening sphere measured with respect to its center in the presence of $\langle E_i \rangle$ tries to disturb the condition of neutrality. To maintain a plasma neutrality f'_2 comes into the picture.

The differential equation of motion of the electron of charge e is given by

$$m_e \frac{\partial^2 r}{\partial t^2} + f'_1 \frac{\partial r}{\partial t} + f'_2 r = e\langle E_i \rangle, \tag{38}$$

here $\partial^2 r / \partial t^2$, $\partial r / \partial t$ and r are the acceleration, velocity and displacement of the electron at time t. The value of r mentioned in eqn. (38) is considered in the vicinity of R_o where $\langle E_i \rangle$ can be considered as a constant.

Equation (38) can be rewritten as follows:

$$\frac{\partial^2 r}{\partial t^2} + \frac{f'_1}{m_e}\frac{\partial r}{\partial t} + \frac{f'_2}{m_e} r = \frac{e}{m_e}\langle E_i \rangle. \tag{39}$$

Here, it is considered that,

$$\frac{f'_1}{m_e} = \nu_E, \tag{40}$$

where ν_E is electron-molecule collision frequency in the presence of $\langle E_i \rangle$ given by eqn. (34).

Using eqn. (40), eqn. (39) gives:

$$\frac{\partial^2 r}{\partial t^2} + \nu_E \frac{\partial r}{\partial t} + \frac{f'_2}{m_e} r = \frac{e}{m_e}\langle E_i \rangle. \tag{41}$$

Complementary function (c.f.) of eqn. (41) is given by:

$$\frac{\partial^2 r}{\partial t^2} + \nu_E \frac{\partial r}{\partial t} + \frac{f'_2}{m_e} r = 0. \tag{42}$$

In the case of low damping, when

$$\frac{f'_2}{m_e} >> \left(\frac{\nu_E}{2}\right)^2, \tag{43}$$

then, the solution of eqn. (42), is given by:

$$r_{c.f.} = \exp\left[\left(-\frac{\nu_E}{2} t\right)\right]\left[C_{eig} \exp(i\,\omega_j\, t) + D_{eig} \exp(-i\,\omega_j\, t)\right], \tag{44}$$

where $i = \sqrt{-1}$ and C_{eig} and D_{eig} are finite constants of the displacement, and

$$f_j = \frac{\omega_j}{2\pi} = \frac{1}{2\pi}\left(\frac{f'_2}{m_e}\right)^{1/2}, \tag{45}$$

where f_j is the eigen-frequency of damped oscillations in the case of low damping as indicated by eqn. (43). ω_j is the angular frequency corresponding to f_j .

Using eqn. (45), eqn. (41) gives:

$$\frac{\partial^2 r}{\partial t^2} + \nu_E \frac{\partial r}{\partial t} + \omega_j^2 r = \frac{e}{m_e}\langle E_I \rangle\,. \tag{46}$$

Particular integrand (p.i.) of eqn. (46) is given by,

$$r_{p.i.} = \frac{e}{m_e}\frac{\langle E_i \rangle}{\omega_j^2}. \tag{47}$$

Hence total solution of eqn. (46), using eqns. (44) and (47) is given by,

$$r = r_{c.f.} + r_{p.i.}$$

or,

$$r = \left[\exp\left(-\frac{\nu_E}{2}t\right)\right]\left[c_{eig}\exp(i\omega_j t) + D_{eig}\exp(-i\omega_j t)\right] + \frac{e}{m_e}\frac{\langle E_j\rangle}{\omega_j^2}. \quad (48)$$

Equation (48) denotes a damped simple harmonic motion of the displacement of the electron with a time constant of $2/\nu_E$ over the displacement of value $\left[(e/m_e)(\langle E_i\rangle/\omega_j^2)\right]$.

In the steady state, when

$$r \rightarrow R_o, \quad (49a)$$

then

$$t >> 2/\nu_E, \quad (49b)$$

gives the boundary condition. With this boundary condition eqn. (48), in the steady state gives:

$$R_o = \frac{e}{m_e}\frac{\langle E_i\rangle}{\omega_j^2}. \quad (50)$$

The constants C_{eig} and D_{eig} are chosen such that r always lies in the vicinity of R_o, where $\langle E_i\rangle$ is constant.

Substituting the value of $\langle E_i\rangle / R_o$ from eqn. (14) in eqn. (50), eqn. (50) gives:

$$\omega_j^2 = \left(\frac{N_e e^2}{m_e \epsilon_0}\right), \quad (51)$$

and then eigen-frequency of damped oscillations is given by,

$$f_j = \frac{\omega_j}{2\pi} = \frac{1}{2\pi}\left(\frac{N_e e^2}{m_e \epsilon_0}\right)^{1/2}. \quad (52)$$

Thus in the presence of the average ionic electric field $\langle E_i\rangle$ at R_o, the electron has an intrinsic tendency to give damped eigen-frequency oscillations in the vicinity of R_o with the time constant of $(2/\nu_E)$ where ν_E is the electron-molecule collision frequency in this case.

Coming to eqn. (46), the electric field $\langle E_i\rangle$ on the electron can be considered as due to the relative polarization of electron-ion pairs in the plasma with respect to free space, in the neighbourhood of distance R_o. Thus,

$$\langle E_i\rangle = P_d / \epsilon_0, \quad (53)$$

where the polarization P_d is given by,

$$P_d = N_e\, e\, r. \quad (54)$$

Using eqns. (53) and (54), eqn. (46) gives:

$$\frac{\partial^2 r}{\partial t^2} + \nu_E\frac{\partial r}{\partial t} + \omega_j^2 r = \frac{N_e e^2}{m_e \epsilon_0} r. \quad (55)$$

Using eqn. (51), eqn. (55) gives,

$$\frac{\partial^2 r}{\partial t^2} + \nu_E\frac{\partial r}{\partial t} = 0, \quad (56)$$

which means the intrinsic damped oscillations denoted by eqn. (48) are not sustained as such, when the steady state approaches.

Now to study f_o, suppose an external d.c. electric field E_{dc} in a given direction be applied to the above mentioned plasma model. To consider the differential equation of motion of the electron under the action of E_{dc}, the following forces acting on the electron are to be taken into account, viz.,

(i) a damping force against the motion of the electron which is proportional to its velocity $\partial r/\partial t$ given by, $-f_1''(\partial r/\partial t)$ and

(ii) a restoring force which is proportional to its displacement r given by $-f_2''r$. The displacement r of the electron in the screening sphere measured with respect to its center in the presence of E_{dc} disturbs the space charge existing there and then to maintain overall charge neutrality of the plasma, f_2'' comes into the picture which tries to restore the original conditions.

The differential equation of the motion of the electron in the present case becomes,

$$m_e\frac{\partial^2 r}{\partial t^2} + f_1''\frac{\partial r}{\partial t} + f_2''r = e\,E_{dc}. \quad (57)$$

Equation (57) can be rewritten as follows:

$$\frac{\partial^2 r}{\partial t^2} + \frac{f_1''}{m_e}\frac{\partial r}{\partial t} + \frac{f_2''}{m_e}r = \frac{e}{m_e}E_{dc}. \quad (58)$$

Here it is considered that,

$$\frac{f_1''}{m_e} = 2g_E, \quad (59)$$

where g_E is the electron-molecule collision frequency in the presence of E_{dc} as given by eqn. (36).

Using eqn. (59), eqn. (58) gives:

$$\frac{\partial^2 r}{\partial t^2} + 2g_E\frac{\partial r}{\partial t} + \frac{f_2''}{m_e} = \frac{e}{m_e}E_{dc}. \quad (60)$$

Solution of the complementary function of eqn. (60), is given by:

$$r_{c.f.} = [\exp(-g_E t)][A'_{do}\exp(i\omega_o t) + B_{do}'\exp(-i\omega_o t)], \quad (61)$$

in the case of low damping when ,

$$(f_2''/m_e) >> g_E^2. \quad (62)$$

In eqn. (61), A'_{do} and B_{do}' are the finite constants of the displacement of the electron, and

$$f_o = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}(f_2''/m_e)^{1/2}, \quad (63)$$

in the case of low damping mentioned by eqn. (62). f_o is the frequency of damped oscillations in the presence of E_{dc} and ω_o is the corresponding angular frequency of the damped oscillations.

Using eqn. (63), eqn. (60) gives:

$$\frac{\partial^2 r}{\partial t^2} + 2g_E \frac{\partial r}{\partial t} + \omega_0^2 r = \frac{e}{m_e} E_{dc}. \quad (64)$$

Solution of particular integrand of eqn. (64), is given by:

$$r_{p.i.} = \frac{e}{m_e}\frac{E_{dc}}{\omega_0^2}. \quad (65)$$

Hence total solution of eqn. (64), using eqns. (61) and (65), is given by:

$$r = r_{c.f.} + r_{pf}$$

or,

$$r = [\exp(-g_E t)][A'_{do}\exp(i\omega_o t) + B_{do}'\exp(-i\omega_o t)] + \frac{e}{m_e}\frac{E_{dc}}{\omega_0^2}. \quad (66)$$

Equation (66) denotes a damped simple harmonic motion of the displacement of the electron with the time constant of $(1/g_E)$ over the displacement of value $[(e/m_e)(E_{dc}/\omega_0^2)]$.

In this case, the steady state approaches, when

$$t >> 1/g_E, \quad (67a)$$

and then,

$$r \to R_{oc}, \quad (67b)$$

where R_{oc} is given by eqn. (22). With the boundary condition given by eqns. (67a) and (67b), eqn. (66) in the steady state gives:

$$R_{oc} = \frac{e}{m_e}\frac{E_{dc}}{\omega_0^2}. \quad (68)$$

Substituting the value of R_{oc} from eqn. (22) in eqn. (68), ω_o is given by the following relationship, viz.,

$$\omega_0 = \left[\left\{\frac{1-\exp(-1)}{(3/4\pi)^{1/3}}\right\}\frac{e}{m_e}\right]^{1/2} E_{dc}^{1/2} N_e^{1/6}. \quad (69)$$

Whence the frequency of damped oscillations is given by,

$$f_o = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\left[\left[\left\{\frac{1-\exp(-1)}{(3/4\pi)^{1/3}}\right\}\frac{e}{m_e}\right]\right]^{1/2} E_{dc}^{1/2} N_e^{1/6}. \quad (70)$$

Experimentally f_o is found to be proportional to $E_{dc}^{1/2}$ and $N_e^{1/6}$ (Bhagavat and Nandedkar, 1968) [7], as predicted by eqn. (70). f_o is called the frequency of sustained damped oscillations.

Now f_j and f_o can be compared as follows:

Equation (52) gives eigen-frequency of damped oscillations, viz., f_j . These damped oscillations are, however, not sustained as such in the practice. Equation (70) gives the frequency of damped oscillations, viz., f_o. These damped oscillations are sustained in practice. The intrinsic electronic damped oscillations characterized by frequency f_j and the sustained electronic damped oscillations characterized by frequency f_o , both have the respective steady state amplitude that exists well inside the screening sphere. The amplitude in either case, is measured with respect to the ion at the center of the screening sphere. The steady state amplitude of the intrinsic damped oscillations, R_o is 63.21% of the steady state amplitude of the sustained damped oscillations, viz., R_{oc}. However, the damping force per unit mass per unit velocity of the electron, i.e. f_1' / m_e which comes in the differential equation of the motion of the electron, viz., given by eqn. (39) determining f_j , is equal to the electron collision frequency, i.e. ν_E [eqn. (40)], whereas the damping force per unit mass pre unit velocity of the electron i.e. f_1''/m_e which comes in the differential equation of the motion of the electron, viz., given by eqn. (58) determining f_o is equal to twice the electron collision frequency, i.e. 2 g_E [eqn. (59)].

5. Behaviour of the Electron Collison Frequency in the Determination of the Values of f_1' / m_e and f_1'' / m_e

In the case of the eigen-frequency damped oscillations f_1' / m_e = ν_E , whereas in the case of the sustained damped oscillations f_1''/m_e = $2g_E$, where ν_E and g_E are given by eqns. (34) and (36) respectively. The significance of f_1' / m_e = ν_E and f_1''/m_e = $2g_E$ in the respective cases, is illustrated as follows:

5.1. Role of the Electron Collisions in the Determination of f_1' / m_e

Now consider the differential equation of motion of the electron in the plasma in the presence of the damped oscillations of eigen angular frequency ω_j, given by eqn. (46) i.e.

$$\frac{\partial^2 r}{\partial t^2} + \nu_E \frac{\partial r}{\partial t} + \omega_j^2 r = \frac{e}{m_e}\langle E_I \rangle .$$

where,

$$\nu_E = f_1' / m_e.$$

which is given by eqn. (40). In equation (46) the displacement of the electron from the ion at the center of the screening sphere is considered in the vicinity of R_o where $\langle E_j \rangle$ is constant. But before attaining the steady state, the electron is assumed to have made sufficient oscillations, since for low damping,

$$\omega_j^2 >> (\nu_E/2)^2, \quad (71)$$

[refer eqns. (43) and (45)] and during that time have collided with neutral gas molecules so, ν_E in eqn. (46) comes into the picture. Multiplying eqn. (46) by N_e e, it can be written down as follows:

$$\frac{m_e}{N_e e^2}\frac{\partial j'}{\partial t} + \frac{m_e \nu_E}{N_e e^2} j' + \frac{\partial j' \partial t}{\left(N_e e^2/m_e \omega_j^2\right)} = \langle E_i \rangle, \quad (72)$$

where $j' = \partial(N_e\, e\, r)/\partial t$ is the corresponding electron current density.

Now consider an imaginary cube (of unit dimensions) of a series $L_p' - C_p' - R_p'$ (Inductance-capacitance-Resistance) circuit to which a potential difference of $\langle V_i \rangle$ is applied across the parallel faces, when J' is the current flowing. The differential equation of current in this case is given by:

$$L_P' \frac{\partial J'}{\partial t} + R_P' J' + \frac{\partial J' \partial t}{C_P'} = \langle V_i \rangle. \quad (73)$$

Comparing eqns. (72) and (73), the equivalent resistivity R_p' of the plasma is given by,

$$R_P' = \frac{m_e \nu_E}{N_e e^2}. \quad (74)$$

R_p' can also be imagined to be coming into the picture as explained in the following steps, in terms of the distribution of the free paths of the electrons in the plasma.

5.1.1. Distribution of the Free Paths of the Electrons in the Plasma

The collisions that determine the free paths of the electrons in the plasma are random events. This being true, some free paths would be long and others would be short. On the basic of a random motion of the electrons, an expression can be obtained for 'distance distribution' of electronic free paths.

If one electron makes an average of ν_E collisions per second with neutral gas molecules in the plasma and has an average thermal velocity $v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}$ at equilibrium temperature T of the plasma, then the average number of collisions made in a unit length of travel would be $a_e = \nu_E / v_e$ and the probable number of collisions made by this electron in travelling a distance ∂x_e would be $a_e\, \partial x_e$. Let N_T be the total number of electrons present in the plasma. Assume n_T be the number of electrons that have travelled a distance x_e without having collisions. The number of these electrons having collisions between x_e and $x_e + \partial x_e$ would be proportional to n_T itself and the length of the path, or the change in n_T due to collisions is given by

$$\partial n_T = - a_e\, n_T\, \partial x_e, \quad (75)$$

where a_e is the constant of proportionality and negative sign indicates that ∂n_T decreases as ∂x_e increases. Eqn. (75) gives the number of electrons having free paths between x_e and $x_e + \partial x_e$, numerically.

Equation (75) can be integrated to give:

$$n_T = A_T \exp(-a_e\, x_e), \quad (76)$$

where A_T is a constant of integration. At $x_e = 0$, since there are no collisions as such, $n_T = N_T$. With this boundary condition eqn. (76) gives,

$$n_T = N_T \exp(-a_e\, x_e). \quad (77)$$

The electron-molecule collision frequency can be related to the electron-molecule mean free path λ_e by the following procedure.

If ∂N_T be the number of electrons having a free path of length between x_e and $x_e + \partial x_e$, then the expression for the electronic mean free path viz., λ_e is given by,

$$\lambda_e = \int_0^{N_T} \frac{x_e\, \partial N_T}{N_T}. \quad (78)$$

As,

$$\partial N_T = |\partial n_T| = a_e n_T\, \partial x_e = a_e N_T \exp(-a_e x_e)\, \partial x_e\,, \quad (79)$$

so eqn. (78) gives:

$$\lambda_e = \int_0^{\infty} \frac{x_e a_e N_T \exp(-a_e x_e)\, \partial x_e}{N_T} = \frac{1}{a_e}. \quad (80)$$

Thus the distribution of electronic free paths is given by eqn. (77), using eqn. (80), as follows:

$$n_T = N_T \exp(- x_e/\lambda_e). \quad (81)$$

5.1.2. Alternative Derivation of R_P'

If the electrons start with zero velocity in presence of $\langle E_i \rangle$ after each collision, then the distance s_e they travel in time t_e with constant acceleration f_e, is given by,

$$s_e = \frac{f_e t_e^2}{2}. \quad (82)$$

The average velocity v_{ed} of an electron between collisions is,

$$v_{ed} = \frac{s_e}{t_e} = \frac{f_e t_e}{2}. \quad (83)$$

The average drift velocity v_d is the average over a large number of such free paths of varying length and duration, since the electrons are distributed at random in the plasma.

The average of s_e over a wide range of electronic free paths x_e is considered with a variable time $t_e = x_e / v_e$ given by the ratio of the free path x_e to the average thermal velocity v_e of the electron.

Now, the acceleration f_e of the electrons in presence of $\langle E_i \rangle$ is given by, $f_e = e \langle E_i \rangle / m_e$. So, eqn. (82) gives:

$$s_e = \frac{e\langle E_i \rangle}{2m_e}\left(\frac{x_e}{v_e}\right)^2 . \quad (84)$$

Further the average of s_e i.e. $\langle s_e \rangle$ is given by,

$$\langle s_e \rangle = \int_0^{N_T} s_e \frac{\partial N_T}{N_T}, \quad (85)$$

where $\partial N_T / N_T$ is the proportion of the electrons having free paths of lengths between x_e and $x_e + \partial x_e$ as given by eqn. (79).

Substituting the values of s_e and $\partial N_T / N_T$, eqn. (85) gives:

$$\langle s_e \rangle = \frac{e\langle E_i \rangle}{2m_e \lambda_e v_e^2} \int_0^{\infty} x_e^2 \exp\left(-\frac{x_e}{\lambda_e}\right) \partial x_e ,$$

which gives

$$\langle s_e \rangle = \frac{e\langle E_i \rangle \lambda_e^2}{m_e v_e^2} . \quad (86)$$

The average drift velocity is taken as the average distance divided by the average time τ_e between the collisions. If a large number of collisions take place in the plasma, then τ_e is given by: $\tau_e = \lambda_e / v_e$. So that,

$$\langle v_d \rangle = \frac{\langle s_e \rangle}{\tau_e} = \frac{e\langle E_i \rangle}{m_e}\left(\frac{\lambda_e}{v_e}\right), \quad (87)$$

where, $\langle v_d \rangle$ is the average drift velocity of the electron in the presence of $\langle E_i \rangle$.

If ν_E is the collision frequency of electrons with gas molecules, then by gas kinetics, ν_E is given by: $\nu_E = v_e / \lambda_e$. Thus eqn. (87) gives,

$$\langle v_d \rangle = \frac{e\langle E_i \rangle}{m_e \nu_E} . \quad (88)$$

If N_e is the electron density in the plasma, then the average drift current density $\langle j_d \rangle$ corresponding to $\langle v_d \rangle$ due to the electrons is given by:

$$\langle j_d \rangle = N_e e \langle v_d \rangle = \frac{N_e e^2}{m_e \nu_E} \langle E_i \rangle , \quad (89)$$

using eqn. (88).

Thus, the equivalent resistivity R_P' of the plasma, using eqn. (89), is given by,

$$R_P' = \frac{\langle E_i \rangle}{\langle j_d \rangle} = \left(\frac{m_e \nu_E}{N_e e^2}\right) . \quad (90)$$

Eqn. (90) is the same as eqn. (74).

Thus,

$$f_1' / m_e = \nu_E ,$$

of eqn. (40) assumed in eqn. (46) means that in the presence of the random $\langle E_i \rangle$ while considering the equivalent resistivity of the plasma, the distribution of electronic free paths at the collisions comes into the picture.

5.2. Role of the Electron Collisions in the Determination of f_1''/m_e

Consider the differential equation of motion of the electron in the plasma in the presence of damped oscillations of angular frequency ω_0, i.e. eqn. (64)

$$\frac{\partial^2 r}{\partial t^2} + 2g_E \frac{\partial r}{\partial t} + \omega_0^2 r = \frac{e}{m_e} E_{dc} ,$$

where

$$2g_E = f_1'' / m_e .$$

is given by eqn. (59).

Now consider the differential equation of motion of an electron in the plasma in the presence of an external d.c. electric field E_{dc} as given in a previous paper (Nandedkar and Bhagavat, 1970 [4], i.e.

$$\frac{\partial^2 r}{\partial t^2} + 2g \frac{\partial r}{\partial t} + \frac{f_2}{m_e} r = \frac{e}{m_e} E_{dc} , \quad (91)$$

where, $\partial^2 r / \partial t^2$, , $\partial r / \partial t$ and r, are the acceleration, velocity and displacement of the electron at time t. $\frac{f_2}{m_e}$ is the restoring force per unit displacement of the electron and g is previously assumed value of electron-molecule collision frequency. In the case of low damping,

$$\frac{f_2}{m_e} >> g^2 , \quad (92)$$

and then,

$$\omega_0 = \left(\frac{f_2}{m_e}\right)^{1/2} , \quad (93)$$

gives the angular frequency of damped oscillations as assumed before.

Using eqn. (93), eqn. (91) gives:

$$\frac{\partial^2 r}{\partial t^2} + 2g \frac{\partial r}{\partial t} + \omega_0^2 r = \frac{e}{m_e} E_{dc} . \quad (94)$$

Comparing eqns. (94) and (64), it is found that,

$$2g_E = 2g\,, \tag{95}$$

where g is given by (Nandedkar and Bhagavat 1969) [9]:

$$g = N_m \pi \left(\frac{D}{2}\right)^2 \left(\frac{8kT}{\pi m_e}\right)^{1/2} \tag{96}$$

here N_m is the number density of gas molecules in the plasma at equilibrium temperature T. D gives previously assumed value of the diameter of the gas molecule. Here k is Boltzmann constant and m_e is the mass of the electron.

Comparing eqns. (36) and (96), i.e.

$$N_m \pi R_m^2 \left(\frac{2kT}{\pi m_e}\right)^{1/2} \equiv N_m \pi \left(\frac{D}{2}\right)^2 \left(\frac{8kT}{\pi m_e}\right)^{1/2},$$

it is found that :

$$R_m = D/\sqrt{2}\,, \tag{97}$$

i.e. the actual classical radius of the gas molecule is $1/\sqrt{2}$ times the diameter of the gas molecule assumed previously.

In a previous paper (Nandedkar and Bhagavat 1970) [8], it is shown that the d.c. resistivity R_P of the plasma in the presence of E_{dc} is given by:

$$R_P = \frac{m_e(2g)}{N_e e^2}\,, \tag{98}$$

where m_e is the mass of an electron of charge e and N_e gives the electron density in the plasma. Here g is given by eqn. (96).

Using eqn. (95), the modified value of the d.c. resistivity of the plasma i.e. $(R_p)_m$ in the present case is given by:

$$(R_P)_m = \frac{m_e(2g_E)}{N_e e^2}\,, \tag{99}$$

where g_E is given by eqn. (36).

Thus,

$$f_1''/m_e = 2g_E$$

of eqn. (59) means that in the presence of the unidirectional E_{dc} while treating the d.c. resistivity of the plasma, the distribution of electronic free paths at the collisions is not considered.

6. Charge to Mass Ratio of an Electron and Classical Radius of a Gas Molecule

The frequency of sustained damped oscillation f_o is given by eqn. (70) i.e.,

$$f_o = \frac{1}{2\pi}\left[\left\{\frac{1-\exp(-1)}{(3/4\pi)^{1/3}}\right\}\left(\frac{e}{m_e}\right)\right]^{1/2} E_{dc}^{1/2} N_e^{1/6}\,.$$

Using eqn. (70), the values of charge to mass ratio of an electron i.e. $\frac{e}{m_e}$ is given by:

$$\frac{e}{m_e} = \left[\frac{2\sqrt[3]{6\pi^5}}{1-\exp(-1)}\right]\frac{f_0^2}{E_{dc}N_e^{1/3}}\,. \tag{100}$$

Further, eqn. (95) gives:

$$g_E = g\,, \tag{101}$$

where (Nandedkar and Bhagavat 1969) [9],

$$g = \nu/2, \tag{102}$$

here ν is the electron-molecule collision frequency assumed in the beginning (Bhagavat and Nandedkar 1968) [7].

Thus,

$$g_E = g = \nu/2\,, \tag{103}$$

using eqns. (101) and (102).

If p is the pressure of the gas at which the plasma at ambient temperature T is obtained, then simple kinetic theory of a gas gives:

$$p = N_m kT\,, \tag{104}$$

where N_m is the number density of gas molecules in the plasma.

Using eqns. (104) and (36), g_E is given by:

$$g_E = \left(\frac{p}{kT}\right)\pi R_m^2 \left(\frac{2kT}{\pi m_e}\right)^{1/2}. \tag{105}$$

From eqn. (105), the value of the classical radius of a gas molecule i.e. R_m is given by:

$$R_m = \left(\frac{kTm_e}{2\pi}\right)^{1/4}\left(\frac{g_E}{p}\right)^{1/2}. \tag{106}$$

When a r.f. wave interacts with the present plasma model in the presence of a d.c. electric field E_{dc}, then the analysis of the curve of the phase constant β_p of the wave versus wave frequency f, in the vicinity of f_o can be used to determine the values of f_o, ν (or $2g_E$) and N_e (Bhagavat and Nandedkar 1968) [10]. Thus knowing experimentally E_{dc}, f_o, N_e, g_E, p and T (Bhagavat and Nandedkar 1968) [7] , the values of e/m_e and R_m can be determined using eqn. (100) and (106) respectively.

7. Measurement of the Phase Constant β_p

In the present investigation, the plasma impedance is measured by terminating it on a slab line. The phase constant is determined from the reflection coefficient at the boundary. The problem of interest is the case of a finite plasma having different media on its either side, the boundary of separation begin parallel to each other (Fig. 1). Here a uniform plane r.f. wave is incident of the plasma (Bhagavat and Nandedkar, 1968 & Nandedkar and Bhagavat 1969) [10] & [9].

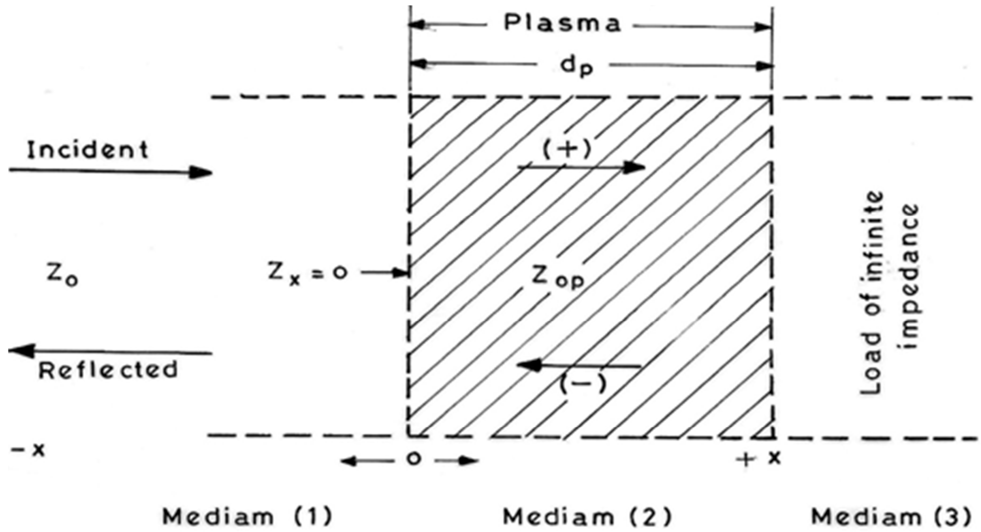

Fig. 1. Reflection and transmission of an electromagnetic wave at the boundary of the finite plasma.

Consider the standing wave pattern of the transverse electromagnetic (T.E.M.) mode in medium (2), i.e. the plasma (slab). The reflection coefficient at the boundary x=o is $\exp(-2\,\gamma_p d_p)$, where d_p is the length of the plasma slab along the x- axis. The input impedance of medium (2) at x = 0, i.e. $Z_{x=o}$ is given by,

$$Z_{x=o} = Z_{op} \coth \gamma_p d_p \,, \qquad (107)$$

where Z_{op} is the characteristic impedance of medium (2).

Coming to medium (1), i.e. the air in a slab line in which standing waves are formed, the output impedance of the medium at x = 0, which is $Z_{x=o}$, is given by:

$$Z_{x=o} = Z_o \frac{s' - i \tan \phi'}{1 - i s' \tan \phi'} , \qquad 108)$$

where Z_o= the characteristic impedance of medium (1), s' = v_{min} / v_{max} , here v_{min} and v_{max} are the minimum and maximum values of the transverse electric voltage in the main slab line and $\phi' = 2\pi x_o / \lambda_o$, where x_o is the position of the first minimum from the boundary x = 0 in medium (1). λ_o is the r.f. wavelength in the same medium.

For the wave in transverse electromagnetic mode,

$$\gamma_p Z_{op} = \gamma_o Z_o \,, \qquad (109)$$

where γ_o = propagation constant of the r.f. wave in medium (1) such that $\gamma_o = i\beta_o$, here β_o is the phase constant of the r.f. wave in medium (1), γ_p = propagation constant of the r.f. wave in medium (2) i.e. the plasma, such that,

$$\gamma_p = \alpha_p + i\,\beta_p \,, \qquad (110)$$

here α_p = attenuation constant and β_p = phase constant of the wave in medium (2).

Equations (107) to (109), give:

$$\gamma_p^2 \frac{d_p}{\gamma_o} = \frac{1 - is' \tan \phi'}{s' - i \tan \phi'}, \qquad (111)$$

where, $\gamma_p\, d_p \le 0.3$.

If

$$\beta_p^2 >> \alpha_p^2 \,, 1 >> s'^2 \text{ and } \tan^2\phi' >> s'^2 \,, \qquad (112)$$

then imaginary part of eqn. (111) gives:

$$\beta_p = \left[\frac{2\pi}{\lambda_o d_p} \cot \frac{2\pi x_o}{\lambda_o}\right]^{1/2} . \qquad (113)$$

From experimental values of x_o and λ_o , the phase constant β_p can be evaluated.

8. Experimental Work

The experimental set up consists of a V.H.F./U.H.F. oscillator type ГСС-12 (U.S.S.R. make), a slab line, a detector unit and a plasma slab (Fig. 2). The slab line is type Ли-3 (U.S.S.R. make) of 75 ohms characteristic impedance. The detector unit consists of a capacitive probe with a magnetic coupling to a diode type ДК-иI, III-60 and a selective amplifier [serial number 32, model number 794 (U.S.S.R. make).

The plasma-slab (Fig. 3) consists of two nickel electrodes sealed in a pyrex glass tube. Near the anode two stainless steel meshes are fitted parallel to the electrodes. The central conductor is fixed with the help of polystyrene spacers. The gas used, in the plasma slab is air. The slab is connected to the main line with appropriate adapters. Discharge of air at a pressure of 2 x 10^{-1} torr is obtained in the plasma slab at ambient temperature of 300 ^{0}K (where ^{0}K is Degree Kelvin). Thermocouple gauge is used for the pressure measurement. The low pressure is obtained with the help of a rotary oil pump (Edwards High Vacuum Ltd., No. W 2293, U.K. make).

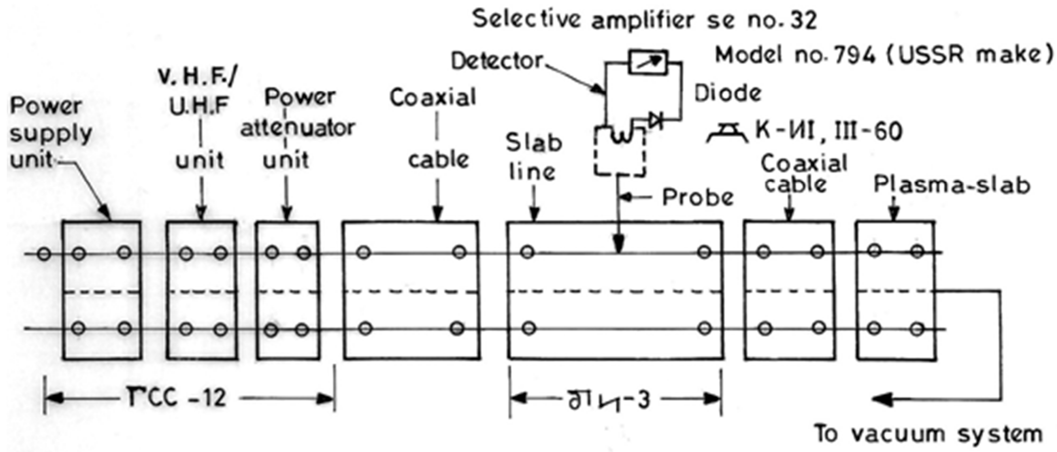

Fig. 2. Experimental set up for measuring the plasma load characteristics.

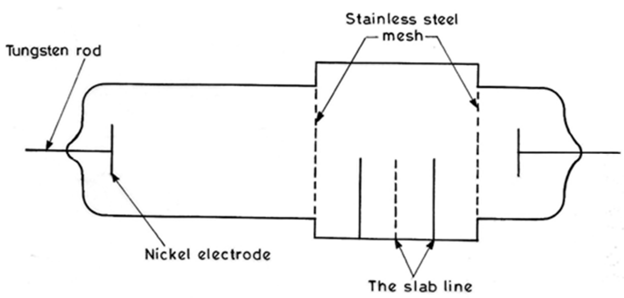

Fig. 3. The plasma-slab.

Table 1. Determination of e/m_e and R_m.

E_{dc} = 16 x 10^2 v/m				
N_e	ν or 2g or $2g_E$	f_o	e/m_e	R_m
x 10^{-12}	x 10^{-7}	x 10^{-6}	x10^{-11}	x 10^{10}
(m^{-3})	(sc^{-1})	(c/s)	(C/kgm)	(m)
3.803	2.511	333	1.7202	1.0744
4.923	2.537	348	1.7238	1.0799
6.776	2.537	367	1.7235	1.0799
10.14	2.519	393	1.7279	1.0761

Table 2. Determination of e/m_e and R_m.

N_e = 10.14 x 10^{12} m^{-3}				
E_{dc}	ν or 2g or $2g_E$	f_o	e/m_e	R_m
x 10^{-2}	x 10^{-7}	x 10^{-6}	x10^{-11}	x 10^{10}
(v/m)	(sc^{-1})	(c/s)	(C/kgm)	(m)
10	2.533	315	1.7761	1.0791
12	2.564	340	1.7244	1.0857
14	2.557	370	1.7504	1.0842
16	2.519	393	1.7279	1.0761

Mean value of e/m_e obtained from Tables 1 and 2 = 1.7352 x 10^{11} C/kgm.
Mean value of R_m obtained from Tables 1 and 2 = 1.0799 x 10^{-10} m.

The discharge current is varied from 5 mA to 20 mA. The potential difference across the stainless steel meshes is varied from 50 volts to 80 volts with the help of an external power supply. Measurements of the reflection coefficient are made in the frequency range 312 Mc/s to 396 Mc/s.

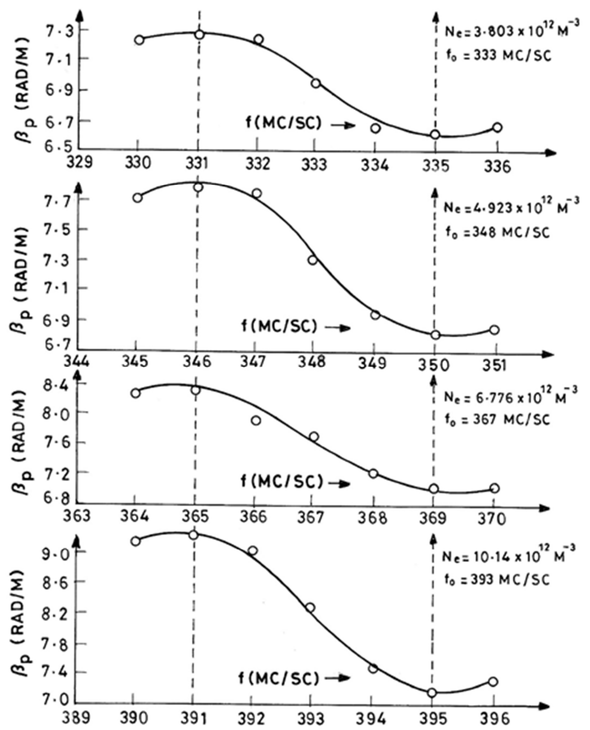

Fig. 4. Phase constant β_p versus frequency f at various electron densities with the field parameter E_{dc} = 16 x 10^2 v/m across the plasma column of ionized air.

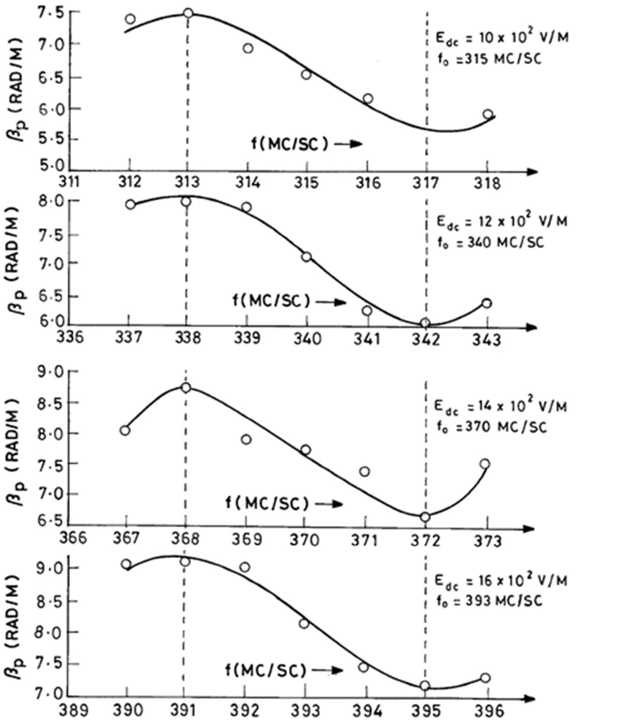

Fig. 5. Phase constant β_p versus frequency f at various d.c. fields across the plasma column of ionized air with the electron density $N_e = 10.14 \times 10^{12}\ m^{-3}$ as a parameter.

Various curves for β_p versus f are drawn (figs. 4 and 5). From these, values of f_o, N_e and g_E are determined for E_{dc} and N_e as parameters, respectively. Variation of f_o versus $N_e^{1/6}$ and $E_{dc}^{1/2}$ as well as of g_E with N_e and E_{dc} are shown in figs. 6 to 9.

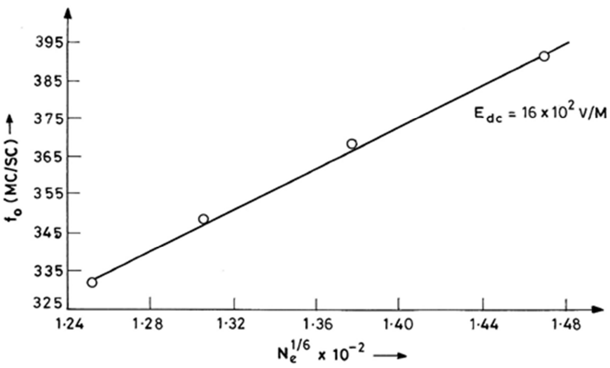

Fig. 6. Variation of f_o with $N_e^{1/6}$.

The adjoining Tables [Tables 1 and 2] record the values of N_e or E_{dc} , ν or 2g or $2g_E$ and f_o as determined from the curves of β_p versus f (figs. 4 and 5) for E_{dc} and N_e as parameters, respectively. Last two columns of each table, give the values of e/m_e and R_m as determined from eqns. (100) and (106) respectively.

Further here,

Corrigenda to papers (Bhagavat and Nandedkar 1968) [10] & [7], are given in [11] & [12].

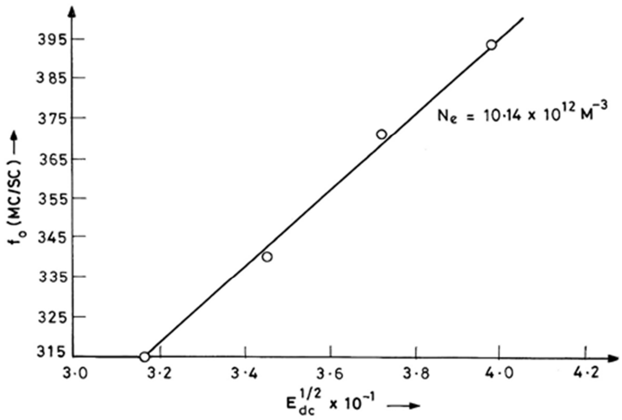

Fig. 7. Variation of f_o with $E_{dc}^{1/2}$.

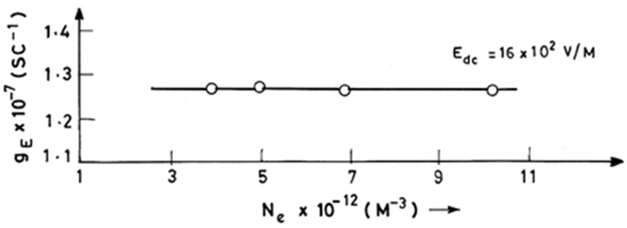

Fig. 8. Variation of g_E with N_e.

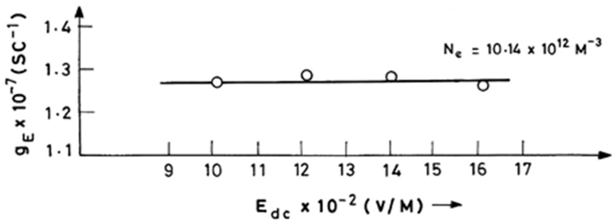

Fig. 9. Variation of g_E with E_{dc}.

Whereas Corrigenda to papers (Nandedkar and Bhagavat 1969 & 1970) [9] & [8], is given in [13].

9. Discussions and Conclusions

Figures 6 and 7 indicate the variation of f_o with $N_e^{1/6}$ and $E_{dc}^{1/2}$ keeping E_{dc} and N_e as constant respectively. Fig. 6 shows a linear relationship between f_o and $N_e^{1/6}$. This means for a given E_{dc} , f_o is directly proportional to $N_e^{1/6}$ [eqn. (70)]. Figure (7) indicates a linear relationship between f_o and $E_{dc}^{1/2}$. Thus, for a given N_e, f_o is directly proportional to $E_{dc}^{1/2}$ [eqn. (70)]. Hence the frequency of damped oscillations f_o is proportional to $E_{dc}^{1/2}$ and $N_e^{1/6}$ as predicted by eqn. (70).

The experimental values of g_E are plotted against N_e and E_{dc} in figs. 8 and 9, keeping E_{dc} and N_e as constant respectively. Electron-molecule collision frequency g_E is found to be independent of both N_e and E_{dc} . These values of g_E are in agreement with the kinetic theory data [eqn. (36)].

The mean value of the charge to mass ratio of an electron viz., e/m_e, as obtained by the analysis mentioned in this paper, is 1.7352 x 10^{11} C/kgm [Tables 1 and 2]. The presently accepted value of e / m_e is 1.7591 x 10^{11} C/kgm. The experimentally determined value of e / m_e is less than the actual value of e / m_e with an error of 1.3586% which is due to the experimental limitations.

The mean value of the classical radius of an air molecule, viz., R_m obtained by the analysis given here is 1.0799 x 10^{-10}m. [Tables 1 and 2]. This is of the same order of the classical radius of any type of a gas molecule which is 10^{-10} m as given by gas-kinetics.

Equation (69) shows that when $E_{dc} \to 0$, then $\omega_o = 2\pi f_o \to 0$, i.e. in the absence of an external d.c. electric field, the frequency of the sustained damped oscillations tends to zero.

However, when E_{dc} is reduced in the limit, then $\langle E_i \rangle$ is approached. $\langle E_i \rangle$ is the average value of the electric field due to the ion at the center of the screening sphere at the distance $R_o = (3/4\pi)^{1/3} N_e^{-1/3}$. This distance acts as the steady state amplitude for the electronic damped oscillations of eigen - frequency f_j. The values of the eigen-frequency f_j and of the corresponding angular eigen-frequency ω_o are given by eqn. (52), viz.,

$$f_j = \frac{1}{2\pi}\left(\frac{N_e e^2}{m_e \epsilon_0}\right)^{1/2},$$

and

$$\omega_j = \left(\frac{N_e e^2}{m_e \epsilon_0}\right)^{1/2}, \quad (114)$$

[refer to eqn. (51)].

The expression for the plasma frequency f_p or the angular plasma frequency ω_p due to Tonks and Langmuir (Tonks and Langmuir 1929) [5] is similar to f_j or ω_j i.e.

$$f_p \to f_j = \frac{1}{2\pi}\left(\frac{N_e e^2}{m_e \epsilon_0}\right)^{1/2}, \quad (115)$$

and

$$\omega_p \to \omega_j = \left(\frac{N_e e^2}{m_e \epsilon_0}\right)^{1/2}. \quad (116)$$

Thus the plasma frequency due to Tonks and Langmuir (Tonks and Langmuir 1929) [5] is similar to the eigen-frequency of damped oscillations, in the case of low damping where electron-molecule oscillations are of main importance. It is interesting to note that the steady state amplitude of f_j exists inside the screening sphere of radius $\frac{\sqrt{2}}{k_{D'}} = \sqrt{\epsilon_0 kT/N_e e^2}$ [refer eqn. (3)], at the distance of $R_o = (3/4\pi)^{1/3} N_e^{-1/3}$ from the center of the sphere having the positive ion.

Now, come to the case of a complex dielectric constant of the plasma. The complex dielectric constant of the plasma, i.e. ϵ_p in general can be expressed as follows:

$$\epsilon_p = \epsilon_p' - \epsilon_p'', \quad (117)$$

where ϵ_p' = real part of the dielectric constant of the plasma, and ϵ_p'' = imaginary part of the dielectric constant of the plasma.

Relative values of ϵ_p' and ϵ_p'' with respect to the permittivity ϵ_o of the free space, in the case of present plasma model biased by a d.c. electric field, are given by (Nandedkar and Bhagavat 1969) [9]:

$$\frac{\epsilon_p'}{\epsilon_0} = 1 + \frac{(N_e e^2/m_e \epsilon_0)(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (2g)^2\omega^2} \quad (118)$$

and

$$\frac{\epsilon_p''}{\epsilon_0} = \frac{(N_e e^2/m_e \epsilon_0)(2g)\omega}{(\omega_0^2 - \omega^2)^2 + (2g)^2\omega^2}, \quad (119)$$

where N_e = electron density in the plasma, e = charge of an electron, m_e = mass of an electron, ω_o = angular frequency of damped oscillations, ω = angular frequency of the interacting r.f. wave and g = previously assumed value of the electron-molecule collisions in the plasma given by eqn. (96).

Using eqn. (95), eqns. (118) and (119) give:

$$\frac{\epsilon_p'}{\epsilon_0} = 1 + \frac{(N_e e^2/m_e \varepsilon_0)(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (2g_E)^2\omega^2}, \quad (120)$$

and

$$\frac{\epsilon_p''}{\epsilon_0} = \frac{(N_e e^2/m_e \varepsilon_0)(2g_E)\omega}{(\omega_0^2 - \omega^2)^2 + (2g_E)^2\omega^2}, \quad (121)$$

where g_E is the electron-molecule collision frequency in the plasma given by eqn. (36).

In the limiting case, when,

$$E_{dc} \to 0 \quad (122a)$$

then,

$$\omega_o \to 0, \quad (122b)$$

[as given by eqn. (69)] and,

$$\frac{N_e e^2}{m_e \epsilon_0} \to \omega_j^2 \to \omega_p^2, \quad (122c)$$

[using eqn. (116)], where ω_j is the angular eigen-frequency of electronic damped oscillations in the case of low damping, which are not sustained. ω_j is similar to ω_p i.e. the plasma angular frequency due to Tonks and Langmuir (Tonks and Langmuir, 1929) [5].

Further in the limit when E_{dc} is removed, then the electric field $\langle E_i \rangle$ at distance R_o due to the ion at the center of the screening sphere, is left. Then in this limiting case, the damping force per unit mass per unit velocity of the electron, in the case of ω_o (or E_{dc}) i.e. f_1''/m_e [refer eqns. (58) and (63)] and in the case of ω_j (or $\langle E_i \rangle$) i.e. f_1' / m_e [refer eqns.. (39) and (45)] tend to each other, i.e.

$$f_1'' / m_e \rightarrow f_1' / m_e$$

or,

$$2g_E \rightarrow \nu_E, \quad (122d)$$

[as given by eqns. (59) and (40)] .

Hence in this limiting case when E_{dc} is removed, eqns. (120) and (121) give:

$$\frac{\epsilon_p'}{\epsilon_o} = 1 - \frac{\omega_p^2}{\omega^2 + \nu_E^2}, \quad (123)$$

and

$$\frac{\epsilon_p''}{\epsilon_0} = \frac{\omega_p^2(\nu_E/\omega)}{\omega^2+\nu_E^2}. \quad (124)$$

The conductivity of the plasma corresponding to eqn. (124), viz., σ_p , is given by:

$$\sigma_p = \omega\epsilon_p'' = \frac{\omega_p^2 \varepsilon_0 \nu_E}{\omega^2+\nu_E^2} = \frac{N_e e^2 \nu_E}{m_e(\omega^2+\nu_E^2)}, \quad (125)$$

using eqn. (122c).

Equations (123) and (125) give the values of relative permittivity with respect to free space and r.f. conductivity of the plasma in the absence of a d.c. electric field. Here the plasma considered, is a weakly ionized medium where electron-molecule collisions are of main importance at thermal equilibrium temperature of the ambient.

Equations (123) and (125) are the forms of the expression for relative permittivity with respect to free space and r.f. conductivity of a weakly ionized plasma, similar to that occurring in the case of the ionosphere {for instance refer Ramo, Whinnery and Van Dauzer 1970 [14] and Ratcliffe 1959 [15]–where electrons make collisions with gas molecules at ambient temperature ~ 300^0 K which are basically due to Appleton and Chapmen (Appleton and Chapmen 1932) [6]}.

In the plasma model proposed in this paper, electrons, ions and neutral molecules are considered to be in thermal equilibrium at ambient temperature T. The electrons in this model, where density is N_e and temperature is T are referred to as ‘slow-electrons’. These slow electrons from about 1% part of the electrons in the other group in the plasma, which are characterized by density N_{eL} and temperature T_{eL} which can be determined by the technique of Langmuir’s probe method. The electrons having density N_{eL} and temperature T_{eL} can be referred to as relatively ‘fast-electrons’ as against the ‘slow-electrons’ which are characterized by density N_e and temperature T. Condition of overall charge neutrally in the plasma is experimentally verified here, where electrons, ions and neutral molecules are in thermal equilibrium at temperature T of the ambient, in the case of positive column of glow discharge of air.

Thus the present model of weakly ionized plasma leads, to the similar expression for plasma frequency due to Tonks and Langmuir (Tonks and Langmuir 1929) [5] and, to the similar expression for the complex dielectric constant of the plasma basically due to Appleton and Chapman (Appleton and Chapman 1932) [6] – as the limiting cases.

Moreover the experimentally determined value, of charge to mass ratio of an electron that is e / m_e is less than the actual value of e / m_e with an error of 1.3586%, which is due to experimental limitations. Further the value of the classical radius of an air molecule viz., R_m obtained by the analysis given here is 1.0799 x 10^{-10}m. This is of the same order of the classical radius of any type of a gas molecule viz., air which is 10^{-10} m as given by gas-kinetics.

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