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Analysis of Mobility of Intrinsic Germanium and Silicon near Room Temperature

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Abstract

Germanium (Ge) and Silicon (Si) are widely used semiconductors in crystal diodes and transistors. So that study of mobility of intrinsic Ge and Si is an important aspect, in electronics/physics. In the present paper, coupled mass-vibrations of atoms in an intrinsic semiconductor, viz., Germanium or Silicon, are analysed near room temperature ~300 ^{0}K. Overall average of resultant mean square amplitudes of an atom in the semiconductor undergoing coupled mass-vibrations is regarded as collision-cross section for conduction electrons or valence holes colliding with the atoms. Here the electrons or the holes are regarded as free and forming a gas with appropriate thermal velocity at the temperature of the semiconductor under consideration. This determines the charge carrier's collision frequency with the atoms. Whence an expression for mobility of the charge carrier is obtained considering distribution of free paths of the charge-carrier. The expression for mobility for isotropic scattering of the charge-carrier at its collision with an atom given here, is modified by comparing the calculated value of mobility at (~300 ^{0}K) with the value given in a physical table, for the case of anisotropic scattering. This analysis shows that the mobility of the charge-carrier viz., the electron or the hole, due to the charge carrier's collisions with atom, varies as $T^{-3/2}$ where T is the temperature of the semiconductor under consideration, a result also confirmed by previous investigators. Present analysis gives fairly well values of motilities near room temperature for Ge and Si. Purpose of this work is to illustrate in a simple manner, how mobility of the semiconductor comes in picture by anisotropic scattering of the electrons and holes at collisions with atoms in the semiconductor. The electrons undergo reverse anisotropic scattering and holes forward anisotropic scattering in Ge whereas both electrons and holes undergo forward anisotropic scattering at collisions with respective atoms in Si.

Keywords

Germanium, Silicon, Electron/Hole, Coupled-Mass-Vibrations, Atoms, Collisions, Mobility, Room-Temperature

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1. Introduction

Germanium (Ge) and Silicon (Si) are widely used semiconductors in crystal diodes and transistors. So that study of mobility of intrinsic Ge and Si is an important aspect, in electronics/physics. The case of mobility of an intrinsic (that is a pure) semiconductor such as Ge or Si near room temperature (~300 ^{0}K, where ^{0}K is Degree Kelvin) has been considered by previous investigators viz., Seitz (1948) [1], Pearson and Bardeen (1949) [2], Pearson and Bardeen (1950) [3], Bardeen and Shockley (1950) [4], Debye and Conwell (1954) [5], Morin and Maita (1954) [6], Fritzsche, (1955) [7], Dush and Newman (1955) [8], Brounstein, Moore and Herman (1958) [9]. Their analysis has shown that the mobility of the charge-carrier, viz., the electron or the hole due to interaction of charge-carrier waves with lattice waves varies as $T^{-3/2}$, where T is temperature of the semiconductor. Here for General Reference, refer also to Ashcroft and Mermin (1976) [10], Ziman (1972) [11], Callway (1974) [12], Sze (1981) [13] & Ibach and Luth (1991) [14].

The present paper considers the analysis of mobilities of charge carriers in the intrinsic semiconductors viz., Ge and Si

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near room temperature ($\sim 300\,^0$K), as an extension of the theory of coupled mass-vibrations of the fixed ions or atoms (which are devoid of one conduction electron each) as already given in the case of noble metals by Nandedkar (2015) [15]. The semiconductor which is dealt herewith is considered as very pure.

The intrinsic semiconductor of this analysis is of macroscopic dimension. Here the intrinsic semiconductor, that is Ge or Si is regarded to consists of fixed (tetravalent) atoms (which are devoid of 4-valence electrons each) in its volume at regular intervals on an average. Each of the tetravalent atoms, shares its outer shell's 4-valence electrons with the (nearest) adjacent 4 atoms forming covalent bonds where each covalent bond contains a pair of two electrons. Thus, here all atoms are linked by covalent bonds. This semiconductor is known as a non-polar semiconductor.

At temperature near room temperature ($\sim 300\,^0$ K), an individual electron of a covalent bond acquires sufficient energy due to the thermal agitation of the atoms to break the bond to become free. Such free electrons move about randomly in the semi-conductor similar to molecules in a gas. These electrons are referred to as conduction electrons in this analysis.

When an electron escapes from a covalent bond to become free, then the empty space left behind is known as a hole which has a charge equal and opposite to that of an electron. The tendency of one remaining electron in the covalent bond is to form again an electron pair; for which it steals one electron from an adjacent pair of another covalent bond because of favourable situation arising due to thermal agitation. In this way the previous hole is now shifted to another place from which the electron is stolen. Thus once a hole is recreated it moves about in the semiconductor in a random fashion as do the free electrons. These free holes are referred to as valence holes in this analysis, where the mass of a hole on an average is considered same as that of an electron here. The number of holes is obviously equal to the number of electrons in the semiconductor.

When the electrons move randomly in the semiconductor, they have encounters with the broken covalent bonds resulting in rejoining of the broken covalent bonds leading to electron-hole recombination of the electrons and holes. The density of electrons and holes in the intrinsic semiconductor rises to such a level that the rate of recombination of electrons and holes equals their rate of generation at the temperature of semiconductor under consideration, which denotes the steady state. The order of electron or hole density which is intrinsic charge carrier density (in the steady state) is known to be 10^{19} m^{-3} in Ge and 10^{16} m^{-3} in Si - refer to Croissettee (1974, table 1-2, pp. 28) [16], near room temperature ($\sim 300\,^0$K). Here the result of mobility of the charge-carrier (viz., the electron or the hole) on an average does not depend on the intrinsic charge-carrier density. In this analysis the intrinsic charge-carrier density is large enough to use method of gas-kinetics for various results analyzed herewith. Here the density of atoms in the intrinsic semiconductor is extremely large as compared to the intrinsic-charge carrier density.

In calculations of mobilities of the charge carriers in this analysis, attention is confined to homogenous and isotropic media whose properties are the same in every part and in every direction. This rules out consideration of directional properties of the crystalline media provided by Germanium and Silicon, and only average values are treated.

Further it is assumed that the fixed atoms of the semiconductor undergo thermal mass-vibrations at temperature T of it. The fixed atoms in the semiconductor are elastically bond with respect to each other. Coupled mass- vibrational modes of the atoms are associated with elastic waves in transverse and longitudinal modes of Debye-type in the semiconductor.

Conduction electrons here form a free electron-gas at temperature T (where an electron of the gas has an average thermal velocity $\mathrm{v_e} = \sqrt{\left[{}^{8\mathrm{kT}}/_{\pi\,\mathrm{m_e}}\right]}$, here k is Boltzmann constant and $\mathrm{m_e}$ is mass of the electron), so also valence holes form a free hole-gas at the same temperature T (where the hole of the gas has an average thermal velocity $\mathrm{v_p} = \sqrt{\left[{}^{8\mathrm{kT}}/_{\pi\,\mathrm{m_p}}\right]}$. Here $\mathrm{m_p}$ is mass of the hole). Atoms, electrons and holes are all in thermal equilibrium at temperature T of the semiconductor.

When a d.c. electric field is applied to the semiconductor, then the conduction electrons or the valence holes acquire drift velocities (since charge of an electron is equal and opposite to that of a hole, so that the respective drift velocities of the electrons and holes are oppositely directed) limited by collisions of the electrons or the holes with the atoms undergoing coupled mass-vibrations. The collision phenomenon is described in terms of electron-atom or hole-atom collisions using gas-kinetics. While dealing with the collision frequency, velocity of the electron or the hole appropriate to average thermal velocity at temperature T of the semiconductor is taken into account. The collision frequency can be determined by gas-kinetics, if effective collision cross-section of electron-atom or hole- atom collisions in the semiconductor be known. In the present paper, an expression for overall average of resultant mean square amplitudes of coupled mass-vibrations of an atom is obtained and is regarded as the effective collision cross-section for the electrons or the holes. Whence knowing the collision frequency, the mobility of electron or hole is obtained,

considering the distribution of free paths of the charge-carrier. This treatment assumes that on an average, the mass of an electron is equal to the mass of a hole which is the free electron mass. The scattering of an electron or a hole with the atom is treated as isotropic. And the expression for mobility of the charge-carrier at the isotropic scattering is modified by comparing the calculated value of mobility at $T = 300\ ^0K$ with the value as given in a physical table from Croissette (1974, table 1-2, pp. 28) [16], for the case of anisotropic scattering. This analysis shows that the mobility of the charge-carrier viz., the electron or the hole, due to the charge-carrier's collisions with atoms, varies as $T^{-3/2}$ where T is the temperature of the semiconductor under consideration.

The Method of Analysis of this research-paper consists of following sections for this article:

2. Mass vibrations of the fixed atom

3. Collision frequency of the charge-carrier

4. Mobility of the charge-carrier

5. Numerical analysis

followed by

6. Conclusions

2. Mass Vibrations of the Fixed Atom

A fixed atom of the semiconductor (of Ge or Si) is assumed to undergo thermal mass-vibrations at temperature T of it. If M_a be the mass of the atom having f as frequency of the mass-vibrations, then its equation of motion, say along x-axis can be written as follows,

$$M_a \frac{\partial^2 x}{\partial t^2} + 4\pi^2 f^2 M_a x = 0, \quad (1)$$

where x is the value of displacement of the atom from its equilibrium position, at the instantaneous time t, when its acceleration is $\partial^2 x/\partial t^2$.

The average potential energy of the atom over a cycle of its vibrations is given by,

$$V_a = 2\pi^2 f^2 M_a x_{oa}^2, \quad (2)$$

where x_{oa} is r.m.s. value of amplitude of the vibrations of the atom along x-axis.

If weight factor to have the average potential energy V_a at temperature T be defined by Boltzmann factor, viz,

$$P(V_a) = \exp\left(-\frac{V_a}{kT}\right), \quad (3)$$

where k is Boltzmann constant, then the average value of x_{oa}^2 i.e .$\bar{x}_{oa}^2$ over various values of x_{oa}^2 for the atomic mass-vibrator with frequency f can be obtained by averaging all values x_{oa}^2 with the weight factor given by eqn. (3). Thus,

$$\bar{x}_{oa}^2 = \frac{\int_0^\infty x_{oa}^2 P(V_a)\, \partial x_{oa}}{\int_0^\infty P(V_a)\, \partial x_{oa}}, \quad (4)$$

Using eqns. (2) and (3), eqn. (4) gives on solving the integral that,

$$\bar{x}_{oa}^2 = \frac{kT}{(2\pi f)^2 M_a} \quad (5)$$

Similarly for the directions of vibrations of the atom along y- and z- axes, it can be shown that,

$$\bar{y}_{oa}^2 = \frac{kT}{(2\pi f)^2 M_a}, \quad (6)$$

and

$$\bar{z}_{oa}^2 = \frac{kT}{(2\pi f)^2 M_a}. \quad (7)$$

Whence resultant average of mean square amplitude of the atomic mass- vibrations averaged with the Boltzmann factor i.e. $\bar{R}_{oa}^2$ is given by,

$$\bar{R}_{oa}^2 = \bar{x}_{oa}^2 + \bar{y}_{oa}^2 + \bar{z}_{oa}^2 = \frac{3kT}{(2\pi f)^2 M_a}. \quad (8)$$

Atoms of the semiconductor are elastically bound with respect to each other. These atoms are assumed to undergo mass-vibrations at temperature T of the semiconductor, in presence of elastic waves existing in the material. Frequency of mass- vibrations of the atoms is considered to be the same as that of elastic waves.

Considering the semiconductor as a continuous medium so far as the propagation of elastic waves is concerned, the number of modes of elastic waves per unit volume in the material denoted by $Z_a(f)\,\partial f$ in the frequency interval f between f and f + ∂f is given by

$$Z_a(f)\,\partial f = 4\pi f^2 \left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right) \partial f, \quad (9)$$

where $Z_a(f)$ = number of elastic wave modes per unit volume per unit frequency interval at frequency f in the semiconductor that means $Z_a(f)$ gives the weight factor to have the elastic waves or atomic mass-vibrations at frequency f, using a similar approach as adopted in the case of the fixed ions or atoms of the noble metals by Nandedkar (2015) [15]. Here v_t denotes the velocity of transverse elastic wave and v_l denotes the velocity of longitudinal elastic wave in the semiconductor. This analysis assumes that the linear dimensions of the semiconductor are extremely large as compared to the inter-atomic distance(s). The semiconductor is of macroscopic dimensions and of conventional sizes.

If N is the density of atoms in the semiconductor then there would be 3N modes of elastic waves per unit volume because there are 3N degrees of freedom per unit volume for mass vibrations of the atoms along three mutually perpendicular axes. This limits the maximum frequency of the wave. If minimum frequency be taken zero for all practical purposes and maximum be denoted by Debye frequency f_D of cut-off, then,

$$\int_0^{f_D} Z_a(f)\,\partial f = 3N, \tag{10}$$

meaning thereby coupled mass-vibrational modes of the atoms take place in presence of the elastic wave modes in the semiconductor. And further, each of the atoms has a band of frequency ranging 0 to f_D for all practical purposes, when linear dimensions of the semiconductor are extremely large as compared to the interatomic distance(s).

Further using eqn. (9) of the present analysis in eqn. (10), eqn. (10) gives that

$$f_D = \left(\frac{9N}{4\pi}\right)^{1/3}\left(\frac{2}{v_t^3}+\frac{1}{v_l^3}\right)^{-1/3}, \tag{11}$$

assuming elastic wave velocities to be independent of frequency.

Coming to eqn. (9), $Z_a(f)$ gives the weight factor to have the elastic waves or atomic mass-vibrations at frequency f. Further average value of $\overline{R}_{oa}^2$ with respect to the weight factor $Z_a(f)$ is given by

$$\langle\overline{R}_{oa}^2\rangle = \frac{\int_0^{f_D}\overline{R}_{oa}^2 Z_a(f)\,\partial f}{\int_0^{f_D} Z_a(f)\,\partial f}. \tag{12}$$

Using eqns. (8) to (11), eqn. (12) gives,

$$\langle\overline{R}_{oa}^2\rangle = \left[\left(\frac{k}{\pi\rho}\right)\left(\frac{2}{v_t^3}+\frac{1}{v_l^3}\right)^{2/3}\left(\frac{9N}{4\pi}\right)^{1/3}\right]T, \tag{13}$$

where

$$\rho = M_a N, \tag{14}$$

gives the density of the semiconductor. Here M_a is mass of atom of the semiconductor.

If characteristic temperature θ_D for the semiconductor be defined by,

$$\theta_D = \frac{hf_D}{k}, \tag{15}$$

where h is Planck constant, then eqn. (15) on using eqn. (11), gives

$$\theta_D = \frac{h}{k}\left[\left(\frac{9N}{4\pi}\right)^{1/3}\left(\frac{2}{v_t^3}+\frac{1}{v_l^3}\right)^{-1/3}\right]. \tag{16}$$

Eliminating the factor $[(2/v_t^3)+(1/v_l^3)]$ from eqns. (13) and (16), the value of $\langle\overline{R}_{oa}^2\rangle$ is given by,

$$\langle\overline{R}_{oa}^2\rangle = \left[\left(\frac{3h}{2\pi}\right)^2\left(\frac{N}{k\rho\theta_D^2}\right)\right]T. \tag{17}$$

Equation (17) gives the overall average of the resultant mean square amplitudes of the mass-vibrations of the atom in the semiconductor undergoing coupled mass-vibrations at temperature T of it.

3. Collision Frequency of the Charge-Carrier

The charge-carriers i.e. either the (conduction) free electrons or the (valence) free holes are in thermal equilibrium with the fixed atoms in the semiconductor, whose temperature T is near room temperature (~300 ^{0}K). Here the free electron-gas or the free hole-gas is similar to the gas of molecules with the average thermal velocity v_c of either charge-carriers having a mass m_c as given by,

$$v_c = \left(\frac{8kT}{\pi m_c}\right)^{1/2}. \tag{18}$$

Here and in the following analysis. the subscript 'c' denotes particular type of the charge carrier i.e. either the electron or the hole. The subscript 'c' is to be read as e for an electron or p for a hole. Thus for the electron and the hole, eqn. (18) gives,

$$v_e = \left(\frac{8kT}{\pi m_e}\right)^{1/2}, \tag{19}$$

and

$$v_p = \left(\frac{8kT}{\pi m_p}\right)^{1/2}. \tag{20}$$

When on an average $m_e = m_p = m_c = m$ (the free electron mass) as assumed in this analysis, then $v_c = v_e = v_p$.

If q_0 be the effective collision cross-section and N be density of atoms in the semiconductor, then the collision frequency ν_{co} of the charge-carriers with the atoms, using gas kinetics is given by,

$$\nu_{co} = Nq_0 v_c. \tag{21}$$

In the present analysis, it is assumed that q_0 is the same as $\langle\overline{R}_{oa}^2\rangle$ which means that the effective collision cross-section is the same as overall average of resultant mean square amplitudes of the fixed atom undergoing coupled mass-vibrations at temperature T of the semiconductor as given by eqn. (17). Thus

$$q_o = \langle\overline{R}_{oa}^2\rangle. \tag{22}$$

Here q_o is the same for either the electron or the hole.

Using eqns. (17), (18) and (22), eqn. (21) gives,

$$\nu_{co} = \left[N^2 \left(\frac{3h}{2\pi}\right)^2 \left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_c}\right)^{1/2}\right] T^{3/2}. \quad (23)$$

Thus for the electron and the hole in the semiconductor, eqn. (23) gives that,

$$\nu_{eo} = \left[N^2 \left(\frac{3h}{2\pi}\right)^2 \left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_e}\right)^{1/2}\right] T^{3/2}, \quad (24)$$

and

$$\nu_{po} = \left[N^2 \left(\frac{3h}{2\pi}\right)^2 \left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_p}\right)^{1/2}\right] T^{3/2}. \quad (25)$$

When on an average $m_e = m_p = m_c = m$ (the free electron mass) as assumed in this analysis, then $\nu_{co} = \nu_{eo} = \nu_{po}$.

4. Mobility of the Charge-Carrier

In the semiconductor the free (conduction) electron-gas or the free (valence) hole-gas is similar to the gas of molecules when temperature T of the semiconductor is near room temperature (~300 ^{0}K). In the present analysis, distribution of free paths of the charge-carriers is taken into account.

The collisions that determine the free-paths of the charge-carriers in the semiconductor, are random events. This being true, some free-paths would be long and other free-paths would be short. On the basis of random motion of the charge-carriers, an expression is obtained for 'distance distribution'.

If one charge-carrier has a collision frequency ν_{co} with the atoms in the semiconductor having an average thermal velocity $v_c = \left(\frac{8kT}{\pi m_c}\right)^{1/2}$ at temperature T (~300^0 K) of the material, then $a_{co} = \nu_{co}/v_c$ gives average number of collisions made by the charge-carrier in its unit length of path and the probable number of collisions made by this charge-carrier in travelling a distance ∂s_{co} would be $a_{co}\,\partial s_{co}$. Let C_{cT} be the total number of charge- carriers in the semiconductor. Out of these charge carriers, let c_{cT} be number of charge carriers that have travelled a distance s_{co} without having collisions. The number of these charge-carriers having collisions between s_{co} and $s_{co} + \partial s_{co}$ would be proportional to c_{cT} itself and the length of the path, or the charge in c_{cT} due to the collisions is given by,

$$\partial c_{cT} = -\, a_{co} c_{cT} \partial s_{co}, \quad (26)$$

where a_{co} is the constant of proportionality in this case and negative sign shows that ∂c_{cT} decreases as ∂s_{co} increases. Eqn. (26) gives the number of charge-carriers with free-paths between s_{co} and $s_{co} + \partial s_{co}$ numerically.

Equation (26) can be integrated to give,

$$c_{cT} = A_{cT}\exp(-a_{co}s_{co}). \quad (27)$$

where A_{cT} is a constant of integration. At $s_{co} = 0$, since there are no collisions, so that $c_{cT} = C_{cT}$. Thus $A_{cT} = C_{cT}$ in eqn. (27). And then eqn. (27) is rewritten as follows:

$$c_{cT} = C_{cT}\exp(-a_{co}s_{co}). \quad (28)$$

If ∂c_{cT} be the number of charge -carriers having a free path of length between s_{co} and $s_{co} + \partial s_{co}$, then the expression for mean free-path λ_{co} of the charge-carrier is given by,

$$\lambda_{co} = \int_0^{C_{cT}} \frac{s_{co}\,\partial C_{cT}}{C_{cT}}. \quad (29)$$

As,

$$\partial C_{cT} = |\partial c_{cT}| = a_{co}\, c_{cT}\, \partial s_{co} = a_{co} C_{cT}\exp(-a_{co}s_{co})\, \partial s_{co}, \quad (30)$$

so that eqn. (29) gives that,

$$\lambda_{co} = \int_0^{\infty} \frac{s_{co} a_{co} C_{cT}\exp(-a_{co}s_{co})\,\partial s_{co}}{C_{cT}} = \frac{1}{a_{co}}. \quad (31)$$

Thus the distribution of free paths of the charge-carrier is given by eqn. (28), using eqn. (31) as follows:

$$c_{cT} = C_{cT}\exp(-s_{co}/\lambda_{co}). \quad (32)$$

Eqn. (32) shows that, the number of free-paths of length greater than a given distance is a decreasing exponential function of the distance and further only 36.78(8) per cent of C_{CT} of charge-carriers have free-paths of length equal to λ_{co}.

Now if one charge-carrier starts with zero initial velocity, in the presence of a d.c. electric field E_x applied along, say parallel to x-axis, in the semiconductor after each collision, then the distance x_{co} it travels in time t_{co} with constant acceleration f_{co}, is given by,

$$x_{co} = \frac{f_{co} t_{co}^2}{2}. \quad (33)$$

The average distance x_{co} over the range of charge-carrier's free-path is now considered with a varying time $t_{co} = s_{co} / v_c$ given by the ratio of the free-path s_{co} to average thermal velocity v_c of the charge-carrier.

Now the acceleration f_{co} of the charge carrier in the presence of field E_x is given by $(e/m_c)E_x$, where e is the magnitude of charge of the charge-carrier. So eqn. (33) gives that,

$$x_{co} = \frac{1}{2}\left(\frac{eE_x}{m_c}\right)\left(\frac{s_{co}}{v_c}\right)^2. \quad (34)$$

Further the average of x_{co} i.e. $\langle x_{co}\rangle$ using eqn. (32) is given by,

$$\langle x_{co} \rangle = \int_0^{C_{cT}} \frac{x_{co}\, \partial C_{cT}}{C_{cT}}, \tag{35}$$

where $\partial C_{cT} / C_{cT}$ is the proportion of the charge-carriers having free-paths of lengths between s_{co} and $s_{co} + \partial s_{co}$ as given eqn. (30).

Using eqns. (30) and (34), eqn. (35) gives that,

$$\langle x_{co} \rangle = \frac{eE_x}{2m_c \lambda_{co} v_c^2} \int_0^{\infty} s_{co}^2 \exp(-s_{co}/\lambda_{co})\, \partial s_{co}, \tag{36}$$

where $a_{co} = 1/\lambda_{co}$ is the relation used as given by eqn. (31). Solution of eqn. (36) gives that,

$$\langle x_{co} \rangle = \frac{eE_x \lambda_{co}^2}{m_c v_c^2}. \tag{37}$$

The average drift velocity $\langle v_{co} \rangle$ of the charge-carrier is taken as the average distance $\langle x_{co} \rangle$ divided by the relaxation time τ_{co} between collisions which is the average time taken between the collisions. Using gas kinetics, the value of τ_{co} is given by,

$$\tau_{co} = \frac{\lambda_{co}}{v_c}, \tag{38}$$

so that,

$$\langle v_{co} \rangle = \frac{\langle x_{co} \rangle}{\tau_{co}} = \frac{eE_x}{m_c}\left(\frac{\lambda_{co}}{v_c}\right). \tag{39}$$

But,

$$\frac{v_c}{\lambda_{co}} = \nu_{co} = \frac{1}{\tau_{co}}, \tag{40}$$

by gas-kinetics where ν_{co} is the charge-carrier's collisions frequency. Thus eqn. (39) gives,

$$\langle v_{co} \rangle = U_{cx} = \frac{eE_x}{m_c \nu_{co}}, \tag{41}$$

where, $\langle v_{co} \rangle = U_{cx}$ is the average drift velocity of the charge-carrier parallel to x-axis.

In the present analysis it is assumed that the average velocity U_{cx} as given by eqn. (41), remains the same before and after scattering of the charge carrier by an obstacle in the form of an atom in the semiconductor at a collision. This is illustrated with reference to Fig. 1(a and b).

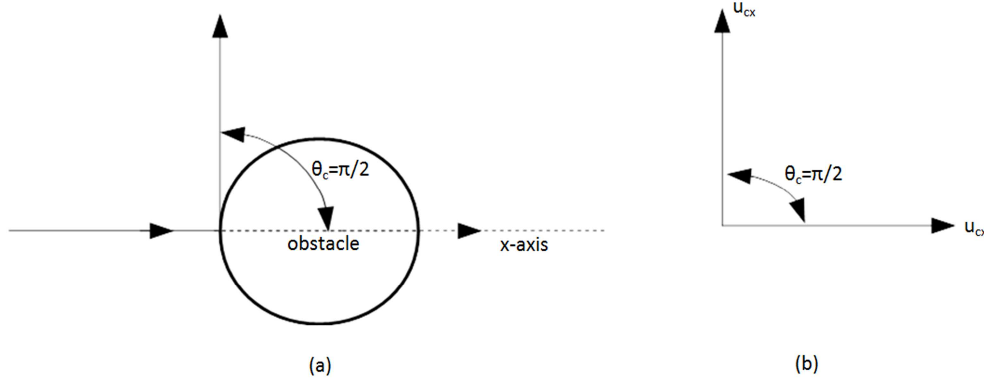

Fig. 1. Isotropic scattering of a charge-carrier by an obstacle in the semiconductor.

With reference to fig.1 (a), $\theta_c = \pi/2$ is the angle of scattering of the charge carrier on its collision with the atom. Fig. 1(b) shows that as a result of the scattering process the velocity of the charge carrier parallel to x axis remains unaltered retaining its original value of as given by eqn. (41). This situation illustrated with reference to Fig. 1(a and b) is referred to as the case of isotropic scattering of the charge-carrier at its collision with an atom characterized by $\theta_c = \pi/2$.

Coming to eqn. (41), the ratio $\frac{U_{cx}}{E_x}$, which is mobility k_{co} of the charge-carrier, is given by,

$$k_{co} = \frac{U_{cx}}{E_x} = \frac{e}{m_c} \frac{1}{\nu_{co}}. \tag{42}$$

For the electron and hole, eqn. (42) gives,

$$k_{eo} = \frac{U_{ex}}{E_x} = \frac{e}{m_e} \cdot \frac{1}{\nu_{eo}}, \tag{43}$$

and

$$k_{po} = \frac{U_{px}}{E_x} = \frac{e}{m_p} \cdot \frac{1}{\nu_{po}}. \tag{44}$$

The average drift velocities of the charge-carriers viz., U_{ex} and U_{px} for the electron and the hole respectively as determined by eqns. (43) and (44) in the presence of electric field E_x in the semiconductor parallel to x-axis, are oppositely directed because the sign of charge of an electron is opposite to that of a hole.

Further substituting the value of ν_{co} from eqn. (23) in eqn. (42), eqn. (42) gives,

$$k_{co} = \left(\frac{e}{m_c}\right)\left[N^2 \left(\frac{3h}{2\pi}\right)^2 \left(\frac{1}{\rho \theta_D^2}\right)\left(\frac{8}{\pi k m_c}\right)^{1/2}\right]^{-1} T^{-3/2}, \tag{45}$$

and for the electron and hole, eqn. (45) gives,

$$k_{eo} = \left(\frac{e}{m_e}\right)\left[N^2\left(\frac{3h}{2\pi}\right)^2\left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_e}\right)^{1/2}\right]^{-1} T^{-3/2}, \quad (46)$$

and

$$k_{po} = \left(\frac{e}{m_p}\right)\left[N^2\left(\frac{3h}{2\pi}\right)^2\left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_p}\right)^{1/2}\right]^{-1} T^{-3/2} \quad (47)$$

when on an average, $m_e = m_p$ = m (the free electron mass) as assumed in this analysis, then eqns. (45) to (47) give that $k_{co} = k_{eo} = k_{po}$.

The differences in the values of electron mobility k_{eo} (or k_{co}) and hole mobility k_{po} (or k_{co}) when temperature T of the semiconductor (that is Ge or Si) is 300 ^{0}K as obtained by the present analysis, and those tabulated in a physical table (Croissette 1974, table 1-2, pp.28) [16] for electron mobility k_{et} and hole mobility k_{pt} at 300 ^{0}K, are attributed to anisotropic scattering of the charge-carriers at their collision with atoms in the semiconductor and are discussed in the next section of this paper here.

5. Numerical Analysis

If N_a is Avogadro's number, ρ and M_A are density and atomic weight of the semiconductor respectively, then

$$N = N_a\rho/M_A \quad (48)$$

where N is the density of atoms in the semiconductor.

Table 1 gives physical constants of Ge and Si viz., atomic weight M_A and density ρ from Croissetee (1974, table 1-2, pp.28) [16] and characteristic (Debye) temperature θ_D from Kittel (1960, table 6.3, pp.132) [17]. Calculated values of N are indicated in the last column of Table 1.

Table 1. Room Temperature (~300 ^{0}K) Values of Physical Constants of Ge and Si.

Seimconductor	M_A	ρ	θ_D	N
	(kgm-at)	(kg/m^3)	(^{0}K)	(m^{-3})
Ge	72.6	5.32 x10^3	366	4.413(5) x 10^{28}
Si	28.1	2.40 x 10^3	658	5.144(2) x 10^{28}

Table 1 gives room temperature (~300 ^{0}K) average values for various parameters in the case of intrinsic Ge and Si. In the present analysis, when temperature T of the semiconductor is near room temperature, then variations of various parameters given in Table 1 with respect to temperature of the semiconductor are neglected and all these parameters are treated as constants.

Knowing N, ρ and θ_D, value of effective collision cross-section is calculated using eqn. (22) and (17). Value of the average thermal velocity of the charge-carrier (electron or hole) i.e. $v_c = v_e = v_p$, when the mass of an electron m_e = the mass of a hole m_p = the mass of a free electron m, is calculated from eqn. (18). Knowing q_0 and v_c, the value of charge-carrier (electron or hole) collision frequency i.e. $\nu_{co} = \nu_{eo} = \nu_{po}$, when $m_e = m_p$ = m, is obtained using eqn. (23). From the values of v_c and ν_{co}, the value of mean free path of the charge-carrier (electron or hole) i.e. $\lambda_{co} = \lambda_{eo} = \lambda_{po}$ when $m_e = m_p$ = m, is obtained using eqn. (40). Knowing ν_{co}, the value of mobility of the charge-carrier (electron or hole) i.e. $k_{co} = k_{eo} = k_{po}$, when $m_e = m_p$ = m, is calculated from eqn. (45). These calculated values of q_0, v_c ($= v_e = v_p$), ν_{co}($= \nu_{eo} = \nu_{po}$), λ_{co} ($= \lambda_{eo} = \lambda_{po}$) and k_{co}($= k_{eo} = k_{po}$) are shown in Table 2 for Ge and Si where temperature T of the semiconductor is 300 ^{0}K.

Table 2. Values of q_0, v_c, ν_{co}, λ_{co} and k_{co} for Ge and Si at T=300 ^{0}K.

Semi-	$q_0 = \langle \bar{R}_{oa}^2 \rangle$	v_c	ν_{co}	$\lambda_{co} = \lambda_{eo} = \lambda_{po}$	$k_{co} = k_{eo} = k_{po}$
conductor	x10^{22}	x 10^{-5}	x10^{-11}	x 10^7	x 10^1
	(m^2)	(m/sc)	(sc^{-1})	(m)	(m^2/V/sc)
Ge	1.346(2)	1.076(1)	6.393(6)	1.683(1)	2.751(3)
Si	1.076(1)	1.076(1)	5.956(9)	1.806(5)	2.953(0)

Table 3. Values of k_{eo}, k_{po}, k_{et} and k_{pt} for Ge and Si at T = 300 ^{0}K.

Semiconductor	k_{eo}	k_{po}	k_{et}	k_{pt}
	(m^2/V/sc)	(m^2/V/sc)	(m^2/V/sc)	(m^2/V/sc)
Ge	2.751(3) x 10^{-1}	2.751(3) x 10^{-1}	3.9 x 10^{-1}	1.9 x 10^{-1}
Si	2.953(0) x 10^{-1}	2.953(0) x 10^{-1}	1.5 x 10^{-1}	0.5 x 10^{-1}

Table 3 gives values of k_{eo} (electron mobility) and k_{po} (hole mobility) of the present analysis together with values of k_{et} (electron mobility) and k_{pt} (hole mobility) as given in a physical table (Croissette, 1974, table 1-2, pp.28) [16] for Ge and Si, when temperature T of the semiconductor is 300^{0}K. Here values of k_{et} and k_{et} are of very pure semiconducting material.

The discrepancy between the two values of motilities viz., k_{eo},

k_{po} and k_{et}, k_{pt} in Table 3 is explained as follows:

In the present analysis it is assumed that the average velocity U_{cx} of the charge-carrier given by eqn. (41), viz.,

$$U_{cx} = \frac{eE_x}{m_c \nu_{co}},$$

remains the same before and after scattering of the charge-carrier on its collisions with an obstacle in the form of an atom of the semiconductor.

The case of isotropic scattering having scattering angle $\theta_c = \pi/2$, is already illustrated in Fig. 1 (a and b).

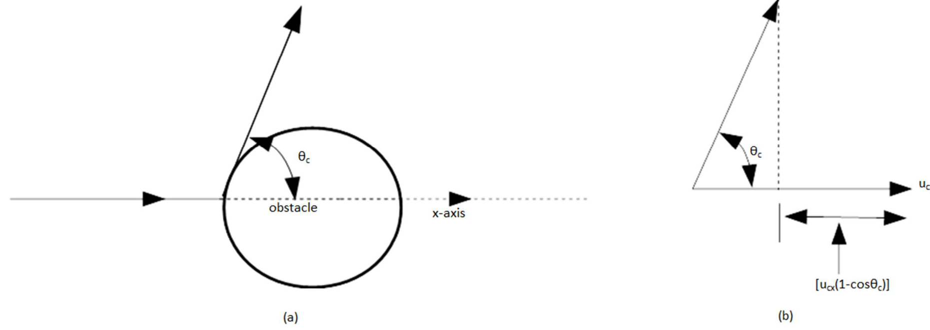

Fig. 2. Anisotropic scattering of a charge-carrier by an obstacle in the semiconductor.

Now another case of scattering, when $\theta_c \neq \pi/2$ is illustrated with reference to Fig. 2 (a and b), and is regarded as the case of anisotropic scattering. Here θ_c is an arbitrary angle chosen. With reference to Fig. 2(a), let $\theta_c \neq \pi/2$ be the angle of scattering of the change-carrier on its collision with the obstacle in the form of the atom. Fig. 2(b) shows that as a result of the scattering process, the velocity U_{cx} as given by eqn. (41) is changed to U_{cx} (1 - cos θ_c). When U_{cx} changes to U_{cx} (1-cos θ_c), then let ν_{cT} be defined as modified collision frequency in these circumstances. Whence eq. (41) is modified to

$$U_{cx}(1 - \cos\theta_c) = \frac{eE_x}{m_c \nu_{cT}}, \quad (49)$$

which gives,

$$U_{cx} = \frac{eE_x}{m_c[\nu_{cT}(1 - \cos\theta_c)]}. \quad (50)$$

If with reference to eqn. (50), U_{cx} is to remain same before and after scattering in comparison with eqn. (41), then comparison of the two expressions gives,

$$\frac{\nu_{co}}{\nu_{ct}} = 1 - \cos\theta_c. \quad (51)$$

This model assumes that θ_c is a constant for the given semiconductor of the present analysis under consideration.

If mobility for a charge-carrier is inversely proportional to collision frequency, then it follows that

$$\frac{k_{ct}}{k_{co}} = \frac{\nu_{co}}{\nu_{cT}} = 1 - \cos\theta_c, \quad (52)$$

using eqn. (51). Here k_{ct} is charge-carrier's mobility when collision frequency is ν_{cT}. And k_{co} is charge-carrier's mobility when collision frequency is ν_{co}.

Further eqn. (52) gives that,

$$\theta_c = \cos^{-1}\left(1 - \frac{k_{ct}}{k_{co}}\right). \quad (53)$$

In the present analysis, θ_c is an arbitrary angle of scattering chosen. The present analysis holds good when θ_c lies in general between 0 to π. When $0 < \theta_c < \pi/2$, then this is considered as a case of forward anisotropic scattering. Whereas when $\pi/2 < \theta_c < \pi$, then this is regarded as a case of reverse anisotropic scattering. And $\theta_c = \pi/2$ gives the case of isotropic scattering.

Coming to eqn. (53), the scattering angle θ_e for an electron and the scattering angle θ_P for a hole, is given by,

$$\theta_e = \cos^{-1}\left(1 - \frac{k_{et}}{k_{eo}}\right), \quad (54)$$

and

$$\theta_p = \cos^{-1}\left(1 - \frac{k_{pt}}{k_{po}}\right). \quad (55)$$

The value of θ_e and θ_p calculated from eqns. (54) and (55) for Ge and Si using Table 3 are given in Table 4.

Table 4. Values of θ_e, θ_p, ν_{et} and ν_{pt} for Ge and Si at T = 300 ^{0}K.

Semiconductor	θ_e	θ_p	ν_{et}	ν_{pt}
	(degrees)	(degrees)	(sc^{-1})	(sc^{-1})
Ge	114.6(8)	71.97(6)	4.510(3) x10^{11}	9.258(2) x 10^{11}
Si	60.52(5)	33.83(1)	1.172(7) x 10^{12}	3.518(2) x 10^{12}

Table 4 shows that, the electrons in Ge undergo reverse anisotropic scattering whereas the holes in it undergo forward anisotropic scattering (since $\theta_e > 90^0$ and $\theta_p < 90^0$ in the case of Ge). Whereas both electrons and the holes in Si undergo forward anisotropic scattering (since $\theta_e < 90^0$ and $\theta_p < 90^0$ in the case of Si).

Further from eqn. (51), the value of ν_{cT} is given by,

$$\nu_{ct} = \frac{\nu_{co}}{1-\cos\theta_c}. \tag{56}$$

For the electron and the hole, eqn. (56) gives,

$$\nu_{et} = \frac{\nu_{eo}}{1-\cos\theta_e}, \tag{57}$$

and

$$\nu_{pt} = \frac{\nu_{po}}{1-\cos\theta_p}. \tag{58}$$

Calculated values for ν_{et} and ν_{pt} using the values of $\nu_{co} = \nu_{eo} = \nu_{po}$ from Table 2 and the values of θ_e and θ_p as given in Table 4 are also given in Table 4 for Ge and Si.

Coming to eqn. (56) the modified expression for mobility k_{mco} of the charge-carrier considering anisotropic scattering at the collision, is given by replacing ν_{co} in eqn. (42) by ν_{ct} as given by eqn. (56), and k_{co} by k_{mco} in eqn. (42), where this model assumes that θ_c is a constant for the given semiconductor of the present analysis under consideration. Thus eq. (42) gives,

$$k_{mco} = \frac{e}{m_c}\frac{1}{\nu_{ct}} = \frac{e}{m_c}\frac{1-\cos\theta_c}{\nu_{co}}. \tag{59}$$

Using eqn. (23), eqn. (59) gives (Nandedkar 1983) [18] that,

$$k_{mco} = \frac{\left(\frac{e}{m_c}\right)(1-\cos\theta_c)}{\left[N^2\left(\frac{3h}{2\pi}\right)^2\left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_c}\right)^{1/2}\right]}T^{-3/2} \tag{60}$$

as an expression for the modified mobility of the charge-carrier considering the anisotropic scattering at the collision with respect to that given by eqn. (45) for the case of isotropic scattering. Eqn. (60) for the electron and hole gives that,

$$k_{meo} = \frac{\left(\frac{e}{m_e}\right)(1-\cos\theta_e)}{\left[N^2\left(\frac{3h}{2\pi}\right)^2\left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_e}\right)^{1/2}\right]}T^{-3/2} \tag{61}$$

and

$$k_{mpo} = \frac{\left(\frac{e}{m_p}\right)(1-\cos\theta_p)}{\left[N^2\left(\frac{3h}{2\pi}\right)^2\left(\frac{1}{\rho\theta_D^2}\right)\left(\frac{8}{\pi k m_p}\right)^{1/2}\right]}T^{-3/2} \tag{62}$$

where on an average, $m_c = m_e = m_p = m$ (the free electron mass) is the assumption in this analysis. Here k_{meo} or k_{mpo} does not depend on an average on the intrinsic charge carrier density. The intrinsic charge carrier density [of which order is known to be 10^{19} m^{-3} in Ge and 10^{16} m^{-3} in Si for the intrinsic semiconductor near room temperature ($\sim 300^0$K) - refer to Croissette (1974, table 1-2, pp. 28) [16], is large enough to use method of gas kinetics for various results analyzed herewith. Here the density of atoms in the intrinsic semiconductor (refer to Table 1) is extremely large as compared to the intrinsic charge-carrier density.

Here the average drift velocities of the charge-carriers as determined by eqns. (61) and (62) in the presence of applied d.c. electric field in the semiconductor are oppositely directed, since the sign of charge of an electron is opposite to that of a hole.

Equation (61) or (62) shows that the mobility of the charge-carrier viz., the electron or the hole due to the charge-carrier's collisions with atoms, varies as $T^{-3/2}$ where T is the temperature of the intrinsic semiconductor (viz. Ge or Si) under consideration. This analysis assumes, that θ_e or θ_p is a constant and does not depend on the temperature T of the non-polar semiconductor (viz., Ge or Si) under consideration and that the semiconductor is very pure. The values of θ_e and θ_p are given in Table 4 for Ge and Si.

6. Conclusions

Equation (61) or (62) shows that the mobility of the charge-carrier viz., the electron or the hole due to the charge-carrier's collisions with atoms, varies as $T^{-3/2}$ where T is the temperature of the intrinsic semiconductor (viz. Ge or Si) under consideration. This analysis assumes, that θ_e or θ_p is a constant and does not depend on the temperature T of the non-polar semiconductor (viz., Ge or Si) under consideration and that the semiconductor is very pure. The values of θ_e and θ_p are given in Table 4 for Ge and Si.

Table 4 shows that, the electrons in germanium undergo reverse anisotropic scattering whereas the holes in it undergo forward anisotropic scattering because scattering angle for electrons i.e. θ_e = 114.6(8) degrees > 90 degrees, and scattering angle for holes i.e. θ_p = 71.97(6) degrees < 90 degrees in the case of Ge. Whereas both electrons and the holes in Si undergo forward anisotropic scattering because scattering angle for electrons i.e. θ_e = 60.525 degrees < 90 degrees, and scattering angle for holes i.e. θ_p = 33.83(1) degrees < 90 degrees in the case of Si.

The present analysis of this paper assumes that for the intrinsic semiconductor viz., Ge or Si under consideration, it is possible

to define the charge-carrier collision frequency by eqns. (23) and (40).

In the present analysis, the average drift velocity of the charge-carrier in the semiconductor is limited due to its finite value of mobility in presence of a d.c. electric field and so that d,c. energy and hence power associated with normal component of the average drift velocity (refer to Figs. 1 and 2) with respect to d.c. electric field parallel to x- axis is scattered over randomly at the collisions with the atoms, thereby increasing temperature of the semiconductor both by the electrons and by the holes, accounting for the heat losses. And the temperature T in the present analysis denotes an average temperature attained by the semiconductor in presence of these heat losses relative to surroundings in equilibrium conditions.

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