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Thermal Radiation and Blackbody Radiation Drag of a Large-Sized Perfectly Black Particle Moving with Relativistic Velocity

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Abstract

Using the fluctuation electromagnetic theory and Lorentz transformations, we have developed a self-consistent description of radiation heat transfer and dynamics of large perfectly black spherical bodies when moving in photonic gas with relativistic velocity. Radiation force acting on a body in the reference frame of the background radiation is found along with the rate of heating (cooling) and the power of emitted and absorbed electromagnetic radiation. Apart from the general physical meaning, the results can be used in studying evolution of cosmic bodies and their thermal radiation.

Keywords

Thermal Radiation Drag, Thermal Radiation, Perfectly Black Bodies

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1. Introduction

The problem of blackbody friction and calculating the drag force acting on a charged particle moving through a thermal bath has attracted attention since pioneering work by Einstein and Hopf [1]. Much later, the problem of blackbody friction in the case of small neutral particles was considered in more detail within the framework of fluctuation electrodynamics [2–6]. In 1968, several authors have calculated the drag force acting on a perfectly black spherical body from the point of view of an observer in arbitrary uniform motion with respect to the c.m. of the radiation [7–10], i. e. in the frame of reference co-moving with particle. This is important in context of the interesting possibility of detecting the Earth motion with respect to the c. m. of the blackbody radiation. However, the corresponding expression for radiation force in the reference frame of blackbody radiation was not obtained, and the important issues concerning the dynamics and intensity of emission and absorption of thermal photons were not discussed.

At the same time, the issues of dynamics and the drag force calculation, thermal radiation and heating (cooling) of moving small and large-sized bodies are closely interrelated. In particular, a moving body has its own local temperature and emits thermal photons into vacuum, whereas the blackbody friction leads to the energy dissipation, heating and radiation of the body. Therefore, a comprehensive solution to this problem requires calculating the intensity of thermal radiation and radiation force both in the reference frame of radiation (RFR) and in the reference frame co-moving with the body. All these characteristics (in RFR) are crucially important for an external observer.

For a small dielectric particle moving in photonic gas (blackbody radiation), the power of emitted and absorbed radiation was calculated in [11, 12], using the formalism of fluctuation electrodynamics and the long-wavelength approximation $R << \lambda$ (R and λ are the particle radius and the characteristic wavelength of radiation). However, for a perfectly black particle in the short-wavelength approximation $R >> \lambda$ (geometrical optics approximation),

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a self-consistent solution to this problem is still absent. This work aims at filling this gap, and the results provide a useful link between the limits $R \ll \lambda$ and $R \gg \lambda$, forming a closed physical picture. Apart from the general physical meaning, the results can be used in studying evolution of cosmic bodies and their thermal radiation.

2. Theory

Figure 1 shows the problem statement and the frames of reference Σ, Σ' used in what follows. The reference frame Σ (RFR) is the frame of reference of blackbody radiation with local temperature T_2, and Σ' is the rest frame of a body with local temperature T_1. In RFR Σ, the body is moving in the direction of the positive x-axis with velocity $V = \beta c$ (c is the speed of light in vacuum).

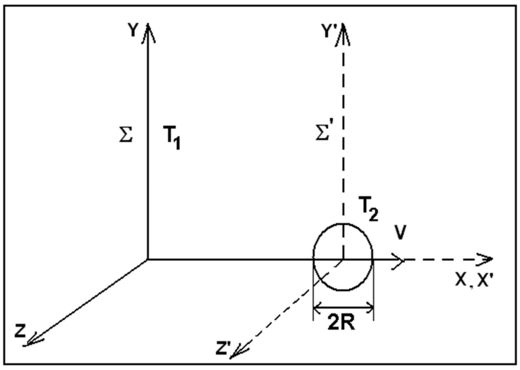

Figure 1. Geometrical configuration and reference frames corresponding to the vacuum background (Σ) and particle (Σ'). Σ' moves along $+x'$ with velocity V.

We adopt the geometrical optics approximation (short-wavelength approximation)

$$R \gg \max\left(\frac{2\pi\hbar c}{k_B T_1}, \frac{2\pi\hbar c}{k_B T_2}\right) \tag{1}$$

Then, according to [7], the dissipative force acting on a particle in Σ', is given by

$$F_x' = -\frac{4}{3}\frac{\beta\gamma^2}{c} a T_2^4 \tag{2}$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $a = 4\pi R^2 \sigma_B$, $\sigma_B = \frac{\pi^2}{60}\frac{k_B^4}{\hbar^3 c^2}$ is the Stefan-Boltzmann constant. Moreover, the energy density of an equilibrium electromagnetic field (photonic gas) is given by [7]

$$\varepsilon' = \frac{4}{c}\sigma_B T_2^4 \gamma^2 \left(1 + \frac{\beta^2}{3}\right) \tag{3}$$

The validity of (2), (3) is easily verified using the energy-momentum tensor of the equilibrium electromagnetic field (written in Σ) [13]

$$T_{\mu\nu} = (p + \varepsilon) u_\mu u_\nu - p g_{\mu\nu} \tag{4}$$

where $p = \varepsilon / 3, \varepsilon = \frac{4}{c}\sigma_B T_2^4$; $\mu, \nu = 0,1,2,3$. By transforming $T_{\mu\nu}$ into Σ', the quantities F_x' and ε' are expressed through the components of tensor $T'_{\mu\nu}$ [7].

The following analysis is based to great extent on the relationships [11, 12]

$$\dot{Q}' = \gamma^2 \dot{Q} \tag{5}$$

$$I \equiv I_1 - I_2 = -\left(\frac{dQ}{dt} + F_x V\right) \tag{6}$$

$$F_x' = F_x - \beta\gamma^2 \dot{Q} / c \tag{7}$$

where I_1 and I_2 denote the power of emission and absorption in Σ; $\dot{Q}$ and $\dot{Q}'$ are the rates of particle heat exchange in Σ and Σ'; F_x and F_x' are the forces acting on the particle in Σ and Σ', respectively. Originally, equations (5)—(7) were obtained using the long-wavelength approximation $R \ll \lambda$ which is opposite to (1). However, as shown in Appendix, these relations are also valid in the case $R \gg \lambda$. It is worth noting that the equations similar to (6), (7) were obtained earlier by Polevoi [14] in the problem of vacuum friction between two semiinfinite half-spaces in relative motion. In the general case, Eqs. (6), (7) appear due to the condition of quasistationarity $dW / dt = 0$, with W being the energy of the fluctuation- electromagnetic field (see [11, 12] for more details), and their validity is independent of the ratio between R and λ.

Further calculations are very simple. According to the Stefan-Boltzmann law, the power (intensity) of thermal radiation in Σ' is given by

$$I_1' = \sigma_B T_1^4 4\pi R^2 = a T_1^4 \tag{8}$$

The power of absorbed radiation in Σ' is obtained using the relation between I_2' and ε':

$$I_2' = \frac{c}{4}\varepsilon' 4\pi R^2 \tag{9}$$

Using (9) and (3) yields

$$I_2' = a T_2^4 \gamma^2 \left(1 + \frac{\beta^2}{3}\right) \tag{10}$$

On the other hand, the rate of particle heating (cooling) in Σ' takes the form

$$\frac{dQ'}{dt'} = Q' = I_2' - I_1' = aT_2^4\gamma^2\left(1+\frac{\beta^2}{3}\right) - aT_1^4 \tag{11}$$

Using (5) and (11) one obtains the equation of heating (cooling) in Σ

$$\frac{dQ}{dt} = \dot{Q} = aT_2^4\left(1+\frac{\beta^2}{3}\right) - \frac{1}{\gamma^2}aT_1^4 \tag{12}$$

Moreover, by inserting (2) and (12) into (7) we obtain the radiation force F_x

$$F_x = -\frac{\beta}{c}a\left(T_1^4 + \frac{1}{3}T_2^4\right) \tag{13}$$

From (13) it follows that, in contrast to F_x', radiation force F_x in Σ contains the contributions from absorbed background radiation (the second term) and that emitted by the particle (the first term). The same situation takes place for dielectric particles in the long-wavelength approximation [11, 12]. Finally, by inserting (12) and (13) into (6) one obtains

$$I = a(T_1^4 - T_2^4) \tag{14}$$

Surprisingly, Eq. (14) does not depend on the particle velocity and formally reduces to the result familiar in the static case. However, paradoxality of this fact vanishes if we take into account that the particle will reache a steady-state temperature as a result of heat exchange with background radiation (assuming that $T_2 = const$), defined by the condition (see the next Section)

$$\dot{Q}' = \dot{Q} = 0 \tag{15}$$

From (12) and (15) the steady-state temperature of a moving perfectly black body is given by

$$T_s = T_2\gamma^{1/2}\left(1+\beta^2/3\right)^{1/4} \tag{16}$$

Inserting (16) into (14) yields the stationary imbalance of radiation and absorption

$$I_s = aT_2^4\left[\gamma^2(1+\beta^2/3) - 1\right] \tag{17}$$

Both T_s and I_s depend on the particle velocity, as it could be expected. Formulas (12)–(14) and (16), (17) are the main results of this work. In the case when a large-sized particle is not perfectly black and is characterized by the absorptivity a_a ($0 < a_a < 1$), and emissivity a_r ($0 < a_r < 1$) these formulas can be trivially modified. In the case when the particle is characterized by the dielectic and magnetic properties, Eqs. (5)–(7) have to be used along with Eqs. (A7) and (A13).

3. Kinetics of Heating/Cooling and Dynamics of the Particle

It is interesting to compare the time-scale of particle deceleration and the time needed to reach a steady-state temperature. The dynamics equation in RFR has the form

$$\frac{d}{dt}\left(\frac{mV}{\sqrt{1-\beta^2}}\right) = F_x \tag{18}$$

Along with the temporal velocity dependence in the left-hand side of (17), we must take into account the change in the particle mass due to the radiation and absorption. With allowance for this fact Eq. (18) takes the form [12]

$$m\frac{dV}{dt} = \gamma^{-3}F_x' \tag{19}$$

Substituting (2) into (19) yields

$$\frac{d\beta}{\beta\sqrt{1-\beta^2}} = -\frac{4aT_2^4}{3mc^2}dt \tag{20}$$

After integrating (20) one obtains

$$\frac{\gamma(t)-1}{\gamma(t)+1} = \frac{\gamma(0)-1}{\gamma(0)+1}\exp\left(-\frac{8aT_2^4}{3mc^2}t\right) \tag{21}$$

where $\gamma(t) = \left(1 - V(t)^2/c^2\right)^{-1/2}$, $\gamma(0) = \left(1 - V(0)^2/c^2\right)^{-1/2}$, $V(0)$ and $V(t)$ are the velocities at the moments $t = 0$ and t. From (21), the characteristic deceleration time is given by

$$\tau_V = \frac{3mc^2}{8aT_2^4} = \frac{R\rho c^2}{8\sigma_B T_2^4} \tag{22}$$

where ρ is the particle density. For example, the time τ_V for an icy H_2O particle at $R = 20\,cm$, $\rho \approx 1\,g/cm^3$ and $T_2 = 100\,K$ is $\tau_V \approx 12\cdot 10^9$ years, i. e. close to the age of the Universe.

To analyze the kinetics of heating, we represent $\dot{Q}'$ in the form (C_s is the specific heat capacity)

$$\frac{dQ'}{dt'} = \frac{d}{dt'}(C_s m T_1) = \gamma C_s m\frac{dT_1}{dt} + \gamma C_s T_1\frac{dm}{dt} + \gamma m T_1\frac{dC_s}{dt} \tag{23}$$

With allowance for the identity $dm/dt = \gamma\dot{Q}/c^2$ from (5) and (23) one obtains

$$C_s m \frac{dT_1}{dt} = \gamma \dot{Q}(1 - C_s T_1 / c^2) - m T_1 \frac{dC_s}{dt} \qquad (24)$$

Since $C_s T_1 / c^2 << 1$ at any possible temperature of solids, the corresponding term in (24) can be omitted. Moreover, assuming $C_s = const$, substituting (12) into (24) and introducing the reduced variables of time $\tau = t / \tau_Q$ (with $\tau_Q = \frac{C_s \rho R}{3\sigma_B T_2^3}$), and temperature $x = T_1 / T_2$, Eq. (24) takes the form

$$\frac{dx}{d\tau} = \gamma(1 + \beta^2 / 3) - \frac{x^4}{\gamma} \qquad (25)$$

As follows from (25), the magnitude of x tends to the steady-state value $x_s = T_s / T_2 = \gamma^{1/2}\left(1 + \beta^2 / 3\right)^{1/4}$ irrespectively of the initial value (provided that the background temperature T_2 is constant). The characteristic time of this process is of order 1 and decreases with increasing γ . Therefore, the time scale of particle heating/cooling is determined by the value of parameter τ_Q . However, at $\gamma \sim 1$ the approximation $C_s = const$ becomes insufficiently correct, and the steady-state temperature should be determined from (24), depending on the behavior of the function $C_s(T_1)$.

Comparing τ_Q and τ_V yields $\tau_Q / \tau_V = \frac{8 C_s T_2}{3c^2} \approx 10^{-10} \div 10^{-14}$ at typical values $C_s = 10 \div 10^3\ J / K \cdot kg$ and $T_2 = 10 \div 10^3 K$. Therefore, the process of heating/cooling proceeds much faster than the process of deceleration.

4. Conclusions

A self-consistent description of radiation heat transfer and dynamics of large particles moving with relativistic velocity in photonic gas has been developed. Radiation force acting on a particle in the reference frame of the background radiation is found. Initially, the difference between emission and absorption intensities does not depend on the particle velocity being described by the Stefan-Boltzmann law. It is shown that the process of heating/cooling proceeds much faster than the process of deceleration and the particle acquires a steady-state temperature proportional to the temperature of the backround radiation. In this case, the intensity of thermal radiation is proportional to the gamma-factor squared being considerably higher than the intensity of absorbed radiation. Further on, the particle is gradually slowing down and its kinetic energy is fully converted into radiation. With slight modification, these results can be applied to not perfectly black bodies. For the bodies with well defined dielectric and magnetic properties the results can be obtained numerically.

Apart from the general theoretical meaning, the results can be used in studying evolution of cosmic bodies and their thermal radiation. Rather typical velocities of cosmic bodies may reach 10^3–10^4 km/s. Though in the state of thermal equilibrium of the system consisting of the solid particles and photons the difference between emission and absorption decreases, the observation of this difference may be an indication of internal dynamics in protostellar clouds to the outside observer.

Appendix

First we recall the known relativistic transformations of the electric density current $\boldsymbol{j}$, the electric and magnetic fields $\boldsymbol{E}$, $\boldsymbol{B}$, polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{M}$ corresponding to the Lorentz transformations from Σ' to Σ :

$$j'_x = \gamma(j_x - V\rho)\,;\ j'_y = j_y\,;\ j'_z = j_z \qquad (A1)$$

$$E'_x = E_x\,;\ E'_y = \gamma(E_y - \beta B_z)\,;\ E'_z = \gamma(E_z + \beta B_y) \qquad (A2)$$

$$B'_x = B_x\,;\ B'_y = \gamma(B_y + \beta E_z)\,;\ B'_z = \gamma(B_z - \beta E_y) \qquad (A3)$$

$$P'_x = P_x\,;\ P'_y = \gamma(P_y + \beta M_z)\,;\ P'_z = \gamma(P_z - \beta M_y) \qquad (A4)$$

$$M'_x = M_x\,;\ M'_y = \gamma(M_y - \beta P_z)\,;\ M'_z = \gamma(M_z + \beta P_y) \qquad (A5)$$

Consider the Joule dissipation integral in Σ'

$$\int \langle \mathbf{j}'\mathbf{E}' \rangle d^3r' = \int \langle j'_x E'_x + j'_y E'_y + j'_z E'_z \rangle d^3r' , \qquad (A6)$$

where the angular brackets $\langle ... \rangle$ imply the total quantum-statistical averaging. In (A6), the integration is performed over the volume of the particle and we extend it to the whole space. This is implicit in what follows for all integrals. Substituting (A1), (A2) into (A6) yields [15]

$$\int \langle \mathbf{j}\mathbf{E} \rangle d^3r = F_x V + \gamma^{-2} \int \langle \mathbf{j}'\mathbf{E}' \rangle d^3r' , \qquad (A7)$$

where F_x is the x-component of the Lorentz force

$$\mathbf{F} = \int \langle \rho\,\mathbf{E} \rangle d^3r + \frac{1}{c} \int \langle \mathbf{j} \times \mathbf{B} \rangle d^3r \qquad (A8)$$

Writing the density current in Σ'

$$\mathbf{j}' = \frac{\partial\,\mathbf{P}'}{\partial\,t'} + c\,\mathrm{rot}\,\mathbf{M}' \qquad (A9)$$

and substituting (A9) into (A6) we get

$$\begin{aligned}\int\langle \mathbf{j}'\mathbf{E}'\rangle d^3r' &= \int\left\langle\left(\frac{\partial \mathbf{P}'}{\partial t'}+c\,\mathrm{rot}\,\mathbf{M}'\right)\mathbf{E}'\right\rangle d^3r' = \\ &= \int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'+c\left(\mathbf{E}'\mathrm{rot}\,\mathbf{M}'-\mathbf{M}'\mathrm{rot}\,\mathbf{E}'+\mathbf{M}'\mathrm{rot}\,\mathbf{E}'\right)\right\rangle d^3r' = \\ &= \int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'+c\,\mathrm{div}\left(\mathbf{M}'\times\mathbf{E}'\right)+c\,\mathbf{M}'\mathrm{rot}\,\mathbf{E}'\right\rangle d^3r' = \\ &= c\oint\langle\mathbf{M}'\times\mathbf{E}'\rangle d\,\mathbf{S}'+\int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'-\mathbf{M}'\frac{\partial \mathbf{B}'}{\partial t'}\right\rangle d^3r' = \\ &= c\oint\langle\mathbf{M}'\times\mathbf{E}'\rangle d\,\mathbf{S}'+\int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'+\frac{\partial \mathbf{M}'}{\partial t'}\mathbf{B}'\right\rangle d^3r'-\int\left\langle\frac{\partial}{\partial t'}(\mathbf{M}'\mathbf{B}')\right\rangle d^3r'\end{aligned} \quad \text{(A10)}$$

In the last line of (A10), the integral over an infinitely remote surface equals zero, while the third term is zero due to stationarity of electromagnetic fluctuations. Then it follows from (A10)

$$\dot{Q}' = \int\langle\mathbf{j}'\mathbf{E}'\rangle d^3r' = \int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'+\frac{\partial \mathbf{M}'}{\partial t'}\mathbf{B}'\right\rangle d^3r' \quad \text{(A11)}$$

Substituting (A2)–(A5) into (A11) and making use simple transformations with allowance for

$dt' = \gamma^{-1}dt$, $d^3r = \gamma^{-1}d^3r'$ one obtains

$$\int\left\langle\frac{\partial \mathbf{P}'}{\partial t'}\mathbf{E}'+\frac{\partial \mathbf{M}'}{\partial t'}\mathbf{B}'\right\rangle d^3r' = \gamma^2\int\left\langle\frac{\partial \mathbf{P}}{\partial t}\mathbf{E}+\frac{\partial \mathbf{M}}{\partial t}\mathbf{B}\right\rangle d^3r \quad \text{(A12)}$$

From (A12) we can see that the quantity

$$\dot{Q} = \int\left\langle\dot{\mathbf{P}}\,\mathbf{E}+\dot{\mathbf{M}}\,\mathbf{B}\right\rangle d^3r$$

is transformed from Σ to Σ' according to Eq. (5), namely:

$$\dot{Q}' = \gamma^2\dot{Q} \quad \text{(A13)}$$

Since we did not use any restrictions on the value of the body radius R, Eq. (A13) is valid irrespectively of the ratio between the radius and characteristic wavelength λ of radiation. As already was said, Eqs. (6), (7) are the consequence of the quasistationarity of the fluctuation-electromagnetic field, and therefore, along with Eq. (7), they have the general character.

It is also worth noting that using (A13), Eq. (A7) takes the form

$$\int\langle\mathbf{j}\mathbf{E}\rangle d^3r = \left(\int\langle\rho\,\mathbf{E}\rangle d^3r+\frac{1}{c}\int\langle\mathbf{j}\times\mathbf{B}\rangle d^3r\right)\cdot\mathbf{V}+\int\left\langle\dot{\mathbf{P}}\,\mathbf{E}+\dot{\mathbf{M}}\,\mathbf{B}\right\rangle d^3r \quad \text{(A14)}$$

Eq. (A14) generalizes the analogous equation for small polarizable particle with fluctuating dipole moments **d** and **m** [15]:

$$\int_{(V)}\langle\mathbf{j}\mathbf{E}\rangle d^3r = \left(\left\langle\nabla\left(\mathbf{dE}+\mathbf{mB}\right)\right\rangle\right)\cdot\mathbf{V}+\left\langle\dot{\mathbf{d}}\mathbf{E}+\dot{\mathbf{m}}\mathbf{B}\right\rangle \equiv F_xV+dQ/dt \quad \text{(A15)}$$

In contrast to (A14), Eq. (A15) is valid under the condition $R << \lambda$.

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