Physics Journal
Vol. 2, No. 1, 2016, pp. 6-10
http://www.aiscience.org/journal/pj

Singular Bound States and Cold Nuclear Fusion

V. K. Ignatovich*

Frank Laboratory of Neutron Physics of the Joint Institute for Nuclear Research, Dubna, Russia

Abstract

It is a commonplace, which can be found in all textbooks on quantum mechanics, to believe that bound states always have discrete spectra. Here, contrary to the belief, the examples of bound states with continuous spectra are demonstrated. They contain singular wave functions. In particular, it is shown, that the hydrogen-like atom can contain the continuous spectrum of singular bound states, when it has zero angular momentum. Relation of these states to possible cold fusion nuclear reactions and to light of stars is discussed.

Keywords

Quantum Mechanics, Bound States, Singular Wave Function, Hydrogen-Like States, Cold Nuclear Fusion

Received: September 9, 2015 / Accepted: November 8, 2015 / Published online: December 6, 2015

1. Introduction

This paper is devoted to consideration of a possibility to explain the cold nuclear fusion (CNF) reactions, which are so much discussed in literature and in internet [1–9]. It is usually claimed [4] that explanation of CNF requires some new physics. However, in opinion of the present author, the old physics is also appropriate for that. It is only necessary to take into account some features, which are usually missed in common approach. The idea is to use singular states in hydrogen-like atoms. In these states the electron can screen the nuclear charge at quite small distances, which, in the case of the proton, creates some neutral particle (like in [6]) able to penetrate another nucleus without overcoming the huge Coulomb barrier. The first time this aspect of singular states in hydrogen-like atoms was considered in [10]. There it was supposed that the singular states can appear only in confined atoms, i.e. atoms inside some matter. The free atom was supposed to be unable to have singular states because the wave function of such states is not only singular near the origin, but also diverges at infinity. Therefore in [11] a possibility of admixture of a singular state to the normal state of a confined hydrogen atom [12] was considered. Further investigations, however, had shown that in the case, when the electron in atom has energy outside of the common discrete spectrum, the singular states with wave functions defined with the help of known hypergeometric functions [13] demonstrate numerically monotonously decreasing logarithmic derivative, which led to a suspicion [14] that these wave functions exponentially decrease at infinity. If so, they represent a continuous spectrum of singular bound states, which is very uncommon for the conventional theory. It was difficult to prove the validity of such a suspicion with the help of functions represented by infinite power series. It was very desirable to represent them in closed integral form, that could be numerically integrated, and their asymptotic found with the help of the classical theory of complex variables [15]. It is just such approach, which is used here.

In the next section a derivation of the radial Schrödinger equation, which will be solved in integral form is reminded, then its solution is derived with the help of Laplace formalism [16]. This solution helps to prove that outside the usual discrete spectrum of hydrogen-like atoms there is a continuous spectrum of singular bound states, while the states regular at the origin are not permissible because they exponentially diverge at infinity and therefore are not normalizable. In section 7 the energy production in a sample with high concentration of hydrogen or deuterium at high

* Corresponding author
E-mail address: v.ignatovi@gmail.com

temperature is estimated. In conclusion it is pointed out that the discussed mechanism can explain how the stars are lighted at high density.

2. General Solution of the Schrödinger Equation in the Coulomb Field

The Schrödinger equation for a hydrogen-like atom looks

$$\left(\frac{\hbar^2}{2m}\Delta+\frac{Ze^2}{r}-E\right)\Psi(\mathbf{r})=0\,, \qquad (1)$$

Where $E>0$, i.e. $-E$ is the bound state energy.

After substitution of $\Psi(r)=\Psi_L(r)Y_L^M(\theta,\varphi)$, which (1) is transformed to

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(L+1)}{r^2}+\frac{2m}{\hbar^2}\left[\frac{Ze^2}{r}-E\right]\right)\Psi_L(r)=0\,, \qquad (2)$$

and after substitution

$$\Psi_L(r)=u(r)/r \qquad (3)$$

eq. (2) for L=0 is transformed to

$$\left(\frac{d^2}{dr^2}+\frac{2m}{\hbar^2}\left[\frac{Ze^2}{r}-E\right]\right)u(r)=0\,. \qquad (4)$$

The radial Schrödinger equation for $u(r)$ can be rewritten in dimensionless units: $r=[r]Z/a_B$, $E=[E]Z^2/E_H$, where square brackets denote usual dimensional values, $a_B=\bar{h}^2/me^2$ =0,525°A is the Bohr radius, and $E^H=e^2/2aB$=13.6eV is the Hartree energy. In dimensionless units the equation is:

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}-E\right)u(r)=0\,. \qquad (5)$$

This Eq. has two solutions $u_{1,2}$, which satisfy a constant Wronskian

$$u'_1u_2-u_1u'_2=1\,. \qquad (6)$$

A solution of (5) can be found by the Laplace method [16]:

$$u(r)=\oint_C dte^{rt}Z(t)\,, \qquad (7)$$

where C is some contour around singular points of the integrand. Substitution into (5) gives an equation for $Z(t)$

$$\begin{aligned}&\oint_C dt\left[r\left(t^2-E\right)+2\right]e^{rt}Z(t)=\left(t^2-E\right)e^{rt}Z(t)\Big|_C\\&+\oint_C e^{rt}dt\left[-\frac{d}{dt}\left(t^2-E\right)+2\right]Z(t)=0\end{aligned}\,, \qquad (8)$$

If the contour C is closed or such that the first term at the end points in the right hand side is zero, then (8) is equivalent to

$$\frac{d}{dt}\left(t^2-E\right)Z(t)=2Z(t)\,. \qquad (9)$$

Solution of this equation is

$$u(r)=\oint_C dte^{rt/\beta}\frac{(t-1)^{\beta-1}}{(t+1)^{\beta+1}}\,, \qquad (10)$$

where $\beta=1/\sqrt{E}$. If $\beta=n$ is an integer, there is only a single pole at $t=-1$ in the complex plane of t, and the integral around this pole gives the common hydrogen wave functions. For instance, at n=1 this wave function is

$$u_0(r)=re^{-r} \qquad (11)$$

So the function $\psi_r(r)=u_0(r)/r$ is regular at the origin and exponentially decreases at $r\to\infty$.

The second independent solution $u_1(r)$ in this case is defined through Wronskian [14]

$$u'_1u_0-u'_0u_1=-1\,. \qquad (12)$$

The result is the function $\psi_s(r)=u_1(r)/r$, which is singular at the origin and exponentially grows at infinity [11, 14].

However in the general case, because of the Rydberg corrections (see, for instance, [17]), the parameter β is not an integer one. The Rydberg corrections appear, for instance, in two electrons atoms. There calculation of spectrum of one electron should take into account screening of the nucleus by another one. This screening leads to deviation of electric field from the Coulomb one. But even in one electron atoms, hydrogen or deuterium, there is a deviation of electric field from the Coulomb one inside the nucleus. Indeed, in the quark model the proton consists of two “up” quarks with charge +2|e|/3 of electronic charge e, and one “down” quark with the charge −|e|/3. This “down” quark plays the same role in the nucleus as the additional electron in, say, Helium atom. It screens the positive charge of “up” quarks and can lead to Rydberg-like correction to the atomic spectrum.

If β is not an integer, then the function under the integral in (10) has two branch points at $t=\pm1$, and two independent solutions of the Eq. (5) can be found by choice of two different cuts through the branch points and two different contours in (10) around these cuts [15, 16]. For instance, two independent solutions can be found with contours shown in Fig-s 1 a) and b).

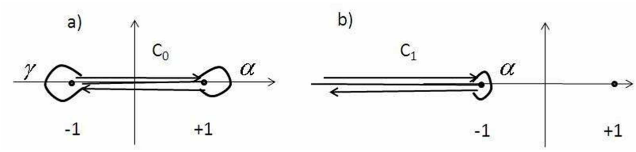

Fig. 1. Two different paths for integration in complex plane t in the integral (10) to find two independent solutions of (5). a) For the wave function regular at the origin. b) For the wave function singular at the origin.

3. The Contour C_0

In the case of Fig. 1a) the contour C_0 consists of two circles of radius α and γ and two straight lines above and below the cut (-1,+1). If radiuses α and γ are not small, then we can integrate by parts. Integration by parts of (10) along the path C_0 transforms (10) to

$$\begin{aligned} u_0(r) &= \oint_{C_0} dt e^{rt/\beta} \frac{(t-1)^{\beta-1}}{(t+1)^{\beta+1}} \\ &= -\frac{e^{-r/\beta}}{2\beta} \frac{(t-1)^{\beta}}{(t+1)^{\beta}}\Bigg|_{-1-\gamma_d}^{-1-\gamma_u} + \frac{r}{2\beta^3} \oint_{C_0} dt e^{rt/\beta} \frac{(t-1)^{\beta}}{(t+1)^{\beta}} \end{aligned} . \quad (13)$$

The first term in the right hand side is zero. Indeed, imagine that the integral starts from the point $t = -1 - \gamma_u$ just above the real axis. Transition to the point $t = -1 + \gamma$ brings the phase factor $\exp(-i\pi\beta)$ because of the denominator. Then follows integration to the point $t = 1 - \alpha$. Transition from the upper line to lower line along α circle adds the phase factor $\exp(2i\pi\beta)$) because of the numerator. After the motion under the cut from $t = 1 - \alpha$ toward $t = -1 + \gamma$ one needs to go to the point $t = -1 - \gamma_d$, just below the real axis. This transition creates additional phase factor $\exp(-i\pi\beta)$ because of the denominator. The total phase factor is zero. Therefore the first term in the right hand side is zero because it is the same at two end points. The left integral is well integrable for $\beta < 1$, and after radiuses α and γ are put to zero the integral becomes

$$u_0(r) = \left(1 - e^{2\pi i\beta}\right) \frac{r}{2\beta^3} \int_{-1}^{+1} dt e^{rt/\beta} \frac{(t-1)^{\beta}}{(t+1)^{\beta}} . \quad (14)$$

If $\beta > 1$, the (13) can be again integrated by parts [15] and reduced to a well numerically integrable function. But in all the cases the integration along the path C_0 gives a function, which is $\sim r$ at $r = 0$, therefore $u_0(r)/r$ is regular function at the origin. However this function exponentially grows at $r \to \infty$. Indeed, in the interval $0 < t < 1$ the integral at

$z \to \infty$ looks like

$$\begin{aligned} u_0(r) &\approx r \int_0^1 dt e^{rt/\beta} (1-t)^{\beta} \approx r \left(\frac{\beta}{r}\right)^{\beta+1} e^{r/\beta} \int_0^{\infty} du e^{-u} u^{\beta} \\ &= \Gamma(\beta+1) r \left(\frac{\beta}{r}\right)^{\beta+1} e^{r/\beta} \end{aligned} , \quad (15)$$

where irrelevant constant factor was omitted. Since the function u_0, which is ~0 in the origin, diverges at $r \to \infty$, one can expect that the linearly independent $u_1(r)$ should be constant at $r = 0$ and exponentially decaying at infinity. Otherwise Wronskian (6) cannot be a constant in the whole range of r.

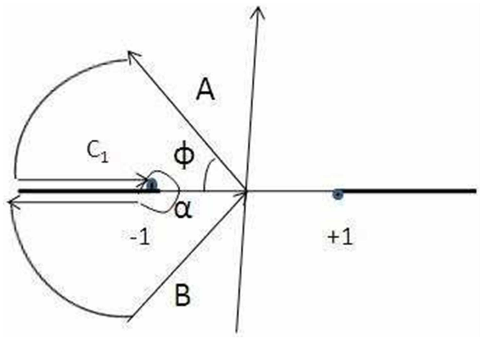

Fig. 2. An integration of (13) over the closed path A, B, C_1 and two arcs of infinite radius is zero, because there is no singularity of the integrand inside this contour. Integration over arcs is zero. Therefore the integral over C_1 is equal to sum of the integrals over two straight lines A and B. They are numerically well integrable.

4. The Contour C_1

In the case of the contour C_1 of Fig. 1b) to find the function $u_1(r)$ one can consider integration along a closed contour as, for instance, is shown in fig. 2. Since the integrand is analytical inside it, the integral over such a closed path is zero. It contains integrals over two arcs, which are zero for infinite radius. Therefore Integral over the cut C_1 is equal to sum of the integrals over two lines B+A, where they are convergent and well numerically integrable. If, for instance, the angle φ of the lines is put to be $\pi/4$ the sum of the integrals over B+A is

$$u_1(r) = c\int_0^{\infty} \mathrm{Im}\left[e^{-i\pi/4} \exp\left(-\frac{r}{\beta} s e^{-i\pi/4}\right) \frac{\left(s e^{-i\pi/4}+1\right)^{\beta-1}}{\left(s e^{-i\pi/4}-1\right)^{\beta+1}} \right] ds , \quad (16)$$

Since the integrand at large s behaves as $1/s^2$, the integral converges for any positive r, and at $r = 0$ the integral is a constant. The factor c normalizes it to unity. Since $u_1(0)$=const≠0, the function $u_1(r)/r$ is singular. However it is normalizable, therefore it can be considered as a singular bound state. It is interesting that the spectrum of such bound states is continuous.

5. Probability of Screening

The function (16) is numerically well integrable and after normalization to $u_1(0) = 1$ is shown in Fig. 3 for different β The contribution of the singular wave function to screening of the nuclear charge can be estimated by the ratio of integrals

$$Q = \int_0^{R_N} dr \left|u_1(r)\right|^2 \Big/ \int_0^{\infty} dr \left|u_1(r)\right|^2 , \qquad (17)$$

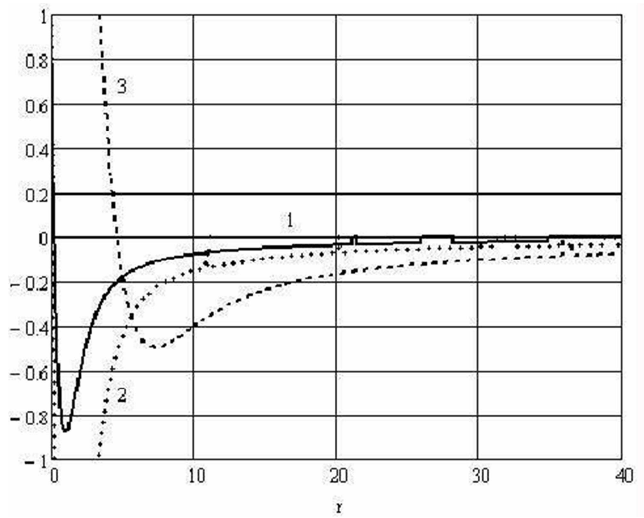

Fig. 3. The function $u_1(r)$ represented by (16) for β equal to: 1—0.6; 2—0.8; 3—1.2. The result does not depend on angle φ of the path A+B in Fig. 2.

where $R_N=10^{-4}a_B$. Numerical calculations with the above parameters gives $Q=6.613\times10^{-5}$ for $\beta=0.6$, $Q=7.401\times10^{-6}$ for $\beta=0.8$, and $Q=2.248\times10^{-6}$ for $\beta=1.2$. This parameter can be considered as a probability of complete screening of the nuclear charge or a probability of production of a compact hydrogen picoatom. Without the singular part the parameter Q is of the order 10^{-12}, i.e. 6-7 orders of magnitude smaller.

The above considerations show that a hydrogen-like atom can have a bound state described by the singular wave function even in vacuum. The singular state inside matter does not require in general introduction of a confining potential, contrary to suggestion in [1]. Only in the case of conventional quantum spectrum some confining potential, which can be readily provided by surrounding atoms, is required for appearance of a singular electron state [11, 14]. So one can conclude that the singular bound electron state is a very general phenomenon. This phenomenon can explain the cold nuclear fusion. Such a fusion is not observable in everyday practice at room temperature, because it has very small probability. Usually atoms in solids are hold apart at some distance of order 3 Angstroms, and the screened nucleus cannot approach another one. Thermal vibrations have the amplitude of the order of 0.1Å. For closer distance between nuclei the vibration amplitude should be an order of magnitude larger, which is possible only at high temperatures above the Debye one. But at such temperatures solids transform to liquids. Only at some special conditions like, possibly, those achieved in the Rossi's generator [9] the nuclear reaction can take place.

6. An Estimation of Heat Release

There are so many unknown parameters that one can make only rough estimation of possible heat production from CNF. Suppose there are 10^{23} of hydrogen or deuterium atoms per cm^3 that can penetrate to a neighbouring site because of self diffusion, and enter neighbouring nucleus. The distance l, which atom can pass during time t, is equal to $l=\sqrt{Dt}$, where D is the diffusion coefficient. Let's suppose that $l=a$, which is the distance between atoms. Then the time needed for an atom to enter the neighboring site, $l=a$, can be estimated as $t=a^2/D$. Probability of a nuclear reaction can be estimated as $N_0\sigma a$, where $N0 \sim 10^{23}$ is atomic density, and σ is the cross section, which can be estimated as10^{-24} cm^2. If probability of approaching the atom to be in picohydrogen state is $Q=10_{-6}$ and the released energy is of the order w=1MeV, then the total energy obtained from a single atom is $E_1 = N_0\sigma a Q w \sim 10^{-28}$ J. The total power released in $1cm^3$ by all the hydrogen and deuterium atoms is therefore

$$P=E_1N_0/t=E_1N_0D/a^2 \sim 10^2 \text{ Watt/cm}^3, \qquad (18)$$

If for D the value $D \sim 10^{-9} cm^2/s$ at the temperature 1000K is substituted [18–21].

7. About Some Objection Against Singular States

In the literature one can find some objection [22] against consideration of singular states with asymptotic $1/r$. The reason to reject such a solution is that substitution of it in the Schrödinger equation creates a δ term [22]:

$$\Delta\frac{1}{r} = -4\pi\delta(\mathbf{r}) , \qquad (19)$$

which, seems, has no physical meaning.

However there are two reasons against such an objection: such a term has a physical meaning.

1) In real physical systems the nucleus is not a point like one. If one takes into account form factor of the nucleus, the asymptotic $1/r$ modifies and δ-function does not appear.

2) It is well known that there exist such a phenomenon, as neutron-electron interaction [23], which can be described by

the Fermi pseudo potential [24]

$$4\pi b_{ne}\delta(\mathbf{r}), \quad (20)$$

where b_{ne} is the point like neutron-electron scattering amplitude defined by quark or meson structure of the neutron. The similar amplitude should exist also in proton-electron scattering. Therefore such a term can be included in the Schrödinger equation for atoms. In fact, the common regular solution of the Schrödinger equation for Hydrogen has a unit amplitude of the electron field on the nucleus, therefore there should be the scattered field with asymptotic 1/r. Moreover, every, even free, electron can scatter on the nucleus with the loss of its energy and become captured into a singular bound state.

8. Conclusion

It is found and mathematically proven that in a hydrogen-like atom electron can have a continuous spectrum of bound singular states. The singular electronic states can screen electronic charge of the nucleus, which helps to overcome the insurmountable Coulomb barrier and initiate fusion reactions. These reactions can explain how the stars light, when their density increases because of gravity forces.

Acknowledgement

I am very grateful to Peter D. Miller, Prof. of University of Michigan, Ann Arbor, MI, USA, for his highly professional consultations in used here mathematics.

References

[1] Cravens D. and R. Gimpel. 2013. Cold Fusion at NI Week 2013. Infinite energy, issue 111, Sept-Oct 2013 http://www.infinite-energy.com/images/pdfs/NIWeekCravens.pdf

[2] Levi G., E. Foschi, T. Hartman, Bo Höistad, R. Pettersson, L. Tegnér, H. Essén. 2013. Indication of anomalous heat energy production in a reactor device containing hydrogen loaded nickel powder. arXiv: 1305.3913 (2013).

[3] Inventor: PIANTELLI, Francesco I-53100 Siena (SI) (IT). European patent EP2368252B1. 2008. Method for producing energy and apparatus there for. Date of publication of application: 28.09.2011 Bulletin 2011/39. Priority: 24.11.2008 IT PI20080119.

[4] Storms E. 2007. The science of low energy nuclear reaction a Comprehensive Compilation of Evidence and Explanations about Cold Fusion. World Scientific (NEW JERSEY-LONDON-SINGAPORE-BEIJING-SHANGHAI-HONGKONG-TAIPEI-CHENNAI).

[5] Biberian Jean-Paul. 2007. Condensed matter nuclear science (coldfusion): an update. Int. J. Nuclear Energy Science and Technology, 3(1): 31-42.

[6] Ratis Yu. L. 2013. On a possibility of existence of a long living exoatom "neutroniy". Journal of Developing directions of science. 1(2): 27-44 (in Russian).

[7] Hora H., et al. 2004. Low Energy Nuclear Reactions resulting as picometer interactions with similarity to K shell electron capture. In: Eleventh International Conference on Condensed Matter Nuclear Science. Marseille, France.

[8] Parkhomov A.G. 2015. Investigation of the heat generator similar to Rossi reactor. Report on Experiments. International Journal of Unconventional Science, 7(3): 68-72. http://www.unconv-science.org/en/n7/parkhomov/.

[9] Levi Giuseppe, Evelyn Foschi, Bo Höistad, et al. 2014. Observation of abundant heat production from a reactor device and of isotopic changes in the fuel. http://www.sifferkoll.se/sifferkoll/wp-content/uploads/2014/10/LuganoReportSubmit.pdf

[10] Ignatovich V.K. 2014. A Missed Solution for an Atom: A Gate Toward Cold Nuclear Fusion. Infinite Energy Magazine, issue117: 33-36.

[11] Ignatovich V.K. 2015. On cold nuclear synthesis. Proc. Of XXIII International Seminar on Interaction of Neutrons with Nuclei, JINR, Dubna May 25-29.

[12] Ferreyra J.M. and C.R. Proetto. 2013. Exact solution of the compressed hydrogen atom. Am. J. Phys 81: 860.

[13] Abramowitz M. and I.A. Stegun. 1970. Handbook of Mathematical Functions. (Dover, New York) The (free) online version of this book is at http://dlmf.nist.gov.

[14] Ignatovich V.K. 2015. A Singular Solution for a Hydrogen Atom as a Way Toward Cold Nuclear Fusion. Open Science Journal of Modern Physics. 2(5): 83-88.

[15] Miller Peter D. 2006. Applied Asymptotic Analysis" Graduate Studies in Mathematics, vol. 75 AMS.

[16] Whittaker E.T. and G.N. Watson. 1927. A course of modern analysis. Cambridge University Press, Fourth edition. Chapter 10.

[17] Landau L.D., E.M. Lifshits. 1989. Quantum mechanics. Nonrelativistic theory. M.: Nauka, Ch.10, eq. (68.2)

[18] Feldman L.C. and J.W. Mayer. 1986. Fundamentals of Surface and Thin Film Analysis. North Holland- Elsevier, N.Y.

[19] Okkerse B. 1956. Self-Diffusion of Gold. Phys. Rev. 103: 1246.

[20] Lazarus D. and C.T.Tomizuka. 1956. Self-Diffusionin Silver-Zinc. Phys. Rev. 103: 1155 .

[21] Tomizuka C.T. and E.Sonder. 1956. Self-Diffusionin Silver. Phys. Rev.103: 1182.

[22] Khelashvii A. A. and T.P. Nadareishvii. 2011. What is the boundary condition for the radial wave function of the Schrodinger equation? Am. J. Phys. 79: 668.

[23] Foldy L.L. 1999. The Electron-Neutron Interaction. Phys. Rev. 87: 693.

[24] Ignatovich V.K., M. Utsuro, Ph. V. Ignatovich 1999. Neutron-electron interaction: Transmission and scattering amplitudes and interference corrections. Phys. Rev. C 59: 1136.