

Form Factor and Cross Section Calculation in Charged Kaon Electroproduction at $Q^2 = 1.9 \& 2.35 (\text{GeV}/c)^2$

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Abstract

In this article it is attempted to separate σ_{tot} into longitudinal, σ_L and transverse, σ_T cross-sections and to calculate charged Kaon form factor. The Jlab 2001 (E98-108) data has been used and Rosenbluth separation technique is applied to charged kaon electroproduction at $Q^2 = 1.9$ and $2.35 (\text{GeV}/c)^2$ in different energies. Data were taken at electron beam energies ranging from 3.40 to 5.75 (GeV). Charged kaon form factor is determined at $Q^2 = 1.9$ and $2.35 (\text{GeV}/c)^2$ by using Chew-low technique and Regge model. Influence of the kaon pole on the cross-sections is investigated by adopting an off-shell form factor in the Regge model, which better describes the observed energy dependence of σ_T and σ_L .

Keyword

Kaon Electroproduction, K^+ Meson, Rosenbluth Separation, Form Factor, Regge Model, Born Model

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1. Introduction

In electroproduction process, high energy electrons collide with stationary proton target and the corresponding cross-section is measured. Let us denote the incoming and outgoing electron's linear momentum vectors by K and K' and the transfer momentum four vector by q . In electroproduction $q^2 < 0$, therefore $Q^2 = -q^2$ is generally used [2, 4, 5].

The following relation describes the kaon electroproduction off a nucleon $N(p_i)$:

$$e(p_e) + N(p_i) \rightarrow e'(p'_e) + K(p_K) + N'(p_f) \quad (1-1)$$

In terms of mass number A for a hadron target we have:

$$e + A \rightarrow e' + K^+ + Y + (A - 1) \quad (1-2)$$

Where Y could be Λ , Σ^0 or Σ^- . Now the target is a proton, then $A = 1$ and we obtain the following relations [6, 7, 8, 9]:

$$\left. \begin{aligned} e + p &\rightarrow e' + K^+ + \Lambda \\ e + p &\rightarrow e' + K^+ + \Sigma^0 \\ e + n &\rightarrow e' + K^+ + \Sigma^- \end{aligned} \right\} \quad (1-3)$$

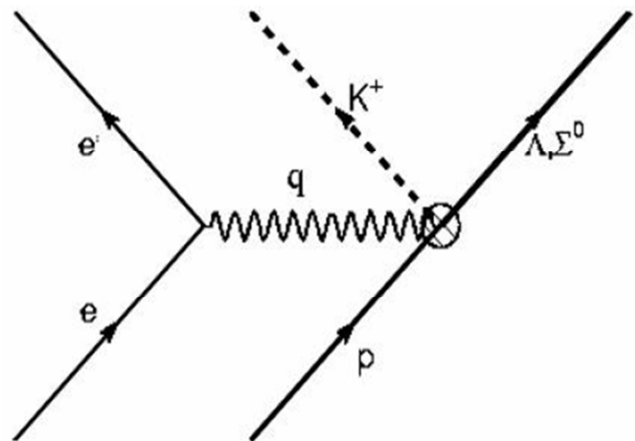


Fig. 1. Feynman diagram representation of kaon electroproduction.

In the one-photon exchange approximation

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When an electron collides with a stationary proton, electron is scattered and depending upon the energy, a hadron Λ or Σ in addition to the charged kaon is produced in the final channel. These typical scatterings in which the identity of the target nucleon changes are known as inelastic scattering given in equation (1-4):

$$e + p \rightarrow \acute{e} + K^+ + Y(\Lambda/\Sigma) \quad (1-4)$$

In fig.1, Feynman diagram is shown for reaction $H(e, \acute{e}, k^+)\Lambda(\Sigma)$. In past, many of such scatterings have been analyzed and published in the literature [1, 3].

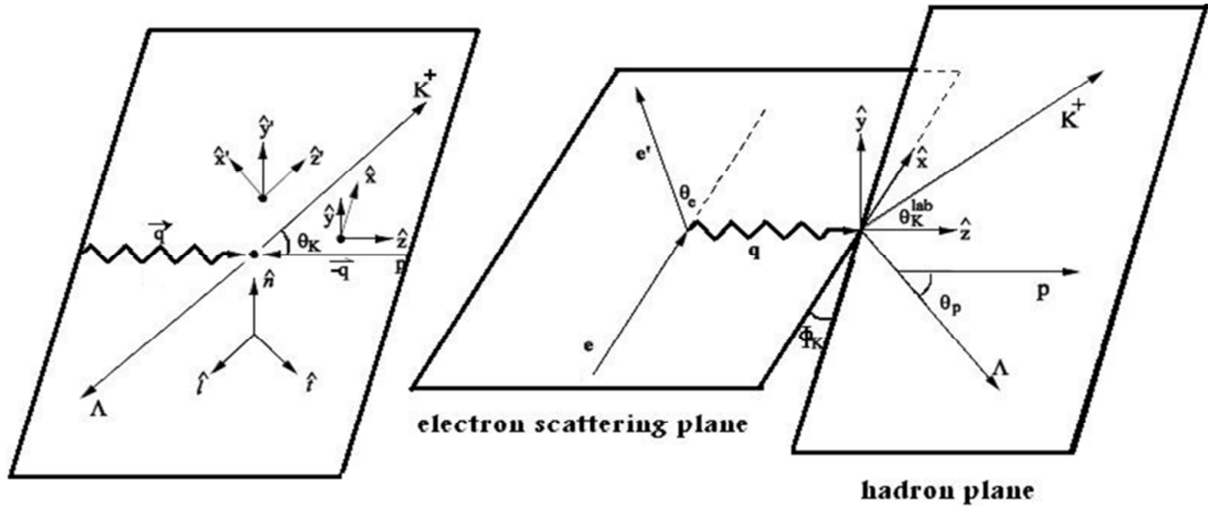


Fig. 2. Reaction planes in $H(e, \acute{e}, k^+)\Lambda$.

The four-momentum involved are:

- $e = (E, \vec{p}_e)$ for the incident electron.
- $\acute{e} = (\acute{E}, \vec{p}_{\acute{e}})$ for the scattered electron.
- $p = (M_p, \vec{0})$ for the target nucleon(proton).
- $k = (E_K, \vec{P}_K)$ for the produced charged kaon.
- $Y = (E_Y, \vec{P}_Y)$ for the unobserved residual system.

A few Lorentz invariants and other kinematic variable defined below as:

- $q = (v, \vec{q})$ is the four-momentum transfer.
- $v = E - \acute{E}$ is the energy of the virtual photon.
- $\vec{q} = (\vec{p}_e - \vec{p}_{\acute{e}})$ is the three-vector-momentum of the virtual photon.
- $q^2 = (e - \acute{e})^2 = -4E\acute{E}\sin^2(\frac{\theta_e}{2}) = -Q^2$ is the square of the four-momentum transfer carried by the virtual photon.
- $W^2 = (q + p)^2 = M_p^2 + 2M_p v - Q^2$ is the mass square of the system recoiling against the electron (i.e. the photon-proton system).
- $t = (k - q)^2 = q^2 + m_k^2 - 2qk$ is the Mandelstam

2. Charged Kaon Electroproduction Theory

In this analysis the following inelastic scattering is studied [1]:

$$e + p \rightarrow \acute{e} + K^+ + \Lambda \quad (2-1)$$

In order to study the reaction given in (2-1), the following quantities are defined and shown in fig. 2.

variable.

- $S = (q + p)^2 = m_p^2 + 2m_p E$ is the Mandelstam variable.
- $u = (k - p)^2$ is the Mandelstam variable.
- $x = \frac{Q^2}{2M_p v}$ is the Bjorken scaling variable(interpreted in the quark - Parton model of the target nucleon's momentum carried by the struck quark).

Available angles in reaction:

- θ_e Scattering angle.
- θ_k Angle between the scattering plane and the hadron plane.
- ϕ_K Angle between kaon Z (or q) axis.

In electroproduction experiments, it is popular to measure $\frac{d^5\sigma}{dE_x d\Omega_x}$, the fifth order differential cross-section. In order to apply partial wave analysis and multipole separation, the existing models, express the angular distribution of the kaon particle in terms of the final hadron center of mass coordinate system. For a Target and an unpolarized beam, this cross-section is expressed as the multiplication of the virtual photon flux Γ_v and the second order differential cross section

of the virtual photon $\frac{d^2\sigma}{d\Omega_k^*}$. Indeed Γ_0 represents the total number of the virtual photons exchanged between electron beam and the hadronic system in the energy range of $dE_{\hat{e}}$ and solid angle of $d\Omega_{\hat{e}}$ [2, 7, 9].

$$\frac{d\sigma^5}{dE_{\hat{e}}d\Omega_{\hat{e}}d\Omega_k} = \Gamma \frac{dcos\theta^*}{dcos\theta} \frac{d^2\sigma}{d\Omega_k} \quad (2-2)$$

$$\left. \begin{aligned} \Gamma &= \frac{\alpha}{2\pi^2} \frac{E_{\hat{e}}}{E_e} \frac{q_l}{Q^2} \frac{1}{1-\varepsilon} \\ q_l &= \frac{(W^2 - M_p^2)}{2M_p} \end{aligned} \right\} \quad (2-3)$$

In relation (2-3), α is the fine structure constant, $E_{\hat{e}}$ is the scattering electron energy and M_p stands for the proton target rest mass.

For the kaon electroproduction, the differential cross-section center of mass is given by:

$$\frac{d^2\sigma}{d\Omega_k^*} = \left(\frac{d^2\sigma_T}{d\Omega_k^*} \right) + \varepsilon \left(\frac{d^2\sigma_L}{d\Omega_k^*} \right) + \sqrt{2\varepsilon(1+\varepsilon)} \left(\frac{d^2\sigma_{LT}}{d\Omega_k^*} \right) \cos\phi_k + \varepsilon \left(\frac{d^2\sigma_{TT}}{d\Omega_k^*} \right) \cos 2\phi_k \quad (2-4)$$

Where (*) represent the C.M system and ε is the virtual photon polarization which is invariant in coordinate system changes and is given as:

$$\varepsilon = \frac{1}{1+2\left(\frac{|q|^2}{Q^2}\right)\tan^2\left(\frac{\theta}{2}\right)} \quad (2-5)$$

Now let us denote $\sigma_x = \frac{d^2\sigma_x}{d\Omega_k^*}$, then equation (2-4) is written in the following form [2, 9]:

$$\frac{d\sigma}{d\Omega_k^*} = \sigma_T + \varepsilon\sigma_L + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT} \cos\phi_k + \varepsilon\sigma_{TT} \cos 2\phi_k \quad (2-6)$$

Where:

1. σ_L : Longitudinal polarization cross-section for virtual photon.
2. σ_T : Transverse polarization cross-section for virtual photon.
3. σ_{LT} and σ_{TT} : Represent the crossed polarization cross-sections for the virtual photon.

σ_x depends upon W , Q^2 and t . If σ is measured at different ε values, while W , Q^2 and θ_k is kept constant, then equation (2-6) can be expressed as following:

$$\left. \begin{aligned} F(\phi) &= A + B\cos 2\phi + C\cos\phi \\ \text{Where:} \\ A &= \sigma_T + \varepsilon\sigma_L \\ B &= \varepsilon\sigma_{TT} \\ C &= \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT} \end{aligned} \right\} \quad (2-7)$$

In equation (2-6) if integration is performed over all

$\phi \in (0, 2\pi)$, then the crossed polarized cross-sections, σ_{LT} and σ_{TT} are vanished due to their dependency upon $\sin\phi_k$ and $\sin^2\phi_k$. Then in equation (2-7), terms B and C equal to zero and only term A remains. As a result, the total cross-section can be expressed in terms of only σ_L and σ_T namely:

$$\frac{d\sigma}{d\Omega_k^*} = \sigma_T + \varepsilon\sigma_L \quad (2-8)$$

Now, if the total cross-section is given in term of virtual photon polarization, then σ_L and σ_T are easily separated as the slope and vertical distance of the plot of $\frac{d\sigma}{d\Omega_k^*}$ versus ε namely, the Rosenbluth separation method [1, 2, 4]. Tables 1 and 2 contain our experimental data taken from [14] and in tables 3 and 4 all our findings are collected and also plotted in figures (3.1) to (3.5). In particular, as seen in Figs. 3.2 and 3.4, the variations of longitudinal and transverse cross sections and their ratio as a function of w , depend upon the value of square momentum transfer.

3. Calculation of Kaon Form Factor

Kaon electroproduction is an inelastic scattering process and its form factor is determined via different technique, namely, Isobaric model and Born model and Regge model. In this paper, the form factor is calculated by using Born and Regge models.

Born model is based upon a photon exchange approximation in which the photon emitted by the scattered electron, interacts with the final hadronic system. In a more precise language, the virtual photon couples with the charged kaon in (-) channel. The important point to be noted is that, only the longitudinally polarized virtual photons participate in the scattering. Therefore the longitudinal cross-section σ_L has the main contribution in the form factor. Particularly in the region of $W > 2 \text{ GeV}$ and small t values, σ_L is dominant in the differential cross-section and following formula is used to calculate the charged kaon form factor, $F_k(Q^2)$ [12]:

$$F_k^2(Q^2) = F_k^2(Q^2, t)|_{t \rightarrow M_k^2} = \frac{(t - M_k^2)^2 \sigma_L}{N(t)} \Big|_{t \rightarrow M_k^2} \quad (3-1)$$

Where:

$$N(t) = (-t)8cg_{KN\Lambda}^2 [k^2 \sin^2\theta_e \frac{1+\varepsilon\cos 2\phi}{2} + \varepsilon \frac{(qE_k - vk\cos\theta_e)^2}{Q^2}] \quad (3-2)$$

If:

$$N(t) = -2tQ^2g_{KN\Lambda}^2 a(t)$$

Then:

$$a(t)g_{KN\Lambda}^2(t)F_K^2(Q^2, t) = \frac{(t - M_K^2)^2 \sigma_L}{-2tQ^2}$$

In Chew-low approximation technique given in (3-3), the value of $g_{KN\Lambda}^2$ is constant, kaon mass is known ($M_K = 0.493677$ (GeV)) and Q^2, t and σ_L are given from data. Therefore we use the following formula to obtain $F_k(Q^2)$ [1, 12]:

$$\sigma_L \approx \frac{-2tQ^2}{(t-M_K^2)^2} g_{KN\Lambda}^2(t) F_K^2(Q^2, t) \quad (3-3)$$

Thh

Now let us consider the Regge model. In this model, the exchange of particles with higher mass and spin are also considered, and instead of one exchange particle, a family of particles with the same quantum numbers is used in the interaction. In the following formula, the kaon form factor is

given in terms of the known quantities and $\Lambda_k^2 = 0.7$ & $C_k = 0.87$ [15].

$$F_k(Q^2, t) = \frac{\Lambda_k^2}{(\Lambda_k^2 + Q^2)} + \frac{C_k(M_k^2 - t)Q^2}{(\Lambda_k^2 + Q^2)^2} \quad (3-4)$$

The second term in equation (3-4) is the genuine off-shell mass term, which is justified by the kaon pole's distance from the physical region (distinct from the pion pole), making this term possibly important. The term C_k determines a slope of the t dependence at the real photon point. For a fixed kaon angle, the momentum transfer t is a function of energy W (t and W are correlated), which introduces an additional energy dependence into the model [15].

If t approaches, M_k^2 ; namely, $t \rightarrow M_k^2$ then, equation (3-4) is reduced to the following form (3-5) [12, 15]:

$$F_k(Q^2) = \frac{\Lambda_k^2}{(\Lambda_k^2 + Q^2)} \quad (3-5)$$

Table 1. Kinematical data for L/T Separation at $Q^2 = 1.9(\text{GeV}/c)^2$, Jlab 2001(E98-108) [14].

$(d\sigma/d\Omega)_{tot}$ (nb/sr)	t_{min} (GeV/c) ²	ϵ	Q^2 (GeV/c) ²	W (GeV)	θ_K Deg.	E (GeV)
177.6±8.6	-0.6183	0.811	1.9	1.91	0.0	5.754
170.7±11.6	-0.6183	0.637	1.9	1.91	0.0	4.238
149.7±11.5	-0.6183	0.401	1.9	1.91	0.0	3.401
178.9± 5.9	-0.579	0.800	1.9	1.94	0.0	5.614
170.0±7.5	-0.579	0.613	1.9	1.94	0.0	4.238
154.4±7.0	-0.579	0.364	1.9	1.94	0.0	3.401
171.8± 7.1	-0.5203	0.792	1.9	2.0	0.0	5.754
162.7± 7.4	-0.5203	0.575	1.9	2.0	0.0	4.238
161.4±5.4	-0.4143	0.726	1.9	2.14	0.0	5.614
152.1±9.4	-0.4143	0.471	1.9	2.14	0.0	4.238

Table 2. Kinematical data for L/T Separation at $Q^2 = 2.35 (\text{GeV}/c)^2$, Jlab 2001(E98-108) [14].

$(d\sigma/d\Omega)_{tot}$ (nb/sr)	t_{min} (GeV/c) ²	ϵ	Q^2 (GeV/c) ²	W (GeV)	θ_K Deg.	E (GeV)
130.3±11.4	-0.9498	0.359	2.35	1.8	0.0	3.401
150.5±9.7	-0.9498	0.796	2.35	1.8	0.0	5.614
134.7±7.36	-0.9498	0.608	2.35	1.8	0.0	4.238
150.1±6.8	-0.8562	0.781	2.35	1.85	0.0	5.614
135.4±6.4	-0.8562	0.579	2.35	1.85	0.0	4.238
129.1±10.6	-0.8562	0.313	2.35	1.85	0.0	3.401
147.4±5.8	-0.6737	0.737	2.35	1.98	0.0	5.614
135.1±8.6	-0.6737	0.494	2.35	1.98	0.0	4.238
137.2 ± 4.1	-0.5716	0.696	2.35	2.08	0.0	5.614
118.1 ± 6.0	-0.5716	0.417	2.35	2.08	0.0	4.238

Table 3. L/T separation result from Fig. 3.1.

Q^2 (GeV/c) ²	W (GeV)	σ_L (nb/sr)	σ_T (nb/sr)	$R = \sigma_L/\sigma_T$
1.9	1.91	67.36 ± 12.87	124.19±8.67	0.54± 0.14
1.9	1.94	56.13±3.85	134.40±2.48	0.42± 0.036
1.9	2.00	41.93± 8.17	138.58± 6.76	0.30± 0.07
1.9	2.14	36.47± 7.61	134.92± 6.06	0.27±0.06

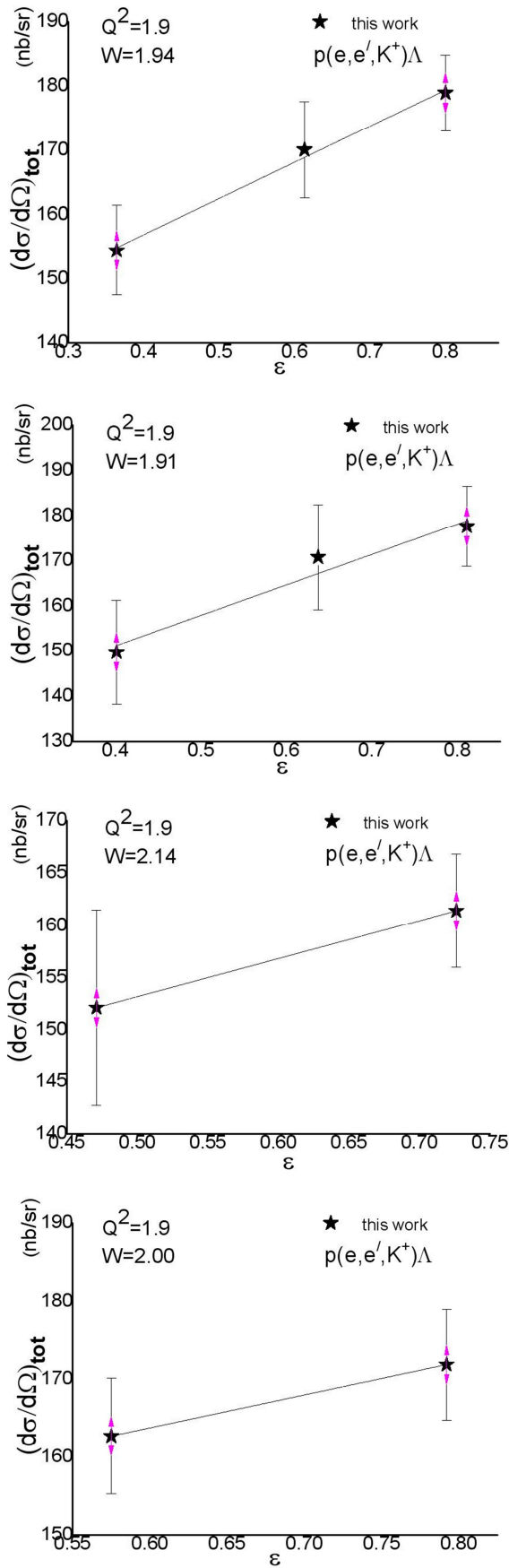


Fig. 3.1. Extraction of L/T separated cross section from Table 1.

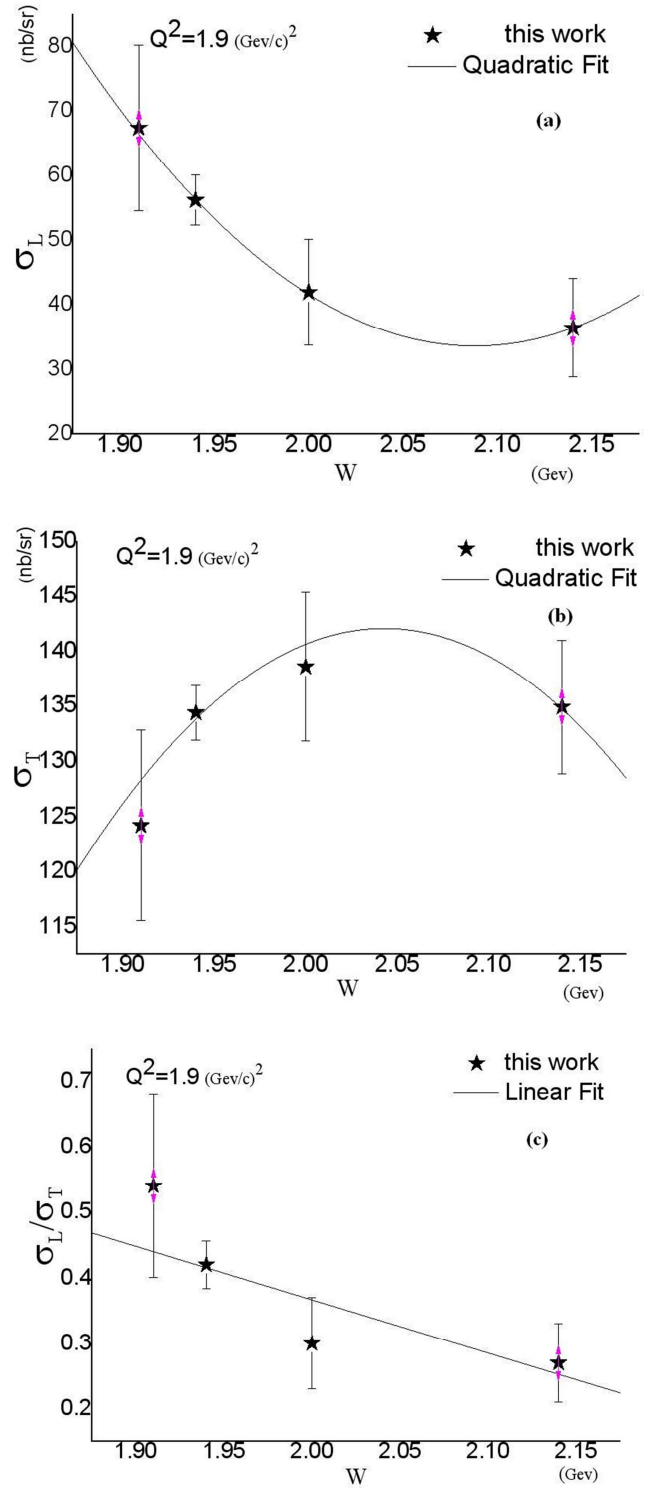

 Fig. 3.2. Longitudinal (a) and transverse (b) cross sections for $H(e, e', k^+) \Lambda$ as a function of W at $Q^2 = 1.9 \text{ (GeV/c)}^2$. The ratio is shown in panel (c).

Table 4. L/T separation result from Fig. 3.3.

$R = \sigma_L / \sigma_T$	σ_T	σ_L	W	Q^2
	(nb/sr)	(nb/sr)	(GeV)	(GeV/c) ²
0.42 ± 0.27	109.93 ± 14.68	46.57 ± 23.35	1.80	2.35
0.44 ± 0.19	110.41 ± 10.78	48.35 ± 16.95	1.85	2.35
0.46 ± 0.09	110.10 ± 6.15	50.62 ± 7.69	1.98	2.35
0.76 ± 0.11	89.55 ± 4.59	68.46 ± 6.01	2.08	2.35

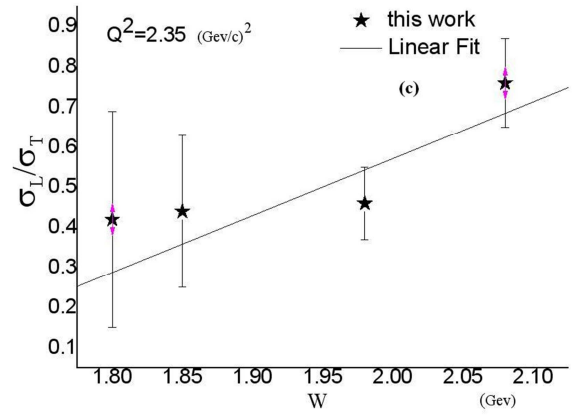
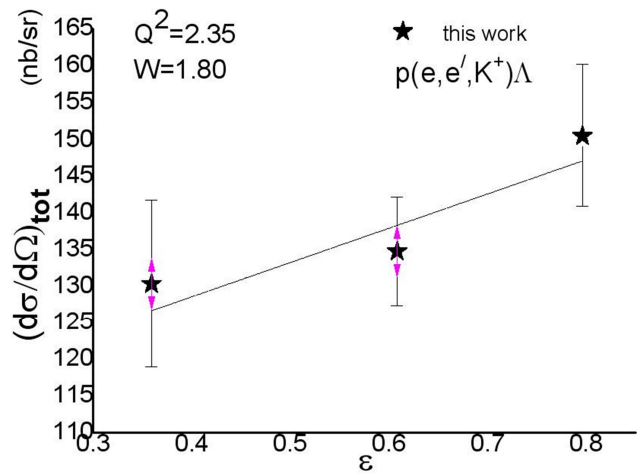
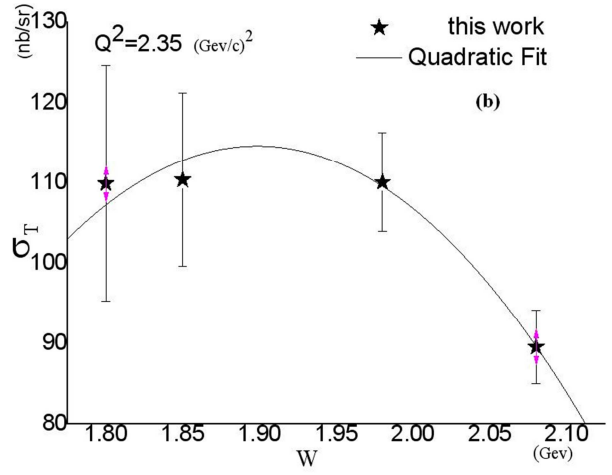
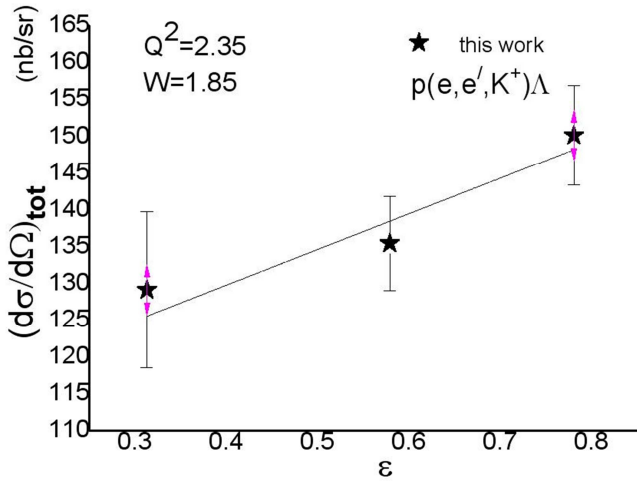


Fig. 3.4. Longitudinal (a) and transverse (b) cross sections for $H(e, e', k^+) \Lambda$ as a function of W at $Q^2 = 2.35 (\text{GeV}/c)^2$. The ratio is shown in panel (c).

Fig. 3.3. Extraction of L/T separated cross section from Table 2.

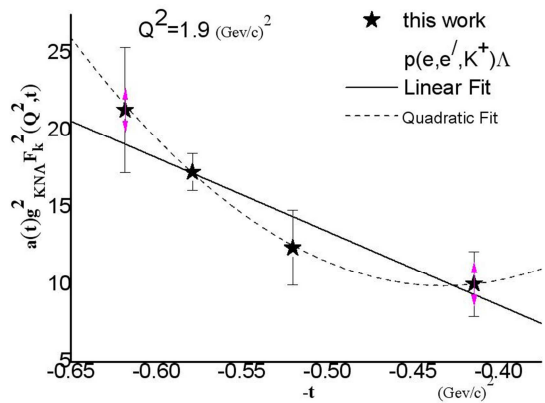
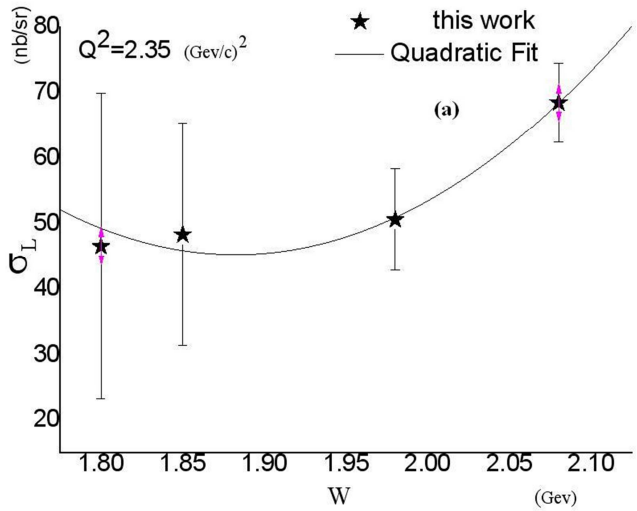


Fig. 3.5. Chew-low extrapolation technique (Born model) used for extraction of the kaon form factor at $Q^2 = 1.9(\text{GeV}/c)^2$.

Table 5. Form factor calculation by using the Born model and Regge model at $Q^2 = 1.9(\text{GeV}/c)^2$.

$Q^2 = 1.9 (\text{GeV}/c)^2$			<i>Born model</i>	<i>Regge model</i>	
W	t	σ_L	$a(t)g_{KNA}^2 F_k^2(Q^2, t)$	$F_k(Q^2, t)$	$F_k(Q^2)$
(GeV)	(GeV/c) ²	(nb/sr)			
1.91	-0.6183	67.36 ± 12.87	21.30 ± 4.07	0.4544	0.2050
1.94	-0.5790	56.13 ± 3.85	17.27 ± 1.18	0.4431	0.2050
2.00	-0.5203	41.93 ± 8.17	12.38 ± 2.41	0.4261	0.2050
2.14	-0.4143	36.47 ± 7.61	10.03 ± 2.09	0.3954	0.2050

Table 6. Form factor calculation by using Born model and Regge model at $Q^2 = 2.35(\text{GeV}/c)^2$.

$Q^2 = 2.35 (\text{GeV}/c)^2$			<i>Born model</i>	<i>Regge model</i>	
W	t	σ_L	$a(t)g_{KNA}^2 F_k^2(Q^2, t)$	$F_k(Q^2, t)$	$F_k(Q^2)$
(GeV)	(GeV/c) ²	(nb/sr)			
1.80	-0.9498	46.57 ± 23.35	14.86 ± 7.45	0.4750	0.1725
1.85	-0.8562	48.35 ± 16.95	14.54 ± 5.9	0.4513	0.1725
1.98	-0.6737	50.62 ± 7.69	13.45 ± 2.05	0.4050	0.1725
2.08	-0.5716	68.46 ± 6.01	16.94 ± 1.49	0.3792	0.1725

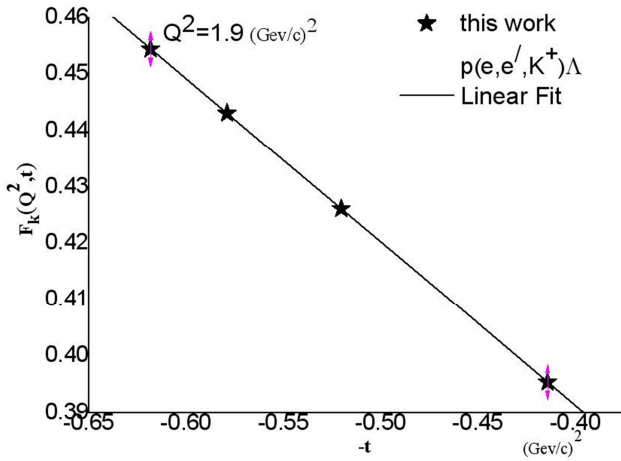


Fig. 3.6. Regge model used for extraction Of the kaon form factor at $Q^2 = 1.9(\text{GeV}/c)^2$.

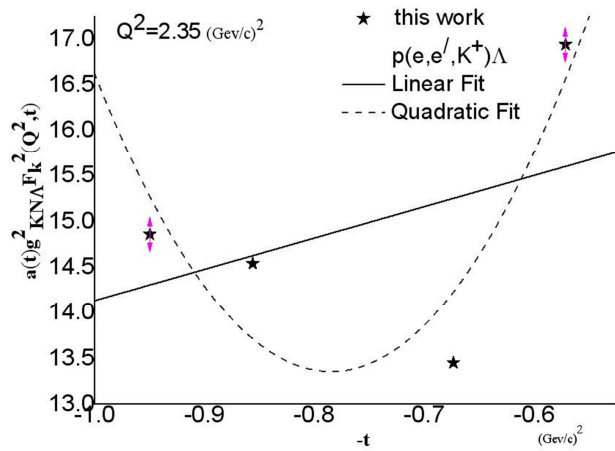


Fig. 3.7. Chew-low extrapolation technique (Born model) used for extraction of the kaon form factor at $Q^2 = 2.35(\text{GeV}/c)^2$.

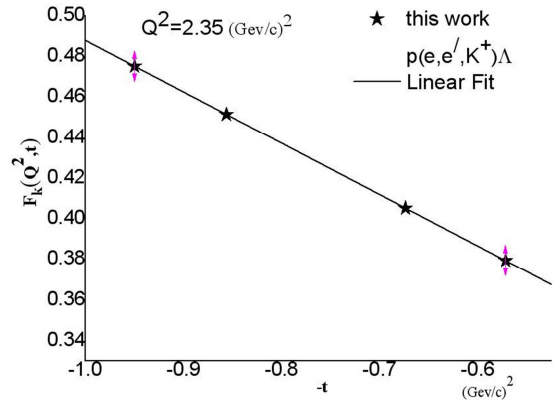


Fig. 3.8. Regge model used for extraction of kaon form factor of the kaon form factor at $Q^2 = 2.35(\text{GeV}/c)^2$.

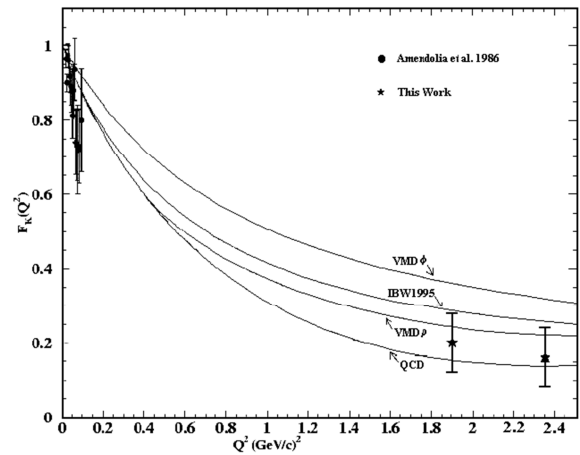


Fig. 3.9. Kaon form factor as a function of Q^2 . The present extraction of $F_K(Q^2)$ is shown at $Q^2 = 1.9$ and $2.35 (\text{GeV}/c)^2$.

All our calculated values of Kaon form factors are collected in tables 5 and 6 and plotted in figures (3.5) to (3.8). In Fig (3.9) our findings for $F_K(Q^2)$ at $Q^2 = 1.9$ and $2.35 (\text{GeV}/c)^2$ are compared with the values given in Ref. [13],

4. Conclusion

In this paper, by using Rosenbluth separation method, the total cross-section of $H(e, e', k^+) \Lambda$ reaction is separated into longitudinal and transverse cross-section at $Q^2 = 1.9$ and $2.35 \left(\frac{\text{GeV}}{c}\right)^2$. The variations of the longitudinal and transverse cross sections and their ratio as a function of w , depend upon the value of square momentum transfer. At $Q^2=1.9$ the ratio has a negative slope and at $2.35 \left(\frac{\text{GeV}}{c}\right)^2$, has a positive slope.

Then, by using our obtained longitudinal cross-section σ_L , and the Chew-low technique in Born model, the charged kaon form factor is determined and is compared to the values, given by using different models. Such comparison indicates that while our findings are within acceptable range of values from other models, yet are closer to the values obtained from QCD model given in ref. [13].

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