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Gap Solitary Pulses Induced by the Modulational Instability and Discrete Effects in Array of Inhomogeneous Optical Fibers

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Abstract

We analytically demonstrate that temporal modulational instability of pulse combined with discrete effects in an array of optical fibers in regime of anomalous dispersion can generate secondary excitations in the gap. These excitations are bound closely to the perturbations of the soliton propagating in the array. They can be of any nature or soliton type and disappear when the modulational instability vanishes. This approach in the research of gap solitons being new, necessitates finding an appropriated method in the research of gap solitons. So the Bogning-Djeumen Tchaho-Kofané method used in this context seems better adapted to determine the analytical shape of gap soliton.

Keywords

Modulational Instability, Gap Soliton, Optical Fibers, Pulse, Kink, Discrete Effects, Soliton of Array

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1. Introduction

The dynamics of high intensity wave propagation through the transmission medium is generally the source of induced phenomena, often generalized in the appellation of nonlinear phenomena. Most often when they are not mastered, they disturb considerably the evolution of the signal during the propagation. One of the most important manifestations of the nonlinear effects is the modulational instability which is mainly due to interplay between linear and nonlinear effects. This significant phenomena was observed for the first time in 1967 in the context of wave propagation in fluids [1, 2]. Since researchers have developed a lot of interest on this exciting phenomena and many interesting results have been reported in fields such as atomic chains, optical fibers, Bose-Einstein condensates, etc [3-7]. Another consequence of the nonlinearity is the presence of the secondary excitations in zones of forbidden propagation regime known as gap zones. These gap zones can be bounded closely to the instabilities in the propagation medium as well as the discrete effects, in a network of fibers [8]. Regarding gap zones, for over the past decade, many works have been published [9, 10-16], where interests were geared towards the investigation of gaps in wave transmission media because of then high potential application in fast and secured communications [8, 18]. However it is worth saying that only a very few works have treated the case of the secondary excitations generated by the modulational instabilities or the discrete effects.

The prime aim of this paper is to study the gap solitons induced by the dual action of the modulational instabilities, and discrete effects in the anomalous dispersion regime of the array of optical fibers. From our detailed derivation, we made it clear that the perturbation of the array soliton leads to a dispersion relation that is often temporal, discrete and can likely generate complex instabilities to be analyzed [8, 17 - 21]. Thus, deviating from the model of modulational

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instability study originally carried out by Alejandro B. Aceves et al, we intend to demonstrate that these types of instabilities in the array of discrete optical fibers produce gap solitons, which vanish when there are no instabilities. The method used to investigate these gap solitons is the method used by Bogning et al. [22-25].

This paper is organized as follows. In the second section, we define globally the model, in the third section we look for the form of array soliton. In the fourth section, we study the modulational instability of array soliton in the limit of the plane waves. In the fifth section, we look for the analytical shape of the gap soliton. We summarize our results in section six by a general conclusion as well as perspectives.

2. Model

The studied model assumes an array of optical fibers where the coating and the cladding are not perfect, so that interactions between the electromagnetic waves propagating in every fiber are possible. Though these interactions are generally weak we assume the case where their effect is not negligible. Under this assumption, we intend to show that these interactions must be taken into account in the model of nonlinear partial differential equations that governs the propagation of the signals in a network of optical fibers. In our analysis, we postulate that the coupling strength between two considered fibers is periodic with a variable part that varies with the fiber index and a homogeneous part. In fact the manner of stacking the fibers is not important because a network can present itself under several shapes; it can be plane, cylindrical, or oblong shape, etc.

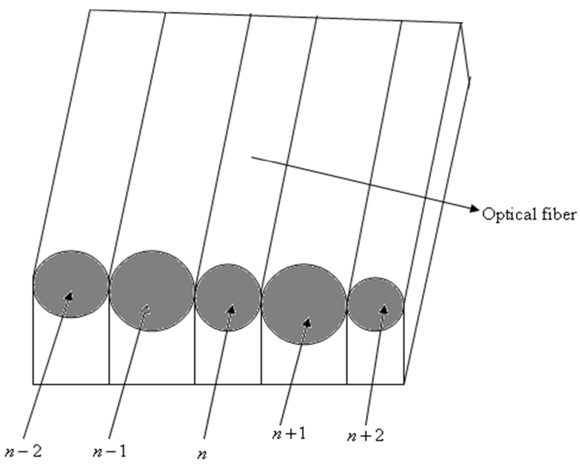

Fig. 1. Plane array of inhomogeneous optical fibers.

The logic that guides the reflection is that when one considers a given propagation of light through a fiber, it undergoes the influence of its neighbors. These influences can vary according to the neighbor's position with respect to the considered light propagation through fiber. Such influencing of the neighboring fiber in the propagation was described by I. Relke [18]. The figure 1 shows a plane array of inhomogeneous optical fibers prototype.

The nonlinear wave propagation in a network of inhomogeneous optical fibers is given by the following differential system of nonlinear partial differential equations [18].

$$i\partial_z\psi_n + C_{n+\frac{1}{2}}\psi_{n+1} + C_{n-\frac{1}{2}}\psi_{n-1} + \partial_{tt}\psi_n + |\psi_n|^2\psi_n = 0, \tag{1}$$

where n is the fiber index, $C_{n+\frac{1}{2}}$ is the coupling strength between the neighboring fibers number n and $n+1$, $C_{n-\frac{1}{2}}$ is the coupling strength between the fiber number n and $n-1$, ψ_n is the complex amplitude of the wave, z is the distance of propagation and t is the time. When we choose the periodic coupling strength of the form [18]

$$C_{n+\frac{1}{2}} = \Omega_0 + \Omega\cos(\pi n), \tag{2}$$

where Ω_0 is the constant part of the coupling strength and $\Omega_n = \Omega\cos(\pi n)$ is the variable part of the coupling strength (Ω is the variable part factor which is constant). While doing the change

$$\psi_n \to \psi_n \exp i\left(C_{n+\frac{1}{2}} + C_{n-\frac{1}{2}}\right)z, \tag{3}$$

Eq. (1) becomes

$$\begin{aligned} &i\partial_z\psi_n + (-1)^n\Omega(\psi_{n+1} - \psi_{n-1}) \\ &+\Omega_0(\psi_{n+1} + \psi_{n-1} - 2\psi_n) + \partial_{tt}\psi_n + |\psi_n|^2\psi_n = 0. \end{aligned} \tag{4}$$

This equation models waves propagation in an array of inhomogeneous optical fibers in a regime of anomalous dispersion with a periodic coupling strength.

3. Array Soliton

The model we have considered in the whole work describes the dynamics of wave propagation in an array of optical fibers in the regime of anomalous dispersion, and governed by the discrete nonlinear partial differential equation (4). The most stable soliton solution that verifies equation (4) whatever the parameters of the array obtained by assuming $\psi_0 = \psi_1 = ... = \psi_n = \psi_{n-1} = \psi_{n+1}$ such that

$$\begin{aligned} \psi_0 &= \sqrt{2}\lambda\,\text{sec}\,h(\lambda t)\exp i\lambda^2 z \\ &= g(t)\exp i\lambda^2 z, \end{aligned} \tag{5}$$

where λ^2 is the spatial wave number of the soliton and $g(t)=\sqrt{2}\lambda \sec h(\lambda t)$ represents its envelope; ψ_0 is also called the array soliton. We are going to propose a survey of the modulational instability of the array soliton obtained previously.

4. Modulational Instability

Modulational instability refers to the growth of side bands at the expense of an intense wave train in nonlinear dispersive medium. Thus, taking the general solution of equation (4) in the form

$$\psi_n=\left(g(t)+\delta\psi_n\right)\exp i\lambda^2 z\,, \tag{6}$$

where $\delta\psi_n$ is a function of the discrete variable n and represents the complex amplitude of perturbation. Taking into account equation (6) in equation (4) leads to a nonlinear partial differential equation which governs the evolution of the amplitude of perturbation

$$\begin{aligned}&i\left(\delta\psi_n\right)_z+\left(\delta\psi_n\right)_{tt}-\left(\lambda^2-2g^2\right)\delta\psi_n\\&+(-1)^n\,\Omega\left(\delta\psi_{n+1}-\delta\psi_{n-1}\right)\\&+\Omega_0\left(\delta\psi_{n+1}+\delta\psi_{n-1}-2\delta\psi_n\right)\\&+g^2\left(\delta\psi_n\right)^*=0,\end{aligned} \tag{7}$$

where $\left(\delta\psi_n\right)^*$ stands for complex conjugate of $\delta\psi_n$. Considering a periodic perturbation of the form

$$\delta\psi_n=\delta\psi\cos nk\,, \tag{8}$$

where $\delta\psi$ is a fixed amplitude of the perturbation, n is the discrete variable and k is the discrete frequency of the perturbation, we obtain

$$\begin{aligned}&i\left(\delta\psi\right)_z+\left(\delta\psi\right)_{tt}-\left(\lambda^2-2g^2\right)\left(\delta\psi\right)\\&-2(-1)^n\,\Omega\sin(k)\tan(nk).\delta\psi\\&-4\Omega_0\sin^2(k/2)\delta\psi+g^2\left(\delta\psi\right)^*=0,\end{aligned} \tag{9}$$

with $nk\neq(2n'+1)\pi/2, n'\in\mathbb{N}-\{0\}$. Writing the non discrete term of perturbation in complex form

$$\delta\psi=u(z,t)+iv(z,t), \tag{10}$$

with $i^2=-1$ and substitution into equation (9) gives the following coupled partial differential equations

$$\begin{aligned}&\frac{\partial u}{\partial z}+\frac{\partial^2 v}{\partial t^2}-\left(\lambda^2-2g^2\right)v\\&-2(-1)^n\,\Omega\sin(k)\tan(nk)v\,,\\&-4\Omega_0\sin^2(k/2)v-g^2v\end{aligned} \tag{11}$$

and

$$\begin{aligned}&\frac{\partial v}{\partial z}-\frac{\partial^2 u}{\partial t^2}+\left(\lambda^2-2g^2\right)u\\&+2(-1)^n\,\Omega\sin(k)\tan(nk)u\,.\\&+4\Omega_0\sin^2(k/2)u-g^2u\end{aligned} \tag{12}$$

Taking the Fourier's transform of equations (11) and (12) with respect to the spatial variable z such that

$$\bar{u}(p,t)=\int_{-\infty}^{+\infty}u(z,t)\exp-ipzdz\,, \tag{13}$$

$$\bar{v}(p,t)=\int_{-\infty}^{+\infty}v(z,t)\exp-ipzdz\,, \tag{14}$$

and

$$u(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\bar{u}(p,t)\exp ipzdp\,, \tag{15}$$

$$v(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\bar{v}(p,t)\exp ipzdp\,, \tag{16}$$

yields the equations

$$iP\bar{u}-\left(L_1+4\Omega_0\sin^2(k/2)\right)\bar{v}=0\,, \tag{17}$$

and

$$iP\bar{v}+\left(L_2+4\Omega_0\sin^2(k/2)\right)\bar{u}=0\,, \tag{18}$$

where, P is the spatial frequency of perturbation, $\bar{u}$ and $\bar{v}$ are respectively the Fourier transforms of u and v, $\omega=-i\frac{\partial}{\partial t}$ the temporal frequency of the perturbation $L_1=\lambda^2-g^2+\omega^2+2(-1)^n\,\Omega\sin(k)\tan(nk)$, and $L_2=\lambda^2-3g^2+\omega^2+2(-1)^n\,\Omega\sin(k)\tan(nk)$. The above equations (17) and (18) admit the non trivial solutions only when the spatial frequency satisfies

$$\begin{aligned}P^2=&\left(L_1+4\Omega_0\sin^2(k/2)\right)\\&\times\left(L_2+4\Omega_0\sin^2(k/2)\right).\end{aligned} \tag{19}$$

4.1. Case of the Spatial Plane Waves

The plane wave limit is obtained when $t=0$; then, the stable solution of equation (5) takes the form

$$\psi_0 = \sqrt{2}\lambda \exp i\lambda^2 z, \tag{20}$$

where, $g(t) = \sqrt{2}\lambda$ is a constant. The dispersion relation given by equation (19) becomes

$$P^2 = \begin{bmatrix} -\lambda^2 + \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{bmatrix} \times \begin{bmatrix} -5\lambda^2 + \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{bmatrix}. \tag{21}$$

We observe that the relation of dispersion depends on the coupling parameters Ω_0, Ω, the discrete frequency k and the fiber index n. Since modulational instability is observed when $P^2 \prec 0$, it is wordwile mentioning that the necessary condition for the occurrence of modulational instability is that the two factors of equation (21) should be of opposite signs. Then modulational instability occurs in certain case when

$$\lambda^2 \succ \begin{pmatrix} \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{pmatrix}. \tag{22}$$

If the variable part coefficient of the coupling strength Ω is zero, Eq. (22) reduces to the result of Alejandro B. Aceves et al. [18]

$$\lambda^2 \succ \omega^2 + 4\Omega_0 \sin^2(k/2) \succ 0. \tag{23}$$

From Eq. (22) it is clear that the variable part coefficient of the coupling strength plays an important role in the appearance of modulational instability in the array, since the contribution of discrete effects to the occurrence of modulational instability is closely linked to Ω_n. From this there is globally modulational instability when the following condition is verified

$$\lambda^2 \prec \begin{pmatrix} \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{pmatrix} \prec 5\lambda^2. \tag{24}$$

The analysis of equations (22) and (24) permits to define the modulational instability gain

$$G(k,n) = \sqrt{\begin{matrix} \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{matrix}}. \tag{25}$$

We can see that the modulational gain given by Eq. (25) is a function of the discrete number n. The evolution of the modulational instability gain $G(k,n)$ in term of the discrete frequency of perturbation k is given in Figure 2. These curves are obtained for different number of optical fibers in the array.

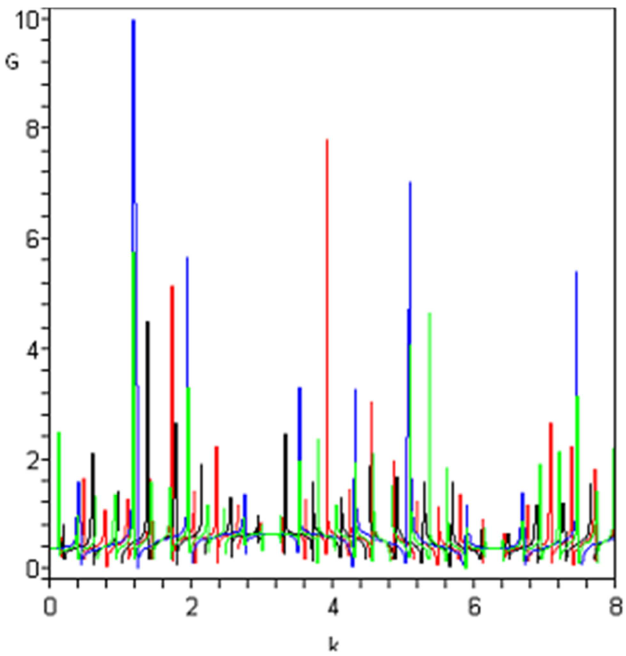

Fig. 2. Evolution of the modulational instability gain G in term of the discrete frequency of perturbation k for $\omega = 0.2$ and $\Omega_0 = 0.2$: blue curve for $n = 4$; Black curve for $n = 8$; red curve for $n = 10$ and green curve for $n = 12$.

4.2. Existence of Soliton Waves

In the case where the wave is a soliton, i.e. $g(t)$ is not a constant, and the condition for the occurrence of modulational instability is given by equation (19). When $\Omega = 0$, the dispersion relation Eq. (19) becomes independent of n. In this case equations (17) and (18) have a non trivial solution for

$$P^2 = \min \left\{ \begin{matrix} \left\langle \psi \left| \left[L_1 + 4\Omega_0 \sin^2(k/2) \right] \right| \psi \right\rangle / \\ \left\langle \psi \left| \left[L_2 + 4\Omega_0 \sin^2(k/2) \right]^{(-1)} \right| \psi \right\rangle \end{matrix} \right\}, \tag{26}$$

Where $\langle \psi | A | \psi \rangle = \int_{-\infty}^{+\infty} \psi A \psi dt$. The average is over the class of functions that degenerates sufficiently fast at $t \to \pm\infty$. It is important to mention that for Eq. (26) to hold, the operator $L_1 + 4\Omega_0 \sin^2(k/2)$ must be invertible, taking into consideration that $\langle \psi | L_0 | \psi \rangle \geq 0$ and is zero only for the ground state $\psi = g(t)$, then the operator is indeed invertible if $k \neq q\pi, q \in \mathbb{N}$. The appropriate conditions for the occurrence of modulational instability is given in [18]. Applying this condition implies

$$4\Omega_0 \sin^2(k/2) \prec 3\lambda^2/2. \tag{27}$$

However, we should note that a complete study of instability necessitates the consideration of the discrete variable n as well as the variable part factor of the coupling force Ω.

5. Gap Solitary Waves

The dispersion relation equation (19), in the case when $\Omega \neq 0$, becomes

$$P^2 = \begin{bmatrix} \lambda^2 - g^2 + \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{bmatrix} \begin{bmatrix} \lambda^2 - 3g^2 + \omega^2 + 2(-1)^n \Omega \sin(k)\tan(nk) \\ +4\Omega_0 \sin^2(k/2) \end{bmatrix}. \tag{28}$$

The dispersion relation given by Eq. (28) depends on n and also time-dependent; this equation involves three variables g , and Ω that can influence the frequencies of the secondary excitations in the gap. To represent the gap zone, it is better to fix t, Ω and n.

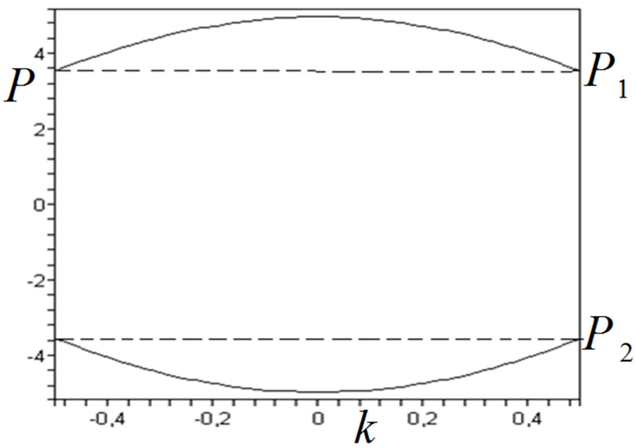

Fig. 3. Representation of the spatial frequency of perturbation P as a function of the discrete wave number k for $\omega = 0.2$, $\lambda = 1$, $\Omega_0 = 0.1$, $t = 0$, $g(t) = \sqrt{2}\lambda$ and $\Omega = 0$.

Figure 3 shows the gap zone in the reduced space for $\omega = 0.2$, $\lambda = 1$, $\Omega_0 = 0.1$, $t = 0$, $g(t) = \sqrt{2}\lambda$ and $\Omega = 0$.

This figure presents naturally an upper branch or high-frequency branch of frequency P_1 and a lower branch or low-frequency branch of frequency P_2. The specificity here is that we have a dispersion relation led by the perturbations whose representation shows zones of gap, i.e. a zone of gap that exists when one perturbs the pulse initially injected in the network.

Having studied about the impact of the perturbation of the array soliton leading to the dispersion relation reveals the gap area. We now look for the solution of Eq. (1) in the form

$$\psi_n = \left[a \operatorname{sec} h\lambda t + U(t)\right] \exp - i(P_1 z + kn), \tag{29}$$

where $U(t)$ is the amplitude of the secondary excitations in the gap and P_1 is the upper branch frequency of the gap spectrum. Hence, while substituting the expression of ψ_n given by Eq. (29) in Eq. (4), we obtain

$$\begin{aligned} &\left[P_1 - 2i(-1)^n \Delta f \sin k + 4f \sin^2 \frac{k}{2}\right] U(t) \\ &+U^3(t) + \partial_{tt} U(t) \\ &+a \begin{bmatrix} P_1 - 2i(-1)^n \Delta f \sin k \\ +4f \sin^2 \frac{k}{2} + \lambda^2 + 3U^2 \end{bmatrix} / \cosh \lambda t \\ &+3a^2 U(t) / \cosh^2 \lambda t \\ &-a\left(2\lambda^2 - a^2\right) / \cosh^3 \lambda t = 0. \end{aligned} \tag{30}$$

The above equation describing the evolution of instabilities in the array and can better describe the secondary excitations in the gap is reduced to

$$\begin{aligned} &\partial_{tt} U(t) + U^3(t) \\ &+ \begin{bmatrix} P_1 - 2i(-1)^n \Delta f \sin k \\ +4f \sin^2(k/2) \end{bmatrix} U(t) = 0. \end{aligned} \tag{31}$$

On the other hand, the equation of factor $1/\cosh \lambda t$ gives

$$\begin{aligned} &P_1 - 2i(-1)^n \Delta f \sin k \\ &+4f \sin^2(k/2) + \lambda^2 + 3U^2 = 0. \end{aligned} \tag{32}$$

It is straightforward to notice from Eq. (32) that $U(t)$ can be real or complex. Thus, taking into account Eq. (32) in Eq. (31), we obtain

$$\frac{d^2U}{dt^2} - 2U^3(t) - \lambda^2 U(t) = 0, \tag{33}$$

Using the quadratic method, the integration of Eq. (31) leads to

$$\frac{dU}{\sqrt{\left(U^2 + \frac{\lambda^2}{2}\right)^2 - \beta}} = \pm dt, \tag{34}$$

with $\beta = \lambda^2/4 - C_1$, where C_1 is the first constant of integration. Looking for the particular solution of Eq. (34), we set $\beta = 0$ i.e. $C_1 = \lambda^2/4$. So, we obtain the general solution of Eq. (34) in the form

$$U=\left(\lambda/\sqrt{2}\right)\tan\left(\pm\frac{\lambda}{\sqrt{2}}t+C_2\right), \quad (35)$$

and

$$U=\frac{i\frac{\sqrt{\lambda}}{2}\left(A\exp\pm i\sqrt{2}\lambda t-1\right)}{1+A\exp\left(\pm i\sqrt{2}\lambda t\right)}, \quad (36)$$

where $A=\exp(-2iC_2)$ and C_2 are arbitrary constants. Equation (36) can admit many particular solutions. For example for $A=1$, we obtain

$$U_1=i\left(\sqrt{\lambda}/2\right)\mathrm{t}anh\left(\pm i\frac{\lambda}{2}t\right), \quad (37)$$

and

$$U_2=-i\left(\sqrt{\lambda}/2\right)\mathrm{t}an\left(\pm i\frac{\lambda}{2}t\right). \quad (38)$$

The resolution of the amplitude equations in $U(t)$, permits to obtain different analytical expressions of the excitation amplitudes in the gap area.

5.1. Dark Gap Soliton Induced by the Bright Soliton Instability

As established in many nonlinear systems, the gap solitons are the topological solitons, in the similar lines we look for the dark gap soliton induced by the perturbation of the bright soliton. Our objective in this subsection is to analyze the impact of the bright soliton instability on the generation of the gap excitations. For this purpose, we look for solitons in the gap with the shape of the form

$$\psi_n=\left(a\sec h\lambda t+b\tanh^2\lambda t\right)\exp-i\left(P_1z+kn\right), \quad (39)$$

where, a and b are time-independent complex coefficients to be determined such that $a=a_r+ia_i$, $b=b_r+ib_i$, P_1 is the spatial frequency of perturbation in the gap which is supposed to be constant, a_i and b_i indicate respectively the imaginary part of a and b. We will use the Bogning-Djeumen Tchaho-Kofané (BDK) method to determine the constants a_r, a_i, b_r, b_i [23 - 25]. Then, substitution of equation (37) in equation (4) gives an equation in terms of power $1/\cosh^j\lambda t$ as

$$\begin{aligned}&\sum_{j=0}^{6}F_j\left(a_r,a_i,b_r,b_i\right)/\cosh^j\lambda t\\&+i\sum_{j=0}^{6}G_j\left(a_r,a_i,b_r,b_i\right)/\cosh^j\lambda t=0,\end{aligned} \quad (40)$$

where $j=0,1,2,\ldots$, $F_j\left(a_r,a_i,b_r,b_i\right)$ and $G_j\left(a_r,a_i,b_r,b_i\right)$ are the functions of a_r, a_i, b_r and b_i such that

$$\begin{aligned}F_0\left(a_r,a_i,b_r,b_i\right)=&\,p_1b_r+2(-1)^n\Omega b_i\sin k\\&-4\Omega_0\sin^2\frac{k}{2}b_r+\left(b_r^2+b_i^2\right)b_r,\end{aligned}$$

$$\begin{aligned}F_1\left(a_r,a_i,b_r,b_i\right)=&\,p_1a_r+2(-1)^n\Omega\sin k\\&-4\Omega_0\sin^2\frac{k}{2}a_r+a_r\lambda^2+2|b|^2a_r\\&+a_r^2\left(b_r^2-b_i^2\right)+2b_rb_ia_i,\end{aligned}$$

$$\begin{aligned}F_2\left(a_r,a_i,b_r,b_i\right)=&-p_1b_r-2b_i\sin k\\&+4\Omega_0\sin^2\frac{k}{2}b_r-4\lambda b_r+2b_r|a|^2\\&+b_r\left(a_r^2-a_i^2\right)+2a_ra_ib_i-3|b|^2b_r\\&+|a|^2a_r-2|b|^2a_r,\end{aligned}$$

$$\begin{aligned}F_3\left(a_r,a_i,b_r,b_i\right)=&-2\lambda^2a_r-2a_r|b|^2\\&-2\left(b_r^2-b_i^2\right)a_r-2a_ib_rb_i,\end{aligned}$$

$$\begin{aligned}F_4\left(a_r,a_i,b_r,b_i\right)=&\,6\lambda^2b_r+3|b|^2b_r\\&-|a|^2b_r+\left(a_r^2-a_i^2\right)b_r+2a_ra_ib_i+b_r|a|^2,\end{aligned}$$

$$F_5\left(a_r,a_i,b_r,b_i\right)=3|b|^2a_r+a_r\left(b_r^2-b_i^2\right)+2b_rb_ia_i,$$

$$F_6\left(a_r,a_i,b_r,b_i\right)=|b|^2b_r,$$

$$\begin{aligned}G_0\left(a_r,a_i,b_r,b_i\right)=&\,p_1b_i-2(-1)^n\Omega\sin kb_r\\&-4\Omega_0\sin^2\frac{k}{2}b_i+b_i|b|^2,\end{aligned}$$

$$\begin{aligned}G_1\left(a_r,a_i,b_r,b_i\right)=&\,p_1a_i-2(-1)^n\Omega\sin ka_r\\&-4\Omega_0\sin^2\frac{k}{2}a_i+a_i\lambda^2+2|b|^2a_i\\&+2a_rb_rb_i-\left(b_r^2-b_i^2\right)a_i,\end{aligned}$$

$$\begin{aligned}G_2\left(a_r,a_i,b_r,b_i\right)=&-p_1b_i+2\sin kb_r\\&+4\Omega_0\sin^2\frac{k}{2}b_i-4\lambda b_i+2b_i|a|^2\\&+2a_ra_ib_r-b_i\left(a_r^2-a_i^2\right)-3|b|^2b_i\\&+|a|^2a_i-2|b|^2a_i,\end{aligned}$$

$$G_3(a_r,a_i,b_r,b_i) = -2\lambda^2 a_i - 2a_i|b|^2 + 2(b_r^2 - b_i^2)a_i + 2a_r b_r b_i,$$

$$G_4(a_r,a_i,b_r,b_i) = 6\lambda^2 b_i + 3|b|^2 b_i - |a|^2 b_i - (a_r^2 - a_i^2)b_i + 2a_r a_i b_i + b_i|a|^2,$$

$$G_5(a_r,a_i,b_r,b_i) = 3|b|^2 a_i - a_i(b_r^2 - b_i^2) + 2b_r b_i a_r,$$

$$G_6(a_r,a_i,b_r,b_i) = |b|^2 b_i.$$

By equating equation (40) to zero, we obtain respectively

$$\sum_{j=0}^{5} F_j(a_r,a_i,b_r,b_i)/\cosh^j \lambda t = 0, \qquad (41)$$

and

$$\sum_{j=0}^{6} G_j(a_r,a_i,b_r,b_i)/\cosh^j \lambda t = 0. \qquad (42)$$

In BDK method, we admit that the best equations permitting us obtain values of the constants a_r, a_i, b_r, b_i are those corresponding to the higher values of j. As we are intending to analyse gap soliton induced by instabilities of array solitons, we postulate that the gap zone being localized in the domain of strong perturbation, the equations of constants that can really describe this reality are those obtained for weak values of $j(j=0;1;2)$. So from Eq. (41), we obtain the following equations:

For $j=0$,

$$p_1 b_r + 2(-1)^n \Omega b_i \sin k - 4\Omega_0 \sin^2\frac{k}{2} b_r + (b_r^2 + b_i^2)b_r = 0. \qquad (43)$$

For $j=1$,

$$p_1 a_r + 2(-1)^n \Omega \sin k - 4\Omega_0 \sin^2\frac{k}{2} a_r + a_r\lambda^2 + 2|b|^2 a_r + a_r^2(b_r^2 - b_i^2) + 2b_r b_i a_i = 0. \qquad (44)$$

For $j=2$,

$$p_1 b_r - 2b_i \sin k + 4\Omega_0 \sin^2\frac{k}{2} b_r - 4\lambda b_r + 2b_r|a|^2 + b_r(a_r^2 - a_i^2) + 2a_r a_i b_i - 3|b|^2 b_r + |a|^2 a_r - 2|b|^2 a_r = 0. \qquad (45)$$

The above system of equations have four unknowns (a_r, a_i, b_r, b_i). Setting $a_i = b_i = 0$, makes Eq. (42) an obvious relation while simplifying the system equations (43) - (45) to

$$p_1 b_r - 4\Omega_0 \sin^2(k/2) b_r + b_r^3 = 0, \qquad (46)$$

$$p_1 a_r + 2(-1)^n \Omega \sin k a_r - 4\Omega_0 \sin^2(k/2) a_r + \lambda^2 a_r + 2b_r^2 a_r + a_r^2 b_r^2 = 0, \qquad (47)$$

and

$$-p_1 b_r + 4\Omega_0 \sin^2\frac{k}{2} b_r - 4\lambda b_r + 3b_r a_r^2 - 3b_r^2 + a_r^3 - 2b_r^2 a_r = 0. \qquad (48)$$

Solving equations (46) and (47), we obtain

$$b_r = \pm\sqrt{4\Omega_0 \sin^2(k/2) - p_1}, \qquad (49)$$

and

$$a_r = \frac{2(-1)^n \Omega \sin k + 4\Omega_0 \sin^2(k/2) + \lambda^2 - p_1}{p_1 - 4\Omega_0 \sin^2(k/2)}, \qquad (50)$$

with $4\Omega_0 \sin^2\frac{k}{2} \succ p_1$. The insertion of equations (49) and (50) in (48) as well as in the rest of the equations of the range of equation (41) can permit us to choose λ for example. Finally, using equations (49) and (50) into (39) gives

$$\psi_n = \left\{ \left[\left(\begin{matrix} 2(-1)^n \Omega \sin k \\ +4\Omega_0 \sin^2(k/2) \\ +\lambda^2 - p_1 \end{matrix} \right) \Big/ \left(\begin{matrix} p_1 \\ -4\Omega_0 \sin^2(k/2) \end{matrix} \right) \right] \sec h\lambda t \atop \pm\left(\sqrt{4\Omega_0 \sin^2(k/2) - p_1}\tanh^2 \lambda t\right) \right\} \exp -i(p_1 z + kn). \qquad (51)$$

We can note that the solution (51) is a function of the coefficients of the coupling strength Ω_0 and Ω, of the frequency of perturbation in the gap zone P_1 and of the discrete variable n. This remark allows us to confirm that the modulational instability of the array soliton combined to the discrete effects generates localized excitations in the gap zone of the perturbed domain. The gap soliton that can be considered here effectively is $\psi_n' = \pm\sqrt{4\Omega_0 \sin^2(k/2) - p_1}\tanh^2 \lambda t \exp -i(P_1 z + kn)$, which is a topological soliton of nature kink and localized in the gap.

5.2. Discussion

In this paper, we have clearly demonstrated that the modulational instability of an array soliton pulse type in a discrete array of nonlinear optical fibers subjected under the action of the periodic coupling strength generated the secondary excitations in the gap. These secondary excitations are of kink type. The investigated case here is closely linked to the array soliton. In other words, our analysis suggests the formation of progressive secondary excitations in the gap as a consequence of the modulational instability. In the following, we would like to explain the importance of this idea through figures.

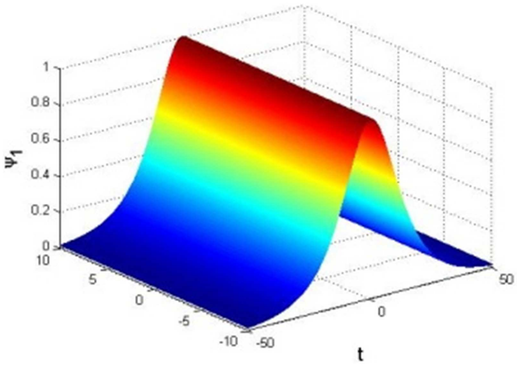

Fig. 4. Profile of the array soliton which is a pulse given by the intensity ψ_1 of the first part of equation (39) for $a=1$ and $b=0$.

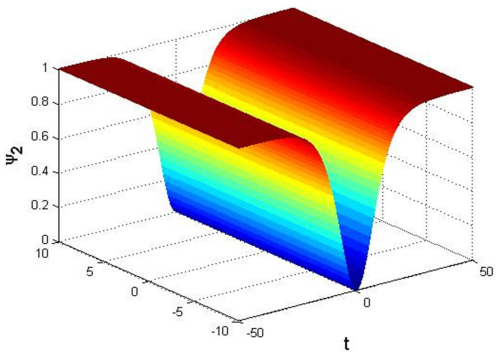

Fig. 5. Profile of the gap soliton which is a dark soliton given by the intensity ψ_2 of the second part of equation (39) for $a=0$ and $b=1$.

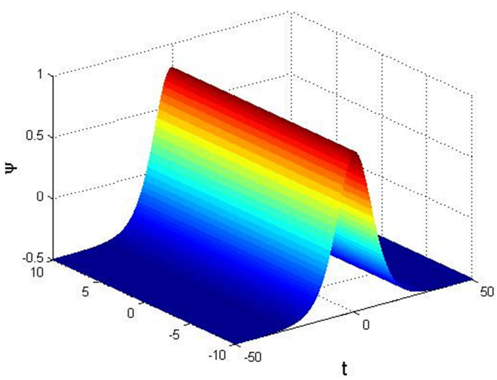

Fig. 6. The resultant wave given by the intensity ψ of the equation (39) for $a \succ b$ ($a=0.8$ and $b=$ -0.5).

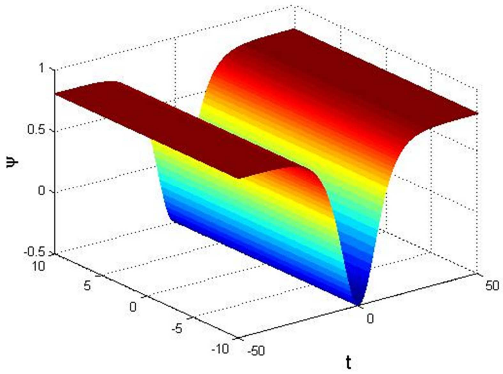

Fig. 7. The resultant wave given by the intensity ψ of the equation (39) for $a \prec b$ ($a=0.8$ and $b=$ -0.5).

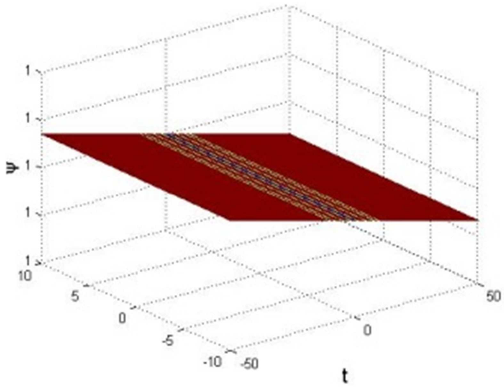

Fig. 8. Profile of the resultant wave of equation (39) when the characteristic coefficients a and b are equal: $a=1$ and $b=1$.

Figure 4 shows the profile of an array soliton of type pulse which must be injected in the network. The Figure 5 gives the profile of the gap soliton which forms in the gap area when there are instabilities; for this studied case, the gap soliton is dark soliton. These two profiles are extracted from Eq. (39). Figures 6 and 7 show the profiles obtained from the combination of the pulse soliton and dark soliton. We note that the resultant profiles in each case depends on the characteristic coefficients of each soliton. From equation (39), we see that a is a characteristic coefficient of the array soliton $\psi_1 = a\sec h\lambda t$ and b is a characteristic coefficient of the gap soliton $\psi_2 = b\tan h^2\lambda t$. As shown in equations (49) and (50) a and b depends on the parameters Ω_0 , Ω , k , n and P_1 . Thus, when $a \succ b$, the resultant wave which propagates in the array has the profile given by Figure 5. We also observe that this profile is the profile of a pulse which is subjected into an external perturbation due to the presence of another wave (dark soliton). When $b \succ a$, the resultant wave which propagates in the array is shown in Figure 6. This resultant wave has the shape of a dark soliton with a reduced high due to the external perturbation (pulse). Figure 8 presents the profile of the resultant wave of equation (39) when the characteristic coefficients a and b are equal; in this case a pulse and kink neutralized. These results suggest that when the characteristic coefficient of the array soliton is high, it resists better to the external perturbations (instabilities). On the other hand, when the characteristic coefficient of the gap soliton is high, the instabilities are very important in the array, hence the dissipation of the pulse induces the formation of the gap soliton. When the instabilities persist, the soliton pulse can completely disappear.

6. Conclusion

This work dealt with the analytical analysis of modulational instability of array soliton in an array of discrete and nonlinear optical fibers. From the analysis that we made, it can be seen that the double action of the modulational instability and the discrete effects are at the origin of another type of gap excitations localized in the domain where the perturbation exist. It was possible thanks to the perturbation of the array soliton through what we obtained the equations that govern the perturbation. Thereafter the exploitation of these perturbation equations leaded to the obtention of the temporal scattering relation. Its survey shows as all relation of dispersion, a upper branch and a lower branch separating a gap zone, which is the seat of the secondary excitations. The interpretation we made is that from the fact these secondary excitations appear under the effect of the instabilities, they also vanish as soon as the instabilities disappear. We suspect that these gap solitons are topological and especially of kink type. The hold in account of the array soliton $g(t)$, in the main equation is not always easy to manage. In this study, we are obliged to assume that $g(t)$ is stationary. But we intend in future numeric studies, to consider $g(t)$ in its entire form to better appreciate its impact. It is vital to note that the main goal of this work is not to study the modulational instability in its totality because modulational instability was already the subject of many works, but to show that another consequence of the modulational instability in an array of optical fibers can be the appearance of the secondary excitations as the soliton of gap. This idea that deserves to be deepened by experimental works remains opened and we think that it will hold the attention of many researchers in the future.

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