

# Static Friction in Adhesive Contact of Rough Surfaces with Soft Coatings

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## Abstract

A numerical study of static friction in adhesive contact between rough surfaces with soft coating is presented using an elastic-plastic model of asperity contact deformation. The analysis considers the elastic and plastic deformation of both the coating and the substrate unlike the work available in literature where the coating is considered to be in pure plastic contact. The JKR (Johnson-Kendall-Roberts) adhesion model is used and the well-established adhesion indices (elastic and plastic adhesion indices) are utilized to consider the different conditions of varying load and material and surface properties of the substrate and the coating. Contact load and friction force are obtained as functions of mean separation between surfaces for different combinations of adhesion parameters, material properties and thickness of the coating. The effects of these parameters on frictional contact behavior of coated surfaces are investigated. For thin coating and light loading, frictional contact behavior is strongly influenced by the existence of soft coating that increases the contact area due to plastic deformation of the coating.

## Keywords

Static Friction, Adhesion, Roughness, Soft Coatings

Received: August 24, 2015 / Accepted: September 13, 2015 / Published online: October 16, 2015

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## 1. Introduction

Friction is inevitable in all machine components and a number of models have been proposed for explaining or predicting friction. Tabor [1] suggested three basic elements to be involved in friction of unlubricated materials. These are a) contact loads, which are related to the true area of contact between rough surfaces b) intermolecular (adhesion) forces relating the strength of the bond formed at the interface where the contact occurs and c) tangential force needed to shear the contact. The static coefficient of friction  $\mu$  is usually defined as the ratio of the tangential force  $Q$ , needed to shear the junctions between the contacting asperities and the external force  $F$ . Equations of the form  $\mu = Q / F = Q / (P - F_s)$  have been used to [2] include the effect of surface roughness and intermolecular adhesion forces ( $F_s$ ) and  $P$  is the contact load. In tribological components surface coatings are used to modify the

tribological properties of the interfaces, in particular to reduce friction and wear. The mechanical properties of the coatings are usually highly process dependent and the reliability of coatings from the tribological perspective depends on a number of factors such as adhesion, coating thickness, surface roughness and relative strength properties of the coating material compared to the bulk etc. In general, good results have been achieved by a trial and error procedure concerning both the choice of material and the tribological coating parameters. In coated contacts four main parameters controlling friction can be identified. They are (i) the coating to substrate hardness relationship (ii) the thickness of the coatings (iii) the surface roughnesses and (iv) the size and hardness of the debris in the contact. The relationship between these four parameters will result in a number of different contact conditions characterized by specific tribological contact mechanisms. It is important to understand the role of these factors on coating performance to help in design or select better coating. Bowden and Tabor

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[3] measured the friction of surfaces with soft metallic coatings and it has since been shown that the coating thickness, substrate surface roughness and normal contact pressure are the three major factors that determine the performance of coating [4, 5]. The models of soft metallic coatings available in the literature were introduced by Dayson[4], Finkin [6], Halling [7], Kato et al. [8], and Ogilvy [9]. Dayson [4] considered a plastic contact model, Finkin [6] considered elastic deformation of coating with rigid substrate, Halling [7] assumed that both the coating and the substrate deformed elastically during contact, Kato et al. [8] considered plastic deformation of the coating with rigid substrate, while Ogilvy [9] concluded that the coating's contribution could be ignored in normal contact analysis. Definitely a complete analysis of coated contact must consider the elastic-plastic deformation of both the coating and the substrate. Chang [10] presented an elastic-plastic analysis but there also it was assumed that the coating deformed plastically with elastic-plastic deformation of the substrate. In all the above models, however, adhesion was not considered though adhesion at the contact of surfaces plays a great role in modifying the tribological behavior. Only recently, Liu et al. [11] have extended Chang's approach to static friction prediction in case of coated contacts considering the DMT (Derjaguin-Muller-Toporov) [12] adhesion model.

In the present work, an elastic-plastic adhesive frictional contact of rough surfaces with soft coatings is presented where the elastic-plastic deformation of the coating as well as the substrate is considered using Roy Chowdhury and Ghosh's approach [13] that uses the JKR (Johnson-Kendall-Roberts) adhesion model [14].

## 2. Loading Analysis

Any adhesive contact is described in terms of two adhesion indices: elastic adhesion index,  $\theta = K\sigma^{3/2}R^{1/2}/(\gamma R)$  and plastic adhesion index,  $\lambda = \pi^2 RH^4\sigma/(18K^2\gamma^2)$ . Here  $\sigma$  is the standard deviation of surface heights,  $K = \frac{4}{3}E$ ,  $E$  being  $\left[ \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{2} \right]^{-1}$ , where  $E_1$  and  $E_2$  are Young's moduli,  $\nu_1$  and  $\nu_2$  are Poisson's ratios of the two surfaces,  $R$  is the asperity radius,  $H$  is the hardness of the softer material and  $\gamma$  the work of adhesion. These indices are merely the ratio of the elastic or plastic force needed to push a sphere of radius  $R$  to a depth  $\sigma$  into an elastic solid of equivalent modulus of elasticity  $E$  to the adhesive force experienced by the sphere. Limiting values of  $\theta$  and  $\lambda$  are usually quoted as 10 and 0.125 respectively, beyond which the effect of surface

adhesion become insignificant indicating that the surfaces are sufficiently rough for the surface forces to be inoperative. Following Roy Chowdhury and Ghosh [13] the contact load may be non-dimensionalized and written in terms of adhesion indices  $\theta$  and  $\lambda$  as

$$\bar{P}_l = \int_{\Delta_0}^{\Delta_{c1}} \left( \Delta^{3/2} - \frac{4.34}{\theta^{1/2}} \Delta^{3/4} \right) \bar{\varphi}(\Delta) d\Delta + \int_{\Delta_c}^{\infty} \left( \frac{7.3\lambda^{1/4}}{\theta^{1/2}} \Delta - \frac{6.28}{\theta} \right) \bar{\varphi}(\Delta) d\Delta \dots \quad (1)$$

Where  $\bar{\varphi}(\Delta)$  is the normalized asperity height distribution function in terms of normalized asperity deformation ( $\Delta$ ),  $\bar{P}_l = \frac{P_l}{KNR^{1/2}\sigma^{3/2}}$ ,  $h = \frac{d}{\sigma}$ ,  $\Delta = \frac{\delta}{\sigma}$ ,  $\Delta_c = \frac{\delta_c}{\sigma}$ ,  $\Delta_{c1} = \frac{\delta_{c1}}{\sigma}$ . Here  $d$  is the mean separation between surfaces and  $N$  is the number of asperities per unit area. In equation (2),  $\Delta_0$  and  $\Delta_{c1}$  are the non-dimensional apparent displacements and  $\Delta_c$  non-dimensional real displacement to be obtained from the following relations [13]

$$\Delta_{c1}^{3/4} - 3.65 \frac{\lambda^{1/4}}{\theta^{1/2}} \Delta_{c1}^{1/4} - \frac{4.34}{\theta^{1/2}} = 0 \dots \quad (2)$$

$$\Delta_0 = \frac{4.125}{\theta^{2/3}} \dots \quad (3)$$

$$\text{and } \Delta_c = \Delta_{c1} - \frac{2.89}{\theta^{1/2}} \Delta_{c1}^{1/4} \dots \quad (4)$$

Traditionally, the Gaussian distribution is used to model the asperity heights and is given in normalized form by

$$\bar{\varphi}(\Delta) = \frac{1}{\sqrt{2\pi}} \exp\{-(h+\Delta)^2/2\} \dots \quad (5).$$

In the present case, it is considered that the coating and substrate are different entity and the analysis has been carried out twice once for substrate and then for coating by considering elastic and plastic adhesion indices for coatings as  $\theta_c = \theta n_E$  and  $\lambda_c = \lambda n_H^4/n_E^2$ , where  $n_E$  is elastic modulus ratio of coating and substrate material and  $n_H$  is hardness ratio of coating and substrate material. It is assumed that each surface is covered with a uniform layer of soft solid coating with small thickness compared to the asperity radius and therefore the topography of the resulting surfaces is the same as the original solid surfaces. The coating on the two contacting rough surfaces can be put onto one surface and the equivalent coating thickness,  $t$ , is equal to the sum of the coating thickness on two surfaces. The total applied load on all the asperities per unit area considering the coating and the

substrate may then be written as [15]

$$\begin{aligned} \bar{P}_t = & \int_{\Delta_{c0}}^{\Delta_{cc1}} \left( \Delta^{3/2} - \frac{4.34}{\theta_c^{1/2}} \Delta^{3/4} \right) \bar{\varphi}(\Delta) d\Delta \\ & + \int_{\Delta_{cc}}^{\bar{t}} \left( 7.3 \frac{\lambda_c^{1/4}}{\theta_c^{1/2}} \Delta - \frac{6.28}{\theta_c} \right) \bar{\varphi}(\Delta) d\Delta \quad \dots \quad (6) \\ & + \int_{\bar{t}+\Delta_0}^{\Delta_{c1}} \left( \Delta^{3/2} - \frac{4.34}{\theta^{1/2}} \Delta^{3/4} \right) \bar{\varphi}(\Delta) d\Delta \\ & + \int_{\Delta_c}^{\infty} \left( 7.3 \frac{\lambda^{1/4}}{\theta^{1/2}} \Delta - \frac{6.28}{\theta} \right) \bar{\varphi}(\Delta) d\Delta \end{aligned}$$

Here the first two integrals represent the elastic and plastic contribution to load by the coating and the last two integrals represent the same for the substrate respectively.  $\bar{t}$  ( $\bar{t} = t / \sigma$ ) is the non-dimensional coating thickness.  $\Delta_{c0}$  and  $\Delta_{cc1}$  are the non-dimensional apparent displacements for the coating similar to  $\Delta_0$  and  $\Delta_{c1}$  for the substrate.  $\Delta_{cc}$  is the non-dimensional real critical displacement for the coating. The values of these variables can be obtained from the following equations [15],

$$\Delta_{cc1}^{3/4} - 3.65 \frac{\lambda_c^{1/4}}{\theta_c^{1/2}} \Delta_{cc1}^{1/4} - \frac{4.34}{\theta_c^{1/2}} = 0 \dots \quad (7)$$

$$\Delta_{c0} = \frac{4.125}{\theta_c^{2/3}} \dots \quad (8)$$

$$\text{and } \Delta_{cc} = \Delta_{cc1} - \frac{2.89}{\theta_c^{1/2}} \Delta_{cc1}^{1/4} \dots \quad (9)$$

### 3. Friction Analysis

The maximum tangential resistance offered by an elastically loaded asperity is governed by two failure criteria. One is slipping and the other is yielding at the surface. Following the work of Savkoor and Briggs [16], the critical value of tangential force ( $\bar{T}_{01}$ ) beyond which slipping will occur is given in the non-dimensional form [3] as

$$\bar{T}_{01} = \frac{4}{(K\beta)^{1/2}} \left( \frac{6.28}{\theta} \Delta^{3/2} - \frac{29.61}{\theta^2} \right)^{1/2} \dots \quad (10)$$

where  $\beta = \frac{2-v_1}{G_1} + \frac{2-v_2}{G_2}$ ,  $G_1$  and  $G_2$  being the shear moduli of the contacting materials. Using Hamilton's stress field [17] and the von-Mises yield criterion, the non-dimensional deformation ( $\Delta_{tc1}$ ) at which yielding will occur at the surface for initially elastically loaded asperities may be obtained from the following condition [13]

$$\begin{aligned} C_1 \Delta^3 + C_2 \Delta^{9/4} + C_3 \Delta^2 + (C_4 + C_5 \bar{T}_{01}) \Delta^{3/2} \\ + C_6 \bar{T}_{01} \Delta^{3/4} + C_7 \bar{T}_{01}^2 = 0. \quad \dots \quad (11) \end{aligned}$$

where the coefficients  $C_1, C_2$  etc. may be found in Ref. [13]. The total non-dimensional frictional force due to all such asperities is given by

$$\bar{T}_1 = \int_{\Delta_0}^{\Delta_{cc1}} \frac{4}{(K\beta)^{1/2}} \left( \frac{6.28}{\theta} \Delta^{3/2} - \frac{29.61}{\theta^2} \right)^{1/2} e^{-(h+\Delta)^{2/2}} d\Delta \dots \quad (12),$$

Where  $\bar{T}_1 = \frac{(2\pi)^{1/2}}{N} \bar{T}_{01}$  Asperities that are not deformed plastically under normal load alone, would resist some tangential force and this traction force  $\bar{T}_{02}$  may be obtained in non-dimensional form by replacing  $\bar{T}_{01}$  by  $\bar{T}_{02}$  in the equation (11) and solving for  $\bar{T}_{02}$ . The total non-dimensional traction force due to all such asperities is

$$\bar{T}_2 = \int_{\Delta_{c1}}^{\Delta_{c1}} \bar{T}_{02} e^{-(h+\Delta)^{2/2}} d\Delta \dots \quad (13).$$

Assuming contribution of plastically deformed asperities to be negligible the sum total of traction forces due to all such asperities is given by  $\bar{T} = \bar{T}_1 + \bar{T}_2$  and coefficient of friction is

$$\mu = \bar{T} / \bar{P}_t \dots \quad (14).$$

For the present case considering the presence of coating and substrate, the  $\bar{T}$  is obtained as

$$\begin{aligned} \bar{T} = & \int_{\Delta_{c0}}^{\Delta_{cc1}} \frac{4}{(K\beta)^{1/2}} \left( \frac{6.28}{\theta} \Delta^{3/2} - \frac{29.61}{\theta^2} \right)^{1/2} e^{-(h+\Delta)^{2/2}} d\Delta \\ & + \int_{\Delta_{cc1}}^{\bar{t}} \bar{T}_{02} e^{-(h+\Delta)^{2/2}} d\Delta \quad \dots \quad (15) \\ & + \int_{\Delta_{c0}+\bar{t}}^{\Delta_{c1}} \frac{4}{(K\beta)^{1/2}} \left( \frac{6.28}{\theta} \Delta^{3/2} - \frac{29.61}{\theta^2} \right)^{1/2} e^{-(h+\Delta)^{2/2}} d\Delta \\ & + \int_{\Delta_{c1}}^{\Delta_{c1}} \bar{T}_{02} e^{-(h+\Delta)^{2/2}} d\Delta \end{aligned}$$

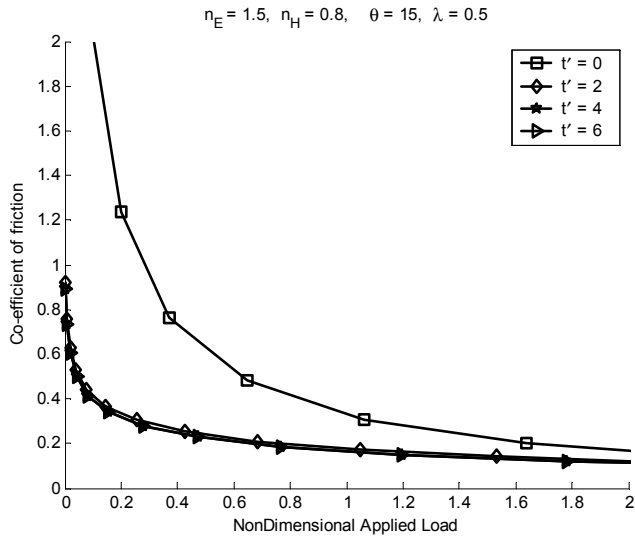
Here also, the first two integrals are corresponding to the coating and the last two integrals are substrate and  $\Delta_{tc1}$  is the non-dimensional deformation of the coating at which yielding occurs in combined loading.

### 4. Results and Discussion

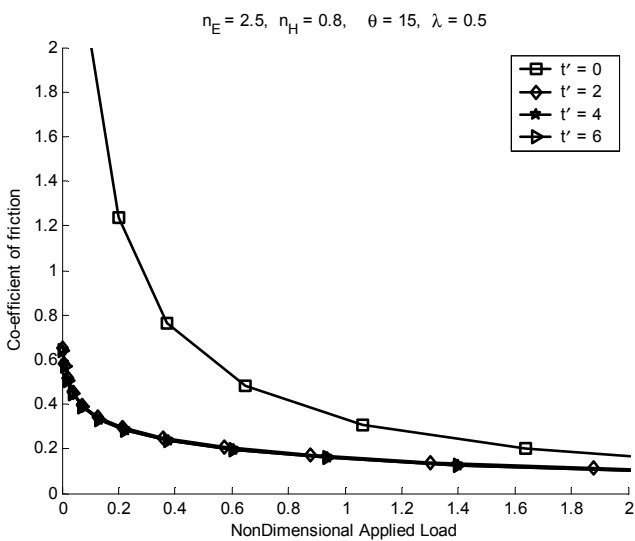
Equations described in the above sections are solved

numerically to obtain the non-dimensional loading and frictional force for different combinations of non-dimensional mean separation,  $h$ , elastic modulus ratio of coating and substrate material,  $n_E$ , hardness ratio of coating and substrate,  $n_H$  and adhesion indices  $\theta$  and  $\lambda$ .

basic assumption of no interaction between them. If  $\sigma$  is assumed to be of the order of  $4.8 \times 10^{-8}$  m [11] and the thickness of coating,  $t$ , in the order of 100–300 nm in accordance with practical values of ion-plated soft metallic coatings [8], then non-dimensional coating thickness  $\bar{t}$ , in figures denoted as  $t'$ , works out to be in the order of 2–6. The values of  $n_E$  are taken as 0.5, 1.5 and 2.5 and  $n_H$  are 0.2, 0.5 and 0.8 in accordance with practical values for soft coatings.



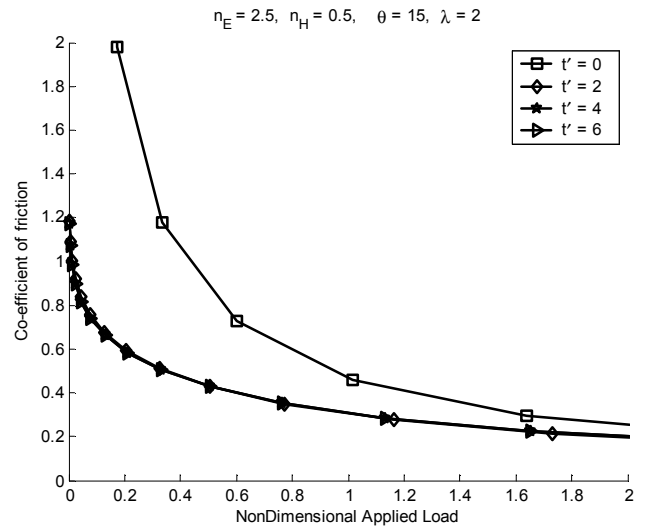
(a)



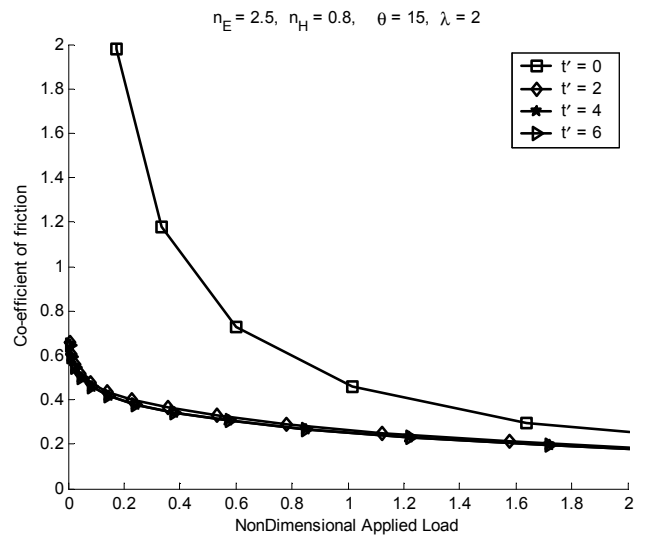
(b)

Fig. 1. Applied load against co-efficient of friction for varying elastic modulus ratio.

In view of the limiting values, typical combinations of  $\theta$  and  $\lambda$  are considered in the present case, in order to analyse the effect of elastic and plastic adhesion. The non-dimensional mean separation ( $h \equiv d / \sigma$ ) is considered between -4 to 4. At larger separations,  $h > 0$ , the number of asperities in contact is small, as evidenced from the magnitude of contact load. On the other hand, for very small separation  $h < -4$ , the present model may give erroneous results since asperities may undergo very large deformation, which may violate the



(a)



(b)

Fig. 2. Applied load against co-efficient of friction for varying hardness ratio.

Figs. 1 and 2 show the effects of variation in the elastic modulus ratio and hardness ratio in frictional behavior respectively. In general the co-efficient of friction is significantly decreases for the coated surfaces. The same effect is prevailing independent of coating thickness. This is

desirable in situations like, ceramic coating on a hard steel to reduce friction and wear. The possible explanation for this behavior can be given from work of Djabellah and Arnell [18], that they have proved in their FEM analysis for this type of situations that the in-built highest shear stress contours are shifting towards the surface which leads to relatively minimum shear stress required for an asperity to fail.

This shifting of contours is more pronounced in the case of relatively high coating thickness. But it is observed in Fig. 3 that, when the elastic modulus ratio is less than one the coefficient of friction becomes higher than the uncoated surface. In practice, many coatings contain residual stress, which often increase with increasing coating thickness under such circumstances, the combined effects of surface fractions and residual stress would need to be considered in choosing an optimum film thickness. So, it requires further investigation. The constant value of the co-efficient of friction at high load probably indicates the deformation of small scale is complete and the frictional behavior approaches the bulk behavior.

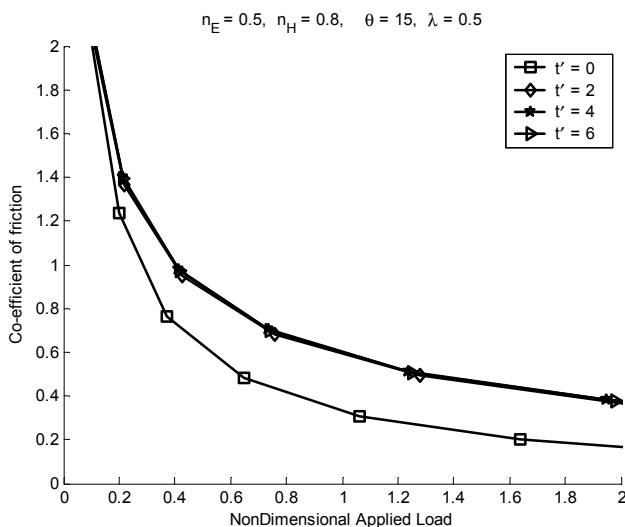


Fig. 3. Applied load against co-efficient of friction for  $n_E=0.5$ ,  $n_H=0.8$ .

The three-point peak (asperity) of the Greenwood-Williamson model has formed the basis of analysis for major analyses. But recently, in spite of the wide acceptance and popularity, adequacy of this model has been questioned by Greenwood [19] himself. According to him his original idea of three point peaks is incorrect as an asperity cannot be defined only as a point higher than its two immediate neighbors. The GW model has been found in good qualitative agreement with experiments, but all attempts to obtain quantitative agreement face the difficulty of obtaining unique values of summit density and curvature. Therefore, the approximation of asperities in terms of peaks or summits is problematic and in an engineering problem, an asperity

should be related to a contact. This paved the way for introduction of a new multiple-point asperity model called the n-point asperity model, put forward by Hariri et al [20]. The n-point asperity model developed by Hariri et al. [20] defines the rough surface in more realistic way as compared to other available techniques. So use of this model is expected to yield comparatively more accurate results of analysis. Based on this new n-point asperity model, the rough surface contact problems for various contact conditions have been considered earlier [21-26]. Future studies will consider the present analysis using n-point asperity model.

## 5. Conclusions

A numerical study of adhesive frictional contact between rough surfaces with soft coating is presented using an elastic-plastic model of asperity contact deformation. The analysis considers the elastic and plastic deformation of both the coating and the substrate. The co-efficient of friction is significantly reduced for the coated surface and it is independent of coating thickness and hardness ratio when elastic modulus of coating is higher than the substrate. However, when elastic modulus of the coating is smaller than the substrate, the friction co-efficient is increased due to the presence of the coating. The above conclusions agree well with available literatures.

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