Physics Journal
Vol. 1, No. 3, 2015, pp. 245-254
http://www.aiscience.org/journal/pj

Dimension of Space Is It constant

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Abstract

Last time cosmological constant (CC) has to be important role in understanding of Universe nature. Explanation of CC appearance described by changing of space dimension is presented. The conditions that determine the dimension of the space, which describes the specific physical interaction, are discussed in various approaches. The dimension of the Universe has not necessarily fixed value. Then it reduces the variation principle framework allow to emphasized the CC as Lagrange's multiplier introduced for search of a conditional extremum of action in this reduction. Cosmological constant is interpreted as the energy density released in the remaining dimensions then space dimension is reduced.

Keywords

Cosmology, Cosmological Constant, Quintessence, Dark Energy, Space-Time Dimension

Received: August 16, 2015 / Accepted: September 18, 2015 / Published online: October 16, 2015

1. Introduction

The development of philosophical ideas about the structure of the world around us leads to the conception of space as a category characterized by the length, width, and depth. It seemed natural that the space acts as only "containers of things" and it is three-dimensional.

Experimental development of electrodynamics at a turn of 19-20 centuries leads G. Lorentz when considering the electron to some convenient transformations of coordinates and time. A. Poincare generalizes them to absolute accuracy and considers as elements of transformations of four-dimensional manifold, the coordinates of which (spatial coordinates and time) preserving the interval [1].

However, the four-dimensionality of space lasted not long. Already the first attempts to unify the gravitational and electromagnetic interactions, undertaken as part of the extension of the General relativity (GR), showed that the dimension must be greater than four. The paper presents a possible interpretation of the nature of the CC and its relationship with the dimensionality of space-time with current approaches to its description.

2. Local Physical Theories

Significant advances in the physics associated with the use of Lagrangian functions with local density. The Lagrange function (Lagrangian) or its density is determined on spatial structures, the simplest of which is the point of space. Such Lagrange functions are called local.

It seems that the currently known interactions – gravitational, electromagnetic, weak and strong are manifestations of a single interaction, which breaks down into these components by decreasing the characteristic energy of interaction of their representatives. The success of A. Salam and S. Weinberg in the unification of electromagnetic and weak interactions [2, 3] strengthened this conviction. It is estimated [4], the unification of the electroweak and strong interactions in such a case should occur at energies of the order of 10^{16} GeV (Grand Unification), and all four interactions at energies of the order of 10^{19} GeV.

One of approaches to unification is to consider multidimensional spaces which structure of a metric tensor allows to describe not only the gravitational field, but also

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other united fields. T. Kaluza [5, 6] approach for the unification of gravity and electromagnetism was the first of such approaches. Fifteen 5-dimensional Hilbert-Einstein's equations (HEE) split into ten ordinary 4-dimensional HEE, four Maxwell's equations and an additional scalar equation. It is believed that the 4th spatial dimension is curtailed to the circle of the small size (compactification), not accessible to direct perception. Low-lying spectra of systems with and without compact curtailed dimensions are the same. The difference of spectra begins with terms of order a/R, where a is the characteristic size of the system, R is the radius of the compact dimension, which collapsed one of the extra coordinates [7]. When R tends to zero, the corrections tend to infinity. While in the experiment is not visible contribution of such corrections.

A similar approach to the unification of gravi-electo-weak interactions leads to a 6-d space of one generation to the 7-d space for three generations of leptons. In the case of unification of gravi-electro-strong interaction space should be 7-d for the three generations of quarks and 8-d with additional accounting tree-color quark nature [6, 8].

But success in the description of the physical picture with local Lagrangians was followed by also conceptual diffficulties of such description which aren't resolved yet. Consider some of them.

The equality of inertial and gravitational masses. The equality of gravitational and inertial masses in GR is considered as an exact law of nature [9]. It is believed that this fact is proved in the theory [9-12], his "evidence" presented for example in [13-15]. But actually, the equality of the masses in the theory is valid only in the systems of a special type, in particular, in Cartesian. In other systems, inert mass can take arbitrary both positive and negative value [16].

Conservation laws in GR. There is also a more key problem of energy-momentum definition of gravitational energy and conservation laws in GR. Great efforts to resolve it has not led to success. The introduction of Einstein pseudo-tensor seemed to chart a path to its solution. But in this case, the energy of the gravitational field changes with spatial-temporal transformation (for example, R. Boyer paradox). C. Möller formulates conditions on pseudo-tensor gravitational field, which should preclude getting absurd results [17, 18], but he also proves the theorem that they can not be satisfied in principle [13].

Gravitational waves. J. Weber announces the registration of gravitational waves in 1969, however, the data have not been confirmed. The theoretical framework for the interpretation of this phenomenon is also encountered some difficulties. Wave carries energy and momentum, but their definition has serious problems (see above). It is difficult to specify general covariant criterion for the wave nature of effect associated with the decisions of the HEE, there are difficulties with using reference systems (reference criterion), with the definition of dynamical (orientational, polarizational) degrees of freedom, it is not even clear what exactly the curvature of space-time should be considered as a gravitational wave. Many problems disappear in a linearized version of the gravitational waves description. But HEE is nonlinear, for them the principle of superposition is not working, which is why it is not clear how to separate wave solutions from non-wave. Quantum theory of linearized gravity is non-renormalizable, the attempt to clarify it causes infinite values which aren't eliminated by the introduction of a finite number of counterterms [6].

Many difficulties in the description of gravity, seems to be connected with the fact that it can not be quantized. Attempts are being made to move the apparatus of quantization developed in electrodynamics, on gravity. So there is a large number of equivalent formulations of GR description of gravitation: geometric, Lagrangian, Hamiltonian, involving different-variables: metric, tetrad, dyadic, monadic, diaric, formalism A. Palatini, way dependent formulation of L. Mandelstam and others, as well as a set of formulations to describe the quantization – canonical quantization, quantization based on the canonical P. Dirac formalism, R. Peierls covariant quantization method, J. Schwinger source theory, etc. [19]. It becomes clear that failures are not random in nature. In fact, the path to the quantization of gravity was developed in two approaches: an attempt to derive quantum theory from geometry, or to obtain the geometry from quantum theory. None of the approaches were successful. Regarding the first approach, noted above, W. Heisenberg wrote that A. Einstein overestimated the possibility of a geometric point of view [20].

In the second approach the unification of interactions on the basis of a multidimensional space using a calibration approach leads to asymmetry of bosonic and fermionic components. It seemed that the use of wave functions in the form of multiplets uniting categories of boson and fermion fields (SUSY) should correct the situation. Some of the divergences associated with the boson fields, partially offset by a divergence associated with fermion fields, but a complete solution was not achieved.

After the failure of gravity quantization is putting forward the idea of a secondary nature of curved space-time ([21] and comments of leading scientists wherein). It is assumed that gravity is not a fundamental interaction and represents a macroscopic (long wavelength) limit of a more General theory due to quantized fields [14].

So, in theories with local Lagrangians dimensionality of space-time is obtained as a consequence of the expansion of the metric tensor to include components that describes the other, not gravitational interaction. The above-mentioned difficulties should be borne in mind in respect to the completeness of this description.

3. Nonlocal Physical Theories

The object following on complexity after a point is the one-dimensional structure – the line. Using theories with local Lagrange functions greatly simplifies the analysis, but it does not follow from anywhere that it is the only possible physical theories. Lines on which Lagrange's function is defined, are called as strings, respectively, we deal with string theories.

Basically, optimism when considering string theory is related to the fact that this approach can lead to quantization of gravity. As it was already noted above, gravitational interaction isn't quantized yet, gravitational waves are also not found yet. However, the gravitational interaction should, apparently, in the case of a positive decision of these tasks, be carried out by particle with spin equal to 2 [6, 22, 23]. In the spectrum of string vibrations was detected particle with spin 2 and this is the main argument that the physics is nonlocal and that further development of the theory to be found in the string approach. Note that while all the advantages of string theory remain without experimental confirmation.

The main focus of research with nonlocal Lagrangians while focused on the examination of the strings. But we should remember that the one-dimensional objects just a special case of non-local objects. In such approach it is necessary to analyze also objects of higher dimension – membranes ($n \geq 2$) [24]. String theory can really be productive if the contribution of membrane theories in the description of physical processes will be small. But the development of membrane theories practically not moving and this is due to their significant nonlinearity [25].

What gives string approach in the question of the dimensionality of our space? Quantization of bosonic strings sector leads to transverse Virasoro operators of the different modes. Through the Virasoro operators there are expressed the generators of the Lorentz coordinate transformations of strings (Lorentz charges), the mass of the string is expressed through the Virasoro operator of zero mode, etc. Calculation of various characteristics containing these operators is rather specific and uses, for example, analytical continuation of Zeta-function of Riemann. Commutation relations for the generators of Lorentz charges depend on both the Virasoro operators and the dimension of the considered space. The correct commutation relations are obtained when the dimension of the space in which strings are considered is equal to 26. The inclusion of the fermion sector leads to superstring theory. A similar calculation in superstring theory leads to a 10-dimensional space-time [7].

There are five types of non-trivial 10-dimensional supersymmetric theories of superstrings: types I, IIA, IIB, and heterodyne $E_8 \times E_8$ and SO (32) types. It turned out, for example, that at a certain dimensional reduction of 11-dimensional membrane ($n = 2$) theories turns out superstrings like IIA [26]. From the 11-dimensional theory containing membranes with $n = 2$ and 5 (M5-brane is magnetically dual to M2-brane in 11 dimensions) in various limits it is possible to receive all 5 types of 10-dimensional theories of superstrings. This theory was called M-theory. Note that the 11-dimensional supergravity theory is also obtained from M-theory in the low-energy limit. The meaning of M-theory is not yet fully understood. It is clear only one that five theories of superstrings and the M-theory are the different parties or limits of one theory [7, 27].

4. The Description of Geometry on the Basis of Physical Structures

There is also an approach to the description of the geometry on the basis of physical structures. This approach is not based on search of the "original matter", i.e., not on traditional historical path of development, but based on search of the "original structure". In particular, it does not contain nor the fields, nor the space. The type of the relations between elements of structure allows to see a prototype of the arising concepts of space and interaction [6, 19, 28-30].

Unary physical structures. The basis of the approach is put a set of elements, the number of which is the rank structure r, and with pair relationships a_{ik} between them. Looking for the general form of the function

$$F_1 = 0 \tag{1}$$

establishes a connection between these relations. All possible laws (1) subjecting the structure elements are searched. These laws are found from the requirement that arbitrary r elements do not coincide with the originally chosen, would lead to the same identity (1), which satisfies the original items. The resulting solution of (1) have a kind of connections between $n = r - 2$ elements of the structure, the number n and is associated with the dimension of the space. These connections have form of the quadratic relationship between $r - 2$ elements of the structure and directly interpreted as the square of the length between n "coordinates" of elements.

This approach, for a set of 5 elements leads to a three dimensional spaces with different geometry: Euclidean, pseudo-Euclidean, the first non-Euclidean geometry (Lobachevsky), the second non-Euclidean geometry (Riemann), a kind of symplectic geometry, etc. Also three virtually unknown exotic geometries are established, mention of which were later found in the forgotten works of geometers.

Binary physical structure or binary system of complex relations (BSCR). In this case binary connections or the relations between r elements of one set and s elements of other set are established, i.e. binary structures of a rank (r, s) are considered. Also as well as in an unary case the general view of the functions establishing connection between these relations is looked for. It turns out that non-trivial are the only structures of rank (4, 2), (2, 4), (r – 1, r), (r, r), (r, r + 1). Unlike a case of unary structures, for system of binary structures the problem about a type of functions F_2 is solved in a general form. Unary relation systems can be obtained from binary systems by gluing certain elements of different sets, and the relationships between them are built from primary binary relations. The resulting unary relations lead to a space prototype (see above).

5. What is Actually the Dimension of the Space

Local theories. In describing of our world by local theories dimension of the space increases with an increasing number of interactions. Thus, the presence along with gravitational also electromagnetic interaction increases the dimension from 4 to 5 and so on, taking into account all known interactions leads to the 11-dimensional space. Now it seems that the evolution of our world is in accordance with the concept of the Big Bang (BB) [31-34]. If there are interactions, we do not know, then the dimension of the space should be more than eleven. This is possible at the time scale, for example, from 10^{-32} to 10^{-12} sec, when in accordance with the concept of BB splitting of interactions was started. To date, representatives of these unknown interactions could disappear (for example, by the processes of annihilation) or their detection is impossible for any reasons. It is also possible that some representatives of these interactions remained in vanishingly small quantities, but, because of its high energy, can not be available to modern measurement technology. But at the moment (13.7 billion years) we can't completely exclude presence of unknown of interactions. It is known that only the baryonic component of matter is not able to explain the origin of galaxies [31]. The difference between the observation and the dynamic mass of matter in the Universe indicates that should be the mass, called the dark mass, which is at least three times exceeding that of the baryon, does not interact with radiation, but interacts with ordinary matter (and to itself) only gravitationally. The most likely candidate for the dark matter was put forward neutrinos because they interact very weakly with each other, with an ordinary baryonic matter and with radiation. However, their too fast motion can not explain the growth of small-scale heterogeneities and by such an explanation had to be abandoned [35]. At this moment there are no candidates for the role of the dark mass carrier [33, 35-37]. So we don't know in what interactions, except gravitational, the dark matter participates. The same can be said about dark energy (see below) – its composition is unknown.

Nonlocal theories. When considering the description of our world by nonlocal theories the received dimension of space (eleven) is based on existence of the M-theory of the same dimension, the corresponding limits of which lead to the known 10-dimensional string theories. In particular, certain reduction of Lagrangian membrane theory (n = 2) leads to the Lagrangian of string theory [26]. The relationship of Lagrangians of membrane theories now is not fully understood. Existing reductions does not exclude the existence of other reductions, thus the dimension of the space in this approach may also increase. Furthermore, spectra of string theories should reproduce the known particles. If we reveal unknown to us interaction, it will lead to reproduce in the spectrum of string theory also their representatives, i.e. we need increase the dimension of the space.

Theory of structures. Classical space-time can be described within BSCR of a rank (3, 3). Respectively, we come to 4-dimensional space with admissible signatures (+, +, +, +), (+, +, +, –), (+, +, – –) and equivalent signatures with replacement of pluses (minuses) by minus (plus). For the construction of a multidimensional theories of Kaluza-Klein it is necessary work with BSCR rank (4, 4) [29], respectively, the dimension of the spatial structures would not exceed nine. Description of leptons and quarks is already possible under BSCR rank (6, 6) [6], resulting dimension can reach to 25. With increasing of number of interactions grows the rank of structure and, respectively, the dimension of the space. Thus, as in the case of describing by means of local and nonlocal Lagrangians a growing number of interactions leads to expansion of the space dimension.

Multidimensional time. The main structures considered above use a one-dimensional time, because they have a direct connection with the Universe in which we live. One should not reject the possibility of worlds in which time is multidimensional. The possibility of the existence of space with signature (+, +, –, –) or (–, –, +, +), noted above, is already evidence of the possible existence of two-dimensional time.

Note. In monadic method of description, reference systems in which the lines of congruence are parallel to temporal coordinate of the considered manifolds (chronometric) represent a special case [6]. In the early works on special relativity temporary coordinate was chosen proportional to the imaginary unit to provide pseudo-Euclidean space. In non-chronometric systems monad vector will not be parallel to temporal coordinate. The coordinates of the event will be presented by complex numbers, real and imaginary parts which are different from zero. The possibility of generalization monadic description from the field of real numbers to description in the field of complex numbers can be associated with the possibility of introducing temporal coordinate. The possibility of generalization to the description in the field of quaternions (there are only two non-trivial generalization (ordinary complex numbers and quaternions [38]) of real numbers on the associative complex structures) can lead to multidimensional temporary structures.

The principal possibility of the existence of different universes is contained in inflation theory proposed by A. Guth and modified A. Linde, P. Steinhardt and A. Albrecht [37, 39, 40]. Fluctuations of the initial scalar field within the Planck scale lead to the conditions for entering the inflationary regime, resulting in the rapid growth of the spatial dimensions of the three-dimensional part of space, leading to the conditions necessary for the evolution of the hot BB. These fluctuations lead to continuous generation of universes with different nature. Transitions between universes with different dimensions of time are still unknown, so we will assume that the dimension of the time continuum of our Universe does not change and is equal to 1.

As we have seen, the dimension of the space in all three approaches to its description is determined by the existence of different kinds of interactions. The more you have, the greater the dimension of the space. Thus, the dimension of our space may be greater than 11. It depends on the presence of unknown to us interactions.

Changing the dimension of the space. The existence of space more or even significantly greater dimension compared to discussed above, follows from the other considerations. String theory due to its nonlocal specifics contains additional symmetries that are not contained in the local theories. One such symmetry is T-duality of free strings, which implies the impossibility of the electric field strength to accept infinitely large values

$$E < E_{\text{crit}} = \frac{1}{2\pi\alpha'}$$

where α` is so called slope parameter in string theory which is proportional to the string tension [7]. Electric energy density therefore can not be more than

$$w_{\text{el\_crit}} = \frac{E_{\text{crit}}^2}{8\pi} = \frac{1}{32\pi^3\alpha'^2}$$

In thermodynamic equilibrium each degree of freedom of the system have on average the same energy (the law of energy equidistribution). If at ultralarge energy densities this law is also true, it will lead to the fact that a finite number of fields will contain only a finite energy density and these fields will not be able to accumulate the released infinite energy density of BB in the initial moments of the explosion. This, in turn, should lead to an infinite or very large (if the energy density in the initial moments of BB is not infinite) dimension of the space N in the initial moments of the explosion

$$N(t) \geq \frac{w(t)}{2w_{\text{el\_crit}}} = 16\pi^3\alpha'^2 w(t) \qquad (2)$$

where *w(t)* is an energy density of BB at some time moment t near initiation of BB (see fig.1).

The given arguments don't exclude possibility of existence of dimension of space, greater eleven which in process of expansion of the Universe gradually decreases. W. Heisenberg noted that the modern physics is close to Heraclitus's doctrine. If to replace his word "fire" with the word "energy", it can be considered as the prime cause of all changes in the world [41]. We would add that perhaps the reason of the space dimension too.

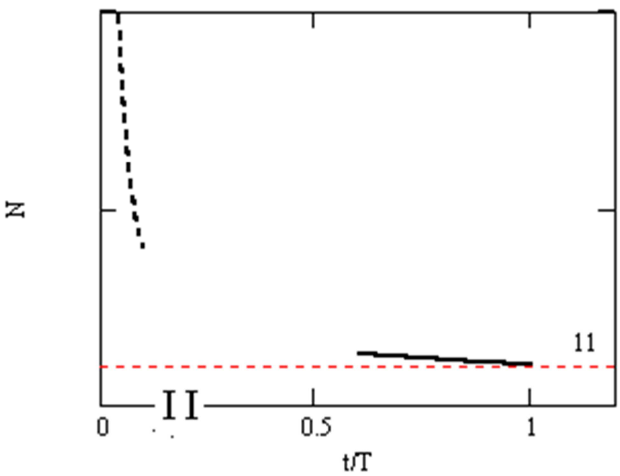

Fig. 1. Possible schematic behavior of space dimension N on time t, T – present age (about 13.7 billion years). At earlier moments of BB it behavior may be described as (2) (dashed line), at later periods, i.e. our age (full line). According to modern concepts dimension of our space is $N(T) = 11$.

6. Cosmological Constant

The current status. Hilbert-Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi k T_{\mu\nu}$$

are obtained from the vanishing of the variational derivative $\delta S_g / \delta g_{\mu\nu}$ of action

$$S_g = S_{gr} + S_m = \int d^4x \sqrt{-g} (L_{gr} + L_m) \quad (3)$$

built using the metric tensor $g_{\mu\nu}$ (μ, ν = 1-n) and its first derivatives with respect to coordinates of space-time, g is the determinant of the metric tensor,

$$L_{gr} = \frac{1}{16\pi k} R = \frac{1}{16\pi k} g^{\mu\nu} R_{\mu\nu}$$

– gravitational scalar Lagrange density, k is the Newtonian gravitational constant, R is the curvature of space, i.e., convolution of the metric tensor with the Ricci tensor $R_{\mu\nu}$ which is $R = g^{\mu\nu} R_{\mu\nu}$, L_m is the Lagrangian density of matter, $T_{\mu\nu}$ is the tensor of energy-momentum of matter,

$$\frac{1}{2}\sqrt{-g} T_{\mu\nu} = \frac{\partial \sqrt{-g} L_m}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha} \frac{\partial \sqrt{-g} L_m}{\partial \frac{\partial g^{\mu\nu}}{\partial x^\alpha}}$$

the integration was is performed over the four-dimensional space-time. HEE are generally covariant and does not depend on the choice of local coordinates. Because the magnitude $\sqrt{-g} d^4x$ is invariant, then it admits in (3) introduction of the term

$$S_\Lambda = -\int d^4x \sqrt{-g} \Lambda$$

to action

$$\tilde{S}_g = S_g + S_\Lambda = S_{gr} + S_m + S_\Lambda = \int d^4x \sqrt{-g} (L_{gr} + L_m - \Lambda) \quad (4)$$

which does not violate the general covariance of HEE, new equations take the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi k (T_{\mu\nu} + \Lambda) \quad (5)$$

Einstein enters CC (Λ) into the equations in 1917 to provide a condition of static character of the Universe. Without the introduction of this term either the energy density or pressure of the matter should be negative. Introduction of Λ or energy density $\rho_{vac} = \frac{\Lambda}{8\pi k}$ allows to avoid such unphysical result. Since the right part of this ratio does not depend on the characteristics of matter, then it is interpreted as the energy density of empty space or vacuum. After the appearance of the models of A. Friedman interest in a static Universe disappears and the vast number of works considered CC equal to zero.

Models with $\Lambda \neq 0$. However, completely this possibility ($\Lambda \neq 0$) is not excluded. Dynamics of expansion of the Universe in such models is investigated, consequences of existence of CC in various astrophysical questions [31, 40, 42] are considered.

Induced theory of gravitation. In this theory, the CC appears as zero term in the expansion of the Lagrangian density in powers of the curvature of space [21].

Propagation of gravitational waves. When considering of a gravitational wave far from a matter only this wave can be the only source supporting its propagation. R. Feynman concludes that non-linear corrections should be added to the action. Then the Feynman analysis has become quite common and has led to the conclusion that a fairly general consistent field equation, which includes no more than two derivatives, is a HEE with CC. Similar approach was developed in the works of C. Gupt and R. Kraichnan [23].

At the end of the 1990s it turns out that the difference of CC from zero in many respects is fundamentally.

The expansion of the Universe. The results on the analysis of red shift of the radiation lines which are carried out by various international astrophysical groups show accelerated expansion of the Universe. This behavior requires a large energy density in the Universe, which was called "dark energy” or quintessence (other names – vacuum-like matter, quintessence, cosmological term, CC) [33, 36, 37, 40, 42]. In contrast to the "dark matter" this component is distributed uniformly and does not tend clustering. It seems that it is a physical field of unknown nature.

The density of matter in the Universe. A convenient parameter in cosmology is the ratio of the density of different structural forms of the material component to its critical density $\Omega = \frac{\rho}{\rho_{cr}}$, $\rho_{cr} = \frac{3H_0^2}{8\pi k}$, where H_0 is the Hubble constant. The contribution of the baryonic components does not exceed the $\Omega_b < 0.023$. The contribution of the whole matter of not the field nature, including "dark matter" does not exceed $\Omega_m < 0.3$. It has long time been a stumbling block because of for the flat geometry of the Universe should be $\Omega_m = 1$. In many works of 1980-1990 it was considered as the statement of fundamental character. The necessity of introduction of "dark energy" was allowed to remove the marked contradiction, it is believed that the remainder of density correspond to it, $\Omega_\Lambda \approx 0.7$ [33, 36, 37, 40, 42].

Modified Newtonian dynamics. Modification of the theory was initially developed to describe the rotation curves of galaxies [43, 44] and rejection of the necessity of the introduction of "dark matter". The first approach is associated with a modification of the gravitational interaction, the second – with modification of Newton's second law. In both cases, the results are modified at large distances (at accelerations greater than a certain value of a_0).

There is a question of an a_0 parameter choice. It would be natural to associate with the Hubble constant ($\propto H_0$), the curvature of space ($\propto 1/R$) or with CC ($\propto \Lambda^{1/2}$) [43].

Note also that the presence of CC affects the age of the Universe, the anisotropy of relic radiation, etc. [33, 36, 37, 40, 42]. Consideration of various questions related to it's the difference its from zero leads to the value $\Omega_\Lambda = 10^{-0.1\pm0.1}$ [40].

On the nature of the cosmological constant. Consider in the space of higher dimension (N) action S_G, where G_{AB} (A, B = 1-N) – the metric tensor in a space of higher dimension. If the dimension of the space is decreased ($N \to n$), it will lead to the fact that the metric tensor space of higher dimension will pass into the metric tensor space of smaller dimension $G_{\mu\nu} \to g_{\mu\nu}$, i.e.,

$$G_{\mu\nu} = g_{\mu\nu},\ \mu,\ \nu = 1\text{-}n, \tag{6}$$

The same transition then must endure and all the other quantities, the Ricci tensor, curvature $R_G \to R_g$, where R_G and R_g – curvatures of spaces, designed by G_{AB} and $g_{\mu\nu}$ and so on. Integrating now the action on space of smaller dimension, we pass from S_G to S_g.

But we can do it in another way, working still with the original action of the S_G and the metric tensor G_{AB}. The transition to a space of smaller dimension is the restriction of the original problem in a certain condition, so we can search for an extremum of the action (i.e., HEE) in the original space under this condition, i.e., to search in the original problem of conditional extremum. We do not impose any conditions on each of the matrix elements of the metric tensor G_{AB} (A, B = 1-n), i.e. still have (6) because they must be obtained as a solution of the variational problem. For A, $B > n$, we can choose

$$G_{AB} = 0,\ A \neq B,\ G_{A\nu} = 0,\ G_{\nu A} = 0,\ \nu = 1\text{-}n,\ G_{AA} = 1/V^2_A \tag{7}$$

(A is not on the summation, V_A – "volume" of space characterized by A coordinate), as a result of the disappearance of the interaction, determining the presence of excess number of dimensions, the off-diagonal elements vanish. Ambiguous is only choice of components G_{AA}, $A > n$. Note that, in the theory of Kaluza and its generalizations when describing the 5-dimensional spaces G_{55} can be chosen constant, thereby leading to a constant ratio of particle charge to mass, or function of space-time coordinates, which is equivalent to the introduction of a scalar field. The manifestations of the scalar potential on changing the ratio of the charge particles to their mass was not detected yet [6]. Therefore we assume that V_A, $A > n$, does not depend on x_μ, μ = 1-n.

The condition is imposed actually only on a determinant of this matrix, therefore

$$\sqrt{-G} = \frac{\sqrt{-g}}{V_{n+1}...V_N} \tag{8}$$

The general Lagrange method for finding an extremum of some function f, in our case of functional S_G, on condition of $F(x_1, \ldots, x_i, \ldots, x_N) = 0$ for the variables $x_1, \ldots, x_i, \ldots, x_N$ consists in introduction to this function of the feature term λF $(x_1, \ldots, x_i, \ldots, x_N)$ and search for it an unconditional extremum for the variables $x_1, \ldots, x_i, \ldots, x_N$, λ [45]. Then according to the general algorithm of search of a conditional extremum using Lagrange's multipliers we have to introduce into the initial action of SG term

$$-\lambda \int \left(\sqrt{-G} - \frac{\sqrt{-g}}{V_{n+1}...V_N} \right) dx_1 dx_2 ... dx_N$$

and to look for not a conditional, but full (unconditional) extremum of new action on the variables G_{AB}, λ. Thus equality of variation of new action

$$\tilde{S}_G = S_G - \lambda \int \left(\sqrt{-G} - \frac{\sqrt{-g}}{V_{n+1}...V_N} \right) dx_1 dx_2 ... dx_N$$

on λ to zero will give us the condition (8) under which we look for a conditional extremum of initial action. Equality of a variation derivative on G_{AB} to zero will lead us to the equations

$$\frac{\delta \tilde{S}_G}{\delta G_{AB}} = \frac{\delta\left(S_G - \lambda \int \sqrt{-G} dx_1 dx_2 ... dx_N \right)}{\delta G_{AB}} = 0$$

thus the condition (6) in sector of the lowered dimension will lead to the equations

$$\frac{\delta\left(S_g - \lambda \int \sqrt{-G} dx_1 dx_2 ... dx_N \right)}{\delta g_{\mu\nu}} = 0$$

as V_A, A, $B > n$, does not depend on x_μ, μ = 1-n, and in view of (7) calculation of the Christoffel symbols, the curvature tensor and Ricci tensor in this sector will not lead to additional contributions from component G_{AB} , $G_{A\nu} = 0$ and $G_{\nu A} = 0$ (A, B = (n + 1)-N, ν = 1-n). Condition (8) will lead to the fact that the integration would be done already by volume with space of dimension n, as $\frac{\int dx_{n+1}...dx_N}{V_{n+1}...V_N} = 1$:

$$\int \sqrt{-G}dx_1...dx_N = \frac{\int \sqrt{-g}dx_1...dx_N}{V_{n+1}...V_N} = \int \sqrt{-g}dx_1...dx_n \ ,$$

i.e., we will come (see (4)) to the equations $\frac{\delta \tilde{S}_g}{\delta g_{\mu\nu}} = 0$, and, respectively, to equations (5).

Thus, CC can be interpreted as Lagrange's multiplier introduced for search of a conditional extremum of action upon transition from space of higher to space of lower dimension as a result of evolution of the Universe. From the physical point of view upon transition to space of lower dimension the energy contained in disappearing degrees of freedom has to be extracted in the remained space. Thus, this extracted energy can be CC or quintessence. While the transition to a space of lower dimension not finished this energy is likely to grow over time, although its specific behavior will depend on the dynamics of the transition, about which we still know nothing. We have no data on the nature of dark energy. However there are attempts of its description in some models of a scalar field. In these models growth of quintessence is necessary for an explanation of growth of observed red shift of emission lines [42] over time.

7. Conclusions

Among the various universes our Universe is characterized by one-dimensional time.

Modern ideas about the nature of space show that its dimension is closely connected with existence of interactions and increases with increasing number of interactions of different nature.

According to modern concepts dimension of our space is equal to eleven. These views are based on the existence of gravitational, electromagnetic, weak and strong interactions (in the standard SU(3) × SU(2) × U(1) model). We don't know the nature of a dark matter and dark energy. What interactions do they represent? If we will not be able to refuse them when describing the Universe, then the dimension of our space in the modern age should be more than eleven.

In the approaching of the BB beginning in addition to well-known could be presented others unknown to us interactions, that entails an increase in the dimension of space. In addition, if the law of equidistribution is fair in the initial moments of BB, the presence of T-duality in string theory can lead to an infinite (or very large) space dimension. Thus, it is possible such a scenario of the Universe when as it cools we move into more and more low-energy sector of interactions, leading to a gradual reduction in the dimension of space.

CC from the mathematical point of view can be interpreted as Lagrange's multiplier describing transition of action in space of higher to space of lower dimension. From the physical point of view upon transition to space of lower dimension the energy associated with disappearing degrees of freedom has to be extracted in the remained space. It extracts as CC or quintessence.

Such view of the nature of dark energy can explain its appearance, but not structure. The transition of dimension of our space to lower values will be followed by change of CC or quintessence in time.

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