Some Algebraic Operations of Intuitionistic Fuzzy Sets on the Basis of Reference Function

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Abstract

In this article, we introduce the concept of reference function in the theory of intuitionistic fuzzy sets. Thereafter, we define some basic operations, namely complement, addition and multiplication of the intuitionistic fuzzy sets on the basis of reference function and some results associated with such operations are discussed.

Keywords

Reference Function, Membership Function, Membership Value

1. Introduction

Intuitionistic fuzzy sets on a universe was introduced by Atanassov [1], as a generalization of fuzzy sets. The concept of intuitionistic fuzzy sets can be viewed as an appropriate / alternative approach in case where available information is not sufficient to define the imprecision by the conventional approach. In the fuzzy set theory, only the degree of acceptance is considered but in case of intuitionistic fuzzy sets both membership function and non membership function is considered so that the sum of these two values add up to one.

2. Preliminaries

In this section, we would like to recall some basic operation of intuitionistic fuzzy set theory like complement, union and intersection of two intuitionistic fuzzy sets.

Let U be an universe of discourse. Then the intuitionistic fuzzy set A is an object having the form

\[ A = \{ x, \mu_A(x), \omega_A(x) : x \in U \} \]

where the function \( \mu_A(x), \omega_A(x) : U \to [0,1] \) define the degree of membership and the degree of non membership of the element \( x \in X \) to the set A with the condition

\[ 0 \leq \mu_A(x) + \omega_A(x) \leq 1 \]

For two intuitionistic fuzzy sets

\[ A = \{ x, \mu_A(x), \omega_A(x) : x \in X \} \]

and

\[ B = \{ x, \mu_B(x), \omega_B(x) : x \in X \} \]

We can find the following operations

i. Complement:

\[ A^0 = \{ x, \omega_A(x), \mu_A(x) : x \in X \} \]

ii. The addition of two intuitionistic fuzzy sets is defined in the following way:

\[ A \oplus B = \{ x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \omega_A(x)\omega_B(x) : x \in X \} \]

iii. The multiplication of two intuitionistic fuzzy sets is defined in the following way:

\[ A \otimes B = \{ x, \mu_A(x)\mu_B(x), \omega_A(x) + \omega_B(x) - \omega_A(x)\omega_B(x) : x \in X \} \]

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3. New Definition of Intuitionistic Fuzzy Sets

Here we shall try to define the concept of intuitionistic fuzzy sets on the basis of reference function defined by Baruah [2]. Later this concept is successfully applied in many of our previous works, see for example Dhar([3]-[27]). If the concept of reference function is introduced in the theory of intuitionistic fuzzy sets then an intuitionistic fuzzy set defined in the way as

\[ A = \{ \langle x, \mu_A(x), \omega_A(x) \rangle, x \in X \} \]  

would be defined as

\[ A = \{ \langle x, (\mu_A(x),0), (1,\mu_A(x)) \rangle, x \in X \} \]  (2)

But it is to be remembered here that here we shall consider the case when

\[ (\mu_A(x),0) + (1,\mu_A(x)) = 1 \]  (3)

Then the operations of complementation, addition and multiplication are defined in the following way for the two intuitionistic fuzzy sets

\[ A = \{ \langle x, (\mu_A(x),0), (1,\mu_A(x)) \rangle, x \in X \} \]
\[ B = \{ \langle x, (\mu_B(x),0), (1,\mu_B(x)) \rangle, x \in X \} \]

The complement is defined as

\[ A^c = \{ \langle x, ((1,\mu_A(x)),(\mu_A(x),0)) \rangle, x \in X \} \]  (4)

Provided

\[ \text{membership value}(1,\mu_A(x)) \geq \text{membership value}(\mu_A(x),0) \]

The addition of two newly defined intuitionistic fuzzy sets is defined as

\[ A \oplus B = \{ \langle x, (\mu_A(x),0) + (\mu_B(x),0) - (\mu_A(x),0)(\mu_B(x),0), (1,\mu_A(x))(1,\mu_B(x)) \rangle, x \in X \} \]  (5)

with the help of some numerical examples.

Let us consider the following example

**Numerical Example:** Assume that the Universe of discourse \( U = \{x_1, x_2, x_3, x_4\} \). It may be further assumed that \( x_1, x_2, x_3, x_4 \in [0,1] \). Then \( A, B \) and \( C \) are intuitionistic fuzzy set of \( U \) such that

\[ A = \{ \langle x_1, 0.6, 0.4 \rangle, \langle x_2, 0.8, 0.2 \rangle, \langle x_3, 0.3, 0.7 \rangle \} \]
\[ B = \{ \langle x_1, 0.1, 0.9 \rangle, \langle x_2, 0.5, 0.5 \rangle, \langle x_3, 0.4, 0.4 \rangle \} \]
\[ C = \{ \langle x_1, 0.3, 0.7 \rangle, \langle x_2, 0.4, 0.6 \rangle, \langle x_3, 0.6, 0.4 \rangle \} \]

This will be expressed in according to the new concept as

\[ A = \{ \langle x_1, (0.6,0),(1,0.6) \rangle, \langle x_2, (0.8,0),(1,0.8) \rangle, \langle x_3, (0.3,0),(1,0.3) \rangle \} \]
\[ B = \{ \langle x_1, (0.1,0),(1,0.1) \rangle, \langle x_2, (0.5,0),(1,0.5) \rangle, \langle x_3, (0.4,0),(1,0.4) \rangle \} \]
\[ C = \{ \langle x_1, (0.3,0),(1,0.3) \rangle, \langle x_2, (0.4,0),(1,0.4) \rangle, \langle x_3, (0.6,0),(1,0.6) \rangle \} \]

Then the complement of the sets \( A \) and \( B \) will be defined as

\[ A^c = \{ \langle x_1, (1,0.6),(0,0.6) \rangle, \langle x_2, (1,0.8),(0,0.8) \rangle, \langle x_3, (1,0.3),(0,0.3) \rangle \} \]
\[ B^c = \{ \langle x_1, (1,0.1),(0,0.1) \rangle, \langle x_2, (1,0.5),(0,0.5) \rangle, \langle x_3, (1,0.4),(0,0.4) \rangle \} \]

Then the addition of these two intuitionistic fuzzy sets defined above will be the following

\[ A \oplus B = \{ \langle x_1, (0.6,0),(0,1.0) \rangle + (0.6,0)(0,1.0), (1,0.6)(1,0.1) \rangle, \langle x_2, (0.8,0),(0,1.0) \rangle + (0.8,0)(0,1.0), (1,0.8)(1,0.5) \rangle, \langle x_3, (0.3,0),(0,1.0) \rangle + (0.4,0)(0,1.0), (1,0.4)(1,0.3) \rangle \} \]  (7)
Again it is obtained that

\[
A \oplus C = \{(0.6,0),(0.3,0),(0.6,0),(0.3,0),(1,0.6),(1,0.3)\},
\]
\[
< (0.8,0),(0.4,0),(0.8,0),(0.4,0),(1,0.8),(1,0.4)>,
\]
\[
< (0.3,0),(0.6,0),(0.3,0),(1,0.3),(1,0.6)\}
\]
\[= \{(0.6,0),(0.3,0),(1,0.6),< (0.8,0),(0.4,0),(1,0.8),(1,0.4)>,< (0.6,0),(0.3,0),(1,0.6)\}
\]
\[= \{(0.6,0),(1,0.6),< (0.8,0),(1,0.8),(1,0.4)>,< (0.6,0),(1,0.6)\}
\]

(8)

Then the following result can be obtained

\[
(A \oplus B) \cup (A \oplus C) = \{(0.6,0),(1,0.6),< (0.8,0),(1,0.8),< (0.6,0),(1,0.6)\}
\]

(9)

Again

\[
B \cup C = \{(0.3,0),(1,0.3),< (0.5,0),(1,0.5),< (0.6,0),(1,0.6)\}
\]

(10)

And then

\[
A \oplus (B \cup C) = \{(0.6,0),(0.3,0),(0.6,0),(0.3,0),(1,0.6),(1,0.3)>,
\]
\[
< (0.8,0),(0.5,0),(0.8,0),(0.5,0),(1,0.8),(1,0.5)>,
\]
\[
< (0.6,0),(0.4,0),(0.6,0),(0.4,0),(1,0.6),(1,0.4)\>
\]
\[= \{(0.6,0),(0.3,0),(1,0.6),< (0.8,0),(1,0.8),< (0.6,0),(1,0.6)\}
\]

(11)

(12)

Hence the result

\[
A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)
\]

(13)

This proves the property (v).

It can be seen that

\[
B \cap C = \{(0.1,0),(1,0.1),< (0.4,0),(1,0.4),< (0.4,0),(1,0.4)\}
\]

(14)

Therefore

\[
A \oplus (B \cap C) = \{(0.6,0),(0.1,0),(0.6,0),(0.1,0),(1,0.6),(1,0.1)>,
\]
\[
< (0.8,0),(0.4,0),(0.8,0),(0.4,0),(1,0.8),(1,0.4)>,
\]
\[
< (0.3,0),(0.4,0),(0.3,0),(0.4,0),(1,0.3),(1,0.4)\>
\]
\[= \{(0.6,0),(0.1,0),(1,0.6),< (0.8,0),(0.4,0),(1,0.8)>,< (0.4,0),(0.3,0),(1,0.4)\>
\]

(15)

and

\[
(A \oplus B) \cap (A \oplus C) = \{(0.6,0),(1,0.6),< (0.8,0),(1,0.8),< (0.4,0),(1,0.4)\}
\]

(16)

Hence the result

\[
A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)
\]

This proves the property (vi).

Again we have
\( A \otimes (B \cup C) = \{(0.6, 0)(0.3, 0), (1, 0.6) + (1, 0.3) + (1, 0.6)(1, 0.3)\}, \\
\{(0.8, 0)(0.5, 0), (1, 0.8) + (1, 0.5) + (1, 0.5)(1, 0.8)\}, \\
\{(0.6, 0)(0.4, 0), (1, 0.6) + (1, 0.4) + (1, 0.6)(1, 0.4)\} \\
= \{(0.3, 0), (1, 0.3) + (1, 0.6)\}, \{(0.5, 0), (1, 0.5) + (1, 0.8)\}, \{(0.4, 0), (1, 0.4) + (1, 0.6)\} \\
\{(0.6, 0)(0.1, 0), (1, 0.6) + (1, 0.1) + (1, 0.6)(1, 0.1)\}, \\
\{(0.8, 0)(0.4, 0), (1, 0.8) + (1, 0.4) + (1, 0.8)(1, 0.4)\}, \\
\{(0.3, 0)(0.6, 0), (1, 0.3) + (1, 0.6)\} \\
= \{(0.3, 0), (1, 0.3)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\}. \\
\{(0.3, 0), (1, 0.3)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\} \\
= \{(0.3, 0), (1, 0.3)\} \sqcup \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\} \sqcup \{(0.3, 0), (1, 0.3)\} \\
\{(0.1, 0), (1, 0.1) + (1, 0.6)\}, \{(0.4, 0), (1, 0.4) + (1, 0.6)\}, \{(0.3, 0), (1, 0.3) + (1, 0.6)\} \\
= \{(0.1, 0), (1, 0.1)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\}

Hence

\((A \otimes B) \cup (A \otimes C) = \{(0.1, 0), (1, 0.1)\}, \{(0.5, 0), (1, 0.5)\}, \{(0.3, 0), (1, 0.3)\}\ |

(19)

and

\((A \otimes B) \cap (A \otimes C) = \{(0.1, 0), (1, 0.1)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\}\ |

(20)

\(A \otimes (B \cup C) = \{(0.3, 0), (1, 0.3) + (1, 0.6)\}, \{(0.5, 0), (1, 0.5) + (1, 0.8)\} + (1, 0.4) \sqcup (1, 0.3) + (1, 0.6)\} \\
= \{(0.3, 0), (1, 0.3)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\} + (1, 0.6)\} \\
= \{(0.1, 0), (1, 0.1)\}, \{(0.4, 0), (1, 0.4)\}, \{(0.3, 0), (1, 0.3)\}

(21)

\((A \otimes B)^c = \{(1, 0.6), (0.6, 0)\}, \{(1, 0.8), (0.8, 0)\}, \{(1, 0.4), (0.4, 0)\}\ |

(22)

Now

\(A^c \otimes B^c = \{(1, 0.6)(1, 0.1), (0.6, 0) + (0.1, 0) + (0.6, 0)(0.1, 0)\}, \\
\{(1, 0.8)(1, 0.5), (0.8, 0) + (0.5, 0) + (0.8, 0)(0.5, 0)\}, \\
\{(1, 0.3)(1, 0.4), (0.3, 0) + (0.4, 0) + (0.3, 0)(0.4, 0)\} \\
= \{(1, 0.6), (0.6, 0)\}, \{(1, 0.8), (0.8, 0)\}, \{(1, 0.4), (0.4, 0)\}\ |

(23)

Hence we get

\((A \otimes B)^c = A^c \otimes B^c

This proves the property (ix)

Again
\[ A^c \oplus B^c = \{ (1,0.6) + (1,0.1) + (1,0.6)(1,0.1),(0,6,0)(0,1,0) >, (1,0.8) + (1,0.5) \\
+ (1,0.8)(1,0.5),(0,8,0)(0,5,0) >, \\
< (1,0.3) + (1,0.4) + (1,0.3)(1,0.4),(0,4,0)(0,3,0) > \} \\
= \{ (1,0.1) + (1,0.6),(0,1,0) >, (1,0.8) + (1,0.5),(0,5,0) >, (1,0.3) + (1,0.3),(0,3,0) > \} \\
= \{ (1,0.1),(0,1,0) >, (0,5,0),(0,5,0) >, (1,0.3),(0,3,0) > \} \tag{24} \]

and

\[ (A \oplus B)^c = \{ (1,0.1),(0,1,0) >, (0,5,0),(0,5,0) >, (1,0.3),(0,3,0) > \} \tag{25} \]

Hence

\[ (A \oplus B)^c = A^c \oplus B^c \]

This proves the property (x)

5. Conclusions

This article deals with the expression of intuitionistic fuzzy sets in terms of reference function and it is seen that the membership values of the newly introduced concept of intuitionistic fuzzy sets on the basis of reference function remains the same, only the method of expression is different. Some operations of newly defined fuzzy sets are put forward and some properties of such operations are studied with numerical examples to illustrate the concept discussed.

References


