Physics Journal
Vol. 1, No. 3, 2015, pp. 295-300
http://www.aiscience.org/journal/pj

Some Algebraic Operations of Intuitionistic Fuzzy Sets on the Basis of Reference Function

Mamoni Dhar*

Department of Mathematics, Science College, Kokrajhar, Assam, India

Abstract

In this article, we introduce the concept of reference function in the theory of intuitionistic fuzzy sets. Thereafter, we define some basic operations, namely complement, addition and multiplication of the intuitionistic fuzzy sets on the basis of reference function and some results associated with such operations are discussed.

Keywords

Reference Function, Membership Function, Membership Value

Received: August 13, 2015 / Accepted: October 10, 2015 / Published online: November 11, 2015

1. Introduction

Intuitionistic fuzzy sets on a universe was introduced by Atanassov [1], as a generalization of fuzzy sets. The concept of intuitionistic fuzzy sets can be viewed as an appropriate / alternative approach in case where available information is not sufficient to define the impreciseness by the conventional approach. In the fuzzy set theory, only the degree of acceptance is considered but in case of intuitionistic fuzzy sets both membership function and non membership function is considered so that the sum of these two values add up to one.

2. Preliminaries

In this section, we would like to recall some basic operation of intuitionistic fuzzy set theory like complement, union and intersection of two intuitionistic fuzzy sets.

Let U be an universe of discourse. Then the intuitionistic fuzzy set A is an object having the form $A = \{< x, \mu_A(x), \omega_A(x) >, x \in U\}$, where the function $\mu_A(x), \omega_A(x): U \to [0,1]$ define the degree of membership and the degree of non membership of the element $x \in X$ to the set A with the condition

$$0 \le \mu_A(x) + \omega_A(x) \le 1$$

For two intuitionistic fuzzy sets

$$A = \{< x, \mu_A(x), \omega_A(x) >, x \in X\}$$

and

$$B = \{< x, \mu_B(x), \omega_B(x) >, x \in X\}$$

We can find the following operations

i. Complement:

$$A^0 = \{< x, \omega_A(x), \mu_A(x) >, x \in X\}$$

ii. The addition of two intuitionistic fuzzy sets is defined in the following way:

$$A \oplus B = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \omega_A(x)\omega_B(x) >, x \in X\}$$

iii. The multiplication of two intuitionistic fuzzy sets is defined in the following way:

$$A \otimes B = \{< x, \mu_A(x)\mu_B(x), \omega_A(x) + \omega_B(x) - \omega_A(x)\omega_B(x) >, x \in X\}$$

* Corresponding author
E-mail address: mamonidhar@rediffmail.com (M. Dhar)

3. New Definition of Intuitionistic Fuzzy Sets

Here we shall try to define the concept of intuitionistic fuzzy sets on the basis of reference function defined by Baruah [2]. Later this concept is successfully applied in many of our previous works, see for example Dhar([3]-[27]). If the concept of reference function is introduced in the theory of intuitionistic fuzzy sets then an intuitionistic fuzzy set defined in the way as

$$A = \{< x, \mu_A(x), \omega_A(x) >, x \in X\} \quad (1)$$

would be defined as

$$A = \{< x, (\mu_A(x), 0), (1, \mu_A(x)) >, x \in X\} \quad (2)$$

But it is to be remembered here that here we shall consider the case when

$$\mu_A(x) + \omega_A(x) = 1 \quad (3)$$

Then the operations of complementation, addition and multiplication are defined in the following way for the two intuitionistic fuzzy sets

$$A = \{< x, (\mu_A(x), 0), (1, \mu_A(x)) >, x \in X\}$$

$$B = \{< x, (\mu_B(x), 0), (1, \mu_B(x)) >, x \in X\}$$

The complement is defined as

$$A^c = \{< x, ((1, \mu_A(x), (\mu_A(x), 0)) >, x \in X\} \quad (4)$$

Provided
$membershipvalue(1, \mu_A(x)) \geq membershipvalue(\mu_A(x), 0)$

The addition of two newly defined intuitionistic fuzzy sets is defined as

$$A \oplus B = \{< x, (\mu_A(x), 0) + (\mu_B(x), 0) - (\mu_A(x), 0)(\mu_B(x), 0), (1, \mu_A(x))(1, \mu_B(x)) >, x \in X\} \quad (5)$$

The multiplication of two newly defined intuitionistic fuzzy sets is defined as

$$A \otimes B = \{< x, (1, \mu_A(x))(1, \mu_B(x)), (1, \mu_A(x))(1, \mu_B(x)) >, x \in X\} \quad (6)$$

4. Algebra Laws on Intuitionistic Fuzzy Sets

1. $A \oplus B = B \oplus A$
2. $A \otimes B = B \otimes A$
3. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
4. $A \otimes (B \otimes C) = (A \otimes B) \otimes C$
5. $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
6. $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
7. $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$
8. $A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$
9. $(A \oplus B)^c = A^c \otimes B^c$
10. $(A \otimes B)^c = A^c \oplus B^c$

Proofs of the above mentioned properties are demonstrated with the help of some numerical examples.

Let us consider the following example

Numerical Example: Assume that the Universe of discourse $U = \{x_1, x_2, x_3, x_4\}$. It may be further assumed that $x_1, x_2, x_3, x_4 \in [0,1]$.Then A , B and C are intuitionistic fuzzy set of U such that

$$A = \{< x_1, 0.6, 0.4 >, < x_2, 0.8, 0.2) >, < x_3, 0.3, 0.7 >\}$$

$$B = \{< x_1, 0.1, 0.9 >, < x_2, 0.5, 0.5 >, < x_3, 0.4, 0.4 >\}$$

$$C = \{< x_1, 0.3, 0.7 >, < x_2, 0.4, 0.6 >, < x_3, 0.6, 0.4) >\}$$

This will be expressed in according to the new concept as

$$A = \{< x_1, (0.6, 0), (1, 0.6) >, < x_2, (0.8, 0), (1, 0.8) >, < x_3, (0.3, 0), (1, 0.3) >\}$$

$$B = \{< x_1, (0.1, 0), (1, 0.1) >, < x_2, (0.5, 0), (1, 0.5) >, < x_3, (0.4, 0), (1, 0.4) >\}$$

$$C = \{< x_1, (0.3, 0), (1, 0.3) >, < x_2, (0.4, 0), (1, 0.4) >, < x_3, (0.6, 0), (1, 0.6) >\}$$

Then the complement of the sets A and B will be defined as

$$A^c = \{< x_1, (1, 0.6), (0.6, 0) >, < x_2, (1, 0.8), (0.8, 0) >, < x_3, (1, 0.3), (0.3, 0) >\}$$

$$B^c = \{< x_1, (1, 0.1), (0.1, 0) >, < x_2, (1, 0.5), (0.5, 0) >, < x_3, (1, 0.4), (0.4, 0) >\}$$

Then the addition of these two intuitionistic fuzzy sets defined above will be the following

$$\begin{aligned} A \oplus B = \{&< (0.6, 0) + (0.1, 0) + (0.6, 0)(0.1, 0), (1, 0.6)(1, 0.1) >, \\ &< (0.8, 0) + (0.5, 0) + (0.8, 0)(0.5, 0), (1, 0.8)(1, 0.5) >, \\ &< (0.3, 0) + (0.4, 0) + (0.3, 0)(0.4, 0), (1, 0.4)(1, 0.3) >\} \end{aligned}$$

$$= \{< (0.6, 0) + (0.1, 0), (1, 0.6) >, < (0.8, 0) + (0.5, 0), (1, 0.8) >, < (0.4, 0) + (0.3, 0), (1, 0.4) >\} \quad (7)$$

Again it is obtained that

$$A \oplus C = \{< (0.6,0)+(0.3,0)+(0.6,0)(0.3,0),(1,0.6)(1,0.3)>,$$
$$< (0.8,0)+(0.4,0)+(0.8,0)(0.4,0),(1,0.8)(1,0.4)>,$$
$$< (0.3,0)+(0.6,0)+(0.3,0)(0.6,0),(1,0.3)(1,0.6)>\}$$

$$= \{< (0.6,0)+(0.3,0),(1,0.6)>,< (0.8,0)+(0.4,0),(1,0.8)(1,0.4)>,< (0.6,0)+(0.3,0),(1,0.6)>\}$$

$$= \{< (0.6,0),(1,0.6)>,< (0.8,0),(1,0.8)(1,0.4)>,< (0.6,0),(1,0.6)>\} \quad (8)$$

Then the following result can be obtained

$$(A \oplus B) \cup (A \oplus C) = \{< (0.6,0),(1,0.6)>,< (0.8,0),(1,0.8)>,< (0.6,0),(1,0.6)>\} \quad (9)$$

Again

$$B \cup C = \{< (0.3,0),(1,0.3)>,< (0.5,0),(1,0.5)>,< (0.6,0),(1,0.6)>\} \quad (10)$$

And then

$$A \oplus (B \cup C) = \{< (0.6,0)+(0.3,0)+(0.6,0)(0.3,0),(1,0.6)(1,0.3)>,$$
$$< (0.8,0)+(0.5,0)+(0.8,0)(0.5,0),(1,0.8)(1,0.5)>, \quad (11)$$
$$< (0.6,0)+(0.4,0)+(0.6,0)(0.4,0),(1,0.6)(1,0.4)>\}$$

$$A \oplus (B \cup C) = \{< (0.6,0)+(0.3,0),(1,0.6)>,< (0.8,0)+(0.5,0),(1,0.8)>,< (0.6,0)+(0.4,0),(1,0.6)>\}$$

$$= \{< (0.6,0),(1,0.6)>,< (0.8,0),(1,0.8)>,< (0.6,0),(1,0.6)>\} \quad (12)$$

Hence the result

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C) \quad (13)$$

This proves the property (v)

It can be seen that

$$B \cap C = \{< (0.1,0),(1,0.1)>,< (0.4,0),(1,0.4)>,< (0.4,0),(1,0.4)>\} \quad (14)$$

Therefore

$$A \oplus (B \cap C) = \{< (0.6,0)+(0.1,0)+(0.6,0)(0.1,0),(1,0.6)(1,0.1)>,$$
$$< (0.8,0)+(0.4,0)+(0.8,0)(0.4,0),(1,0.8)(1,0.4)>,$$
$$< (0.3,0)+(0.4,0)+(0.3,0)(0.4,0),(1,0.3)(1,0.4)>\}$$
$$= \{< (0.6,0)+(0.1,0),(1,0.6)>,< (0.8,0)++(0.4,0),(1,0.8)>,< (0.4,0)+(0.3,0),(1,0.4)>\}$$

$$= \{< (0.6,0),(1,0.6)>,< (0.8,0),(1,0.8)>,< (0.4,0),(1,0.4)>\} \quad (15)$$

and

$$(A \oplus B) \cap (A \oplus C) = \{< (0.6,0),(1,0.6)>,< (0.8,0),(1,0.8)>,< (0.4,0),(1,0.4)>\}$$

Hence the result

$$A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$$

This proves the property (vi).

Again we have

$$A\otimes(B\cup C)=\{<(0.6,0)(0.3,0),(1,0.6)+(1,0.3)+(1,0.6)(1,0.3)>,\\ <(0.8,0)(0.5,0),(1,0.8)+(1,0.5)+(1,0.5)(1,0.8)>,\\ <(0.6,0)(0.4,0),(1,0.6)+(1,0.4)+(1,0.6)(1,0.4)>\}$$

$$=\{<(0.3,0),(1,0.3)+(1,0.6)>,<(0.5,0),(1,0.5)+(1,0.8)>,<(0.4,0),(1,0.4)+(1,0.6)>\}$$

$$=\{<(0.3,0),(1,0.3)>,<(0.5,0),(1,0.5)>,<(0.4,0),(1,0.4)>\}$$

$$=\{<(0.1,0),(1,0.1)+(1,0.6)>,<(0.5,0),(1,0.5)+(1,0.8)>,<(0.3,0),(1,0.3)+(1,0.4)>\}$$

$$=\{<(0.1,0),(1,0.1)>,<(0.5,0),(1,0.5)>,<(0.3,0),(1,0.3)>\} \tag{16}$$

$$A\otimes B=\{<(0.6,0)(0.1,0),(1,0.6)+(1,0.1)+(1,0.6)(1,0.1)>,\\ <(0.8,0)(0.5,0),(1,0.5)+(1,0.8)+(1,0.8)(1,0.5)>,\\ <(0.3,0)+(0.4,0),(1,0.3)+(1,0.4)+(1,0.3)(1,0.4)>\}$$

$$=\{<(0.1,0),(1,0.1)>,<(0.5,0),(1,0.5)>,<(0.3,0),(1,0.3)>\} \tag{17}$$

$$A\otimes C=\{<(0.6,0)(0.1,0),(1,0.6)+(1,0.1)+(1,0.6)(1,0.1)>,<(0.8,0)(0.4,0),(1,0.4)+(1,0.8)\\ +(1,0.8)(1,0.4)>,<(0.3,0)(0.6,0),(1,0.3)+(1,0.6)+(1,0.3)(1,0.6)>\}$$

$$=\{<(0.1,0),(1,0.1)+(1,0.6)>,<(0.4,0),(1,0.4)+(1,0.8)>,<(0.3,0),(1,0.3)+(1,0.6)>\}$$

$$=\{<(0.1,0),(1,0.1)>,<(0.4,0),(1,0.4)>,<(0.3,0),(1,0.3)>\} \tag{18}$$

Hence

$$(A\otimes B)\cup(A\otimes C)=\{<(0.1,0),(1,0.1)>,(0.5,0),(1,0.5)>,<(0.3,0),(1,0.3)>\} \tag{19}$$

and

$$(A\otimes B)\cap(A\otimes C)=\{<(0.1,0),(1,0.1)>,(0.4,0),(1,0.4)>,<(0.3,0),(1,0.3)>\} \tag{20}$$

$$A\otimes(B\cup C)=\{<(0.3,0),(1,0.3)+(1,0.6)>,<(0.5,0),(1,0.5)\\ +(1,0.8)>,<(0.4,0),(1,0.4)+(1,0.6)>\}$$

$$=\{<(0.3,0),(1,0.3)>,<(0.5,0),(1,0.5)>,<(0.4,0),(1,0.4)>\} \tag{21}$$

$$(A\oplus B)^c=\{<(1,0.6),(0.6,0)>,<(1,0.8),(0.8,0)>,<(1,0.4),(0.4,0)>\} \tag{22}$$

Now

$$A^c\otimes B^c=\{<(1,0.6)(1,0.1),(0.6,0)+(0.1,0)+(0.6,0)(0.1,0)>,\\ <(1,0.8)(1,0.5),(0.8,0)+(0.5,0)+(0.8,0)(0.5,0)>,\\ <(1,0.3)(1,0.4),(0.3,0)+(0.4,0)+(0.3,0)(0.4,0)>\}$$

$$=\{<(1,0.6),(0.6,0)>,<(1,0.8),(0.8,0)>,<(1,0.4),(0.4,0)>\} \tag{23}$$

Hence we get

$$(A\oplus B)^c=A^c\otimes B^c$$

This proves the property (ix)

Again

$$A^c \oplus B^c = \{< (1,0.6)+(1,0.1)+(1,0.6)(1,0.1),(0.6,0)(0.1,0)>,< (1,0.8)+(1,0.5) +(1,0.8)(1,0.5),(0.8,0)(0.5,0)>, < (1,0.3)+(1,0.4)+(1,0.3)(1,0.4),(0.4,0)(0.3,0)>\}$$

$$= \{< (1,0.1)+(1,0.6),(0.1,0)>,< (1,0.8)+(1,0.5),(0.5,0)>,< (1,0.3)+(1,0.3),(0.3,0)>\}$$

$$= \{< (1,0.1),(0.1,0)>,< (0.5,0),(0.5,0)>,< (1,0.3),(0.3,0)>\} \quad (24)$$

and

$$(A \otimes B)^c = \{< (1,0.1),(0.1,0)>,< (1,0.5),(0.5,0)>,< (1,0.3),(0.3,0)>\} \quad (25)$$

Hence

$$(A \otimes B)^c = A^c \oplus B^c$$

This proves the property (x)

5. Conclusions

This article deals with the expression of intuitionistic fuzzy sets in terms of reference function and it is seen that the membership values of the newly introduced concept of intuitionistic fuzzy sets on the basis of reference function remains the same, only the method of expression is different. Some operations of newly defined fuzzy sets are put forward and some properties of such operations are studied with numerical examples to illustrate the concept discussed.

References

[1] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems 20(1), (1986) 87-96.

[2] Hemanta K. Baruah, Theory of Fuzzy sets Beliefs and Realities, International Journal of Energy, Information and Communications, 2011, 2(2): 1-22.

[3] On Hwang and Yang's definition of Entropy of Fuzzy sets, *International Journal of Latest Trend in Computing*, 2(4), 496-497, 2011.

[4] Fuzzy Sets Towards Forming Boolean Algebra, *International Journal of Energy, Information and communications*, 2(4), 137-142, 2011.

[5] A Note on existing Definition of Fuzzy Entropy, *International Journal of Energy, Information and communications*, Vol.3, Issue 1, 17-21,2012.

[6] A Note on the Coherence between Probability and Possibility Measures, International *Journal of Computer Applications*, 43(7), 28-29, 2012.

[7] On Separation Index of Fuzzy Sets, *International Journal of Mathematical Archive*, 3(3), 932-934, 2012.

[8] On Geometrical Representation of Fuzzy Numbers, *International Journal of Energy, Information and communications*, 3(2), 29-34, 2012.

[9] On Fuzzy Measures of Symmetry Breaking of Conditions, Similarity and Comparisons: Non Statistical Information for the Single Patient, *International Journal of Mathematical Archive*, 2516-2519, 3(7), 2012.

[10] Probability- Possibility Transformations: An Overview, *International Journal of Mathematical Archive*, 3(8), 2887-2890, 2012.

[11] On Yager's Probability-Possibility Transformations, *International Journal for Basic Sciences and Social Sciences*, 1(3), 2012.

[12] A Note on Subsethood measure of Fuzzy sets, *International Journal of Energy, Information and communications*, 55-61, 3(3), 2012.

[13] Probability- Possibility Transformation: A Brief revisit, *Annals of Fuzzy Mathematics and Informatics*, 301-308, 5(2), 2013.

[14] A Note on Variable transformations, *Annals of Fuzzy Mathematics and Informatics*, 5(2), 371-376, 2013.

[15] Representation of fuzzy matrices Based on Reference Function, *International Journal of Intelligent systems and Applications*, 5(2), 84-90, 2013.

[16] On Some properties of entropy of fuzzy numbers, *International Journal of Intelligent systems and Applications*, 5(3), 66-73, 2013.

[17] A Revisit to Probability - Possibility Consistency Principles, *International Journal of Intelligent systems and Applications*, 90-99, 5(4), 2013.

[18] A Note on Determinant and Adjoint of Fuzzy Square Matrix , *International Journal of Intelligent systems and Applications*, 5(5), 58-67, 2013.

[19] On Cardinality of Fuzzy sets, *International Journal of Intelligent systems and Applications*, 5(6), 47-52, 2013.

[20] On Some results based on geometrical representation of fuzzy sets, *International Journal of Intelligent systems and Applications*, 5(7), 57-62, 2013.

[21] The Complement of Normal Fuzzy Numbers: An Exposition, *International Journal of Intelligent Systems and Applications*, 5(8), 73-82, 2013.

[22] Cardinality of fuzzy sets: An Overview, *International Journal of Energy, Information and communications*, 4(1), 15-21, 2013.

[23] A note on Determinant of square fuzzy matrix, *International Journal of Information Engineering and Electronic Business*, 5(1), 26-32, 2013.

[24] Theory of fuzzy sets: An overview, *International Journal of Information Engineering and Electronic Business,* 5(3), 22-33, 2013.

[25] A note on entropy subsethood relationship, *International Journal of Intelligent Systems and Applications in Engineering*, 1(3), 44-46, 2013.

[26] A note on fuzzy relational matrices, *International Journal of Intelligent systems and Applications*, 5(10), 43-49, 2013.

[27] On Fuzzy Soft matrix based on reference function, *International Journal of Information Engineering and Electronic Business*, 5(2), 52-59, 2013.