Solitary Waves and Modification of the Characteristic Coefficients of a Single-mode Optical Fiber

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Abstract

We study in this article the conditions to be fulfilled by the properties of a single-mode optical fiber so that certain types of waves of our choice and in particular solitary waves propagate there. What guides our thinking in this work starts from the fact that we asked the question of knowing if it is possible to boost a transmission medium and more precisely the optical fiber so as to propagate exactly the type of signal that we wish. We have estimated that such a thing can be possible only by the modification of the constitutive properties of this waveguide. But in nonlinear partial differential equations which describe the dynamics in the waveguides, the properties of materials are embodied by the coefficients of the terms. Thus, the principle of work is to establish the constraint relationships between the scattering, dissipation and nonlinear coefficients for the proposed wave type to propagate in the fiber or simply that the nonlinear partial differential equation that governs the propagation dynamics in a single-mode optical fiber accepts the solution we need. Once these constraint relations are obtained, we rewrite the corresponding nonlinear partial differential equations. The reliability of the results is tested through the study of the propagation of the solutions obtained. The partial differential equations which describe the dynamics of propagation in the support being of Schrödinger type, in order to easily manipulate the necessary calculations, we make use of the Bogning Djumen-kofané method extended to the implicit functions to obtain the analytical solutions and the Split-step Fourier programming method for numerical study.

Keywords


1. Introduction

Wave propagation dynamics are closely related to different propagation media that may be fluid or solid. Thus, the propagation of any wave or signal in these media is subjected to many effects such as nonlinear effects, dispersion and dissipation. The impact of these effects on the wave in the transmission medium is so great that if these effects are not controlled, the signal is subject to much instability that eventually completely attenuates the wave. This is valid for all waveguides that exist such as power lines, atomic chains, optical fibers, etc. Of all these waveguides, the one that holds our attention as part of the study conducted in this work is the single-mode optical fiber, probably because it is the medium of transmission which is at the center of all the technological exploits currently. By entering into a futuristic dimension, we

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also choose the solitary wave which by its properties [1-4] is considered as the wave whose wide use as signal of information and energy transport will have a lot of benefits on improving the quality of communication and energy by humans.

Returning to the single-mode optical fiber, the propagation of a wave within it, faces nonlinear phenomena, chromatic dispersion of order 1, chromatic dispersion of order 2 and also dissipation phenomena [5, 6].

In the light of all these facts, we have asked the question of how we should analytically choose these non-linear coefficients of dispersion and dissipation so as to easily propagate the signal we want to propagate in the fiber. In other words, what are the relations that bind these different coefficients so that the nonlinear partial differential equation that describes the propagation dynamics in the single-mode optical fiber admits the type of solution we want?

To answer this question, we propose to find the links between the characteristic coefficients of the single-mode optical fiber so as to propagate the type of signal we want. Another intention is to determine exactly the nonlinear partial differential equation which admits for exact solution that which we wish [7-36].

This work, which will have a double analytical and numerical dimension, is organized as follows: section 2 models the equation to be solved, section 3 proposes different constraints between the coefficients as well as the solutions; Section 4 presents the resulting nonlinear partial differential equations as well as the propagation of the solutions obtained and finally we end the work with a conclusion.


The nonlinear partial differential equation which governs the dynamics of propagation of the wave in single-mode optical fiber is given by

\[
\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A - i \gamma |A|^2 A = 0, \tag{1}
\]

where \(A(z,t)\) is the envelope of the wave, \(\beta_1\) the coefficient of chromatic dispersion of order one, \(\beta_2\) coefficient of chromatic dispersion of order two, \(\alpha\) the coefficient of dissipation, \(\gamma\) the coefficient of nonlinearity, \(z\) the spatial variable and \(t\) the time variable.

We propose to construct the solution of equation (1) in the form

\[
A(z,t) = U(t) \exp[-i(kz - \omega t)], \tag{2}
\]

where \(U(t)\) is the temporal envelope, \(k\) the wave number and \(\omega\) the angular frequency. The insertion of relation (2) into equation (1) leads to

\[
\begin{align*}
\frac{\partial^2 U}{\partial t^2} + \left( \beta_1 - \omega \beta_2 \right) \frac{\partial U}{\partial t} + \frac{\alpha}{2} U(t) - i \left( k - \beta_1 \omega + \frac{\beta_2 \omega^2}{2} \right) U(t) - i \gamma |U|^2 U = 0.
\end{align*}
\]

By setting \(\lambda_1 = \frac{\beta_1}{2}, \lambda_2 = \beta_1 - \omega \beta_2, \lambda_3 = \frac{\alpha}{2}\) and \(\lambda_4 = k - \beta_1 \omega + \frac{\beta_2 \omega^2}{2}\), equation (3) becomes

\[
\begin{align*}
&i \lambda_1 \frac{\partial^2 U}{\partial t^2} + \lambda_2 \frac{\partial U}{\partial t} + \lambda_3 U(t) - i \lambda_4 U(t) - i \gamma |U|^2 U = 0. \tag{4}
\end{align*}
\]

Equation (4) is the one that will allow conducting the different analyzes.

### 3. Building the Solutions of Equation (4)

We search the solutions of equation (4) in the form

\[
U(t) = a J_{n,m}(\Omega t), \tag{5}
\]

where, \(a \in C\), \(\Omega \in C\) such that \(J_{n,m}(\Omega t)\) is the Bognin implicit function of power \(n, m\) and parameter \(\Omega\). \(n\) and \(m\) find their values in the set of real \(R\). Taking into account the ansatz (5) in equation (4) leads for \(a \neq 0\), to the range equation of coefficient.

\[
\begin{align*}
&i \lambda_1 \Omega^2 \left[ m(m-1) J_{n-2,m-2} - (m(n-1)+n(m+1)) J_{n,m} + n(n+1) J_{n+2,m+2} \right] \\
&+ \lambda_2 \Omega \left( m J_{n-1,m-1} - n J_{n+1,m+1} \right) + (\lambda_3 - i \lambda_4) J_{n,m} - i \gamma |U|^2 J_{3n,3m} = 0. \tag{6}
\end{align*}
\]
The field of possible solutions of equation (4) and (6) is given in this case by the values of \( n \) and \( m \) for which some terms of equation (6) are grouped together. Thus, some terms in equation (6) are grouped together for the following values of \( n \) and \( m \):

\[
\begin{align*}
 n, m & \in \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}. \\
\end{align*}
\]

The different combinations between \( n \) and \( m \) which define the fields of possibilities of obtaining solutions are given by the following table.

**Table 1.** Fields of possibilities of solutions.

<table>
<thead>
<tr>
<th>( (n,m) )</th>
<th>(-1)</th>
<th>(-\frac{1}{2})</th>
<th>(0)</th>
<th>(\frac{1}{2})</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>((-1,-1))</td>
<td>((-\frac{1}{2},\frac{1}{2}))</td>
<td>((-1,0))</td>
<td>((-\frac{1}{2},\frac{1}{2}))</td>
<td>((-1,1))</td>
</tr>
<tr>
<td>(-\frac{1}{2})</td>
<td>((-\frac{1}{2},-\frac{1}{2}))</td>
<td>((-\frac{1}{2},-\frac{1}{2}))</td>
<td>((-\frac{1}{2},0))</td>
<td>((-\frac{1}{2},\frac{1}{2}))</td>
<td>((-\frac{1}{2},1))</td>
</tr>
<tr>
<td>(0)</td>
<td>((0,-1))</td>
<td>((0,-\frac{1}{2}))</td>
<td>((0,0))</td>
<td>((0,\frac{1}{2}))</td>
<td>((0,1))</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>((\frac{1}{2},-\frac{1}{2}))</td>
<td>((\frac{1}{2},-\frac{1}{2}))</td>
<td>((\frac{1}{2},0))</td>
<td>((\frac{1}{2},\frac{1}{2}))</td>
<td>((\frac{1}{2},1))</td>
</tr>
<tr>
<td>(1)</td>
<td>((1,-1))</td>
<td>((\frac{1}{2},-\frac{1}{2}))</td>
<td>((\frac{1}{2},0))</td>
<td>((\frac{1}{2},\frac{1}{2}))</td>
<td>((\frac{1}{2},1))</td>
</tr>
</tbody>
</table>

1. Case \((n,m) = (-1,-1)\): We obtain from equation (6) the equation

\[
\left(2\lambda\Omega^2 - \gamma|a|^2\right)J_{-3,-3} - \Omega\lambda_2 J_{-2,-2} + \left(\lambda_3 - 2i\lambda_4\Omega^2 - i\lambda_4\right)J_{-1,-1} + \lambda_2\Omega = 0.
\]

Equation (8) is checked if we have the following relationships

\[
\beta_1 = \omega\beta_2, \quad \alpha = i\alpha_0, \quad \alpha_0 = 2\beta_2\Omega^2 + 2k - \beta_2\omega^2.
\]

and

\[
|a|^2 = \frac{2\lambda\Omega^2}{\gamma} = \frac{\lambda_2\Omega^2}{\gamma} \Rightarrow |a| = \Omega \sqrt{\frac{\beta_2}{\gamma}}.
\]

We get from the relationship (11)

\[
a = \Omega \sqrt{\frac{\beta_2}{\gamma}} \exp i\theta, \quad \Omega > 0, \quad \theta \in R, \quad \gamma\beta_2 > 0.
\]

The solution in this case is given by

\[
A(z,t) = \Omega \sqrt{\frac{\beta_2}{\gamma}} J_{-1,-1}(\Omega t) \exp i(\theta - kz + \alpha t).
\]

This solution is indeed the exact solution of the equation (1) when the constraints joining the coefficients of the terms are taken into account such that equation (1) is modified as follows

\[
\frac{\partial A}{\partial z} + \omega\beta_2 \frac{\partial A}{\partial t} + \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i\frac{\alpha_0}{2} A - i\gamma|A|^2 A = 0,
\]

where \(\alpha_0 = 2\beta_2\Omega^2 + 2k - \beta_2\omega^2\).

2. Case \((n,m) = (1,0)\): We obtain from equation (6) the equation

\[
\left(-i\lambda J_{3,0} + \lambda_3 - 2i\lambda_4\Omega^2 - i\lambda_4\right)J_{1,0} + \lambda_2\Omega = 0.
\]

Considering the transformation \(J_{3,2} = J_{1,0} - J_{3,0}\) in equation (15), we obtain

\[
\left(i\lambda J_{3,2} + \lambda_3 - 2i\lambda_4\Omega^2 + i\gamma|a|^2\right)J_{1,0} - \lambda_2\Omega J_{3,2} = 0.
\]

Equation (16) is checked if we have the following relationships

\[
\beta_1 = \omega\beta_2, \quad \alpha = i\alpha_0, \quad \alpha_0 = \beta_2\Omega^2 - 2k + \beta_2\omega^2.
\]

and

\[
|a| = \Omega \sqrt{\frac{\beta_2}{\gamma}} \exp i\theta, \quad \beta_2\omega^2 > 0, \quad \Omega > 0, \quad \theta \in R.
\]

The solution in this case is given by
The solution (20) is indeed the exact solution of the equation (1) when the constraints joining the coefficients of the terms are taken into account such that the equation (1) is modified as follows

\[
\frac{\partial A}{\partial z} + a \beta_2 \frac{\partial A}{\partial t} + i \beta_2 \frac{\partial^2 A}{\partial t^2} - i \Gamma_0 A - i \gamma |A|^2 A = 0, \tag{28}
\]

with \( \Omega_0 = 2k - \beta_2 \omega^2 + 2\beta_2 \Omega^2 \).

The pairs \((n,m)\) globally define the field of investigation of the solutions. But at the end of the calculations and verifications, except the solutions obtained above, we obtain the trivial solutions for the other cases.

### 4. Study of the Propagation

to verify that the solutions obtained are likely to propagate, we use the Split-step Fourier programming method, which is one of the most widely used to materialize the movement of signals in the optical fiber [37]. Thus, splitting various nonlinear partial differential equations into nonlinear parts and taking into account the corresponding analytical solution for \( z = 0 \), as initial condition in each case study in this article, we obtain the propagation of the wave along distance \( z \) according to the values of the chosen parameters. We have chosen for this purpose to numerically verify the propagation of three of the solutions obtained analytically above.

**First case**

The nonlinear partial differential equation (14) is discretized so that the envelope \( A(z,t) \) is given by the relation (13).

Among the many forms of profiles that we have obtained, we retain the following

**Figure 1.** Propagation of the envelope (13) in equation (14): the left profile is obtained for: \( \omega = 5 \times 10^5 \), \( \beta_2 = 6 \times 10^{-7} \), \( \gamma = 1.6667 \times 10^3 \), \( \Omega = 0.005 \), \( \theta = \pi \), \( k = 1 \), \( \alpha_0 = 1.5 \times 10^{-4} \) the right profile is obtained for \( \omega = 4.5 \times 10^5 \), \( \beta_2 = 0.001 \), \( \gamma = 0.1 \), \( \Omega = 0.01 \), \( \theta = \pi \), \( k = 1 \), \( \alpha_0 = 2.259 \times 10^{-4} \).

**Second case**

The nonlinear partial differential equation (21) is discretized so that the envelope \( A(z,t) \) is given by the relation (13). We retain the following curves
Third case

The nonlinear partial differential equation (28) is discretized so that the envelope $A(z,t)$ is given by the relation (27). We retain the following curves

![Figure 3](image3.png)

Figure 3. Propagation of the solitary wave (27) in equation (28): the left profile is obtained for: $\omega = 0.1 \beta_2 = 10^{-5} , \gamma = 10^5 , \Omega = 0.2 , \theta = \pi , k = 1 , \Omega_0 = 2$, right profile is obtained for $\omega = 0.005 \beta_2 = 10^{-6} , \gamma = 10 , \Omega = 0.2 , \theta = \pi , k = 1 , \Omega_0 = 2$.

5. Conclusion

In this work, we first study how to choose the parameters that characterize the single-mode optical fiber so that the nonlinear partial differential equations that govern the propagation dynamics in this transmission medium admit the desired forms of solutions. The computational techniques used for the cause enabled us not only to establish the constraint relations between the characteristic coefficients of the fiber so that it is possible to obtain solutions, but to establish a field of possibilities for search progressive wave and solitary wave solutions defined by the pairs $(n,m)$. For some values of the pairs $(n,m)$, the solutions obtained are trivial and of less importance in physics. In the same logic of search for solutions, the pairs $(n,m)$ given respectively by $(-1,-1); (1,0); (1,1)$ allowed to get the solutions that are presented in this article. In each case studied, the nonlinear partial differential equation has been modified in accordance with the constraint relations established for this purpose. Using the split-step Fourier method, we have verified the effective propagation of the solutions obtained. The lesson we want to share with our readers is that the wave propagation quality in a waveguide is closely related to the quality of the waveguide by its characteristic properties in general and particularly in the optical fiber. This analysis reveals the possibility of adapting each transmission medium according to the type of signal that must propagate during its manufacture.

References


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