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Multistable Dynamic Response Behavior of Two-dimensional Discrete Duffing System

Luman Wang, Haixia Fu, Dongmei Yan, Youhua Qian*

College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua, P. R. China

Abstract

Multi-time scale problems are ubiquitous in natural science. While slow-varying parameter is one of the typical symbols of multiple-time scale. However, there is few research on the phenomenon of periodic catastrophe. In this paper, we study the multistable dynamic response behavior of the discrete fast-slow coupled Duffing system. In addition, we observe a pair of critical parameter values, which result in the disappearance of period-1 attractor under some certain parameters and the bistable dynamic behavior appears in which the periodic attractor and the chaotic attractor coexisted near the critical value. When the bifurcation parameter passes through critical points, the system will jump, which may lead to the transition from period-1 attractor to previous coexisting attractor, thus bistability is destroyed and system gets into mono-stasis. We obtain the bifurcation charts and time history curve of the bistable dynamic system for the coexistence of period-1 attractor and periods-1, 2, 4 attractors and chaos in the critical range. When the critical value range is exceeded, the period-1 attractor disappears, which leads to the bistable imbalance. Our results enrich the bistable dynamical mechanisms in discrete systems.

Keywords

Bistable Dynamic System, Bifurcation Parameter, Periodic Attractor, Chaos

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1. Introduction

Nonlinear dynamics plays an important role in mathematics, physics, mechanics, biology, economics and other fields. The emergence of many complicated phenomena is related to nonlinear factors in nature and the research of nonlinear dynamics has been concentrated in changes in the orientation properties of systems such as periodic solutions, bifurcation and chaotic motion. In addition, chaos is a unique vibration form for the nonlinear systems.

Multistable dynamic systems have been widely applied to physics [1, 2], biology [3], mechanical engineering and other fields. The generalized multistability was first put forward by Arecchi et al. [1982] in nonlinear dynamics experiments. Luo et al. [2003] presented a new hybrid control strategy, which used state feedback and parameter perturbation to control periodic bifurcation and stabled the unstable periodic orbit of the chaotic attractor embedded in discrete nonlinear dynamical systems. The simulation results indicate that it can lower the stable $2m$-periodic orbit by means of controlling highly stable $2n$-periodic orbit $(n > m)$. Jing and Wang [2005] discussed the complex dynamics in Duffing system with two external forcing. On the basis of the Second Order Averaging Method and Melnikov Method, they obtained the critical value of the chaotic motion under periodic and quasi-periodic perturbations, the numerical simulation results not only show the consistency of theoretical analysis, but also exhibit interesting bifurcation phenomena and more new complex dynamics behaviors. Yang and Chen [2005] investigated the bifurcation and chaos of an axially accelerating viscoelastic beam. Jing and Wang [2006] obtained the discrete-time predator-prey system by using

* Corresponding author
E-mail address: qyh2004@zjnu.edu.cn (Youhua Qian)

Euler Method and applied central popular theory and bifurcation theory to derive the existence conditions of reversal bifurcation and Hopf bifurcation. Shrimali et al. [2008] studied the systems with multiple coexisting attractors, where attractors evolve in a specific way and established each new attractor. A new 4-D smooth quadratic autonomous system was proposed by Cang et al. [2010] which presents periodic orbits, chaos and hyperchaos with different parameters. The system is of tremendous importance in generating four-wing hyper-chaotic attractors with two symmetrical initial conditions and two-wing hyper-chaotic attractors coexisting. Hens et al. [2012] introduced an approach for designing a coupling scheme suitable for two dynamic systems to achieve the extreme multistability. By using the concept of partial synchronization based on Lyapunov function stability, the coexistence of infinite attractors in a given set of parameters is realized. A general scheme was proposed by Pal et al. [2016] for designing multistable state continuous dynamic systems. The scheme applies the concepts of the state-based partial synchronization and motion constants to derive the time evolution diagram, phase diagram, maximum Lyapunov index variation and bifurcation diagram of the system to display the multistable properties of coupled systems. Euler method was employed by Chen et al. [2017] to convert the Duffing-Holmes equation into the discrete nonlinear dynamical system and obtained Standard Holmes Mapping. It also considered the existence and stability conditions of fixed points of the mapping. In addition, it was proved that there exists chaos in the sense of Marotto for mappings. Han et al. [2017] studied the bursting dynamic of Duffing system with multi-frequency excitation. The oscillations was proven in the quasi-static process. Han et al. [2017] reported a new route of complex blasting models based on strong cubic mapping. Boundary-Crisis-Induced complex blasting patterns indicate that chaotic attractors on stable branches may suddenly disappear owing to the boundary crisis, causing a rapid transition from chaos to other attractors and switching between stable branches of cubic graphs. A scheme concerning designing extreme multistable discrete systems with two identical dynamical systems was introduced by Chakraborty [2017]. For a given set of parameters, the existence of a boundless number of attractors is gained by partial synchronization between two systems. Zhang et al. [2017] analyzed dynamic characteristics and generation mechanisms of various mixed mode oscillations in fast-slow coupled systems with different frequency ratios under multi-frequency excitation. The research shows that by translating two external excitations into a slow variable, the system is divided into a fast slow subsystem. It also analyzed the equilibrium point and bifurcation condition of a fast subsystem, explored the impact of different frequency ratios on the mixed mode oscillation structure. Wiggers and Rech [2017] discussed the dynamic response behavior of van der Pol-Duffing forced oscillator and the existence, periodicity, quasi-periodicity and chaos of different attractors. Furthermore, they considered the occurrence of multistableness for some fixed parameter sets in the system. Taking the Duffing system with slow-variable periodic excitation as an example, Chen et al. [2017] considered a class of relaxation oscillations with complex bifurcation structures. A simple and effective method proposed namely MFSPM by Han et al. [2018]. The validity of the method is proved by several examples and this method does not rely on a particular system or branch, therefore it is a universal method.

This paper aims at employing the Euler method to study the multistable response behavior of a discrete Duffing system, and find the bistable dynamic system with periodic attractors coexisting with periodic attractors and chaotic attractors under certain parameters. The structure of this paper is as follows. In section 2, the fast-slow coupling dynamic response behavior of discrete Duffing system will be investigated. In section 3, further conclusions are presented.

2. Study on Fast Slow Coupling Dynamic Response Behavior

Due to the Duffing system has abundant dynamic properties, it has always been an example of the study of nonlinear systems. In this paper, we consider the Duffing equation as follows

$$\begin{aligned} x' &= y, \\ y' &= a_1 x - a_2 x^3 - a_3 y + f\cos\Omega t, \end{aligned} \tag{1}$$

where $x(t)$, $y(t)$ are the real functions, f is the excitation amplitude, Ω is the external excitation frequency, $a_i\,(i=1,2,3)$ are the physical parameter. That is, when the excitation frequency is much less than the inherent frequency of the system, there is a multi-time scale in the frequency domain to exhibit some typical fast-slow behaviors.

Let $\beta = f\cos\Omega t$, by using the Euler method, the discrete system is converted as follows

$$\begin{aligned} x_{n+1} &= x_n + \Delta t y_n, \\ y_{n+1} &= a_1\Delta t x_n - a_2\Delta t x_n{}^3 + (1-a_3\Delta t) y_n + \Delta t\beta. \end{aligned} \tag{2}$$

Let $z_n = x_n + \Delta t y_n$, the discrete system is written as

$$x_{n+1} = z_n,$$
$$z_{n+1} = (a_1\Delta t^2 + a_3\Delta t - 1)x_n - a_2\Delta t^2 x_n^{\ 3} + (2 - a_3\Delta t)z_n + \Delta t^2\beta. \quad (3)$$

We define a new scale transformation equation as follows

$$x^*_{n+1} = \frac{x_{n+1}}{\Delta t^2},$$
$$y^*_{n+1} = \frac{z_{n+1}}{\Delta t^2}. \quad (4)$$

Using the scale transformation equation, we have

$$x^*_{n+1} = y^*_n,$$
$$y^*_{n+1} = ax^*_n - bx^{*3}_n + ky^*_n + \beta, \quad (5)$$

where

$$a = \frac{a_1\Delta t^2 + a_3\Delta t - 1}{\Delta t^2},$$
$$b = a_2\Delta t^6,$$
$$k = 2 - a_3\Delta t. \quad (6)$$

For the sake of simplicity, we remove the asterisk in equation (5) and the system is given by

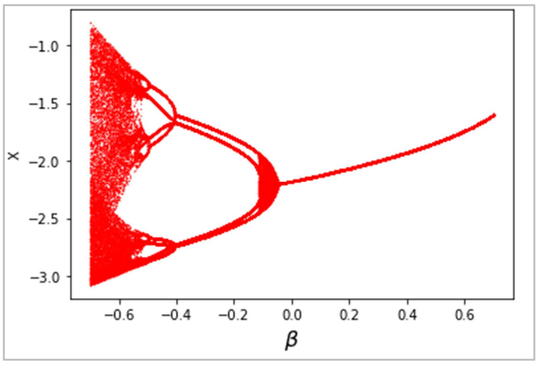
$$x_{n+1} = y_n,$$
$$y_{n+1} = ax_n - bx_n^{\ 3} + ky_n + \beta. \quad (7)$$

In order to explain the nonlinear dynamic response of the system with different parameters, the fast subsystem corresponding to equation (6) is analyzed in this paper due to the existence of slow periodic perturbations, and slow subsystem is characterized by

$$\beta = f\cos\Omega t\,. \quad (8)$$

This paper mainly reveals the nonlinear dynamic response behavior of (6). Taking β as the bifurcation parameter, when other parameters are in certain intervals, there is a critical value for β. For systems in the vicinity of β, a bistable state in which a periodic an attractor coexists with a periodic attractor or a chaotic attractor can be exhibited. In addition, a sudden change in the attractor occurs at the critical value, which leads to the disappearance of the bistable state, so the system will migrate to another attractor. Therefore, we obtained several typical transition modes when a, b, k set for different values.

Setting $a = 1.92$, $b = 0.2$, $k = 0.03$, Figure 1 shows the time history curve and bifurcation diagram of the system. As shown in the Figure 1, when $\beta < -0.58$, the system generates chaos; when $-0.5 < \beta < -0.4$, the system does 8-fold periodic motion; when $-0.4 < \beta < -0.18$, the system becomes 4-fold periodic motion. But when $\beta = -0.14$, the system turns into chaos again. And when $\beta > 0$, the system moves in a single period.



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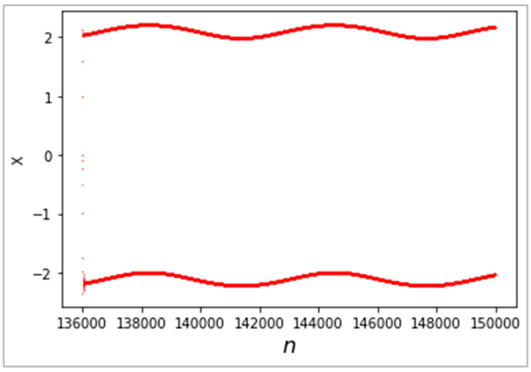

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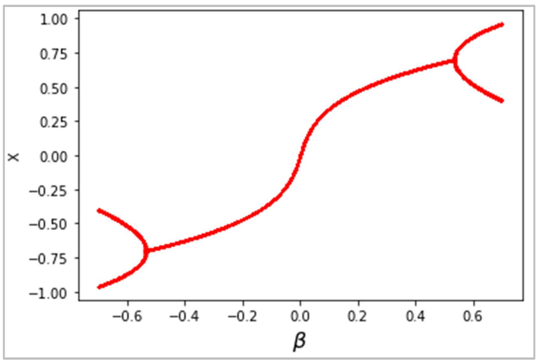
Figure 1. (a) Bifurcation diagram and (b) time history curve for $a = 1.92$, $b = 0.2$, $k = 0.03$.

Figure 2 shows the time history curve and bifurcation diagram of the system by setting $a = 0.85$, $b = 1.26$, $k = 0.003$. It can be seen that when $-0.5 < \beta < 0.5$, the system is in single periodic motion; when $\beta < -0.5$ or $\beta > 0.5$, the system makes period-doubling motion. And we could also obviously see that the nonlinear dynamic response behavior of the system without the jumping phenomenon.



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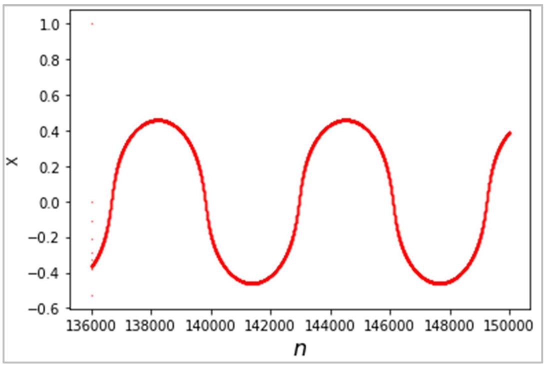

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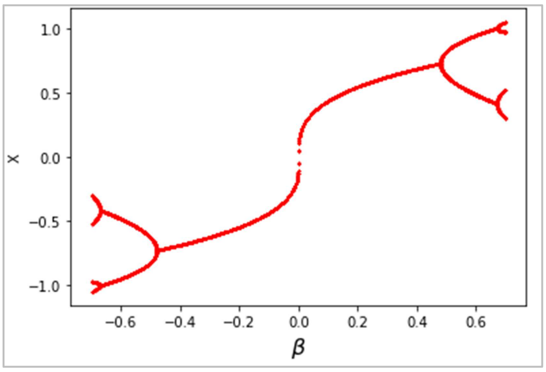
Figure 2. (a) Bifurcation diagram and (b) time history curve for $a = 0.85$, $b = 1.26$, $k = 0.003$.

Setting $a = 1.01$, $b = 1.26$, $k = 0.003$, Figure 3 obviously shows that when $0.6 < \beta < 0.5$ or $0.5 < \beta < 0.6$, the system does period-doubling motion. When $0.5 < \beta < 0$ or $0 < \beta < 0.5$, the system runs in a single period. However when $\beta = 0$, the system is unstable and there will be a jump phenomenon, which is a common phenomenon in nonlinear systems.



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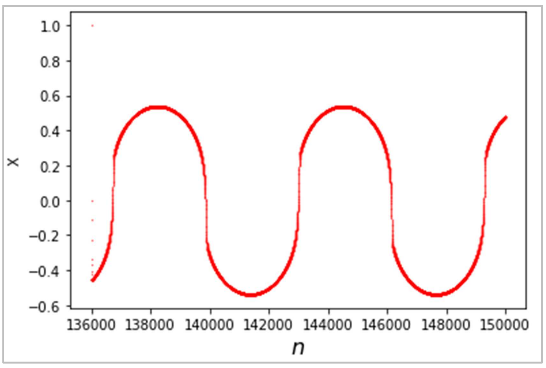

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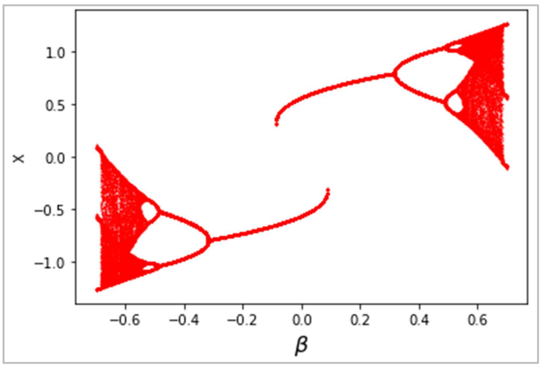
Figure 3. (a) Bifurcation diagram and (b) time history curve for $a = 1.01$, $b = 1.26$, $k = 0.003$.

Taking $a = 1.4$, $k = 0.003$, we find that the system appears bistable dynamic behavior. Figure 4 shows the bistability of the system when period 1 attractor coexists with period 1 attractor when $-0.1 < \beta < 0.1$. Once β crosses the critical value, bistability will be destroyed, therefore the system produces a transition to the periodic-1 attractor.



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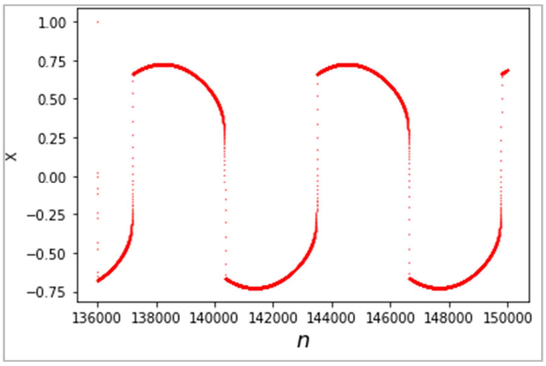

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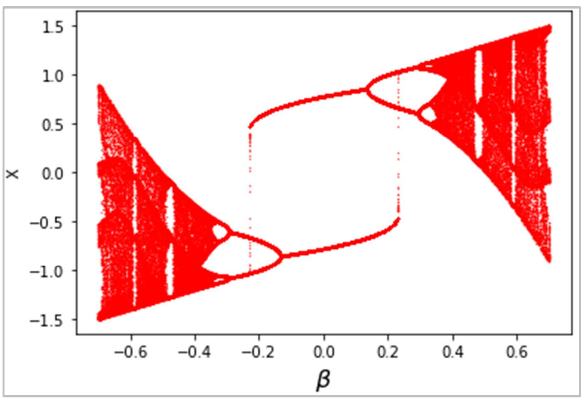
Figure 4. (a) Bifurcation diagram and (b) time history curve for $a = 1.4$, $b = 1.26$, $k = 0.003$.

We obtained a new bistable behavior with the increase of a. Setting $a = 1.76$, $b = 1.26$, $k = 0.003$, we discovered that when $0.2 < \beta < 0.1$ or $0.1 < \beta < 0.2$, the attractor of period-1 coexists with the attractor of period 2, but when β exceeds the critical value, the attractor of periodic 1 disappears, leading to the imbalance of bistability, and the attractor of periodic-1 jumps to the attractor of periodic-2.



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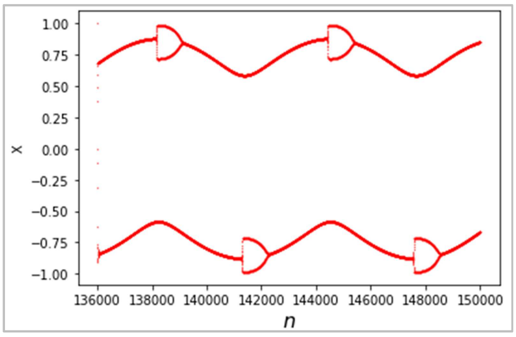
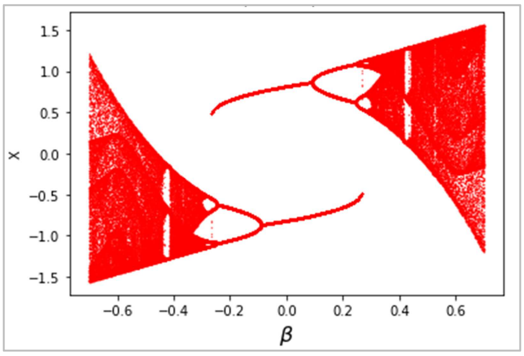


(b)

Figure 5. (a) Bifurcation diagram and (b) time history curve for $a=1.76$, $b=1.26$, $k=0.003$.

The new time history curve and bifurcation diagram are gained by increasing the value of parameter a continuously while keeping b and k unchanged. Setting $a=1.89$, Figure 6 shows when $0.3<\beta<$�0.25 or $0.25<\beta<0.3$, the system is bistable, namely, the attractor of period-1 coexists with the attractor of period-4. When $\beta=0.3$, the system jumps from the haploid periodic motion to the quadruple periodic motion. Once β outstrips the critical value, the periodic-1 attractor disappears, wrecking the bistability, and the system transits from the periodic-1 attractor to the periodic-4 attractors.



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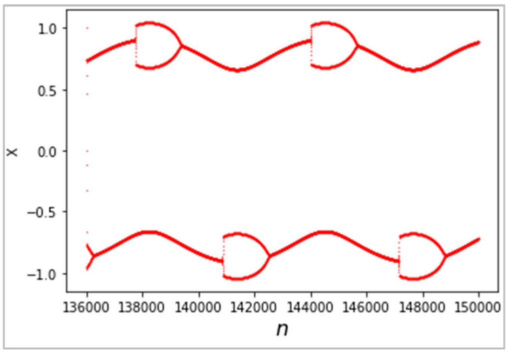
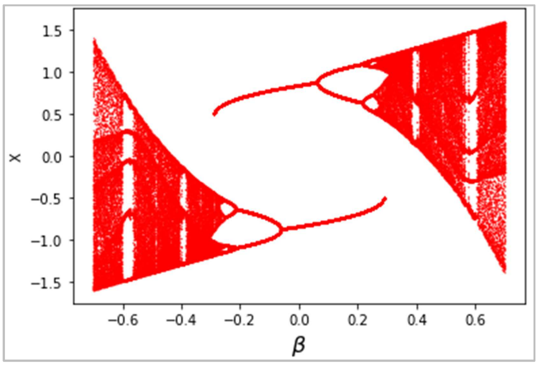


(b)

Figure 6. (a) Bifurcation diagram and (b) time history curve for $a=1.89$, $b=1.26$, $k=0.003$.

Setting $a=1.86$, $b=1.26$, $k=0.003$, we can see that when $-0.32<\beta<-0.1$ or $0.1<\beta<0.32$, the system is bistable and when $\beta=0.32$, it jumps from the haploid periodic motion to chaos. In addition, the jumping phenomenon disappears with the increase of β and the system transits from the periodic-1 attractor to chaos motion.

In summary, when the bifurcation parameter is close to the critical value, the system will have bistable dynamic response. And when the bifurcation parameter exceeds the critical value, the system will jump, leading to the disappearance of periodic-1 attractor, the bistable imbalance and the transits to periodic attractor or the chaotic attractor.



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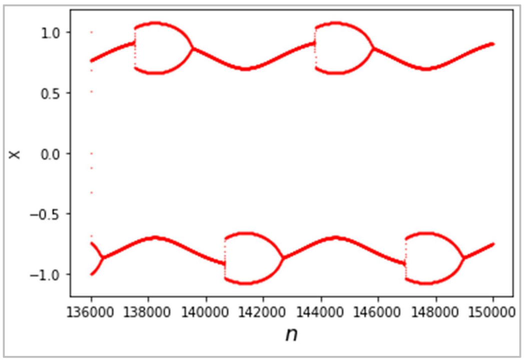

(b)

Figure 7. (a) Bifurcation diagram and (b) time history curve for $a=1.86$, $b=1.26$, $k=0.003$.

3. Conclusions

In this paper, we studied the multistable dynamic response behavior of the discrete fast-slow coupled Duffing system and explored the bistable dynamic behavior in which the periodic attractor coexists with the periodic attractor and the chaotic attractor under certain parameters. Firstly, we transformed the system into a discrete nonlinear dynamic system by employing the Euler method. Secondly, when $f\neq 0$, the system contains parametric excitation. Figure 1 and 2 are the bifurcation charts and time history curve where we can see there is no jump in the system, namely there is no bistable coexistence in the system. Setting $a = 1.01$, $b = 1.26$, $k = 0.003$, it can be seen from Figure 3 that when $\beta = 0$, jumping phenomena occurs in the system. Keeping b, k unchanged, the system exhibits bistable response behavior with the increase of a. And within the critical range of bifurcation parameter, period-1 attractor coexists with periods-1, 2, 4 attractors and chaos. But when it exceeds the critical value, the attractor of period 1 disappears, which leads to the imbalance of bistability and makes the system mono-stable.

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