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Analysis of Heat Transfer on Magnetohydrodynamic Convective Flow Past a Vertical Plate in the Presence of Heat Source/Sink

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Abstract

This work examines the heat transfer analysis of a convective flow over a vertical plate under the combined influence of viscous dissipation and thermal radiation in the presence of heat source/sink with the plate being subjected to a variable surface temperature. The governing boundary layer equations are formulated, simplified and non-dimensionalised. The dimensionless equations were solved by employing Crank Nicolson's implicit finite difference scheme. The effects of dimensionless numbers affecting the flow are shown graphically on the dimensionless temperature profile. Increasing thermal radiation reduces temperature profile while there was an increase on temperature profile with an increase in dissipation parameter.

Keywords

Heat Transfer, Convective Flow, Radiation, Viscous Dissipation, Heat Source, Magnetic Field

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1. Introduction

Free convective flow of a viscous, incompressible fluid past semi-infinite or infinite flat plates with thermal radiation effects is of great importance due to their numerous applications in engineering and other processes involving high temperatures such as nuclear power plants, gas turbines and thermal energy storage. It is often assumed that in such flows, the viscous dissipative heat is negligible. It was shown by Gebhart [1] that the viscous dissipative heat is important when the natural convection flow field is of extreme size or the flow is at extremely low temperature or in high gravity field. Gebhart and Mollendorf [2] also investigated the effects of viscous dissipative heat on free convective flow past semi-infinite plates. Analysis on the radiation effects of free convection flow of a gas past a semi-infinite flat plate was considered by Soundalgekar et al. [3]. Sakiadis [4] studied the steady flow on a moving continuous flat surface and developed a numerical solution using a similarity transformation. Sparrow and Cess [5] investigated the effects of magnetic field on the natural convection heat transfer. Romig [6] considered the effect of electric and magnetic fields on the heat transfer to electrically conducting fluids. The heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation was analysed by Vajravelu and Hadjinicolaou [7]. Muthucumaraswamy and Ganesan [8] worked on the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. The radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical porous plate with viscous dissipation was examined by Gnaneshwara and Bhaskar [9]. Alam, Rahman and Sattar [10], tried transient magnetohydrodynamic free convective heat and mass transfer flow with thermophoresis past a radiative inclined permeable plate in the presence of variable chemical reaction and temperature dependent viscosity. Anjali Devi and Ganga

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[11] considered the problem of dissipation effects on MHD nonlinear flow and heat transfer past a Porous surface with prescribed heat flux. Kumar [12] investigated the radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Gribben [13] investigated the boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of pressure gradient where he obtained solutions for large and small magnetic Prandtl number using the method of matched asymptotic expansion. Unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting fluid through a porous medium bounded by infinite vertical plane surface of constant temperature was carried out by Helmy [14]. The radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method was investigated by Takhar [15]. Chamkha et al. [16] examined the radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer. The radiative and free convections effects on the oscillatory flow past a vertical plate was carried out by Mansour [17]. Raptis and Perdikis [18] studied the effects of thermal radiation and free convective flow past moving plate.

Gregantopoulos et al. [19] considered two-dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate.

However, convective flow over a vertical plate under the combined influence of viscous dissipation and thermal radiation in the presence of heat source/sink with the plate being subjected to a variable surface temperature has not been considered so far in literature.

2. Mathematical Analysis

Consider an unsteady flow of an electrically conducting, viscous, incompressible and radiating fluid flowing past a semi-infinite vertical plate. The x axis is taken along the plate in the vertical direction while the y axis is taken normal to it. A uniform magnetic field with strength B_o is applied perpendicular to the plate along the y axis and the effect of viscous dissipation is taken into account. All the fluid properties are taken to be constant except the influence of the density variation in the body force term that are caused by changes in temperature which are approximated by Boussinesq. The governing boundary layer equations are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T^*-T^*_\infty)-\frac{\sigma B_o^2}{\rho}u \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}+u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}=\alpha\frac{\partial^2 T^*}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{Q_s(T^*-T^*_\infty)}{\rho C_p}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The initial and boundary conditions are given as

$$u=0, v=0, T^*=T^*_\infty\ for\ all\ x,y,t\le 0$$

$$u=u_0, v=0, T^*=T^*_w at\ y=0\ for\ t>0$$

$$u\to 0, T^*\to T^*_\infty\ as\ y\to\infty \tag{4}$$

where u is the velocity in x direction, v is the velocity in the y direction, t^* is the time, g is the acceleration due to gravity, T^* is the fluid temperature, T^*_∞ is the free stream temperature, β is the coefficient of thermal expansion, u_0 is the velocity of the plate, υ is the kinematic viscosity, B_0 is the magnetic field, q_r is the radiative heat flux, σ is the electrical conductivity, C_p is the specific heat at constant pressure, ρ is the fluid density, q_r is the radiative heat flux and Q_s is the heat source/sink parameter.

For an optically thin limit, the radiating gas is said to be non-gray near equilibrium and it is defined according to Oyelami and Dada [20] as follow

$$\frac{\partial q_r}{\partial y^*}=4(T^*-T^*_0)L \tag{5}$$

where L= $\int_0^\infty k\lambda_w\left(\frac{de_{b\lambda}}{dT^*}\right)_w d\lambda$, $k\lambda_w$ is the absorption coefficient and $e_{b\lambda}$ is the plank function.

By introducing these non-dimensional quantities,

$$X=\frac{x}{l}, Y=\frac{y}{l}, U=\frac{u}{u_o}, V=\frac{v}{u_o}, t=\frac{t^*u_o}{l}, T=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty}, Q=\frac{Q_s l^2}{\rho C_p u_o} \tag{6}$$

$$M=\frac{\sigma B_0^2 l}{\rho u_o}, Ec=\frac{u_0^2}{C_p(T^*_w-T^*_\infty)}, Pr=\frac{\nu}{\alpha}, N=\frac{4Ll}{\rho C_p u_o}, G_c=\frac{g\beta T(T^*_w-T^*_\infty)l}{u_0^2}$$

the dimensional equations (1), (2) and (3) becomes

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{1}{Re}\frac{\partial^2 U}{\partial Y^2}+G_rT-MU \tag{8}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{PrRe}\frac{\partial^2 T}{\partial Y^2}+Ec\left(\frac{\partial U}{\partial Y}\right)^2-N(T-1)+QT \tag{9}$$

together with the dimensionless initial and boundary conditions

$$U=0, V=0, T=0 \; for\; all\; X,Y,t\leq 0$$

$$U=1, V=0, T=1 \; at\; Y=0 \; for\; t>0$$

$$U\rightarrow 0, T\rightarrow 0 \; as\; Y\rightarrow\infty \tag{10}$$

Where U and V are the dimensionless velocities in X and Y directions, t is the time, T is the temperature of the surrounding fluid, Re is the Reynold number, G_r is the thermal Grashof number, M is the Magnetic field parameter, Q is the heat absorption/generation, Ec is the Eckert number, N is the thermal radiation parameter and Pr is the Prandtl number.

3. Numerical Approach

The set of unsteady non-linear partial differential equations under the boundary conditions are solved by employing an implicit finite difference scheme of Crank-Nicolson type. This method converges faster and it is unconditionally stable. The corresponding finite difference equations are given in the form below:

$$\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}+U_{i,j-1}^{k+1}-U_{i-1,j-1}^{k+1}+U_{i,j-1}^{k}-U_{i-1,j-1}^{k}}{4\Delta X}+\frac{V_{i,j}^{k+1}-V_{i,j-1}^{k+1}+V_{i,j}^{k}-V_{i,j-1}^{k}}{2\Delta Y}=0 \tag{11}$$

$$\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\frac{\left(U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}\right)}{2\Delta X}+V_{i,j}^{k}\frac{\left(U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}\right)}{4\Delta Y}$$

$$=\frac{1}{PrRe}\frac{U_{i,j-1}^{k+1}-2U_{i,j}^{k+1}+U_{i,j+1}^{k+1}+U_{i,j-1}^{k}-U_{i,j}^{k}+U_{i,j+1}^{k}}{2(\Delta Y)^2}+Gr\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}-M\frac{U_{i,j}^{k+1}+U_{i,j}^{k}}{2} \tag{12}$$

$$\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\frac{\left(T_{i,j}^{k+1}-T_{i-1,j}^{k+1}+T_{i,j}^{k}-T_{i-1,j}^{k}\right)}{2\Delta X}+V_{i,j}^{k}\frac{\left(T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}\right)}{4\Delta Y}$$

$$=\frac{1}{PrRe}\frac{T_{i,j-1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j+1}^{k+1}+T_{i,j-1}^{k}-T_{i,j}^{k}+T_{i,j+1}^{k}}{2(\Delta Y)^2}+Ec\left(\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2\Delta Y}\right)^2-N\left[\left(\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right)-1\right]+Q\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2} \tag{13}$$

In the finite difference equations, the co-ordinates of the mesh points of the solution are defined by $X=i\Delta X, Y=j\Delta Y, t=k\Delta t$ where i, j and k are positive integers and the value of U and V at the mesh point is given as $U_{i,j}^{k}$ and $V_{i,j}^{k}$.

The region of integration is considered to be a rectangle with sides X_{max}=1 and Y_{max}=1 whereby regarding Y_{max}=1 as Y=∞ which lie very well outside the momentum, thermal and concentration boundary layers. The subscript i, j and k relates to grid points along X, Y and t respectively. We therefore divide X and Y into M and N grid spacing respectively. The mesh sizes are ΔX=0.05, ΔY=0.05 and Δt=0.01.

The coefficients $U_{i,j}^{k}$ and $V_{i,j}^{k}$ appearing in the difference equation are treated as constants in any one time step. The values of T, U and V are known at all grid point from the initial conditions at t=0. The computations of T, U and V at time level (k+1) using the known values at previous time level k are carried out as follows.

At every internal nodal point, equation (13) on a particular i-level forms a tri-diagonal system of equations which is solved by Thomas algorithm as discussed in Carnahan et al [21].

In this way, the values of T are found at every nodal point for a particular i at (k + 1) time level. Using T values at (k + 1) time level in equation (12), the values of U at (k + 1) time level are found in the same way we calculated T values. Thus, the values of T and U are known on a particular i-level at every nodal point. The values of V are calculated explicitly using equation (11) at every nodal point on a particular i-level at (k + 1) time level. This process is repeated for various i-levels. Thus, the values of T, U, and V are known at all grid points at (k + 1) time level. Computations are carried out until the steady state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of U as well as temperature T at two consecutive time steps are infinitely small.

4. Discussion of Results

To report on this work, calculations are carried out for different values of fluid parameters to show their influence on the flow field. Default values are Gr=5.0, Re=1.0, M=1.0, Pr=0.7, N=0.04, Q=1.0, Ec=0.002.

As observed from figure 1, the temperature distribution decreases as heat absorption parameter Q increases because when heat is absorbed, the buoyancy force decreases the temperature profiles.

A similar observation is seen by considering various values of Eckert number Ec as shown in Figure 2. To observe the effect of Eckert number, other parameters are kept fixed, i.e Gr=5.0, Re=1.0, M=1.0, Pr=0.7, N=0.04, Q=1.0, n=0.5 and the effect of Eckert number is observed for Ec=0.0, 0.1, 0.4, 1. It's graphical representation for the temperature distribution revealed that by increasing Eckert number Ec, the temperature profile increases.

Figure 3 illustrates the numerical results for various values of Prandlt number Pr=0.04, 0.7, 3. It is observed that by increasing Prandtl number Pr, the temperature distribution decreases. This is as a result of the presence of Lorentz force that has power to slow down the motion of the fluid.

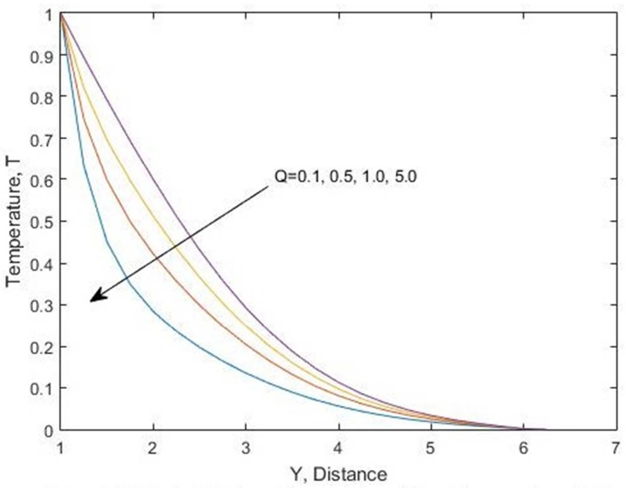

Figure 1. Effect of variable temperature (Q) on temperature profile.

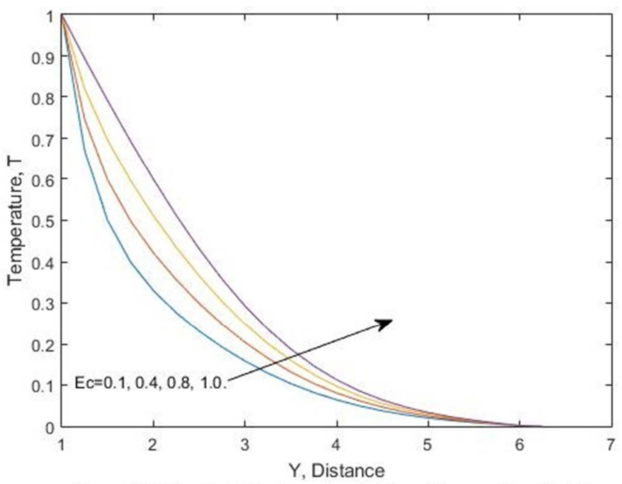

Figure 2. Effect of Eckert number (Ec) on temperature profile.

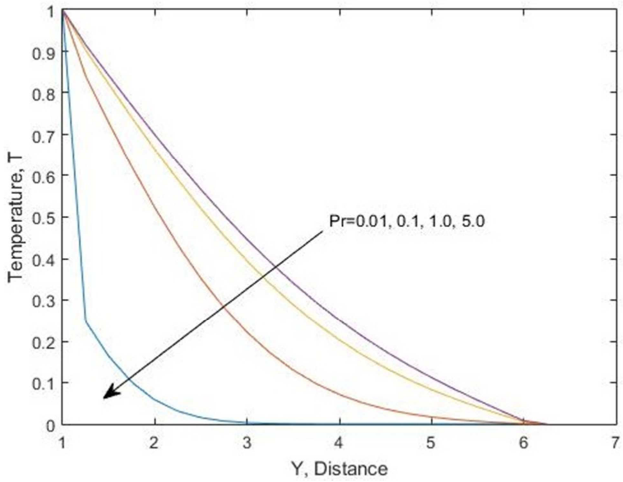

Figure 3. Effect of Prandtl number (Pr) on temperature profile.

Figure 4 represents the influence of radiation on the temperature profile. Increasing radiation value reduces temperature profile.

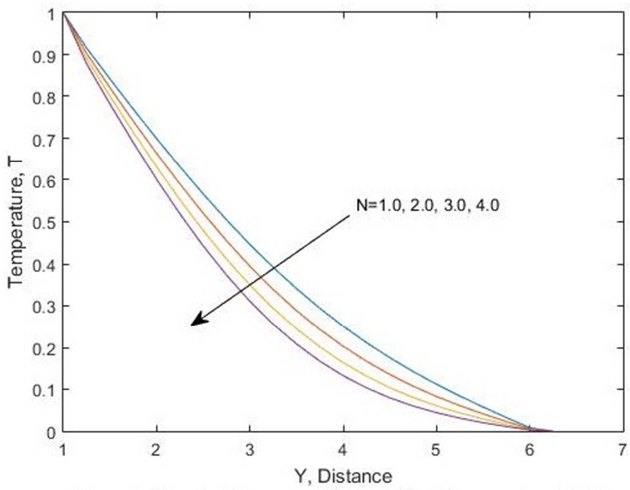

Figure 4. Effect of thermal radiation (N) on temperature profile.

5. Conclusion

The heat transfer analysis of a convective flow over a vertical plate under the combined influence of viscous dissipation and thermal radiation in the presence of heat source/sink with the plate being subjected to a variable surface temperature is considered in this analysis. The governing boundary layer equations are formulated, simplified and non-dimensionalised. The dimensionless equations were solved with the help of Crank Nicolson's implicit finite difference scheme. The effects of dimensionless numbers affecting the flow are shown graphically on the dimensionless temperature profile. Increasing thermal radiation reduces temperature profile while there was an increase on temperature profile with an increase in dissipation parameter.

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