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Origin of the Cusp Phenomenon of the Coercivity Curve of the Stoner-Wohlfarth Magnet Under Stress

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Abstract

In this paper, we study a Stoner-Wohlfarth (SW) magnet under stress and explain the origin of the cusp phenomenon of its coercivity curve. With the help of the method of vector composition, the uniaxial magneto-crystalline anisotropy field and stress anisotropy field of the magnet are combined into an equivalent uniaxial anisotropy field (the resultant of the two fields), therefore, the magnet can be resolved using the SW model to analyze its stress-dependent nature. Based on that, the cusp phenomenon of the coercivity curve as the function of stress intensity is reproduced successfully. This phenomenon was reported as early as 1999, but has not been explained satisfactory. It is shown that the cusp phenomenon is the result of the shift of the equivalent easy axis, or the competing between the stress anisotropy and the uniaxial anisotropy. The existence of double-cusp phenomenon in the systems with exchange bias is also predicted.

Keywords

The Stoner-Wohlfarth Model, Coercive Force, Stress, Cusp Phenomenon

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1. Introduction

The concept of strain engineering was popular in the semiconductor industry in its early years, for example, the strain silicon technology was well known in all fields related with semiconductor CMOS circuits. Now various strain engineering techniques have been widely used in many fields of material science and engineering. As an important mean of controlling or modulation for spintronic devices, the stress or strain effects of magnetic materials, especially the magnetic multilayer film materials have attracted widespread interest [1-6].

Studies have shown that stress has a certain impact on the physical properties of magnetic materials, such as magnetic coercivity, remanence, anisotropy, magnetostriction, magnetoresistance, ferromagnetic resonance, spin waves, etc. Till now, many authors have presented their reports on the stress effects on coercivity H_c and remanence M_r, and with more in-depth ,the dependence of field orientation on H_c. As for the exchange bias system, in addition to H_c, the exchange bias field H_e was also very attractive. The investigations on the phenomena of angle-dependent exchange bias (ADEB) under stress deepen one's understanding about the relationship of H_c with field orientation. However, at present, the study of the relationship of H_c along with the change of stress field intensity is not thorough. Some results reported show that，in the single domain approximation ，H_c tends to be a monotonically increasing function of stress, whereas M_r tends to be a monotonically decreasing function of stress. However, other results indicate a more complex relation between H_c and the stress intensity.

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The most earliest report on the non-monotonically behavior of H_c as the function of stress intensity was given by Sablik and Jiles [7]. In 1999, they studied the stress effects of a kind of ferromagnetic composite rod with the Stoner-Wohlfarth (SW) model and found that a cusp or fold appeared in the coercivity curve in the process of the stress intensity increasing and they called this phenomenon as the "cusp phenomenon ". After that, Zhu et al. [8] simulated the stress properties of magnetic materials with the Landau–Lifshitz–Gilbert (LLG) equation and they showed the similar phenomenon. Work also was done in the aspect of the experiments. Permiakov et al. [9] performed the electrical steel stress experiment in 2004 and their measured results showed the non- monotonicity of the stress-dependent H_c. In the experiment, they focused on the magnetic hysteresis loss. As the coercive force is proportional to the area of the hysteresis loop, the properties of the area of the hysteresis loop can also be applied to coercive force. The most recent report on the cusp phenomenon was given by Yamamoto et al. [10] in 2011 (details can be seen from figure 1 and figure 2 of literature 10).

The above review tells us that the cusp phenomenon of coercive force curve is an interesting phenomenon, and can be achieved under relatively loose conditions. However, to the best of our knowledge, no satisfactory explanations for this phenomenon have been presented till now. Sablik and Jiles explained this phenomenon as a first- and second-order transition in their paper published in 1999. Obviously, this explanation is not satisfactory.

The SW model is widely used in simulations and theoretical calculations for all kinds of magnetic materials under the single domain approximation. The most important characteristics of this model is that one can obtain the analytical forms of some important physical quantities ,such as coercivity, and that makes the analysis simple and clear. In this paper, we analysis the stress properties of magnetic materials using the SW model. We reproduced successfully the cusp phenomenon for the intensity-dependent coercivity curve and showed that it is the result of the shift of the equivalent easy axis, or the competing between the stress anisotropy and the uniaxial anisotropy.

The existence of double-cusp phenomenon in the systems with exchange bias is also predicted.

2. The Coercivity of the SW Model

Consider a SW particle with its easy axis (EA) along the x direction and its magnetization direction shifting from the EA with angle θ. The external field (with the intensity H_0) is applied in the plane (x-y plane) formed by the EA and the magnetization vector of the particle , and its orientation angle to the EA is denoted as θ_0.

The total free energy density can be written as:

$$F = -K_F \cos^2\theta - M_F H_0 \cos(\theta - \theta_0) \quad (1)$$

where the first term represents the uniaxial anisotropy energy with the anisotropy constant K_F, the second term is the Zeeman energy with M_F the saturation magnetization . If we are making the notations, $E = 2F / M_F, H_K = 2K_F / M_F$, the total free energy density may be rewritten in the reduced form, as:

$$E = -H_K \cos^2\theta - 2H_0 \cos(\theta - \theta_0)\,. \quad (2)$$

The coercivity is determined from the stable equilibrium conditions $\frac{\partial E}{\partial \theta} = 0$ and $\frac{\partial^2 E}{\partial \theta^2} > 0$,as [11，12]

$$H_{cr} = \begin{cases} H_{sr} \ (0 \le \theta_0 \le \frac{\pi}{4}) \\ \left|H_K \sin\theta_0 \cos\theta_0\right| \ (\frac{\pi}{4} < \theta_0 \le \frac{3\pi}{4}) \\ H_{sr} \ (\frac{3\pi}{4} < \theta_0 \le \pi) \end{cases} \quad (3)$$

Here, H_{cr} is the coercivity at the ascending branch of the hysteresis loop, the expression of the coercivity at the descending branch of the hysteresis loop, H_{cl}, is not presented.

In Eq. (3) $H_{sr} = \dfrac{H_K}{[\cos^{\frac{2}{3}}\theta_0 + \sin^{\frac{2}{3}}\theta_0]^{\frac{3}{2}}}$ is the switching field at the ascending branch of the hysteresis loop， which is obtained with the equilibrium conditions $\frac{\partial E}{\partial \theta} = 0$ and $\frac{\partial^2 E}{\partial \theta^2} = 0$.

3. The Composition of Two Vectors

When there are two kinds of anisotropic field (such as magnetocrystalline uniaxial anisotropy and stress anisotropy), one can use the method of vector composition to combine them into an equivalent anisotropic field. The principle is shown in figure 1.

Figure 1 shows the vector resultant $\vec{A}$ and its two components $\vec{A}_1$ and $\vec{A}_2$, $\vec{A} = \vec{A}_1 + \vec{A}_2$. The projections for a certain direction (x direction) of these vectors satisfies

$A_x = A_{1x} + A_{2x}$ .From figure 1, let $A_{1x}=A_1\cos\phi_1 = A_1\cos(\phi_2-\psi)$, $A_{2x}=A_2\cos\phi_2$, $A_x=A\cos(\phi_2-\beta)$, we have

$$A_1\cos(\phi_2-\psi)+A_2\cos\phi_2=A\cos(\phi_2-\beta) \quad (4)$$

where ϕ_1 and ϕ_2 stand for the angles between $\vec{A}_1$, $\vec{A}_2$ and the reference axis (x axis), with $\psi=\phi_2-\phi_1$, $\beta=\phi_2-\phi$, $A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\psi)}, \tan(\beta)=\dfrac{A_1\cos(\psi)}{A_2+A_1\sin(\psi)}$.

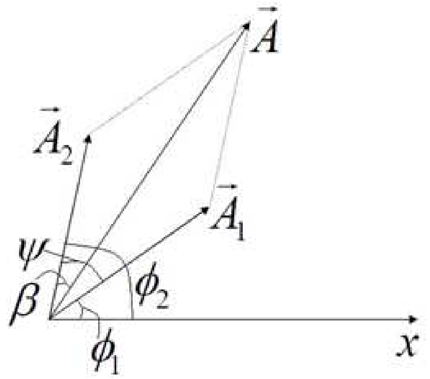

Figure 1. Schematic illustration for the composition of two vectors.

4. The Coercivity of the SW Magnet under Stress

Take a FM film as an example. The film is taken to lie in the x-y plane, and the z axis is normal to the film plane. We have assumed that the magnetization vector $\vec{M}$ is along the x direction and θ is the orientation angle of $\vec{M}$ to the easy axis. The external field is applied in the film plane and its orientation angle to the easy axis is denoted as θ_0 . We consider only in-plane stress, with σ the mechanical stress vector directed at angle ψ to the easy axis.

The total free energy density can be written as:

$$F=-K_Ft_F\cos^2\theta-M_FH_0t_F\cos(\theta-\theta_0)$$
$$-\frac{3}{2}t_F\lambda_s\sigma\cos^2(\theta-\psi) \quad (5)$$

where the first term represents the uniaxial anisotropy energy of the FM layer with the anisotropy constant K_F and thickness t_F , the second term is the Zeeman energy with M_F the saturation magnetization . Finally, the last term stands for the magneto-elastic free energy, in which λ_s is the saturation magnetostriction coefficient. If we are making the notations, $E=2F/M_Ft_F, H_K=2K_F/M_F, H_\sigma=3\lambda_s\sigma/M_F$, the total free energy density may be rewritten in the reduced form, as:

$$E=-H_K\cos^2\theta-2H_0\cos(\theta-\theta_0)-H_\sigma\cos^2(\theta-\psi) \quad (6)$$

By using of the trigonometric function relationship $\cos^2\alpha=\dfrac{1+\cos2\alpha}{2}$ and the transformation of Eq. (4), Eq. (6) can be transformed into [13-16]

$$F=-K_Ft_F\cos^2\theta-M_FH_0t_F\cos(\theta-\theta_0)$$
$$-\frac{3}{2}t_F\lambda_s\sigma\cos^2(\theta-\psi) \quad (7)$$

where $H^*=\sqrt{H_K^2+H_\sigma^2+2H_KH_\sigma\cos(2\psi)}$ is the equivalent total anisotropy field and θ^* is the angle between this equivalent total anisotropy field and the EA of the particle, satisfying $\tan2\theta^*=\dfrac{H_\sigma\sin2\psi}{H_K+H_\sigma\cos2\psi}$.

The third term of Eq. (7) is independent on θ ,therefore, similar to the derivation process of Eq.(3), we can obtain the coercivity which is corresponding to Eq.(7) as

$$H_{cr}=\begin{cases} H_{sr} \ (0\le|\theta_0-\theta^*|\le\frac{\pi}{4}) \\ |H^*\sin(\theta_0-\theta^*)\cos(\theta_0-\theta^*)| \ (\frac{\pi}{4}<|\theta_0-\theta^*|\le\frac{3\pi}{4}) \\ H_{sr} \ (\frac{3\pi}{4}<|\theta_0-\theta^*|\le\pi) \end{cases} \quad (8)$$

in which the switching field at the ascending branch of the hysteresis loop is $H_{sr}=\dfrac{H^*}{[\cos^{\frac{2}{3}}(\theta_0-\theta^*)+\sin^{\frac{2}{3}}(\theta_0-\theta^*)]^{\frac{3}{2}}}$.

With the help of (8), we calculate the coercivity as the function of stress intensity and plot the results in figure 2. We take $\psi=\pi/4$ in the calculations, implying that a biaxial in-plane strain takes place and the stresses along both x and y directions are approximately the same. In addition, we have taken the notation $s=H_\sigma/H_K$.

The cusp phenomenon appears in figure 2, which is similar to that shown in literature [7]. From figure 2, one can see that the cusp position shifts right as the external field orientation angle increases. When the external field orientation angle exceeds a certain range, the cusp phenomenon disappears and the coercivity becomes a monotonic function of the stress intensity. Later, we will determine the cusp position exactly by analyzing the origin of this phenomenon.

It should be pointed out that the generation of the cusp phenomenon is conditional. In other words, the cusp phenomenon will disappear if ψ takes some special values (for example, $\psi=0$).

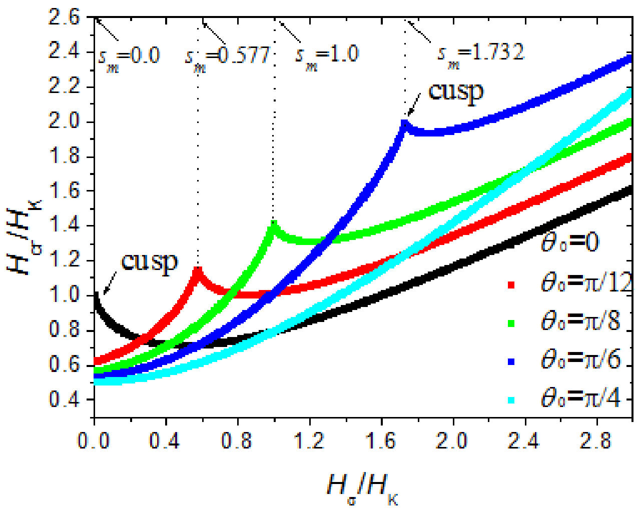

Figure 2. Coercive force changing as the function of stress intensity, s_m stands for the value of H_σ / H_K at the peak position.

5. The Coercivity of the FM/AFM Exchange Bias System under Stress

The method developed above can be used to deal with the systems with exchange bias, such as FM/AFM bilayer films [17-20]. Assuming the FM film still follows the assumptions given in the above section, and the AFM layer is thick enough so that its anisotropy energy can be neglected, the free energy density of the system can be written in the reduced form as:

$$\begin{aligned} E = &-H_K \cos^2\theta - 2H_0\cos(\theta-\theta_0) \\ &-H_\sigma\cos^2(\theta-\psi) - J\cos\theta \end{aligned} \tag{9}$$

in which the last term is the so-called unidirectional anisotropy energy characterized by $J = 2J_E / M_F t_F$ with J_E the exchange-coupling constant. In Eq. (9) , we have assumed that the easy axes of the unidirectional and uniaxial anisotropies are collinear. The other parameters of Eq. (9) are the same as in Eq. (6). Similarly, Eq. (9) can be transformed into

$$\begin{aligned} E = &-H^*\cos^2(\theta-\theta^*) - 2H_0\cos(\theta-\theta_0) \\ &-J\cos\theta + \frac{H^*-H_K-H_\sigma}{2} \end{aligned} \tag{10}$$

The coercivities derived from Eq. (10) are in the following forms

$$H_{cr} = \begin{cases} H_{sr} \ (0 \le \theta_0 \le \theta_1^{**}) \\ H_{c1}(\theta_1^{**} < \theta_0 \le \frac{\pi}{2}+\theta^*) \\ H_{c2}\ (\frac{\pi}{2}+\theta^* < \theta_0 \le \theta_3^{**}) \\ H_{sr}\ (\theta_3^{**} < \theta_0 \le \pi) \end{cases} \tag{11a}$$

$$H_{cl} = \begin{cases} H_{sl} \ (0 \le \theta_0 \le \theta_2^{**}) \\ H_{c2}\ (\theta_2^{**} < \theta_0 \le \frac{\pi}{2}+\theta^*) \\ H_{c1}(\frac{\pi}{2}+\theta^* < \theta_0 \le \theta_4^{**}) \\ H_{sl}\ (\theta_4^{**} < \theta_0 \le \pi) \end{cases} \tag{11b}$$

in which the switching fields H_{sr} and H_{sl} are determined by the equation $\begin{aligned}&[H_s\cos(\theta_0-\theta^*)+\frac{J}{2}\cos\theta^*]^{\frac{2}{3}} \\ &+[H_s\sin(\theta_0-\theta^*)-\frac{J}{2}\sin\theta^*]^{\frac{2}{3}} = (H^*)^{\frac{3}{2}}\end{aligned}$, θ_1^{**} and θ_4^{**} are the two roots of the equation $-2H^*\cos 2(\theta_0-\theta^*) - J\sin\theta_0 = 0$, θ_2^{**} and θ_3^{**} are the two roots of the equation $-2H^*\cos 2(\theta_0-\theta^*) + J\sin\theta_0 = 0$. $H_{c1} = [H^*\sin 2(\theta_0-\theta^*) - J\cos\theta_0]/2$, $H_{c2} = -[H^*\sin 2(\theta_0-\theta^*) + J\cos\theta_0]/2$. In Eq.(11), H_{c1} and H_{c2} are used alternatively to ensure $H_{cr} > H_{cl}$, since H_{c1} and H_{c2} will change the magnitude relationship in the process of change with θ_0 (see figure 3).

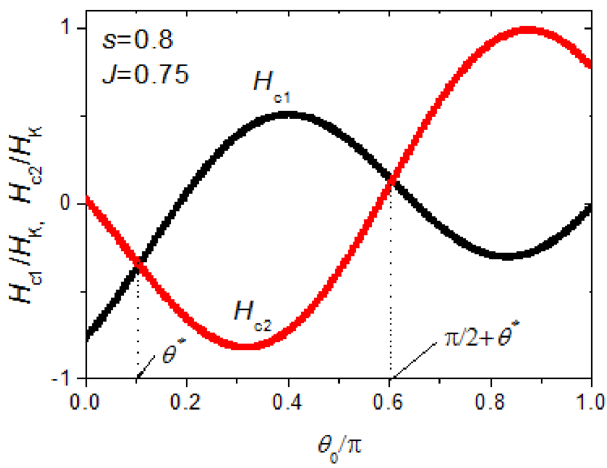

Figure 3. The angular dependence of H_{c1} and H_{c2} ($s = H_\sigma / H_K$).

For the exchange bias systems, the coercivities of the ascending and descending branches of the hysteresis loop are not the same. Then the coercivity is generally defined as $H_c = (H_{cr} - H_{cl})/2$, in which H_{cr} and H_{cl} are the coercivities of the ascending and descending branches of the hysteresis loop, respectively. We calculated the coercivities changing with stress intensity (see figure 4), and we took $\psi = \pi/4$ in the calculation.

Figure 4 also shows the cusp phenomenon in the curves of coercivity changing with the stress intensity, which is similar to that shown in figure 2. However, unlike the single-peak curve structure of figure 2, we see a double-peak curve structure or a doublet splitting phenomenon in figure 4. Moreover, with the increase of J and s, doublet splitting becomes more and more obvious.

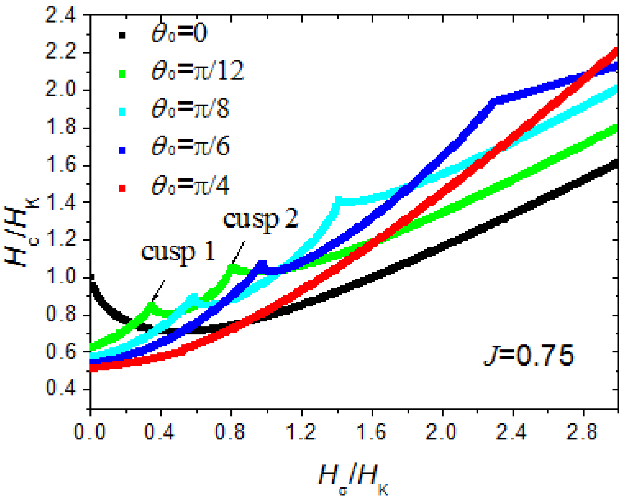

Figure 4. Coercivity curves as the function of stress intensity for an exchange bias system.

6. Origin of the Cusp Phenomenon

It should be noted that this cusp phenomenon is similar to the jump phenomenon reported in [19,20]. However, no reasonable explanations for the origin of this phenomenon have been given for this jump phenomenon till now. In the following, we will calculate the coercive force as the function of the external field orientation (figure 5) to analyze the origin of the cusp phenomenon.

In figure 5, the angular dependence of the coercivity for s=0.0,J=0.0, s=1.5,J=0.0 and s=1.5,J=0.75.($s = H_\sigma / H_K$) are shown. When s=0, we see a standard coercivity curve of the SW model, which is symmetrically distributed with the maximum at θ_0=0 and the minimum $\theta_0=\pi/2$ .After the stress applied, or s>0, the coercivity curve is not symmetric and shifts right, its maximum shifts from θ_0=0 to $\theta_0=\theta^*$ and its minimum shifts from $\theta_0=\pi/2$ to $\theta_0=\pi/2+\theta^*$. The cusp phenomenon appears. Obviously, this cusp phenomenon is related quantitatively with the angle θ^*, implying that the reason for this phenomenon is caused by the offset of the EA - the equivalent EA shifts from the original EA of the degree of θ^*.

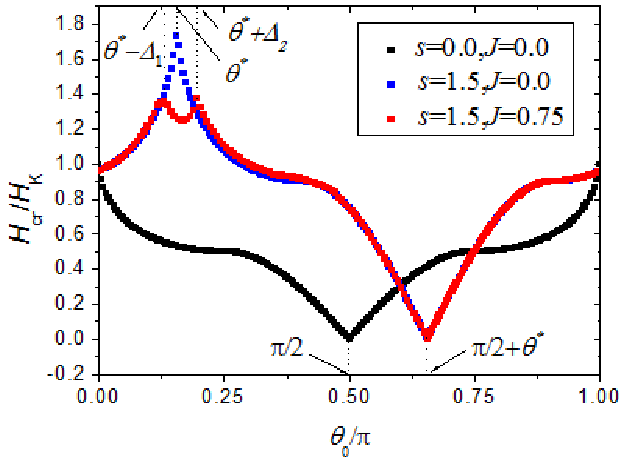

Figure 5. The angular dependence of the coercivity for s=0.0,J=0.0, s=1.5, J=0.0 and s=1.5,J=0.75.($s = H_\sigma / H_K$).

It is easy to see from figure 5 that, when the peak occurs , the external field should be on the direction of the equivalent EA, in other words, the condition for the peak occurring is $\theta_0 - \theta^* = 0$. That is also the condition that the external field orientation meets when the switching field of the ascending branch takes its maximum. Assume $s = s_m$ at which the peak occurs. From the equation $s_m = \tan(2\theta^*) = \tan(2\theta_0)$ we can know that, θ_0=0 when s_m=0 ; $\theta_0=\pi/12$ when s_m=0.577 ; $\theta_0=\pi/8$ when s_m=1.0 ; $\theta_0=\pi/6$ when s_m=1.732 ; and $\theta_0=\pi/4$ when $s_m=\infty$. These results are in good agreement with the numerical work.

If $J > 0$, the curve at the maximum point $\theta_0=\theta^*$ will split and a singlet will change into a doublet. The positions of the two peaks or the critical angles at which the two branches of the coercivity reach their peaks, $(\theta_0)_c$, are determined from the equations

$$\begin{cases} |H_s|\sin[\theta^* - (\theta_0)_c] - \frac{J}{2}\sin\theta^* = 0 \\ |H_s|\cos[\theta^* - (\theta_0)_c] = H^* + \frac{J}{2}\cos\theta^* \end{cases}$$

(for $(\theta_0)_c < \theta^*$, at this case HS<0) and

$$\begin{cases} H_s \sin[(\theta_0)_c - \theta^*] - \frac{J}{2}\sin\theta^* = 0 \\ H_s \cos[(\theta_0)_c - \theta^*] + \frac{J}{2}\cos\theta^* = H^* \end{cases}$$

(for $(\theta_0)_c > \theta^*$, at this case HS>0). After a direct derivation, one can obtain $(\theta_0)_{c1} = \theta^* - \Delta_1$ and $(\theta_0)_{c2} = \theta^* + \Delta_2$, in which

$$\Delta_1 = \tan^{-1}\frac{\frac{J}{2}\sin\theta^*}{H^* + \frac{J}{2}\cos\theta^*} \text{ and } \Delta_2 = \tan^{-1}\frac{\frac{J}{2}\sin\theta^*}{H^* - \frac{J}{2}\cos\theta^*}$$

7. Summary

The cusp phenomenon of the coercivity curve of the SW-type magnets under stress is an interesting phenomenon. Although this phenomenon has been reported both theoretically and experimentally since 1999, it has not been well explained. In this paper, with the help of the SW model, we make a detailed analytical analysis of the stress properties of coercivity, and reproduce successfully the cusp phenomenon of the coercivity curve. The peak positions are derived analytically for the case of $\psi = \pi / 4$. It is shown that the cusp phenomenon is the result of competing between stress anisotropy and uniaxial anisotropy and this competing leads to the offset of the equivalent EA of the system.

The existence of double-cusp phenomenon in the systems with exchange bias is also predicted.

Although the above results are obtained by the SW model, they are universal properties of the magnets under uniaxial stress. The cusp phenomenon has been observed experimentally several times till now. Of course, this phenomenon can not be observed all cases, for example, the peak disappears when $\psi = 0$. It is noteworthy that all of the experiment observations for the cusp phenomenon were obtained for the bulk materials. In addition, there has not been experimental reports for the double-cusp phenomenon in the systems with exchange bias.

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