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Finite Element Analysis of Convective Heat and Mass Transfer Through a Parallel Plate Reactor with Heat of Reaction Effect

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Abstract

In the present work, the heat and mass transfer analysis for laminar double-diffusive mixed convection flow in a parallel plate reactor with four heated cylindersis performed. The reactant fluid enters from the left inlet and exits from the right outlet. All solid walls of the reactor are assumed to be thermodynamically isolated. After entering the reactor, the fluid passes four heated cylinder. Two-dimensional continuity, momentum, energy and concentration equations govern the developed mathematical model. The governing non-dimensional equations are solved by using Galerkin's finite element method with triangular grid discretization system. Numerical simulations are carried out for different combinations of the heat of reaction parameter and results are presented in terms of streamlines, temperature and concentration distributions. The results indicate that the average Nusselt and Sherwood numbers at the heat and contaminant sources strongly depend on the mentioned parameter. This study will become helpful for designing the parallel plate reactor considering the heat released from the chemical reaction.

Keywords

Heat Transfer, Mass Transfer, Parallel Plate Reactor, Heat of Reaction, Finite Element Method

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1. Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, in chemical reaction engineering heat and mass transfer occur simultaneously. During a chemical reaction between two species heat is also generated. Therefore combined heat and mass transfer problems with heat generation received a considerable amount attention in recent years.

Brown and Lai [1] numerically examined combined heat and mass transfer from a horizontal channel with an open cavity heated from below. Since heat and contaminant sources usually co-exist indoors, the present work is to numerically study the double-diffusive mixed convection in a vented cavity due to the discrete heat and contaminant sources. Parvin et al. [2] analyzed numerically the effect of double-diffusive natural convection of a water–Al_2O_3nanofluid in a partially heated enclosure with Soret and Dufour coefficients. Double diffusive natural convection with MHD and joule heating effect in a chamber is studied by Parvin et al. [3]. Azad et al. [4] investigated double diffusive mixed convection in an open channel with a circular heater on the bottom wall. They found that, average Nusselt number at the heat source decreases and overall mass transfer rate in terms of average Sherwood number increases with the rising of Lewis number. Muthucumaraswamy and Ganesan [5] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Das et al. [6] studied the effect of the first

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order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamkha [7] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. He assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Ibrahim et al. [8] have studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. Kesavaiah et al [9] have studied the effect of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Heat and mass transport in tubular packed reactors at reacting and non-reacting conditions was analyzed by Koning [10] where the most common models of wall-cooled tubular packed bed reactors were presented. The two dimensional axial plug flow model was used for a water gas shift reactor to compare heat conduction or mass diffusion with convective effect. Kugai [11] studied Heat and Mass Transfer in Fixed-bed Tubular Reactor. The two dimensional axial plug flow model was used for a water gas shift reactor to compare heat conduction or mass diffusion with convective effect in his study. Recently Parvin et al. [12] studied the heat and mass transfer due to double diffusive mixed convection in a parallel plate reactor in presence of chemical reaction and heat generation.

The objective of the present work is to investigate the effect of heat of chemical reaction on the characteristics of the flow and heat/contaminant transport mechanism inside a chemical reactor channel in terms of streamlines, isotherms and iso-concentration lines.

2. Analysis

2.1. Physical Model

The domain under analysis is, as sketched in Figure 1(a)-(b), a two-dimensional cross section of a reactor channel of length L and height h with four heated tubes each of radius r, suffering the influence of a gravitational field. The centers of the heaters are located at (L/5, h/2), (2L/5, h/2), (3L/5, h/2) and (4L/5, h/2). The heaters are maintained at constant and uniform temperature T_h. The fluid flow is entering from the left with velocity U_i, temperature T_i and concentration C_i, then passes the tubes and then the fluid exhausted from the outlet opening at the right.

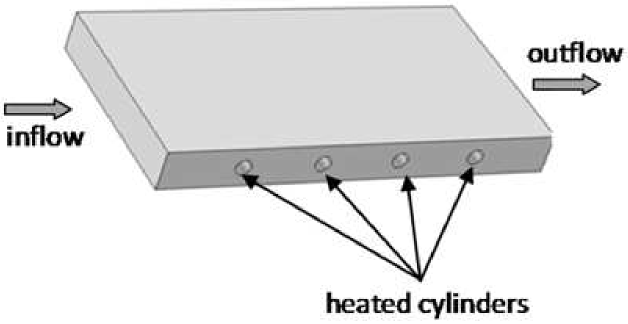

Fig. 1(a). 3-D geometry of the considered reactor.

2.2. Mathematical Model

The governing mass, momentum, energy and species conservation equations have been presented by Deng et al. [13] for double-diffusive mixed convective flows driven by the combined effect of the internal buoyancy induced from temperature and concentration differences and the external mechanical driven forced flow from the inlet port. With use of the Boussinesq approximation, the dimensionless governing equations under steady-state condition are given by:

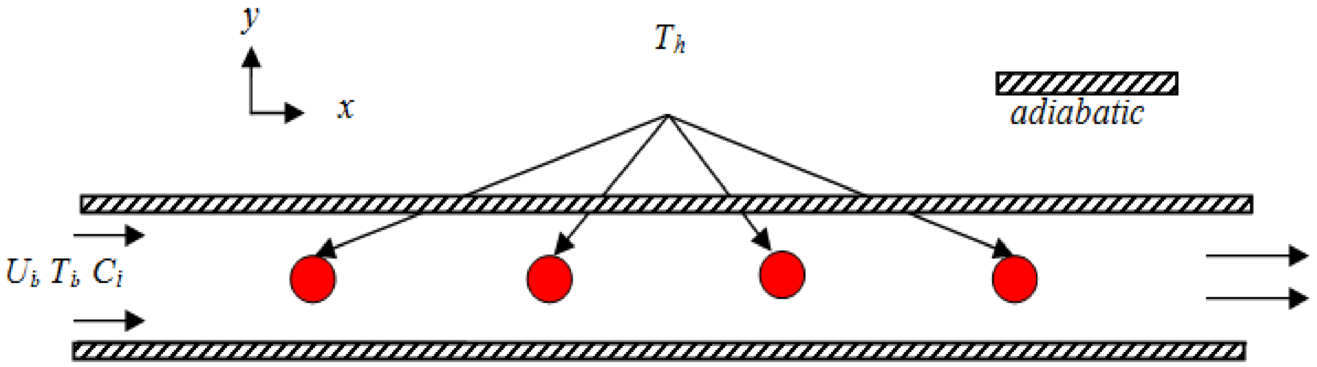

Fig. 1(b). Schematic diagram of the problem.

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\left(\theta+NC\right) \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)-H\theta \quad (4)$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{RePrLe}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right)-KC \quad (5)$$

The above equations are non-dimensionalized by using the following dimensionless variables

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{Ui},\ V=\frac{v}{Ui},\ P=\frac{p}{\rho U_i^{\,2}},$$
$$\theta=\frac{T-T_i}{T_h-T_i},\ C=\frac{c-C_i}{\Delta c} \quad (6)$$

and the dimensionless parameters are Reynolds number (*Re*), Grashof number (*Gr*), Richardson number (*Ri*), Prandtl number (*Pr*), Lewis number (*Le*), the buoyancy ratio (*N*) and Chemical reaction parameter (*K*) and heat of reaction (*H*) they are defined as follows:

$$Re=\frac{U_iL}{\nu},\ Gr=\frac{g\beta_T\left(T_h-T_i\right)L^3}{\nu^2},\ Ri=\frac{Gr}{Re^2},\ Pr=\frac{\nu}{\alpha},$$
$$Le=\frac{\alpha}{D},\ N=\frac{\beta_c\Delta c}{\beta_T\left(T_h-T_i\right)},\ K=\frac{kL^2}{D},\ H=\frac{QL^2}{\alpha} \quad (7)$$

where ν, α, *D*, *k* and *Q* are kinematic viscosity, thermal diffusivity, solutal diffusivity, reaction coefficient and strength of heat generating source respectively. The buoyancy ratio measures the relative importance of solute and thermal diffusion in creating the density difference to drive the flow. It is clear that *N* is zero for pure thermally driven flows and infinity for pure solute driven flows.

The boundary conditions are

at the inlet: $U=1,\ V=\theta=C=0$

at the circular tube walls: $\theta=1,\ \frac{\partial C}{\partial n}=0$

at other surfaces: $\frac{\partial \theta}{\partial n}=0,\frac{\partial C}{\partial n}=0$

at all solid boundaries: $U=V=0$

at the outlet: convective boundary condition, $P=0$

The average Nusselt and Sherwood number may be expressed as

$$Nu=-\frac{1}{S}\int_0^S\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}\,dS \text{ and}$$

$$Sh=-\frac{1}{S}\int_0^S\sqrt{\left(\frac{\partial C}{\partial X}\right)^2+\left(\frac{\partial C}{\partial Y}\right)^2}\,dS$$

where *S* is the non-dimensional length of the heated/contaminant surface

3. Computational Methodology

In the present study, governing equations (1) – (5) are solved by using Galerkin’s weighted residual method of finite element formulation. The application of this technique is well documented by Zienkiewicz and Taylor [14] and is used in Parvin and Chamkha [15]. The nonlinear parametric solution technique is chosen to solve the governing equations. This approach will result in substantially fast convergence assurance. In addition, the absolute convergence criteria are set to be 10^{-4} for velocities, energy and concentration.

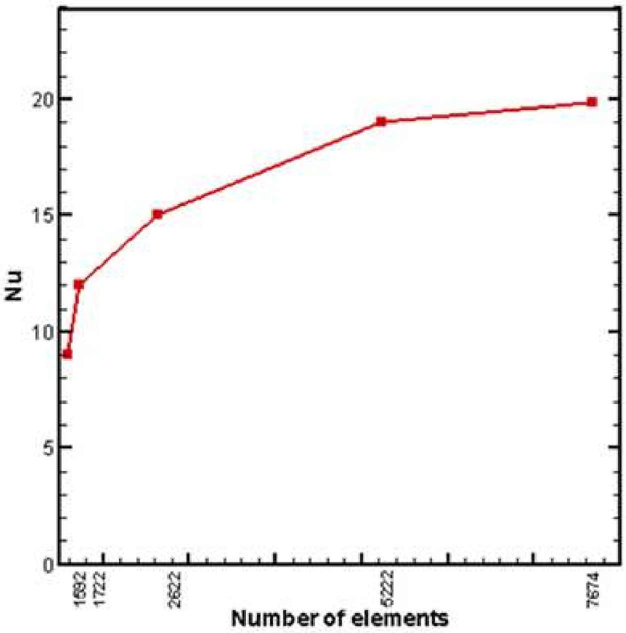

Fig. 2. Grid independent test for the geometry.

3.1. Grid Independent Test

To guarantee a grid-independent solution, an extensive mesh testing procedure is conducted with *Ri*= 1, *Re* = 100, *Pr* = 0.7, *Le* = 1, *N* = 1, *H* = 5 and *K* = 1 in the considered geometry. In the present work, we examine five different non-uniform grid systems with the following number of elements within the resolution field: 1592, 1722, 2622, 5222 and 7674. The numerical scheme is carried out for highly precise key in the average Nusselt (*Nu*) number at the first heater for the aforesaid elements to develop an understanding of the grid fineness as shown in Fig. 2. The scale of the average Nusselt numbers for 5222 elements shows a little difference with the results obtained for the higher elements. Hence, the non-uniform grid system of 5222 elements is preferred for the whole computation for saving the computation time.

3.2. Model Validation

The model validation is an important part of a numerical investigation. Hence, the outcome of the present numerical code is benchmarked against the numerical results of Ajibade and Umar [16] which were reported for combined effect of diffusion-thermo and chemical reaction on the unsteady MHD double diffusive flow between two inclined parallel plates with heat and radiation absorption. The comparison is conducted while employing the dimensionless parameters Prandtl number Pr = 0.71, chemical reaction parameter γ = 0.6, Schmidt number Sc = 0.4, heat absorption parameter λ = 0.2, magnetic field parameter M = 0.2, radiation absorption parameter Q = 1, Dufour parameter Df= 4, buoyancy ratio parameter N = 2, permeability parameter K = 2and inclination angle $\alpha = \frac{\pi}{4}$. Present results for Average Nusselt number (Nu_0)at y = 0 and (Nu_1) at y = 1 are shown in Table 1 which is good agreement with those of Ajibade and Umar [16]. This validation boosts the confidence in our numerical code to carry on with the above stated objectives of the current investigation.

Table 1. Comparison of Nusselt number for the present numerical technique with that of Ajibade and Umar [16].

	Present result	Ajibade and Umar [16]	Error(%)
Nu_0	0.3298	0.3310	0.36%
Nu_1	1.5507	1.5723	1.37%

4. Results and Discussion

The current computation was carried out on laminar double diffusive mixed convection in a parallel plate reactor containing heated tubes for various reaction heat parameter H (= 0, 1, 5 and 10) with Ri= 1, Re = 100, Pr = 0.7, Le = 1, N = 1, K = 1. Now in the following section, a detailed description of mixed convection with heat and mass transfer in a parallel plate reactor is given in terms of streamline, thermal and concentration contours for different H. In addition, the results for both average Nusselt and average Sherwood numbers at various H will be presented.

Figure 3-5 exhibits the effect of H on the streamlines, isotherms and iso-concentrations. In fact, the analysis is performed at pure mixed convection regime by fixing Ri= 1. The values of reaction heat parameters 0, 1, 5 and 10 are chosen to examine the evolution of streamline, isotherm and concentration patterns.

From Figures 3, it is observed that there is a common trend of the development of streamlines with increasing heat generation parameter. The streamlines are almost parallel to the channel wall and condensed in region between the circular heater and the channel wall. The streamlines become more condensed along the middle of the channel due to increasing heat of chemical reaction effect. This indicates higher velocity.

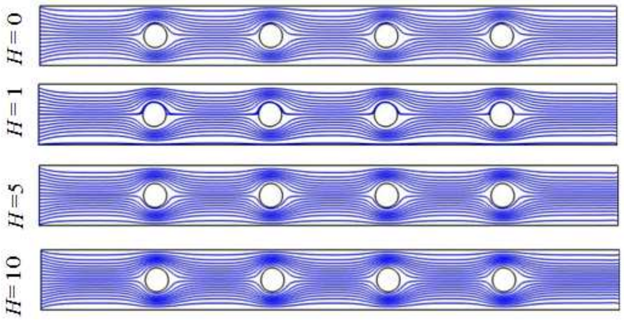

Fig. 3. Effect of H on streamlines.

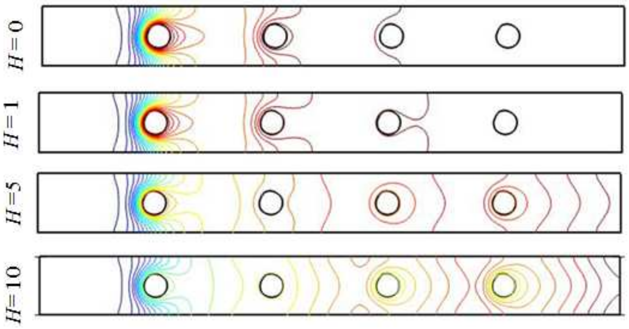

Fig. 4. Effect of H on isotherms.

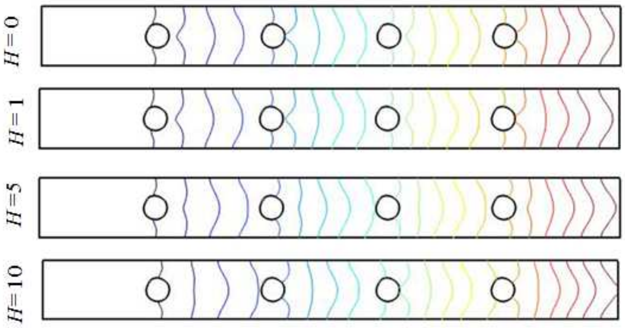

Fig. 5. Effect of H on iso-concentrations.

As in Figure 4, isothermal lines have significant change due to the variation of H. At H = 0, that is producing heat is neglected, isothermal lines appear at the inlet portion of the channel. But for higher values of H, these lines spread all over the channel. It is seen from the figure that, at the highest value of H, the lower temperature lines remain at the left potion where as the higher temperature lines at the right exit port. Temperature gradient at the heat source becomes lower for increasing heat generation in the fluid. This happens because higher temperature of the fluid produces lower temperature difference between the heat source and the fluid. It is also clear that the higher temperature gradient exists at the first heater from the inlet and sequentially it reduces for the second, third and fourth.

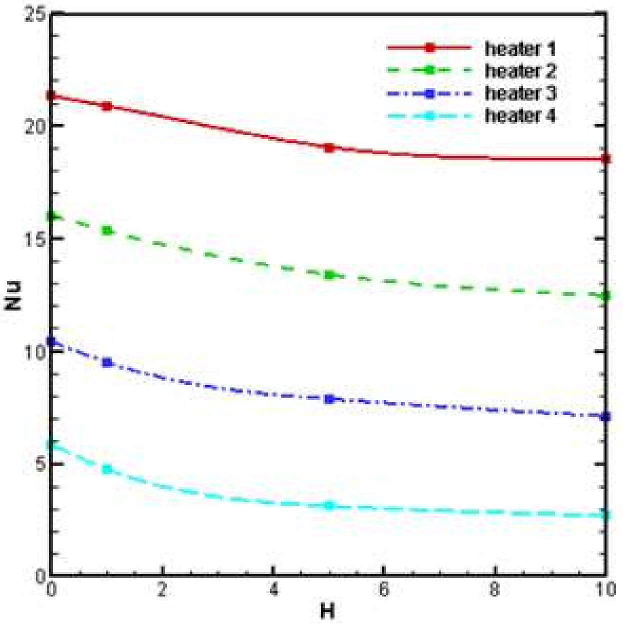

Fig. 6. Effect of H on heat transfer.

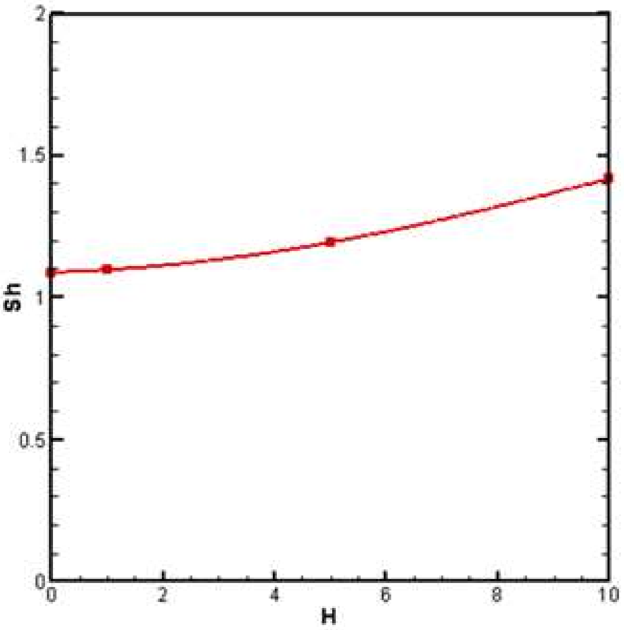

Fig. 7. Effect of *H* on mass transfer.

Iso-concentration lines have also considerable change due to generating heat as shown in Figure 5. Iso-concentration lines spreads all over the channel. As heat generation increases these lines depart to the exit port which indicates higher mass transportation. This phenomenon is logical because higherheat generation causes higher velocity which leads to more concentration transfer.

Figure 6 depicts the average heat transfer *Nu* at the four consecutive heaters for different *H*. Highest heat transfer rate is observed for the first heater and sequentially these values reduce for second, third and fourth heater. This phenomenon is very logical because the flow intensity becomes lower for the last heater due to the obstacles. Increasing *H* decreases the value of *Nu* due to lowering the temperature difference.

The average mass transfer *Sh* at the inlet port for different heat generation parameter is shown in Figure 7. Enhanced mass transfer rate are observed for increasing the heat generation. This is because; the heat of reaction causes rapid movement of the fluid which carries more concentrations.

Average reaction rate versus heat of reaction is depicted in Figure 8.It is seen that rate expression is enhancing with the increment in values of reaction heat parameter. Reaction rate is about linearly increasing with *H*.

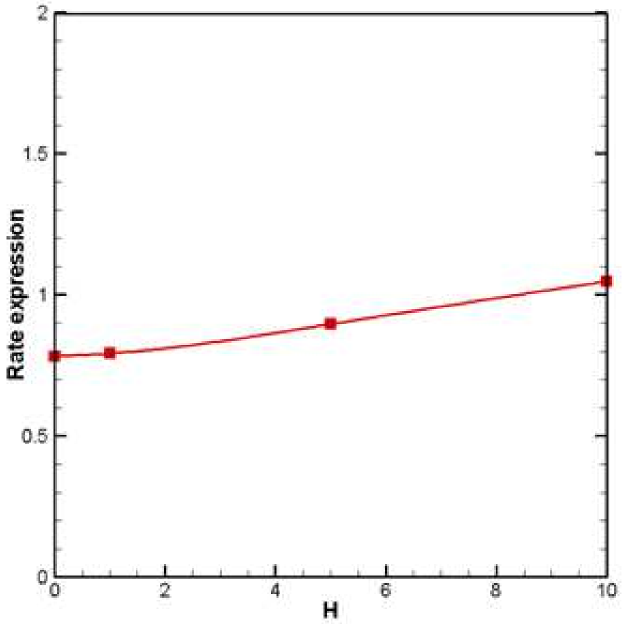

Fig. 8. Effect of *H* on reaction rate.

5. Conclusion

Laminar double-diffusive mixed convection flow in a parallel plate reactor with four heated cylinders for various reaction heat parameters has been studied. The following major conclusions are drawn:

- Increasing *H* has significant effects on flow, temperature and concentrations.
- Lower temperature and higher concentration gradient observe for higher *H*.
- Heat transfer reduces where as mass transfer enhance for rising values of *H*.

- Reaction rate is found almost linearly proportional to the heat of reaction.
- The heater placed near the inlet and outlet gives respectively the highest and lowest heat transfer rate.

In general the effect of heat of chemical reactionplays an important role in both heat and mass transfer forthe considered reactor. This study will become helpful for designing the parallel plate reactor considering the heat released from the chemical reaction.

Nomenclature

α	thermal diffusivity
β	thermal expansion coefficient
ν	kinematic viscosity
ρ	density
θ	nondimensional temperature
C	nondimensional concentration
c	concentration
D	mass diffusivity
g	gravitational acceleration
Gr	Grashof number
H	reaction heat parameter
h	height of the reactor
K	Chemical reaction parameter
L	length of the reactor
Le	Lewis number
N	buoyancy ratio
Nu	average Nusselt number
P	nondimesional pressure
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
Sh	average Sherwood number
T	temperature
U,V	nondimensional velocity components
u,v	velocity components
X,Y	nondimensional coordinate
x,y	Cartesian coordinate

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