

# Using LBM to Investigate the Effects of Solid-Porous Block in Channel

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## Abstract

In this paper the fluid flow and heat transfer in a channel with hot solid block which is located inside a square porous block was carried out numerically. The lattice Boltzmann method with nine velocities, D2Q9 was employed in all numerical simulations. The effects of some parameters including ratio of the thermal conductivities of fluid and solid structure of porous media, and Darcy number on flow pattern and thermal field were investigated. The result shows that with increasing the thermal conductivity ratio and decreasing Darcy number, the fluid temperature will reduce.

## Keywords

Heat Transfer, Porous Block, Effective Thermal Conductivity, Darcy Number

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## 1. Introduction

The fluid flow and heat transfer in a porous medium have been investigated numerically and experimentally in many years, due to its importance in many fields of science and engineering, such as filtering, food processing, fuel cells, petroleum processing, catalytic and chemical reactors, insulation, drying processes, lubrication, nuclear reactors, and many others.

Many researches were performed studies on fluid flow and convective heat transfer in channels fully and partially filled with porous medium. Hadim [1] investigated forced convection in a channel with porous layers above the heat sources. He reported that the pressure drop is 50% lower than the totally porous channels. Pavel and Mohamad [2] developed a numerical investigation of heat transfer enhancement inside a gas heat exchangers fitted with porous medium. The numerical method was based on the control volume finite difference scheme. Miranda and Anand [3] numerically investigated the fluid flow and heat transfer in a channel with constant heat flux on both walls with staggered porous blocks. Different parameters were investigated for the

configuration. Chen et al. [4] simulated numerically steady free convection in a cavity filled with a porous medium. In their work they used the finite-volume method. Elakkad et al. [5] numerically simulated the flow in porous media by Nodal Method and Finite Volume Method. Wu et al. [6] studied numerically the unsteady flow and convection heat transfer over a heated square porous cylinder in a channel. Brinkman-Forchheimer model was used to simulate fluid flow in porous media. The effects of different parameters were studies on heat transfer performance.

The lattice Boltzmann method (LBM) is a powerful numerical technique based on kinetic theory for simulation of fluid flows and modeling the physics in fluids [7-9]. The method was successfully applied to simulate incompressible flows through porous media by Guo and Zhao [10]. Seta et al. [11] used the lattice Boltzmann method for simulation of natural convection in porous media. Delavar et al. [12] studied numerically the effect of heater location and entropy generation, in a cavity on flow pattern and heat transfer, using lattice Boltzmann method. In this study the fluid flow and heat transfer over a hot solid block inside a square porous block located in a channel were investigated

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numerically by lattice Boltzmann method. Hot solid block inside a square porous block may be seen in some engineering application such as porous media as an active layer in reacting chemical flows, fuel cell, heat exchangers, and electronic devices cooling.

## 2. Numerical Model

The general form of lattice Boltzmann equation with nine velocities, D2Q9, with external force can be written as [8]:

$$f_k(\vec{x} + \vec{c}_k \Delta t, t + \Delta t) = f_k(\vec{x}, t) + \frac{\Delta t}{\tau} [f_k^{eq}(\vec{x}, t) - f_k(\vec{x}, t)] + \Delta t \vec{F}_k$$

$$f_k^{eq} = \omega_k \cdot \rho \left[ 1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} + \frac{1}{2} \frac{(\vec{c}_k \cdot \vec{u})^2}{c_s^4} - \frac{1}{2} \frac{\vec{u} \cdot \vec{u}}{c_s^2} \right] \tag{1}$$

where  $\Delta t$  is the lattice time step,  $\vec{c}_k$  denotes the discrete lattice velocity in direction  $k$ ,  $\tau$  denotes the lattice relaxation time,  $f_k^{eq}$  is the equilibrium distribution function,  $\vec{F}_k$  is the external force,  $\omega_k$  is weighting factor,  $\rho$  is the lattice fluid

density. To consider both the flow and the temperature fields, the thermal LBM utilizes two distribution functions,  $f$  and  $g$  for flow and temperature fields, respectively. The  $f$  distribution function is as same as discussed above; the  $g$  distribution function is as below [8]:

$$g_k(\vec{x} + \vec{c}_k \Delta t, t + \Delta t) = g_k(\vec{x}, t) + \frac{\Delta t}{\tau_g} [g_k^{eq}(\vec{x}, t) - g_k(\vec{x}, t)], \quad g_k^{eq} = \omega_k T \cdot \left[ 1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} \right] \tag{2}$$

The flow properties are defined as ( $i$  denote the component of the Cartesian coordinates):

The Brinkman-Forchheimer equation was used for flow in porous regions that is written as [10,12]

$$\rho = \sum_k f_k, \quad \rho u_i = \sum_k f_k c_{ki}, \quad T = \sum_k g_k \tag{3}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \left( \frac{\vec{u}}{\varepsilon} \right) = -\frac{1}{\rho} \nabla(\varepsilon p) + \nu_{eff} \nabla^2 \vec{u} + \left( -\frac{\varepsilon \nu}{K} \vec{u} - \frac{1.75}{\sqrt{150 \varepsilon K}} |\vec{u}| \vec{u} + \varepsilon \vec{G} \right) \tag{4}$$

where  $\varepsilon$  is the porosity,  $K$  is the permeability,  $\nu_{eff}$  is the effective viscosity,  $\nu$  is the kinematic viscosity, and  $G$  is the acceleration due to gravity. The last term in the right hand in the parenthesis is the total body force,  $F$ , which was written

by using the widely used Ergun's relation [13]. For porous medium the corresponding distribution functions are as same as Eq. 1. But the equilibrium distribution functions and the best choice for the forcing term are [10-12]:

$$f_k^{eq} = \omega_k \cdot \rho \cdot \left[ 1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} + \frac{1}{2} \frac{(\vec{c}_k \cdot \vec{u})^2}{\varepsilon c_s^4} - \frac{1}{2} \frac{\vec{u} \cdot \vec{u}}{\varepsilon c_s^2} \right], \quad F_k = \omega_k \rho \left( 1 - \frac{1}{2 \tau_v} \right) \left[ \frac{\vec{c}_k \cdot \vec{F}}{c_s^2} + \frac{(\vec{u} \cdot \vec{F} : \vec{c}_k \vec{c}_k)}{\varepsilon c_s^4} - \frac{\vec{u} \cdot \vec{F}}{\varepsilon c_s^2} \right] \tag{5}$$

The permeability is related to Darcy number ( $Da$ ), and the characteristic length ( $H$ ) with:

$$K = Da H^2 \tag{6}$$

The forcing term  $\vec{F}_k$  defines the fluid velocity  $\vec{u}$  as:

$$\vec{u} = \sum_k c_k F_k / \rho + \frac{\Delta t}{2} \vec{F} \tag{7}$$

According the equations (5) and (6)  $\vec{F}$  is related to  $\vec{u}$ , so Eq. (7) is nonlinear for the velocity. To solve this nonlinear problem a temporal velocity  $\vec{v}$  is used [9]:

$$\vec{u} = \frac{\vec{v}}{c_0 + \sqrt{c_0^2 + c_1 |\vec{v}|}}, \quad \vec{v} = \sum_k c_k f_k / \rho + \frac{\Delta t}{2} \varepsilon \vec{G}, \quad c_0 = \frac{1}{2} \left( 1 + \varepsilon \frac{\Delta t}{2} \frac{\nu}{K} \right), \quad c_1 = \varepsilon \frac{\Delta t}{2} \frac{1.75}{\sqrt{150 \varepsilon^3 K}} \tag{8}$$

The effective thermal conductivity,  $k_{eff}$  of the porous medium should be recognized for proper investigation of

conjugate convection and conduction heat transfer in porous zone, which was calculated by [14]:

$$k_{eff} = k_f \left[ (1 - \sqrt{1 - \epsilon}) + \frac{2\sqrt{1 - \epsilon}}{1 - \sigma B} \left( \frac{(1 - \sigma)B}{(1 - \sigma B)^2} \ln \left( \frac{1}{\sigma B} \right) - \frac{B + 1}{2} - \frac{B - 1}{1 - \sigma B} \right) \right], B = 1.25 \left[ \frac{1 - \epsilon}{\epsilon} \right]^{10/9}, \sigma = \frac{k_f}{k_s} \tag{9}$$

### 3. Boundary Condition

Regarding the boundary conditions of the flow field, the solid walls are assumed to be no slip, and thus the bounce-back scheme is applied. From the streaming process the distribution functions out of the domain are known as a consequence the unknown distribution functions are those

$$g_{2,n} = T_h (\omega_2 + \omega_4) - g_{4,n} \quad g_{5,n} = T_h (\omega_5 + \omega_7) - g_{7,n} \quad g_{6,n} = T_h (\omega_6 + \omega_8) - g_{8,n} \tag{11}$$

where  $n$  is the lattice on the boundary. For adiabatic boundary condition such as bottom wall the unknown distribution

$$g_{2,n} = g_{2,n-1}, \quad g_{5,n} = g_{5,n-1}, \quad g_{6,n} = g_{6,n-1} \tag{12}$$

### 4. Computational Domain

Figure 1 shows the computational domain which consists of a hot solid block inside a square porous block located in a channel. The simulation parameters are illustrated in the Table 1.

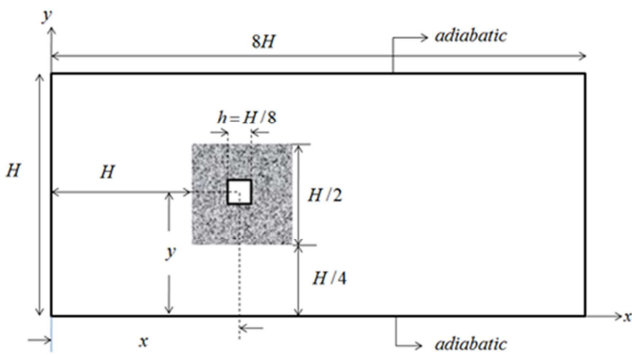


Figure 1. The computational domain.

Table 1. Simulation Parameters.

|          |                           |
|----------|---------------------------|
| H (cm)   | 1.0                       |
| L (cm)   | 8.0                       |
| $\sigma$ | 0.0001, 0.001, 0.01, 0.1  |
| Da       | 0.001-0.005-0.01-0.05-0.1 |

### 5. Validation and Grid Independent Check

In this study, fluid flow and heat transfer were simulated in a channel with a hot solid block inside a porous block by using

toward the domain. For example for flow field in the north boundary (the upper solid wall of channel) the following conditions is used:

$$f_{4,n} = f_{2,n}, \quad f_{5,n} = f_{7,n}, \quad f_{8,n} = f_{6,n} \tag{10}$$

For isothermal boundaries such as a bottom hot wall the unknown distribution functions are evaluated as:

functions are evaluated as:

lattice Boltzmann method. In the Fig. 2 the velocity profile in a porous channel for different Darcy numbers compares well with Mahmud and Fraser [15]. Table 2 shows the computed Nusselt number by LBM and Kays and Crawford [16], good agreement is observed.

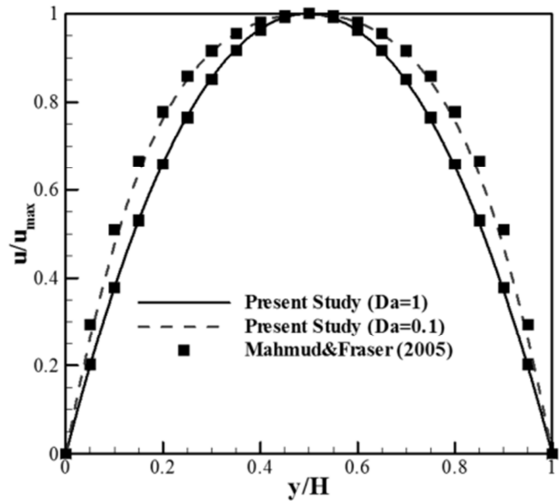


Figure 2. Comparison of velocity profile for porous channel flow between LBM and Mahmud and Fraser [15].

Table 2. Comparison of averaged Nusselt number between LBM and Kays and Crawford [16].

| $q''_2/q''_1$ |                   | 0.5   | 1    | 1.5   |
|---------------|-------------------|-------|------|-------|
| Nu1           | Kays and Crawford | 17.48 | 8.23 | 11.19 |
|               | LBM               | 17.25 | 8.16 | 11.1  |
| Nu2           | Kays and Crawford | 6.51  | 8.23 | 7     |
|               | LBM               | 6.49  | 8.16 | 6.91  |

## 6. Results and Discussion

In the present work the thermal lattice Boltzmann model with nine velocities, is employed to investigate the force convection heat transfer in a channel containing hot solid block inside a square porous block. The effects of Darcy number and thermal conductivity ratio on the flow field and heat transfer were studied. The effect of thermal conductivity ratio ( $\sigma$  in Eq. 9) on the thermal performance of the channel is illustrated in Fig. 3. In this figure temperature contours for different values of thermal conductivity ratio is illustrated. It is found that the temperature was increased for lower thermal conductivity ratio. According to Eq. 9 with increasing the thermal conductivity ratio, the effective thermal conductivity decreases. As a result in higher value of thermal conductivity ratio, the heat transfer between fluid and solid block decrease due to lower values of thermal conductivity. So the temperature of fluid will decrease. In this figure the velocity contours for different value of  $\sigma$  are not displayed because of the fact that in forced convection and constant properties, the velocity and temperature fields are independent. The variation of thermal conductivity ratio ( $\sigma$ ) only affects on temperature field, so the velocity contour for all models is the same.

The velocity and temperature contours for different Darcy numbers are shown in Fig. 4. From this figure the effects of

the Darcy number on flow field and heat transfer are illustrated. According to Eq. 6 the Darcy number is directly related to permeability of the porous block. The permeability is the capability of the porous media to transmit fluid from porous block. In higher permeability as well as higher Darcy number, fluid can move easier through porous block. So with increasing the Darcy number the velocity inside the porous block increases, while it decreases at the up and bottom of porous block. Maximum velocity in clear passages varies from 4.2 at  $Da=0.001$  to 3.2 at  $Da=0.1$ . By increasing the Darcy number, more fluid flows through porous block and absorbs heat from solid block. So the heat transfer will be enhanced at higher Darcy number, and causes the temperature of fluid increases.

In Fig. 5 the temperature profile at  $x/H=1.75$  and average fluid temperature for different values of thermal conductivity ratio ( $\sigma$ ) are shown. In Fig. 5a the temperature profile rises for lower values of  $\sigma$ . From Fig. 5b, it is found that lower value of  $\sigma$  leads to higher value of average fluid temperature. The average fluid temperature varies from  $26^\circ\text{C}$  at  $\sigma=0.0001$  to  $25.2^\circ\text{C}$  at  $\sigma=0.1$ . By decreasing the thermal conductivity ratio ( $\sigma$ ), the effective thermal conductivity increases. Higher values of thermal conductivity in the porous block will enhance heat transfer and higher average temperature of fluid is obtained.

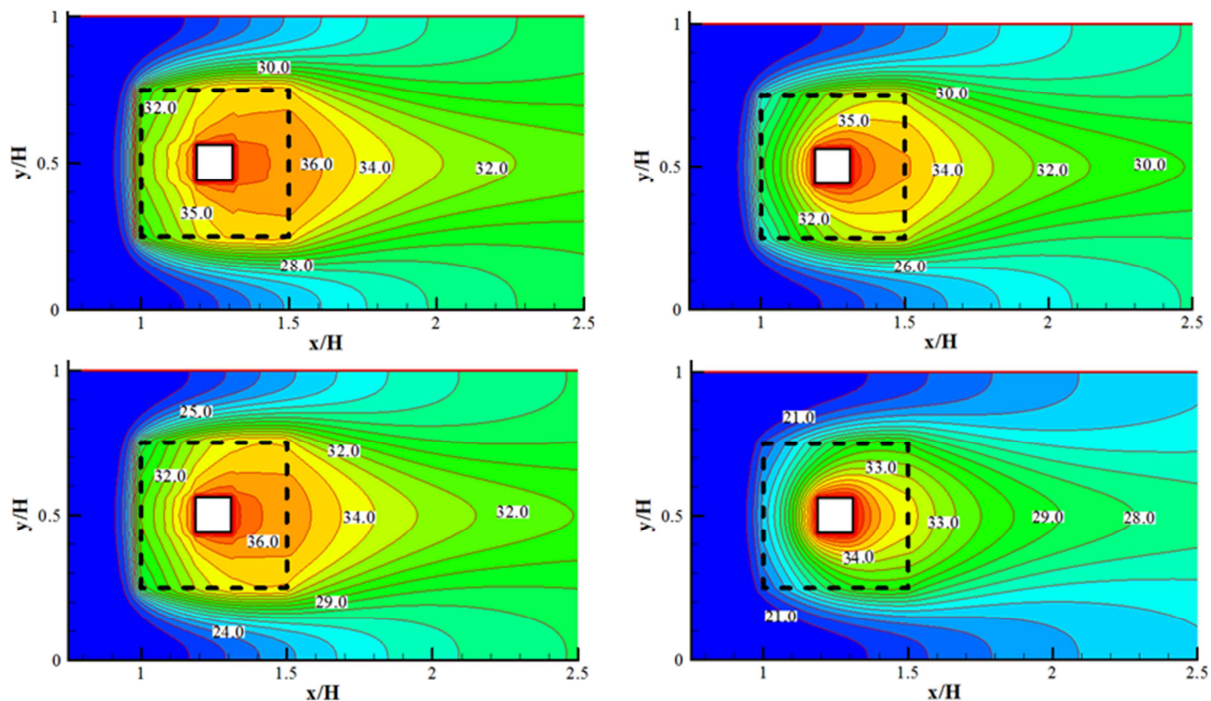


Figure 3. Temperature contours ( $^\circ\text{C}$ ) for different thermal conductivity ratio at  $Da=0.001$ .

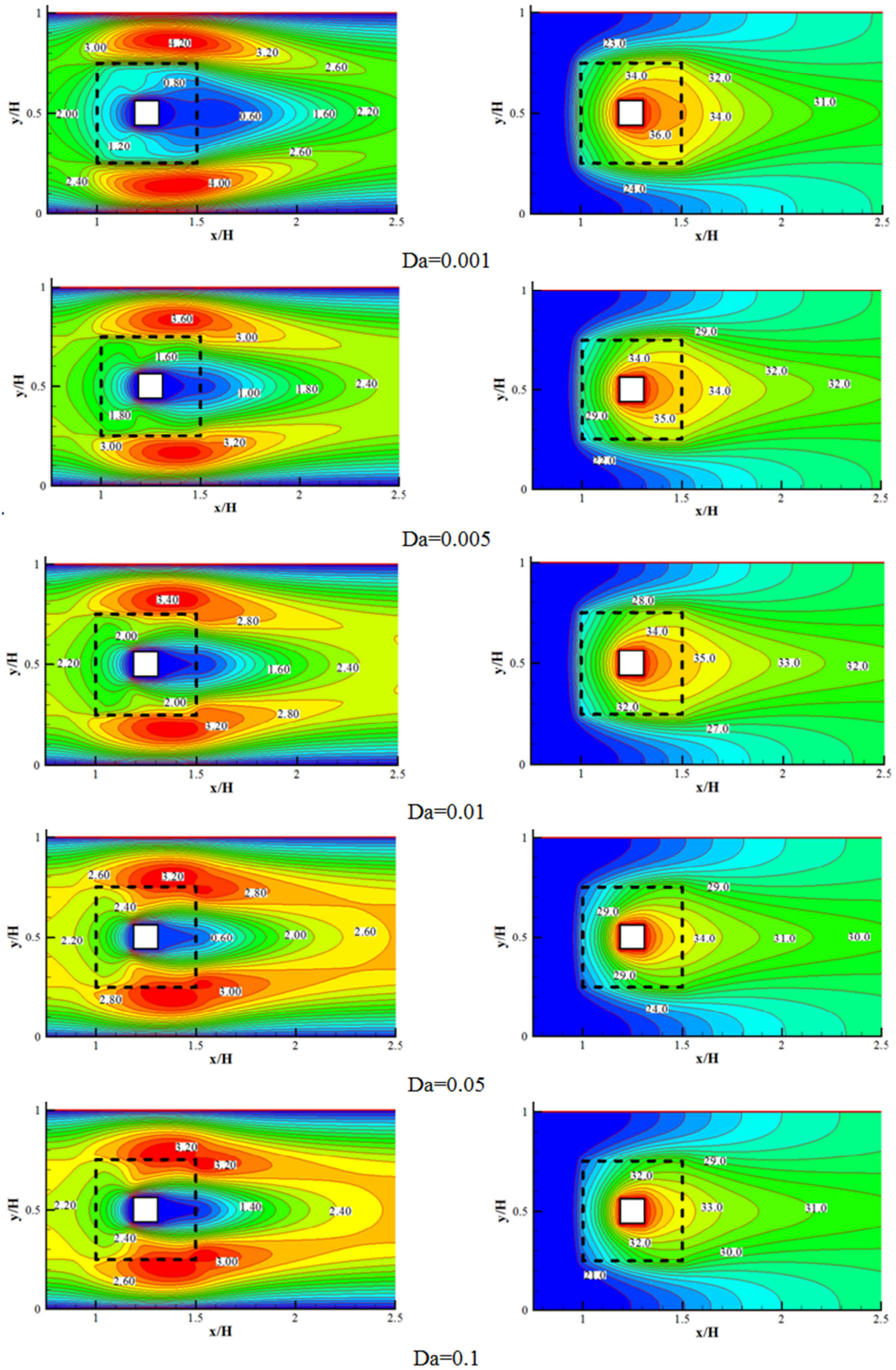


Figure 4. Velocity contours ( $U/U_{inlet(Re=25)}$ ), temperature contours ( $^{\circ}C$ ) for different Darcy number at  $\sigma=0.0001$ .

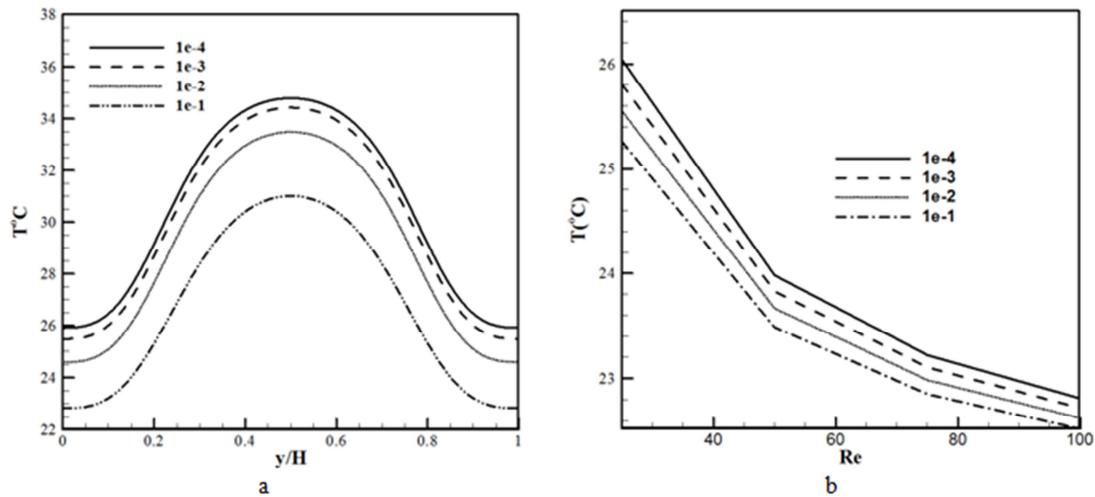


Figure 5. a) Temperature profile at  $x/H=1.75$ , b) Average fluid temperature at different thermal conductivity ratio.

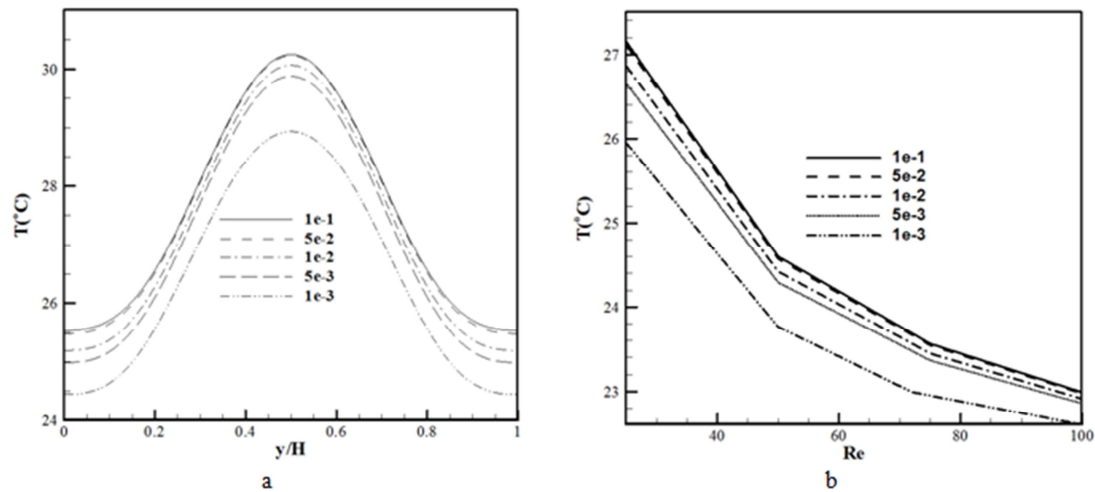


Figure 6. a) Temperature profile at  $x/H=1.75$ , b) Average fluid temperature at different Darcy number.

The variation of Darcy number for temperature profile at  $x/H=1.75$  and average fluid temperature are drawn in Fig. 6. With increasing the Darcy number, the temperature profile has greater values (Fig. 6a) and the average fluid temperature will increase (Fig. 6b). The average fluid temperature varies from  $25.8^{\circ}C$  at  $Da=0.001$  to  $27.5^{\circ}C$  at  $Da=0.1$ . As mentioned by increasing the Darcy number, the permeability of porous block increases (Eq. 6). At higher permeability the resistance of porous block on fluid flow reduces as a consequence more fluid pass through porous block and has heat transfer with solid block. So the heat transfer increases and then the average temperature of the fluid increases.

## 7. Conclusion

Flow pattern and thermal field over a solid block inside a square porous block located in a channel were simulated numerically using lattice Boltzmann method. The effect of parameters including Darcy number and thermal conductivity

ratio on the velocity and thermal fields were investigated. The results illustrated that that by increasing the Darcy number, the velocity inside the porous block increases and at the upper and bottom clear areas decreases. The fluid temperature increases as Darcy number increases due to fact that the fluid flow moves easier through the porous block and heat transfer improves between solid block and fluid. Increasing the thermal conductivity ratio reduces the convective heat transfer from solid block to working fluid due to lower values of effective thermal conductivity. As a result the temperature will decrease.

## Nomenclature

$c$  : discrete lattice velocity

$Da$  : Darcy number ( $KH^{-2}$ )

$F$  : external force

$f$  : distribution function for flow

$g$  : distribution function for temperature

$H$  : characteristic Height, m

$K$  : Permeability

$k$  : thermal conductivity,  $W/m^2.K$

$T$  : Temperature, K

$u$  : velocity component in x direction, m/s

$v$  : velocity component in y direction, m/s

#### Greek Symbols

$\mathcal{E}$  : is the porosity of porous media

$\nu$  : Kinematic viscosity,  $m^2/s$

$\rho$  : Density,  $kg/m^3$

$\tau$  : relaxation time

$\mu$  : Dynamic viscosity, Pa.s

#### Superscripts

$eq$ : equilibrium distribution function

#### Subscript

$eff$ : Effective

$i$ : dimension direction

$k$  : lattice model direction

$\omega$ : weighting factor

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