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Mechanical Properties of SWNT Within the Framework of Gradient Theory of Adhesion

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Abstract

The model of two-dimensional defectless medium is formulated as a special case of the general theory of three-dimensional medium with fields of conserved dislocations with adhesive properties of a surface limiting it. Potential energy in the general theory is the sum of volume and superficial integral from the corresponding densities of energy. In a limit case, when thickness of shell is equal to zero, volume part of potential energy become equal to zero. As a result the potential energy of such an object is defined only by the surface potential energy. A Single Wall Nano Tube (SWNT) is examined as an example of such two-dimensional medium. The problem statement of a SWNT axial deforming, and the torsion one, are examined. The general statement of an axisymmetric problem within the gradient theory of adhesion is formulated. Special cases are studied: a case of ideal and purely gradient adhesion, quasiclassical case, cases at big and small sizes of radius of SWNT. It is shown that the case of ideal adhesion corresponds to correct statement of the membrane theory of cylindrical shell. The case of purely gradient adhesion corresponds to correct statement of the theory of edge effect of cylindrical shell. It is shown, that particular case of the quasiclassical theory of cylindrical shell is not consecutive approach of the general theory when moduli of ideal adhesion are partially considered, and partially - moduli of purely gradient adhesion. The characteristic feature of all statements is the fact that the mechanical properties of SWNT are not defined by “volumetric” moduli but by adhesive ones which have different physical dimension which coincides with the dimension of the corresponding stiffness of classical and nonclassical shells.

Keywords

Gradient Theories of Elasticity, Ideal Adhesion, Gradient Adhesion, Mechanical Properties of SWNT, Nonclassical Moduli

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1. Introduction

The generalization of Mindlin’s model built in [1] is under study. Unlike the “classical” models of Mindlin [2] and Toupin [3] its generalization takes into consideration not only the curvatures connected with the gradient of the free distortion in the volumetric density of potential energy but also the curvatures connected with the gradient of the restricted distortion, as well as their interaction. Another difference is considering the generalized model of the surface potential energies (the energy of adhesion interactions) in the Lagrangian, the surface edges energy U_s and the energy of specific points of the surface edges U_p . Particularly, the Lagrangian of the generalized model can be presented as follows:

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$$
\begin{aligned}
L = A - \frac{1}{2}\iiint \; [C^{11}_{ijmn}D^1_{ij}D^1_{mn} + 2C^{12}_{ijmn}D^1_{ij}D^2_{mn} \\
+C^{22}_{ijmn}D^2_{ij}D^2_{mn} + C^{11}_{ijkmnl}D^1_{ijk}D^1_{mnl} + 2C^{12}_{ijkmnl}D^1_{ijk}D^2_{mnl} + \\
+C^{22}_{ijkmnl}D^2_{ijk}D^2_{mnl}]dV - \frac{1}{2}\oiint \; [A^{11}_{ijmn}D^1_{ij}D^1_{mn} + \\
+2A^{12}_{ijmn}D^1_{ij}D^2_{mn} + A^{22}_{ijmn}D^2_{ij}D^2_{mn} + A^{11}_{ijkmnl}D^1_{ijk}D^1_{mnl} + \\
+2A^{12}_{ijkmnl}D^1_{ijk}D^2_{mnl} + A^{22}_{ijkmnl}D^2_{ijk}D^2_{mnl}]dF - \\
-\sum\oint \; U_s ds - \sum U_p
\end{aligned}
\tag{1}
$$

The kinematic variables of the Lagrangian are:

- the continuous part of the displacement vector R_i

- the distortions of two types D^1_{ij} , D^2_{ij} (restricted and free distortions)

- the curvatures of two types D^1_{ijk} , D^2_{ijk} (the gradients of the corresponding distortions).

Between these kinematic variables there are restrictions defining the kinematic model of such medium:

$$
D^1_{ij} = R_{i,j} \quad D^1_{ijk} = R_{i,jk} \quad D^2_{ijk} = D^2_{ij,k} \tag{2}
$$

The tensors of moduli C^{pq}_{ijmn} and C^{pq}_{ijkmnl} define the mechanical properties of medium in volume and the tensors A^{pq}_{ijmn} and A^{pq}_{ijkmnl} define them on the medium surface.

This model demonstrates some new qualitative results which are impossible to be obtained in the frames of simpler models. One of such results is studied in this work, and namely the opportunity to explain the mechanical properties of two-dimensional medium and to make applied theories of SWNT axial deforming, and the torsion one, as a two-dimensional medium.

Actually, the Lagrangians of both the classical mechanics of continuous medium and well known gradient models of Mindlin and Toupin contain the potential energy defined only through the volumetric density of the potential energy. Formally the Lagrangians of these models cannot be applied to the two-dimensional medium. This statement follows from the fact that in these models the potential energy of the zero volume medium equals null. In these models all plate and shell theories are formulated as the three-dimensional body models with a small, when compared to others, size in the third direction. Nevertheless, the same substantial mistake exists in the plate/shell theory of these models, i.e. the object of zero volume (due to its zero thickness) will have zero potential energy. One cannot consider the example of a graphene sheet or SWNT as a volumetric structure having the thickness of about a carbon atom diameter as correct [4]. For the formulation of theories of two-dimensional structures the Lagrangian required, directly containing the surface density of potential energy. In work [5] the variant of the theory of thin films with face adhesive properties was examined. However, the bending equation degenerated into a second-degree equation in the extreme case which is considered here at a zero volume film. It was defined by the fact that then only ideal, not gradient face adhesive properties were taken into consideration. The definition of a more general theory [1] allows to turn back to this problem and to formulate a non-degenerated case [6]. In the same way, as well as in article [6], in this work will construct the theory of SWNT and its special cases are investigated.

2. Geometry for SWNT

Coordinates on a cylindrical surface:

$$
\begin{cases} x = x \\ y = rCos\phi \\ z = rSin\phi \end{cases}
\quad
\begin{cases} X_i = X_i \\ s_i = Sin\phi Y_i - Cos\phi Z_i \\ n_i = Cos\phi Y_i + Sin\phi Z_i \end{cases}
$$
$$
\begin{cases} X_i = X_i \\ Y_i = Sin\phi s_i + Cos\phi n_i \\ Z_i = -Cos\phi s_i + Sin\phi n_i \end{cases}
\tag{3}
$$

Here are: x, y,z - the Cartesian coordinates, x,ϕ,r - cylindrical coordinates, X_i - ort of axial coordinate, s_i - ort of district coordinate, n_i - ort of a normal to a cylindrical surface.

Kronecker's "flat" tensor:

$$
\delta^*_{ij} = (\delta_{ij} - n_i n_j) = X_i X_j + s_i s_j \tag{4}
$$

Gradient on a cylindrical surface:

$$
\begin{aligned}
&\frac{\partial(...)}{\partial x_j} = \frac{\partial(...)}{\partial y_k}\frac{\partial y_k}{\partial x_j} = \\
&= \frac{\partial(...)}{\partial x}\frac{\partial x}{\partial x_j} + \frac{\partial(...)}{\partial r}\frac{\partial r}{\partial x_j} + \frac{\partial(...)}{\partial \phi}\frac{\partial \phi}{\partial x_j} = \\
&= \frac{\partial(...)}{\partial x}X_j + \frac{\partial(...)}{\partial \phi}\frac{1}{r}[Sin\phi Y_j - Cos\phi Z_j] = \\
&= \frac{\partial(...)}{\partial x}X_j + \frac{\partial(...)}{\partial \phi}\frac{1}{r}s_j
\end{aligned}
\tag{5}
$$

Derivatives of orts on a cylindrical surface:

$$
s_{i,j} = \frac{1}{r}n_i s_j \quad n_{i,j} = -\frac{1}{r}s_i s_j \tag{6}
$$

Displacements vector is:

$$R_i = U(x,\phi)X_i + V(x,\phi)s_i + W(x,\phi)n_i \quad (7)$$

Restricted distortion (displacements derivatives) tensor is:

$$\begin{aligned} R_{i,j} &= [\frac{\partial U}{\partial x}X_i + \frac{\partial V}{\partial x}s_i + \frac{\partial W}{\partial x}n_i]X_j + \\ &+ [\frac{\partial U}{\partial \varphi}X_i + (\frac{\partial V}{\partial \varphi} - W)s_i + (V + \frac{\partial W}{\partial \varphi})n_i]\frac{1}{r}s_j \end{aligned} \quad (8)$$

Curvatures (distortion derivatives) are:

$$\begin{aligned} R_{i,jk} &= [\frac{\partial^2 U}{\partial x^2}X_i + \frac{\partial^2 V}{\partial x^2}s_i + \frac{\partial^2 W}{\partial x^2}n_i]X_jX_k + \\ &+[\frac{\partial^2 U}{\partial x \partial \phi}X_i + (\frac{\partial^2 V}{\partial x \partial \phi} - \frac{\partial W}{\partial x})s_i + \\ &(\frac{\partial V}{\partial x} + \frac{\partial^2 W}{\partial x \partial \phi})n_i]\frac{1}{r}(X_js_k + s_jX_k) + \\ &+[\frac{\partial^2 U}{\partial \phi^2}X_i + (\frac{\partial^2 V}{\partial \phi^2} - V - 2\frac{\partial W}{\partial \phi})s_i + \\ &+(2\frac{\partial V}{\partial \phi} + \frac{\partial^2 W}{\partial \phi^2} - W)n_i]\frac{1}{r}\frac{1}{r}s_js_k + \\ &+[\frac{\partial U}{\partial \phi}X_i + (\frac{\partial V}{\partial \phi} - W)s_i + (V + \frac{\partial W}{\partial \phi})n_i]\frac{1}{r}\frac{1}{r}n_js_k \end{aligned} \quad (9)$$

3. Variational Statement of SWNT Theory

In case of Lagrangian (1) if the medium volume equals null, the Lagrangian becomes a nontrivial specifically simple type:

$$\begin{aligned} L = A - \frac{1}{2}\oiint [&A^{11}_{ijmn}D^1_{ij}D^1_{mn} + 2A^{12}_{ijmn}D^1_{ij}D^2_{mn} + \\ +&A^{22}_{ijmn}D^2_{ij}D^2_{mn} + A^{11}_{ijkmnl}D^1_{ijk}D^1_{mnl} + 2A^{12}_{ijkmnl}D^1_{ijk}D^2_{mnl} + \\ +&A^{22}_{ijkmnl}D^2_{ijk}D^2_{mnl}]dF \end{aligned} \quad (10)$$

If we consider SWNT as an ideal two-dimensional structure, then we should put aside all terms containing the free distortion tensor D^2_{ij} from expression (10) due to the fact that this tensor determines the defectiveness of the medium under study. The Lagrangian becomes as follows:

$$L = A - \frac{1}{2}\oiint [A^{11}_{ijmn}D^1_{ij}D^1_{mn} + A^{11}_{ijkmnl}D^1_{ijk}D^1_{mnl}]dF \quad (11)$$

Besides as SWNT is a two-dimensional structure, the surface density of potential energy should not depend on normal derivatives of displacements. In connection with this fact, we should demand that the tensors of adhesive moduli have the following properties:

$$\begin{aligned} &A^{11}_{ijmn}n_j = A^{11}_{ijmn}n_n = 0 \\ &A^{11}_{ijkmnl}n_j = A^{11}_{ijkmnl}n_k = A^{11}_{ijkmnl}n_n = A^{11}_{ijkmnl}n_l = 0 \end{aligned} \quad (12)$$

To simplify the task let us accept the idea that the mechanical properties are isotropic on the surface of SWNT. The result of (12) and (11) is the next structure of adhesive tensors, a simpler in comparison with (1):

$$A^{11}_{ijmn} = a^{11}_1\delta^*_{ij}\delta^*_{mn} + a^{11}_2(\delta^*_{im}\delta^*_{jn} + \delta^*_{in}\delta^*_{jm}) + a^{11}_6 n_i n_m \delta^*_{jn}$$

$$\begin{aligned} &A^{11}_{ijkmnl} = \\ &= A^{11}_1(\delta^*_{ij}\delta^*_{km}\delta^*_{nl} + \delta^*_{mn}\delta^*_{li}\delta^*_{jk} + \delta^*_{ij}\delta^*_{kn}\delta^*_{ml} + \delta^*_{mn}\delta^*_{lj}\delta^*_{ik} + \\ &+ \delta^*_{ik}\delta^*_{jm}\delta^*_{nl} + \delta^*_{ml}\delta^*_{ni}\delta^*_{jk} + \delta^*_{in}\delta^*_{km}\delta^*_{jl} + \delta^*_{mj}\delta^*_{li}\delta^*_{nk} + \\ &+ \delta^*_{ij}\delta^*_{mn}\delta^*_{kl} + \delta^*_{in}\delta^*_{jm}\delta^*_{kl} + \delta^*_{jn}\delta^*_{ik}\delta^*_{ml} + \delta^*_{jn}\delta^*_{il}\delta^*_{km} + \\ &+ \delta^*_{im}\delta^*_{nj}\delta^*_{kl} + \delta^*_{im}\delta^*_{kj}\delta^*_{nl} + \delta^*_{im}\delta^*_{lj}\delta^*_{nk}) + \\ &+ A^{11}_3 n_i n_m(\delta^*_{nj}\delta^*_{kl} + \delta^*_{kj}\delta^*_{nl} + \delta^*_{lj}\delta^*_{nk}) \end{aligned} \quad (13)$$

The structure of tensor A^{11}_{ijmn} for the first time is received in [5] and defines much more adhesive properties of an ideal surface, than in [7]. The structure of tensor A^{11}_{ijkmnl} for the first time is received in [8] and defines much more adhesive properties of surface with gradient adhesion, than in [9]. The expanded structure of the potential energy for an axisymmetric problem becomes as follows:

$$\begin{aligned} U_F &= \frac{1}{2}\{A_{ijmn}R_{i,j}R_{m,n} + A_{ijkmnl}R_{i,jk}R_{m,nl}\} = \\ &= \frac{1}{2}\{[a^{11}_1(U' - W\frac{1}{r})(U' - W\frac{1}{r}) + \\ &+ a^{11}_6(W'W' + \frac{1}{r^2}VV) + \\ &+ a^{11}_2(2U'U' + V'V' + \frac{1}{r^2}VV + W'W' + 2\frac{1}{r^2}WW)] + \\ &+ [15A^{11}_1U''U'' - 12A^{11}_1W'U''\frac{1}{r} + \\ &+ 3A^{11}_3W''W'' + 12A^{11}_1W'W'\frac{1}{r^2} - 2A^{11}_3WW''\frac{1}{r^2} + \\ &+ 3A^{11}_3WW\frac{1}{r^4} - 2A^{11}_3WV'\frac{1}{r^3} + 3A^{11}_1V''V'' + \\ &+ 4A^{11}_3V'V'\frac{1}{r^2} - 6A^{11}_1VV''\frac{1}{r^2} + 15A^{11}_1VV\frac{1}{r^4}]\} \end{aligned} \quad (14)$$

We can define the force factors using the Green's formulae:

$$\begin{aligned} \sigma_{ij} &= \frac{\partial U_F}{\partial R_{i,j}} = A_{ijmn}R_{m,n} \quad m_{ijk} \\ &= \frac{\partial U_F}{\partial R_{i,jk}} = A_{ijkmnl}R_{m,nl} \end{aligned} \quad (15)$$

The corresponding variation equation follows from Lagrange's principle.

The variation equation in the force factors is:

$$\begin{aligned}&\delta L = 2\pi r\int_0^l \{\sigma_{ij,j} - m_{ijk,kj} + P_i^F\}\delta R_i dx - \\ &+2\pi r(-m_{ijk}X_kX_j)\delta(R_{i,p}X_p)\ \Big|_{x=0}^{x=l} + \\ &+2\pi r[P_i - (\sigma_{ij} - m_{ijk,k})X_j]\delta R_i\ \Big|_{x=0}^{x=l} = 0\end{aligned} \tag{16}$$

The variation equation in displacements is:

$$\begin{aligned}&\delta L = 2\pi r\int_0^l \{[-15A_1^{11}U'''' + (a_1^{11}+2a_2^{11})U'' + 6A_1^{11}\frac{1}{r}W''' - \\ &-a_1^{11}\frac{1}{r}W' + P_x^F]\delta U - +[-3A_1^{11}V'''' + (a_2^{11}r^2 + 6A_1^{11} + \\ &+4A_3^{11})\frac{1}{r^2}V'' - ((a_2^{11}+a_6^{11})r^2 + 15A_1^{11})\frac{1}{r^4}V - A_3^{11}\frac{1}{r^3}W' + \\ &+P_\phi^F]\delta V + [-6A_1^{11}\frac{1}{r}U''' + a_1^{11}\frac{1}{r}U' + A_3^{11}\frac{1}{r^3}V' + \\ &-3A_3^{11}W'''' + ((a_2^{11}+a_6^{11})r^2 + 12A_1^{11} + 2A_3^{11})\frac{1}{r^2}W'' - \\ &-(3A_3^{11} + (a_1^{11}+2a_2^{11})r^2)\frac{1}{r^4}W + P_r^F]\delta W\}dx + \\ &+2\pi r\{-[15A_1^{11}U'' - 6A_1^{11}\frac{1}{r}W']\delta U' - \\ &-[3A_1^{11}V'' - 3A_1^{11}\frac{1}{r^2}V]\delta V' - [3A_3^{11}W'' - A_3^{11}\frac{1}{r^2}W]\delta W' + \\ &+[P_x^e + 15A_1^{11}U''' - (a_1^{11}+2a_2^{11})U' - 6A_1^{11}\frac{1}{r}W'' + \\ &+a_1^{11}\frac{1}{r}W]\delta U + [P_\phi^e + 3A_1^{11}V''' - (a_2^{11}r^2 + 3A_1^{11} + 4A_3^{11})\frac{1}{r^2}V' + \\ &+A_3^{11}\frac{1}{r^3}W]\delta V + [P_r^e + 6A_1^{11}\frac{1}{r}U'' + [3A_3^{11}W''' - A_3^{11}\frac{1}{r^2}W'] - \\ &-((a_2^{11}+a_6^{11})r^2 + 12A_1^{11})\frac{1}{r^2}W']\delta W\}\ \Big|_{x=0}^{x=l} = 0\end{aligned} \tag{17}$$

The equilibrium equations' system:

$$\begin{cases}-15A_1^{11}U'''' + (a_1^{11}+2a_2^{11})U'' + 6A_1^{11}\frac{1}{r}W''' - a_1^{11}\frac{1}{r}W' + P_x^F = 0\\ -3A_1^{11}V'''' + (a_2^{11}r^2 + 6A_1^{11} + 4A_3^{11})\frac{1}{r^2}V'' - \\ -((a_2^{11}+a_6^{11})r^2 + 15A_1^{11})\frac{1}{r^4}V - A_3^{11}\frac{1}{r^3}W' + P_\phi^F = 0\\ -6A_1^{11}\frac{1}{r}U''' + a_1^{11}\frac{1}{r}U' + A_3^{11}\frac{1}{r^3}V' + \\ -3A_3^{11}W'''' + ((a_2^{11}+a_6^{11})r^2 + 12A_1^{11} + 2A_3^{11})\frac{1}{r^2}W'' - \\ -(3A_3^{11} + (a_1^{11}+2a_2^{11})r^2)\frac{1}{r^4}W + P_r^F = 0\end{cases} \tag{18}$$

We will pay attention to the fact that in the received theory torsion doesn't separate from tension/compression. Nevertheless, it is interesting to consider these cases separately, demanding the absence of torsion or absence of deviations. Such consideration will allow us to carry out comparison with the classical theory of cylindrical shells.

4. The Mechanical Properties of SWNT While Axial Deforming

Let us assume, that the torsion is absent. Then displacement V is equal to zero. Then the equilibrium equations' system becomes:

$$\begin{cases}-15A_1^{11}U'''' + (a_1^{11}+2a_2^{11})U'' + 6A_1^{11}\frac{1}{r}W''' - a_1^{11}\frac{1}{r}W' + P_x^F = 0\\ -6A_1^{11}\frac{1}{r}U''' + a_1^{11}\frac{1}{r}U' + \\ -3A_3^{11}W'''' + ((a_2^{11}+a_6^{11})r^2 + 12A_1^{11} + 2A_3^{11})\frac{1}{r^2}W'' - \\ -(3A_3^{11} + (a_1^{11}+2a_2^{11})r^2)\frac{1}{r^4}W + P_r^F = 0\end{cases} \tag{19}$$

Particular case, when moments are absent, take place when tensor $A_{ijkmnl}^{11} = 0$. This case (really membrane theory) was under study in [10]:

$$\begin{cases}(a_1^{11}+2a_2^{11})U'' - a_1^{11}\frac{1}{r}W' + P_x^F = 0\\ a_1^{11}\frac{1}{r}U' + (a_2^{11}+a_6^{11})W'' - (a_1^{11}+2a_2^{11})\frac{1}{r^2}W + P_r^F = 0\end{cases} \tag{20}$$

Particular case, when ideal adhesion is absent, take place when tensor $A_{ijmn}^{11} = 0$:

$$\begin{cases}-15A_1^{11}U'''' + 6A_1^{11}\frac{1}{r}W''' + P_x^F = 0\\ -6A_1^{11}\frac{1}{r}U''' - 3A_3^{11}W'''' + \\ +(12A_1^{11} + 2A_3^{11})\frac{1}{r^2}W'' - 3A_3^{11}\frac{1}{r^4}W + P_r^F = 0\end{cases} \tag{21}$$

Particular case, when $r \to \infty$. This case was under study in [6]:

$$\begin{cases}-15A_1^{11}U'''' + (a_1^{11}+2a_2^{11})U'' + P_x^F = 0\\ -3A_3^{11}W'''' + (a_2^{11}+a_6^{11})W'' + P_r^F = 0\end{cases} \tag{22}$$

We will exclude axial displacement U from system (19) and receive the allowing equation on a deflection. Let's pay attention that the first equation of system (19) is easily integrated. Therefore it is possible to write down and solve system (19) as algebraic relatively U''' and U' and if the determinant of this system is other than zero. We will consider a case when it is equal to zero:

$$\begin{aligned} Det &= -15A_1^{11}a_1^{11}\frac{1}{r} + 6A_1^{11}\frac{1}{r}(a_1^{11} + 2a_2^{11}) \\ &= -3A_1^{11}\frac{1}{r}(3a_1^{11} - 4a_2^{11}) = 0 \end{aligned} \tag{23}$$

Apparently (23) determinant is equal to zero in three cases: $\frac{1}{r} = 0$, $A_1^{11} = 0$ и $a_2^{11} = 3a_1^{11}/4$.

The first case corresponds to the breaking-up system (22).

The second case corresponds to a special case of system (19) at $A_1^{11} = 0$:

$$\begin{cases} U' = \dfrac{a_1^{11}}{(a_1^{11} + 2a_2^{11})}\dfrac{1}{r}W - \dfrac{1}{(a_1^{11} + 2a_2^{11})}p_x(x) \\ 3A_3^{11}W'''' - ((a_2^{11} + a_6^{11})r^2 + 2A_3^{11})\dfrac{1}{r^2}W'' + \\ +(3A_3^{11} + (a_1^{11} + 2a_2^{11} - \dfrac{a_1^{11}a_1^{11}}{(a_1^{11} + 2a_2^{11})})r^2)\dfrac{1}{r^4}W = \\ = [P_r^F - \dfrac{a_1^{11}}{(a_1^{11} + 2a_2^{11})}\dfrac{1}{r}p_x(x)] \end{cases} \tag{24}$$

$$p_x(x) = p_x(0) + \int_0^x P_x^F(t)dt$$

The third case corresponds to a special case of system (19) at $a_2^{11} = 3a_1^{11}/4$:

$$\begin{cases} 3A_1^{11}U''' - (a_1^{11}/2)U' = \dfrac{6}{5}A_1^{11}\dfrac{1}{r}W'' - \dfrac{1}{5}a_1^{11}\dfrac{1}{r}W + \dfrac{1}{5}p_x(x) \\ 3A_3^{11}W'''' - ((3a_1^{11}/4 + a_6^{11})r^2 + 48A_1^{11}/5 + 2A_3^{11})\dfrac{1}{r^2}W'' + \\ +(3A_3^{11} + (3a_1^{11}/5)r^2)\dfrac{1}{r^4}W = \\ = -\dfrac{2}{5}\dfrac{1}{r}p_x(x) + P_r^F \end{cases} \tag{25}$$

The first case (22) leads to the theory of a graphene and look-like-graphene flat 2D-structures. Two other cases (24) and (25) are the simplified theories of nanotubes and correspond to the generalized theory of cylindrical shells. Generalization consists in emergence composed with the second derivative of a deflection. Moreover, unlike the classical theory of cylindrical shells, the simplified theories of nanotubes (24) and (25) contain five physical parameters A_1^{11}, A_3^{11}, a_1^{11}, a_2^{11}, a_6^{11}, restricted according to (23).

We will show now that the system (19) at the determinant (23) other than zero, leads to the equation of the sixth order concerning deflections. For this purpose it is necessary to solve system (19) as algebraic relatively U''' and U', twice to differentiate the first derivative: $(U')''$ and to equate the right parts of U''' and $(U')''$.

$$\begin{cases} U''' = -\dfrac{r}{3A_1^{11}(3a_1^{11} - 4a_2^{11})}\{-a_1^{11}\dfrac{1}{r}[6A_1^{11}\dfrac{1}{r}W'' - a_1^{11}\dfrac{1}{r}W - p_x] + \\ +(a_1^{11} + 2a_2^{11})[-3A_3^{11}W'''' + ((a_2^{11} + a_6^{11})r^2 + 12A_1^{11} + 2A_3^{11})\dfrac{1}{r^2}W'' - \\ -(3A_3^{11} + (a_1^{11} + 2a_2^{11})r^2)\dfrac{1}{r^4}W + P_r^F]\} \\ U' = -\dfrac{r}{3A_1^{11}(3a_1^{11} - 4a_2^{11})}\{-6A_1^{11}\dfrac{1}{r}[6A_1^{11}\dfrac{1}{r}W'' - a_1^{11}\dfrac{1}{r}W - p_x] + \\ +15A_1^{11}[-3A_3^{11}W'''' + ((a_2^{11} + a_6^{11})r^2 + 12A_1^{11} + 2A_3^{11})\dfrac{1}{r^2}W'' - \\ -(3A_3^{11} + (a_1^{11} + 2a_2^{11})r^2)\dfrac{1}{r^4}W + P_r^F]\} \end{cases}$$

As a result we will receive that the theory of tension/compression of SWNT is defined by the ordinary differential equation of the sixth order concerning deflections. The nanotube is defined by five physical parameters playing a role of the generalized stiffnesses in theory of cylindrical shells:

$$\begin{aligned} &15A_1^{11}3A_3^{11}W'''''' - \\ &+[-3(a_1^{11} + 2a_2^{11})A_3^{11} + 36A_1^{11}A_1^{11}\frac{1}{r^2} - 15A_1^{11}((a_2^{11} + a_6^{11})r^2 + \\ &+12A_1^{11} + 2A_3^{11})\frac{1}{r^2}]W'''' + [(a_1^{11} + 2a_2^{11})((a_2^{11} + a_6^{11})r^2 + \\ &+12A_1^{11} + 2A_3^{11})\frac{1}{r^2} - 12a_1^{11}A_1^{11}\frac{1}{r^2} + \\ &+15A_1^{11}(3A_3^{11} + (a_1^{11} + 2a_2^{11})r^2)\frac{1}{r^4}]W'' - \\ &+[-(a_1^{11} + 2a_2^{11})(3A_3^{11} + (a_1^{11} + 2a_2^{11})r^2)\frac{1}{r^4} + a_1^{11}a_1^{11}\frac{1}{r^2}]W + \\ &+[(a_1^{11} + 2a_2^{11})P_r^F - 15A_1^{11}(P_r^F)''] + [a_1^{11}\frac{1}{r}P_x^e - 6A_1^{11}\frac{1}{r}(p_x)''] = 0 \end{aligned} \tag{26}$$

It isn't difficult to be convinced that the structure of the equation (26) contains structure of the classical theory of cylindrical shells. For this purpose it is necessary to neglect the items in the first, third and fourth line (26). However, even at classical structure of the equation (existence only of the fourth derivative and the function of a deflection) it essentially differs from the classical equation of deflections as multipliers at derivatives has the other physical sense - there are some combinations of moduli of ideal and gradient adhesion which dimension coincides with dimension of a stiffness of cylindrical shell.

Generally the equation of balance (26) can be presented as product of three operators of the second order over a deflection. For physical reasons it is possible to claim that the deflection fading from edges either three exponents, or one exponential and two aperiodic, complex interfaced decisions will be own functions of these operators. Attenuation indicators of exponents will be functions of physical parameters and radius. Therefore in the theory of SWNT it is impossible to share edge effects (when the indicator of attenuation depends only on sizes, in this case – from SWNT radius) and multiscale-effects (when the indicator of attenuation depends only on the relations of adhesive modules of different dimension, in this case – on the relations $\sqrt{A/a}$ type).

5. The Mechanical Properties of SWNT While Torsion

The case of torsion assumes that for an axisymmetric problem warping have to be absent. From here:

$$U = 0 \tag{27}$$

As well as in the theory of thin-walled cores, the hypothesis of an invariance of cross section is accepted. That for an axisymmetric problem is equivalent to absent of deflections. From here:

$$W = 0 \tag{28}$$

Thus, the problem of torsion is reduced to one differential equation of the fourth order concerning one unknown function V :

$$\begin{gathered} 3A_1^{11}V'''' - (a_2^{11}r^2 + 6A_1^{11} + 4A_3^{11})\frac{1}{r^2}V'' + \\ +((a_2^{11} + a_6^{11})r^2 + 15A_1^{11})\frac{1}{r^4}V - P_\phi^F = 0 \end{gathered} \tag{29}$$

In torsion problem the limit cases, similar to the cases considered in the previous section, also take place. Particular case, when moments are absent, take place when tensor $A_{ijkmnl}^{11} = 0$:

$$a_2^{11}V'' - (a_2^{11} + a_6^{11})\frac{1}{r^2}V + P_\phi^F = 0 \tag{30}$$

At rather big radiuses of nanotubes $r \to \infty$ the classical case takes place:

$$a_2^{11}V'' + P_\phi^F = 0 \tag{31}$$

At small values of radius of SWNT torsion has nature of edge effect and fades at order distances

$r\sqrt{a_2^{11}/(a_2^{11} + a_6^{11})}$. Particular case, when ideal adhesion is absent, take place when tensor $A_{ijmn}^{11} = 0$:

$$3A_1^{11}V'''' - (6A_1^{11} + 4A_3^{11})\frac{1}{r^2}V'' + 15A_1^{11}\frac{1}{r^4}V - P_\phi^F = 0 \tag{32}$$

In models of torsion of SWNT (29) or (32) two fundamental decisions, represented two nonclassical edge effects with the corresponding characteristic lengths of attenuation. This equation (29), as well as in case of tension/compression (26), don't allow to separate nonclassical edge effects and multiscale-effect.

6. Conclusions

The applied theories of SWNT tension and torsion formulated in this work offer possibilities to study the mechanical properties of 2D-objects, to set and solve test problems, the solutions of which can be tested experimentally. In particular the tension problem defined the SWNT mechanical properties by five non-classical moduli, correspondingly the torsion problem defined by four ones.

Special cases are studied: a case of ideal and purely gradient adhesion, quasiclassical case, cases at big and small sizes of radius of SWNT. It is shown that the case of ideal adhesion corresponds to correct statement of the membrane theory of cylindrical shell. The case of purely gradient adhesion corresponds to correct statement of the theory of edge effect of cylindrical shell. It is shown, that particular case of the quasiclassical theory of cylindrical shell is not consecutive approach of the general theory when moduli of ideal adhesion are partially considered, and partially - moduli of purely gradient adhesion. The characteristic feature of all statements is the fact that the mechanical properties of SWNT are not defined by "volumetric" moduli but by adhesive ones which have different physical dimension which coincides with the dimension of the corresponding stiffness of classical and nonclassical shells.

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