

# Aerodynamics Performance of a Variable-Speed Variable-Pitch Wind Turbine Blade Using the BEM Theory at Off-Design Condition

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## Abstract

A small scale wind turbine blade is studied using blade element momentum (BEM) theory. A difficult goal in the implementation of the BEM theory is the correct representation of the lift and drag coefficients at post-stall regime. A method based on the Viterna equations is implemented for producing airfoil data at the post-stall regime and results are compared with various mathematical models. Results showed the high capability of this method to predict the lift and drag coefficients for airfoils, resulting in better power curve estimation. The implemented model for wind turbine load estimation is applied for a the variable-speed variable-pitch (VSVP) wind turbine and the results show a considerably increased in turbine annual energy production (AEP) about 16.78% compared with the NREL phase VI turbine.

## Keywords

Variable-Speed Variable-pitch Wind Turbine, BEM Theory, Annual Energy Production, Post-stall Region

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## 1. Introduction

The ever-increasing demand for energy, the limited resources of fossil fuels, and the rise of environmental concerns over chemical pollution and water usage have forced energy engineers to push the limits and introduce methods to increase the efficiency of energy conversion process [1]. In this process renewable energy has drawn unprecedented attention to itself [2-3]. Wind energy has proved to be an important source of clean and renewable energy in order to produce electrical power. Nowadays, wind energy is one the fast-growing renewable energy sources [4]. A wind turbine is a device that converts kinetic energy from the wind into mechanical energy. It is shown that the major aspects of wind turbine performance (mean power output and mean loads) are determined by the aerodynamic forces generated by the wind. Therefore, wind turbine aerodynamics is one of the most important factors in the developing of the wind energy

industry.

Various experimental tests have been carried out regarding the aerodynamic performance of the wind tunnel-scale wind turbines. A two-bladed turbine of NREL [5] with a simple geometry rotor was tested in the 24 m\*36 m wind tunnel for pressure measurements along the blade. A detail investigation of aerodynamic coefficients for the S809 airfoil was reported in the stall region. The rotor turned clockwise at a constant 71.6 rpm, was stall regulated, and blade pitch was held constant at 3°. The MEXICO turbine with a three-bladed upwind rotor (4.5 m diameter) was tested in German Dutch wind tunnel for surface pressure measurements to specify the blade loading distribution [6]. The MEXICO experiment blades were both twisted and tapered. The rotor was turned counterclockwise (viewed from downwind) at constant speeds 324 rpm and 424 rpm. A three-bladed small HAWT rotor of 2.2 m in diameter was tested in the BLWT2 at the University of Western Ontario. The overall scope of this

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work was to assess the feasibility of wind tunnel testing for small HAWT to provide reliable power curves for validating computational models [7].

Numerous computational approaches have been applied to aerodynamic design of wind turbine such as Blade Element Momentum theory (BEM) [8-9], Vortex Wake Method [8], and Computational Fluid Dynamic (CFD) [10-11]. CFD has gained tremendous amount of attention globally in the past few decades and is being widely implemented in studies regarding investigation of aerodynamics and optimization [12-13]. However, the BEM theory is one of the most popular methods in wind turbine blade design and analysis [14-15], due to its simplicities and accuracies. It applies the conservation of mass, momentum and energy equations on annual rings of flow volume through the blade and assumes that there is no interaction between those rings. The BEM theory employs airfoil aerodynamics data which can be obtained from experiments or simulations. It is not cost-effective to do experiment for every airfoil at different flow conditions (such as Re number), therefore using numerical simulations becomes very popular. Different models are developed to determine the airfoil lift and drag coefficients before stall region and they have been successfully validated with experiments. However, wind turbines are exposed to various wind conditions, thus sometimes working at post-stall region. A difficult task in the implementation of the design codes such as BEM theory in the post-stall region is the correct representation of the lift and drag coefficients for airfoils to determine the loading on the blade. Therefore, developing a model that can provide reliable airfoil data at the post-stall region is crucial. The reliable airfoil data will lead to better characterization of wind turbine loading and estimation of wind turbine performance. Several models have been presented for determining airfoil aerodynamics coefficients at post stall region [16-19]. The developed models are used in BEM theory and results showed good agreements with experiment. The implemented methods have been used to design, analysis and optimizations of wind turbine blades [20-21].

In the present work, a mathematical model based on Viterna equations [22] is developed to better airfoil data estimation at the post stall region. This region is found interesting when turbine works at off-design condition. The implemented model is validated with the experimental data and compared with different other mathematical models. The developed model is used to model the NREL Phase VI wind turbine and analyze the data from power coefficient and power points of view. In addition, it is used to analyse the variable speed wind turbine and the results are compared with the fixed speed turbine.

## 2. The Mathematical Model

The blade element momentum (BEM) theory is used for the fluid dynamic design of a wind turbine. The BEM theory can be subdivided into two parts. In the first part, the model divides the blade into several independent elements. It further assumes that the aerodynamic forces on each element can be calculated as a two dimensional airfoil subjected to the flow conditions.

A cross section of a rotor blade at radius  $r$  of the blade element is shown in figure 1. Velocities and forces related to the blade are demonstrated in this cross section of the rotor blade. Thrust and torque can be obtained from aerodynamic forces on the cross section of the rotor blade as a function of tangential and axial induction factors,  $a$  and  $a'$  [21]:

$$dT = \frac{\rho U^2 (1-a)^2}{2 \sin^2 \phi} N_b (C_L \cos \phi + C_D \sin \phi) c dr \quad (1)$$

$$dM = \frac{\rho U (1-a)}{2 \sin \phi} \cdot \frac{\omega r (1+a')}{\cos \phi} N_b (C_L \sin \phi - C_D \cos \phi) c r dr \quad (2)$$

where  $\theta_{p,0}$ ,  $\theta_p$ ,  $\theta_r$ ,  $\alpha$ ,  $\phi$  are blade pitch angle at the tip, section pitch angle, section twist angle, angle of attack and angle of relative wind, respectively.

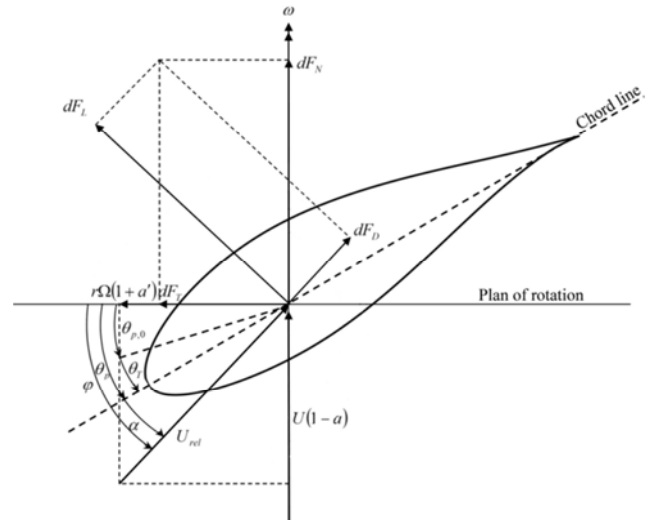


Figure 1. Blade element geometry, velocities, and forces [21].

In the second part, by applying the conservation of momentum, the net force on the actuator disk comes from the change of air momentum passing through the disk; and the rate of momentum is equivalent to the air velocity difference across the rotor plane times the mass flow rate. Hence, the thrust and torque of the blade can be obtained as follow:

$$dT = 4\pi r \rho U^2 a (1-a) dr \quad (3)$$

$$dM = 4\pi r^3 \rho U \omega (1-a) a' dr \quad (4)$$

By equalizing the equation 1 with equation 3 and equation 2 with equation 4, axial and tangential induction factors can be found as follows:

$$a = \frac{1}{\frac{4F \sin^2 \phi}{\sigma (C_L \cos \phi + C_D \sin \phi)} + 1} \quad (5)$$

$$a' = \frac{1}{\frac{4 \sin \phi \cos \phi}{\sigma (C_L \sin \phi - C_D \cos \phi)} + 1} \quad (6)$$

Where F is the Prandtl tip loss correction factor. An approximate formula for the Prandtl tip loss function was introduced by Glauert [23], namely

$$F = \frac{2}{\pi} ar \cos \left[ \exp \left( \frac{N_b (r - R)}{2r \sin \phi} \right) \right] \quad (7)$$

$\sigma$  is the local solidity, defined as:

$$\sigma = \frac{cN_b}{2\pi r} \quad (8)$$

Equation (5) only gives reliable results for axial induction factor values between 0 and 0.4. For axial induction factors greater than 0.4 the BEM theory does not yield reliable results. To correct the axial induction factor when  $a > 0.4$ , equation (13) was proposed in ref [24]:

$$a = \frac{18F - 20 - 3\sqrt{C_N(50 - 36F) + 12F(3F - 4)}}{36F - 50} \quad \text{For } a > 0.4 \quad (9)$$

Airfoil behavior can be categorized into three flow regimes: the attached flow regime, the high lift/stall development regime, and the flat plate/fully stalled regime [25]. The dimensionless coefficients for the S809 airfoil data resulting from tests taken at Delft University of Technology (DUT) are used for BEM analysis at angle of attack below stall [26]. A complete table of airfoil coefficients for angle of attacks from  $-180^\circ$  to  $180^\circ$  is needed, since the airfoils may encounter high angles of attack at off-design condition. In order to estimate airfoil behavior at high positive and negative angles of attack other methods must be employed. A method based on the Viterna equations, is used for extrapolating airfoil data into the high lift/stall development regime. The extrapolation of drag and lift coefficients are calculated for  $\alpha_{stall} \leq \alpha \leq 90^\circ$

$$\text{From } C_D = B_1 \sin^2 \alpha + B_2 \cos \alpha \quad (10)$$

where:

$$B_1 = C_{Dmax} \quad (11)$$

$$C_{Dmax} = 1.11 + 0.018AR \quad (\alpha = 90^\circ) \quad (12)$$

$$B_2 = \frac{C_{Dstall} - C_{Dmax} \sin^2 \alpha_{stall}}{\cos \alpha_{stall}} \quad (13)$$

$$C_L = A_1 \sin 2\alpha + A_2 \frac{\cos^2 \alpha}{\sin \alpha} \quad (14)$$

$$A_1 = B_1 / 2 \quad (15)$$

$$A_2 = (C_{Lstall} - C_{Dmax} \sin \alpha_{stall} \cos \alpha_{stall}) \frac{\sin \alpha_{stall}}{\cos^2 \alpha_{stall}} \quad (16)$$

For angles of attack between  $90^\circ$  and  $180^\circ$  positive or negative, lift and drag coefficients are approximated as the flat plate, fully stalled regime, values [27]:

$$C_{Lplat} = 2 \sin \alpha \cos \alpha \quad (17)$$

$$C_{Dplat} = 2 \sin^2 \alpha \quad (18)$$

### 3. Annual Energy Production (AEP)

A wind turbine is a device that converts kinetic energy from the wind into mechanical energy. A mechanical torque is produced when air passes through the turbine blades. This torque is used to produce power [28-32]. The mechanical power of the wind turbine extracted from the wind is expressed as:

$$P = \frac{1}{2} C_p \rho A U^3 \quad (19)$$

Where  $\rho$  is air density, A is the swept area of rotor, U is wind speed and  $C_p$  is the power coefficient. Wind turbines having the control of pitch angle will have the power coefficient described by a function that depends on pitch angle ( $\theta_p$ ) and tip speed ratio ( $\lambda$ ). The tip speed ratio is defined as the ratio between the blade tip speed and the wind speed

$$\lambda = \frac{R\omega}{U} \quad (20)$$

The power curve P(U) is used to calculate the AEP by assuming the probability of the occurrence of wind is Weibull distribution [25]:

$$p(U) = \left( \frac{k}{c'} \right) \left( \frac{U}{c'} \right)^{k-1} \exp \left[ - \left( \frac{U}{c'} \right)^k \right] \quad (21)$$

In this equation,  $c'$  is Weibull scale parameter and k is

Weibull shape parameter. Thus, the AEP is calculated using the following formula [25]

$$AEP = 8760 \int_0^{\infty} P(U) p(U) dU \quad (22)$$

This integral is numerically approximated with a trapezoidal rule over the range of wind speed from cut-in to cut-out velocity.

### 4. Results

The BEM theory applies the lift and drag coefficients of the turbine blade’s airfoils to evaluate the performance of the rotor. The airfoil data resulting from experiment is used for BEM analysis at the attached flow regime, while the mathematical models are utilized to find the aerodynamic

coefficients at the post-stall regions. The accuracy of the results obtained from the implemented code is validated by the experimental data from the NREL Phase VI wind turbine reported in [5]. The fundamental parameters of the wind turbine are listed in Table 1. The obtained findings in Fig 2 and 3 adequately demonstrate the high capability of the implemented approach in predicting the power coefficient and power of wind turbines.

Table 1. The specifications of the NREL phase VI wind turbine [5].

Parameters	Unit	Value
Rotor diameter	m	10.6
Rotational speed	r/min	72
Rated power	kW	10
Blade pitch	degree	3
Number of blades	-	2
Blade profile	-	S809

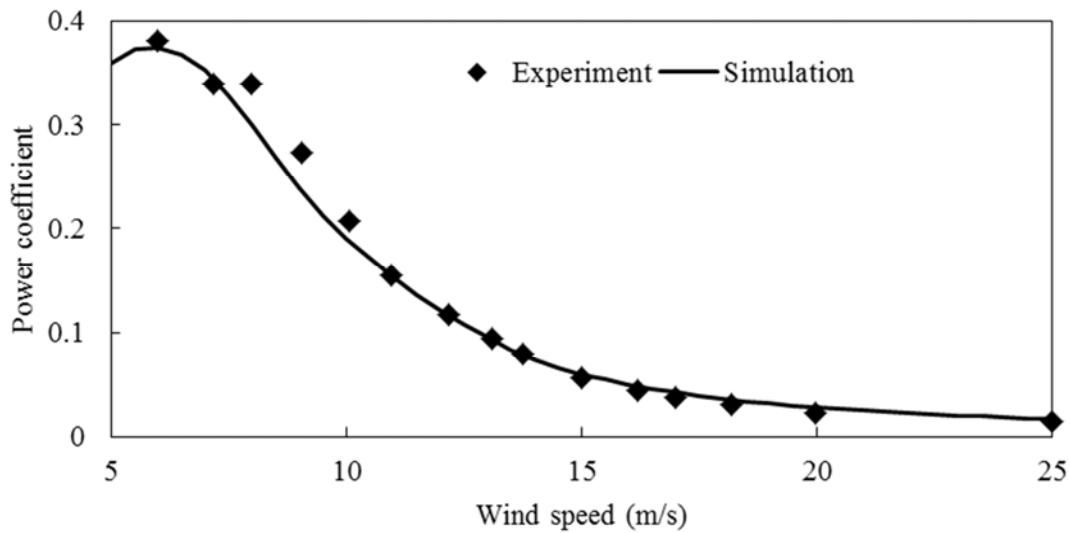


Figure 2. Comparison of power coefficient for experimental data [5] and simulation.

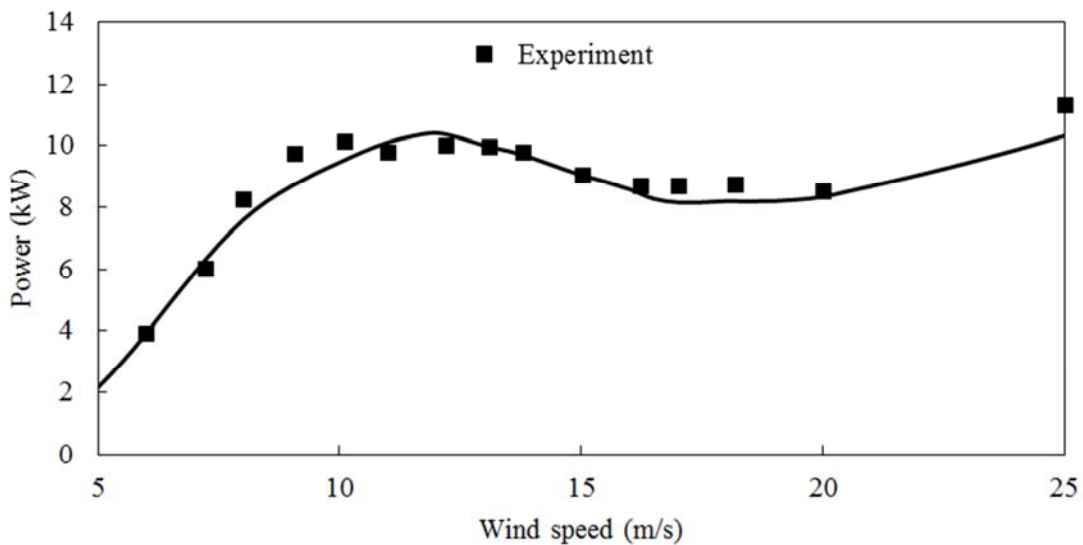


Figure 3. Comparison of power output for experimental data [5] and simulation.

The differences between the predicted and measured power are calculated at different wind speeds. It showed that the overall discrepancy between the simulated results and the experimental data from the NREL Phase VI turbine is small and the minimum discrepancy is found to be 0.02% happening at  $V=13.1$  m/s.

In addition to experimental data, the current model results are compared with those from different mathematical models.

Various mathematical models have been presented to describe the correct trend of lift and drag coefficients at the stall regions [16-19]. The results of comparison in Fig 4 shows that the implemented approach has high capability for prediction the performance of wind turbines, especially after rated wind speed region. This region is important to be captured precisely, as turbines are mostly work in this region in some days of the year.

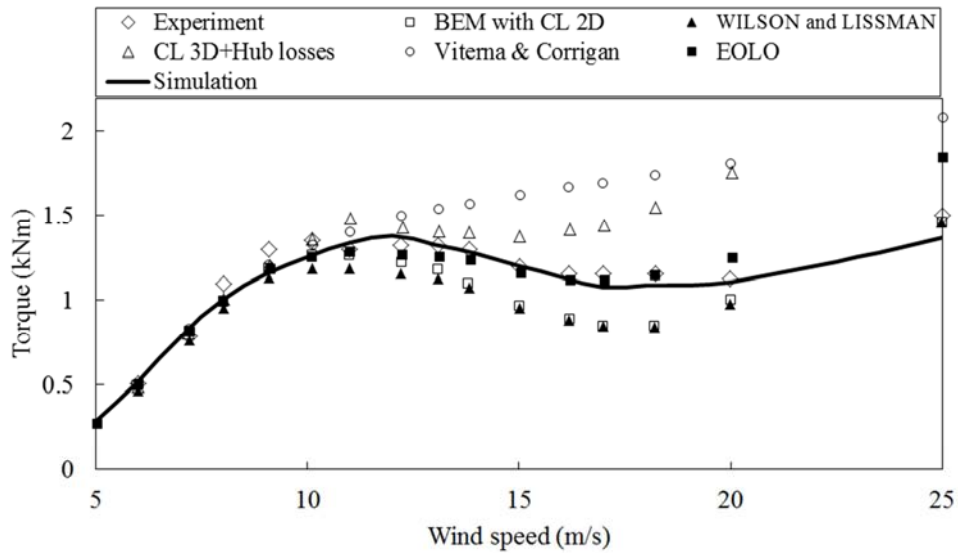


Figure 4. Comparison of torque between experiment and different mathematical models representing the lift and drag coefficients.

Comparison between the results of fix-speed fix-pitch (the NREL phase VI turbine) and the same turbine but working at variable-speed variable-pitch condition is shown in Figure 5. It can be seen that the power coefficient is considerably higher for variable-speed variable-pitch than fixed speed-fixed pitch at ranges of wind speeds lower than the rated wind speed. In addition, the power coefficients is higher at the velocities from 15 m/s to 20 m/s and in this region the flow separation is delayed due to the increase in pitch angle

(decrease of angle of attack), therefore more power can be achieved.

The most interesting parameter which can be compared for the FSFP and VSVP turbines is power output. It directly determines how much power one turbine can produce. As shown in Figure 6, the VSVP turbine produces much more power than the FSFP. This happens because of the variable rotation speed at velocities lower than rated wind speed, and variable pitch angle at the above rated wind speeds.

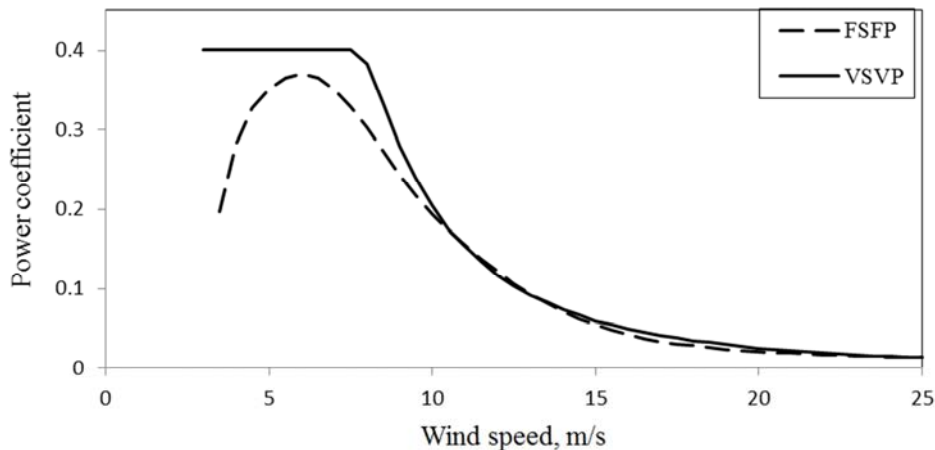


Figure 5. Power coefficient versus wind speed.

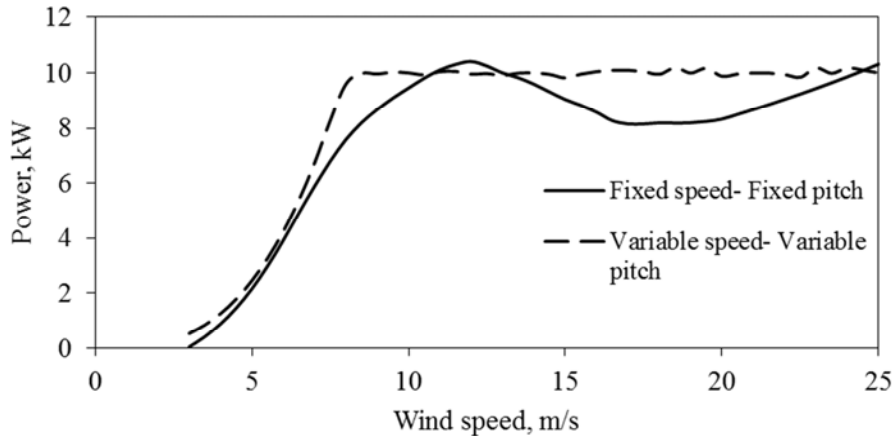


Figure 6. Wind turbine power at different wind speeds.

In order to calculate how much increase in annual energy production (AEP) occurs for the VSVP turbine versus FSFP turbine, some calculations are required to be carried out. The AEP is related to the mechanical power of the wind turbine and the probability of wind speed occurrence. The AEP values for the FSFP and the VSVP turbines are calculated to be 36413 kW-hr/yr and 42520 KW-hr/year, respectively. Therefore, the AEP has increased about 16.78% for the VSVP turbine. This is one of the promising ways to increase the wind turbine power production and decrease the wind turbine long term expenses.

## 5. Conclusion

The BEM theory was applied for the evaluation of wind turbine rotor performance. A difficult goal in the implementation of the BEM theory is the correct representation of the lift and drag coefficients at post-stall regime. In this research, the method based on the Viterna equations was applied for airfoil data at the post-stall regime. Guidance for the Viterna method input parameters was provided by the NREL phase VI wind turbine data. The results were compared with NREL phase VI wind rotor for which experimental mechanical power measurements are reported in scientific literature and showed that implemented method has high capability to predict the rotor performance.

The BEM code along with the airfoil data from the implemented approach is used to study the variable-speed variable-pitch wind turbines. The NREL Phase VI (FSFP) data is used as a case study for the specific region in Iran which is highly recommend for installing small wind turbines. The turbine data compared with the VSVP version of the NREL turbine and results showed the increase in the annual energy production. Therefore, employing VSVP turbines is one of the promising ways to increase the wind turbine power production and decrease the wind turbine long

term expenses.

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