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Simplified Approach of Turbulent Film Boiling on a Horizontal Elliptical Tube with Surface Tension Effects

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Abstract

A theoretical investigation is performed into the heat transfer efficiency of turbulent film boiling on a horizontal, isothermal elliptical tube immersed in upward flowing liquid. The high velocity of the flowing saturated liquid at the boundary layer is determined by potential flow theory. The vapor-liquid interfacial shear induced in forced convection turbulent film boiling process is developed in the present research. The results shows, the surface tension effects increase in the forced convection boiling system under high velocities of flowing liquid, the mean Nusselt will very slightly decrease. Besides, turbulent film boiling under the high velocity of upward flowing liquid, the increase in the Froude number will bring out an increase in the mean Nusselt number. Finally, a comparison between the results of the present study and those reported in previous experimental studies is provided.

Keywords

Froude Number, Surface Tension, Eccentricity, Interfacial Shear

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1. Introduction

The problem of laminar film boiling has received extensive attention in the literature. Bromley [1] investigated laminar film boiling on a horizontal tube under the simplifying assumptions of a linear temperature distribution within the vapor and negligible inertia effects within the vapor film. In 1966, Nishikawa and Ito. [2] performed a theoretical investigation into free-convection film boiling on an isothermal vertical plate and a horizontal cylinder, respectively, using a two-phase boundary-layer treatment in which the effects of radiation were ignored. Jordan [3] analyzed the heat transfer characteristics of the laminar film boiling and transition boiling regimes, respectively, and examined the heat flux properties at the boundary between them. Sakurai et al. [4] presented the pool film boiling on a horizontal cylinder with theoretical solutions. The analytical heat transfer model is based on laminar boundary theory including radiation effects. Yang and Hsu [5] investigated free convection film boiling on an isothermal elliptical tube with surface tension, and showed that the effect of surface tension, superheating and liquid viscosity on mean Nusselt number. Laminar film boiling on horizontal tubes has been widely discussed in published literature, and there is also some development on the researches of turbulent film boiling. For example, Sarma et al. [6] presented turbulent film boiling with consideration to thermal radiation for the vertical surface. In the research, the assumption of equal shear condition both at the wall and the vapor-liquid interface is reasonable. About the turbulent film boiling on a horizontal isothermal circular cylinder, Sarma et al. [7] presented some theoretical results. The analysis compared the theoretical results with previous experimental results, and found that their results were in a good agreement with the experimental data.

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The published literature contains many investigations into laminar film boiling on horizontal tubes or elliptical tubes, or turbulent film boiling on horizontal cylinders or vertical surface. However, the problem of turbulent film boiling on a horizontal elliptical tube with surface tension effects under high velocity flowing liquid has attracted relatively little attention. Accordingly, this study performs a theoretical investigation into the heat transfer efficiency of turbulent film boiling on a horizontal, isothermal elliptical tube. Taking cryogenic liquid nitrogen as the working fluid, the analyses investigate the distribution of the dimensionless film thickness on the elliptical tube and examine the respective effects of the Froude number, Bond number and the eccentricity on the mean Nusselt number. Finally, the present results obtained for the correlation between the mean Nusselt number and the Rayleigh number are compared with the experimental results presented in the literature.

2. Description of Physical Model

Consider a horizontal elliptical tube immersed in an up flowing LN_2 of the high velocity u_∞ at saturated temperature T_s. The wall temperature T_w is assumed high enough to induce turbulent film boiling on the surface of the elliptical tube, and then a continuous film of vapor runs upward over the surface. The physical model and the coordinate system adopted in the present study are shown in figure 1, where the coordinates is to use two-dimensional orthogonal curvilinear coordinate system.

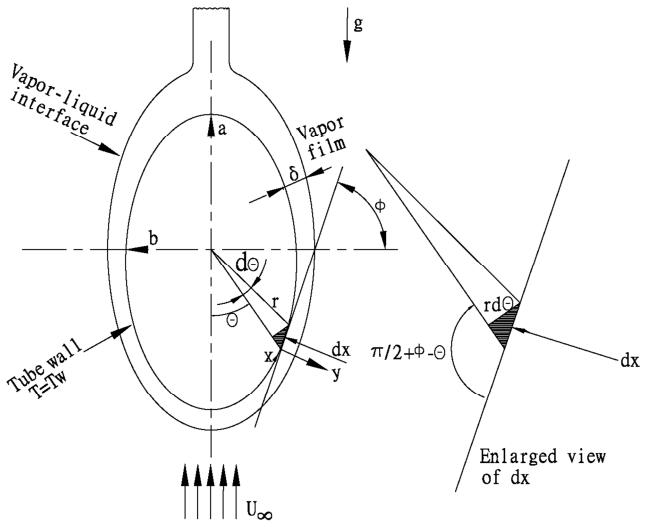

Figure 1. Physical model and coordinate system.

For thin film flow of turbulent film boiling under the forced convection, the viscosity component and the buoyancy effect are assumed more significant than the inertia force. Then the force balance equation for the vapor film can be expressed as:

$$\tau_w = \tau_\delta + g\delta(\rho_l - \rho_s)[\sin\phi + Bo(\phi)] \tag{1}$$

Where $Bo(\varphi)$, the surface tension effect is as follows:

$$Bo(\phi) = \frac{3e^2}{2Bo}\left(\frac{1-e^2\sin^2\phi}{1-e^2}\right)^2 \sin 2\phi \tag{2}$$

It is assumed the thickness of vapor film is much thinner than the diameter of the tube ($\delta << D_e$). And it's further assumed the turbulent conduction term across the vapor layer is more significant than the convective term, and hence the convective term can be neglected. The energy equation can be expressed as:

$$\frac{\partial}{\partial y}\left\{\left(1+\frac{\varepsilon_m}{\nu}\Pr\right)\frac{\partial T}{\partial y}\right\} = 0 \tag{3}$$

The boundary conditions of energy equation under isothermal condition are as follows:

$$T = T_w \text{ at } y = 0$$

$$T = T_s \text{ at } y = \delta \text{ for } 0 \le \phi \le \pi \tag{4}$$

For a pure substance, the thermal energy balance equation of the vapor film can be expressed as:

$$\frac{d}{dx}\int_0^\delta \rho_v u\, dy = -\frac{k_w}{h_{fg}}\frac{dT}{dy}\bigg|_{y=0} + \frac{\varepsilon\sigma\left(T_w^4 - T_s^4\right)}{h_{fg}} \tag{5}$$

The differential arc length for ellipse can be expressed with the following equation:

$$dx = I(\phi)\frac{D_e}{2}d\phi \tag{6}$$

where

$$D_e = \frac{2a}{\pi}\left(1-e^2\right)\int_0^\pi \left(1-e^2\sin^2\phi\right)^{-3/2} d\phi \tag{7}$$

$$I(\phi) = \pi\left(1-e^2\sin^2\phi\right)^{-3/2} \Big/ \int_0^\pi \left(1-e^2\sin^2\phi\right)^{-3/2} d\phi \tag{8}$$

where D_e is an equivalent diameter based on the equal outside surface area, which is compared with circular tubes.

Substitute the dx into thermal energy balance equation (Eq. 5) can be modified as follows:

$$\frac{d}{I(\varphi)d\varphi}\int_0^{\delta}\rho_v u\ dy=-\frac{k_w}{h_{fg}}\frac{D_e}{2}\frac{dT}{dy}\bigg|_{y=0}+\frac{\varepsilon\sigma D_e\left(T_w^4-T_s^4\right)}{2h_{fg}} \quad (9)$$

In the turbulent region the semi-empirical equation which describes heat transfer in the flow parallel to a moderately curved surface may also be used to describe the heat transfer in the flow parallel to an elliptical surface. Jakob [8] proposed that this situation may be described for any fluid by the following expression:

$$Nu_l = CRe_l^{\,n}Pr^{1/3} \quad (10)$$

Where C is a constant in flow configuration, C=0.034.

According to Colburn analogy, the friction factor can be written as the following equation:

$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3} \quad (11)$$

The mean friction coefficient in the streamwise direction may then be calculated as:

$$C_f = \frac{1}{\pi}\int_0^{\pi} f_\varphi d\varphi \quad (12)$$

Furthermore, the local friction can be obtained as:

$$f_\varphi = C\pi\,\mathrm{Re}_l^{\,n-1}\sin\varphi \quad (13)$$

The turbulent boundary layer exerts a friction force on the liquid-vapor boundary. The shear stress is estimated by considering the external flowing liquid across the surface of the tube when there is no vapor film on the surface. The local shear stress is defined as:

$$\tau_\delta = f_\varphi\left(\frac{1}{2}\rho_l u_l^{\,2}\right) \quad (14)$$

According to potential flow theory, when the uniform liquid flow of velocity u_∞ passing a tube, the liquid velocity at the edge of the boundary is as follows:

$$u_e = u_\infty(1+\sqrt{1-e^2})\sin\varphi \quad (15)$$

Combining Eq. (13-15), the local shear stress can be expressed as:

$$\tau_\delta = \frac{1}{2}\pi C\rho_l u_\infty^{\,2}\,\mathrm{Re}_l^{\,n-1}\left(1+\sqrt{1-e^2}\right)^2\sin^3\varphi \quad (16)$$

Incorporating the interfacial vapor shear stress τ_δ given by Eq. (16) into the elemental forced balance equation enables Eq. (1) to be rewritten in the following form:

$$\tau_w = 2c\pi\rho_l u_\infty^{\,2}\,\mathrm{Re}_l^{\,n-1}\frac{\left(1+\sqrt{1-e^2}\right)^2}{4}\sin^3\varphi + g\delta(\rho_l-\rho_s)\left[\sin\varphi+Bo(\varphi)\right] \quad (17)$$

The forced balance equation Eq. (17) yields the following dimensionless equation:

$$R_e^{*3} = \frac{\left(1+\sqrt{1-e^2}\right)^2}{4}R_e^{*}\phi Fr^{\frac{n+1}{2}}\sin^3\varphi+\delta^{+}\left[\sin\varphi+Bo(\varphi)\right] \quad (18)$$

It's further assuming the pressure across the boundary layer is constant and the density variation across the boundary layer is given by the following equation:

$$\rho T = \rho_w T_w = \rho_s T_s \quad (19)$$

The energy equation Eq. (3) yields the following dimensionless energy equation:

$$\frac{d}{dy^+}\left[\left(1+\frac{\varepsilon_m}{\nu_s}\frac{\mathrm{Pr}}{\mu^+(T^+)\left(1+T^+(T_r-1)\right)}\right)\frac{dT^+}{dy^+}\right]=0 \quad (20)$$

The dimensionless boundary conditions of Eq. (20) are:

$$\begin{aligned}&(1)\,\mathrm{at}\,y^+=0;T^+=1\\&(2)\,\mathrm{at}\,y^+=\delta^+;T^+=0\end{aligned} \quad (21)$$

Where the absolute viscosity equation μ^+ in dimensionless energy equation Eq. (20) is expressed as the vapors of liquid nitrogen at the saturation temperature corresponding to a system pressure under 1 atm. i. e.

$$\mu^+ = \frac{\mu}{\mu_s} = 2.656-3.804\left(\frac{T}{T_s}\right)+2.148\left(\frac{T}{T_s}\right)^2 \quad (22)$$

Besides, the thermal energy balance equation Eq. (9) can be rewritten in dimensionless form as follows:

$$\frac{d}{I(\varphi)d\varphi}\int_0^{\delta^+}\frac{u^+}{1+T^+(T_r-1)}dy^+ =$$
$$T_rS\left\{(Tr-1)\left(\frac{Gr}{2}\right)^{\frac{1}{3}}\left(R_e^{*3}-\frac{\left(1+\sqrt{1-e^2}\right)^2}{4}R_e^*\phi Fr^{\frac{n+1}{2}}\sin^3\varphi\right)^{1/3}k_w^+\left.\frac{dT^+}{dy^+}\right|_{y^+=0}+NR\left(T_r^4-1\right)\right\} \quad (23)$$

Where the absolute conductivity equation k^+ in thermal energy balance equation Eq. (23) is expressed as the vapors of liquid nitrogen at the saturation temperature corresponding to a system pressure under 1 atm.:

$$k^+=\frac{k}{k_s}=236.1-811.5\left(\frac{T}{T_s}\right)+1045.4\left(\frac{T}{T_s}\right)^2-596.2\left(\frac{T}{T_s}\right)^3+127.2\left(\frac{T}{T_s}\right)^4 \quad (24)$$

Furthermore, the dimensionless thermal energy balance equation Eq. (23) requires the velocity profile u^+ in the vapor film. And u^+ can be obtained by following equation:

$$\frac{du^+}{dy^+}=\frac{1-2\frac{y^+}{\delta^+}}{\frac{T}{T_s}\mu^+(T^+)\left[1+\frac{\varepsilon_m}{\nu_s}\frac{1}{\mu^+(T^+)\left(1+T^+(T_r-1)\right)}\right]} \quad (25)$$

The boundary condition is:

$$u^+=0,\quad y^+/\delta^+=0 \qquad at \qquad y^+=0 \quad (26)$$

The eddy diffusivity distribution presented by Kato et al. [9] is expressed as:

$$\frac{\varepsilon_m}{\nu_s}=0.4y^+[1-\exp(-0.0017y^{+2})] \quad (27)$$

The heat transfer of turbulent film boiling can be given by the following equation:

$$h(T_w-T_s)=-k_w\left.\frac{dT}{dy}\right|_{y=0}+\sigma\varepsilon\left[T_w^4-T_s^4\right] \quad (28)$$

Obviously, the local Nusselt number can be expressed as:

$$Nu=-k_w^+R_e^*\,Gr^{1/3}\left.\frac{dT^+}{dy^+}\right|_{y^+=0}+2NR\left(\frac{T_r^4-1}{T_r-1}\right) \quad (29)$$

The mean Nusselt number for the entire surface of the tube can be obtained:

$$Nu_m=\frac{1}{\pi}\int_0^{\pi}\left[-k_w^+R_e^*Gr^{1/3}\left.\frac{dT^+}{dy^+}\right|_{y^+=0}+2NR\left(\frac{T_r^4-1}{T_r-1}\right)\right]d\varphi \quad (30)$$

According to the governing equations, the mean Nusselt number can obtained as a

function of the following parameters:

$$Nu_m=f\left(Fr,Gr,T_r,NR,S,e,1/Bo\right) \quad (31)$$

3. Numerical Method

The dimensionless governing Eqs. (18), (20-27) and (29-30) subject to the relevant boundary conditions given can be used to estimate δ^+, R_e^* and Nu for the vapor film by means of the following procedures by using C^{++}:

Suitable dimensionless parameters, such as $1/Bo$, e, T_r, S, NR, Fr and Gr are specified.

The boundary conditions of velocity and temperature are as follows:

$$at\ y^+=0;T^+=1;u^+=0;\mu^+=2.656-3.804T_r+2.148T_r^2$$
$$k^+=236.1-811.5T_r+1045.4T_r^2-596.2T_r^3+127.2T_r^4 \quad (32)$$
$$at\ y^+=\delta^+;T^+=0$$

(3) Since the u^* at the bottom of the tube ($\varphi=0$, i=0) is zero, the dimensionless film thickness δ^+ is also zero ($\delta^+=\frac{\delta u^*}{\nu}=\frac{\delta 0}{\nu}=0$). At the next node, i.e. i=i+1, the value of φ is given by $\varphi_{i+1}=\varphi_i+\Delta\varphi$, where $\Delta\varphi=(\pi/360)$.

Guess an initial value of δ^+; substitute Eqs. (22), (27) into Eqs. (20) and (21) and then get the value of $\left.\frac{dT^+}{dy^+}\right|_{y^+=0}$.

Substitute $\left.\frac{dT^+}{dy^+}\right|_{y^+=0}$, Eqs (24), (25) and (26) into Eq. (23), and get the value of $R_e^{*3} - \frac{\left(1-\sqrt{1-e^2}\right)^2}{4} R_e^* \phi Fr^{\frac{n+1}{2}} \sin^3\varphi$.

After calculating, the values of R_e^* can be gotten, and then substitute the values of R_e^* into Eq. (18).

The criterion for the accuracy of δ^+ is assessed by Eq. (18), and it can be expressed as the following unequal equation:

$$\delta^+ - \frac{R_e^{*3} - \frac{\left(1+\sqrt{1-e^2}\right)^2}{4} R_e^* \phi Fr^{\frac{n+1}{2}} \sin^3\varphi}{\sin\varphi} \le 10^{-6}$$

If the calculation is a convergence, process the film thickness of next angular position. If the calculation is not a convergence, guess a new thickness and repeat processes (4)-(6).

The process above is repeated at the next node position, i.e. $\varphi_{i+1} = \varphi_i + \Delta\varphi$, and then subsequently at all nodes within the range $0 \le \varphi \le \pi$.

The local Nusselt number and mean Nusselt number are then calculated.

4. Results and Discussion

Figure 2 plots the dimensionless local velocity distributions in vapor film for *Fr*=500. The results show that a zero velocity exists at the wall as a result of the no slip boundary condition. Besides, for each angular position φ on entire tube surface, because of both the effects of buoyancy and interfacial shear, it is shown that with an increase in y^+, u^+ will first increase to a maximum value, and then it will slightly decrease. Furthermore, the velocity also increases as the angular positions increase.

Figure 3 shows the temperature distribution on entire elliptical tube. For the boundary conditions prescribed in the proposed model, the dimensionless temperature on the tube surface is 1, and is 0 on the vapor-liquid interface, respectively. Furthermore, the assumption of the stagnation flow is imposed at the bottom of the tube. As a result, the temperature variation along the vapor film thickness at φ=0 is linear. In addition, the interfacial shear with high velocity of liquid, and the effects of turbulence are considered in this wok. As the angular position increases, the effects of eddy diffusivity get stronger, the non-linear temperature profile of the vapor film appears.

Figure 4 displays the variation of the dimensionless vapor film thickness on the elliptical tube along φ. Specifically, the film thickness increases continuously from a minimum value at the bottom of the tube ($\varphi = 0°$) and reaches its maximum value at the top of the tube ($\varphi = 180°$). Besides, according to potential theory, the increase in the eccentricity value will lead to a decrease in the liquid velocity. And the decrease in the liquid velocity will lead to the decrease in heat transfer efficiency and evaporative rate. The former phenomenon will bring about the decrease in the film thickness. Furthermore, the film thickness increases with *Fr*. This can be explained from the fact that increasing *Fr* will bring out the increase of the interfacial shear stress and lead to the increase of the evaporative rate. Consequently, the increasing evaporative rate may cause an increase of the vapor film thickness.

Figure 5 presents the relationship between the mean Nusselt number and the 1/Bo for eccentricity $e = 0.0$, $e = 0.2$, $e = 0.4$ $e = 0.6$ and $e = 0.8$. The figure presents the results of the forced convection film boiling. According to equation 2, when the eccentricity of the elliptical tube is zero and as a special case of a circular tube ($e = 0.0$), the surface tension effects, $Bo(\varphi)$ values will be also zero and the surface tension effects won't influence the mean Nusselt number. Besides, when the values of eccentricity are larger, the $Bo(\varphi)$ values will be larger and the mean Nusselt is influenced by the surface tension. The surface tension effects increase in the forced convection boiling system under high velocities of flowing liquid, the mean Nusselt will very slightly decrease. Besides, under the case of *e*=0.8, when the 1/Bo increase from 0.0 to 0.1, the mean Nusselt will decrease from 529.077 to 528.743.

Figure 6 presents the effects of eccentricity on mean Nusselt number under five different wall temperature ratio, T_r. According to the potential flow theory, the larger the eccentricity parameter is, the smaller the liquid velocity and interfacial shear are. As a result, the vapor velocity will decrease, and then the mean Nusselt number will also decrease. In addition, it is evident that for a given radiation emissivity, the mean Nusselt number increases with an increasing temperature ratio. Since Nu_m varies as a function of T_r^4, the enhancement in the heat transfer performance is particularly apparent at higher values of the temperature ratio.

Figure 7 presents the relationship between the mean Nusselt number and the Froude numbers for five values of Grash of number. The figure shows the results of the forced convection film boiling. A higher Froude number will bring an increase in

the mean Nusselt. Besides, the Grashof number *Gr* is also one of the dominant factors, and therefore increasing the Grashof number will bring out an increase in the mean Nusselt.

To validate the present model, a comparison is made between this work and previous studies for different cases. For this reason, a modified Rayleigh number is introduced as $Ra=\frac{D^3(\rho_l-\rho_s)g}{\mu_s{}^2}\rho_s\frac{C_p\mu}{k}\left[0.5+\frac{h_{fg}}{C_p\Delta T}\right]$ which can be further nondimensionalized in the form of $Ra=\mathrm{GrPr}[0.5+\frac{1}{S(T_r-1)}]$.

Figure 8 depicts the effects of *Ra* on the mean Nusselt numbers on an elliptical tube under $e=0$ with special case of a tube subject to turbulent film boiling first. It shows that the mean Nusselt number of the present study has a good agreement with previous experimental data [10] under the condition of quiescent liquid (i.e. $Fr=0$ or $u_\infty=0$). The figure also shows the correlation of Rayleigh number and mean Nusselt number under five different Froude number. According the figure, the increase the Rayleigh number will bring out the increase the mean Nusselt number. Besides, the larger the Froude number will increase in the liquid velocity and also increase the mean Nusselt number.

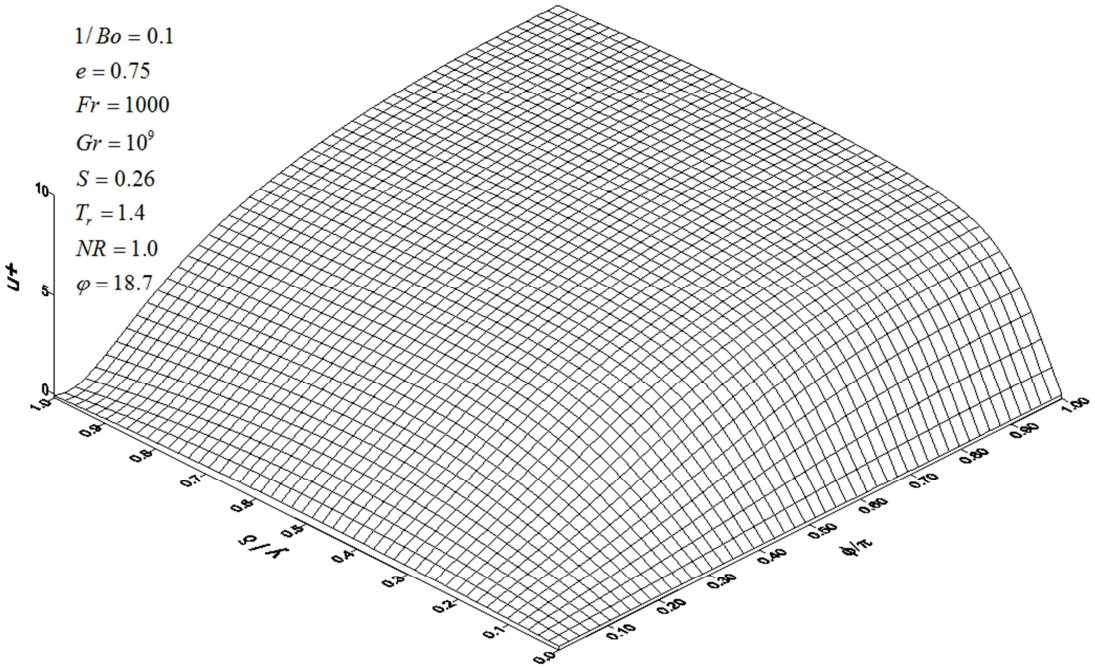

Figure 2. Local velocity distribution in vapor film.

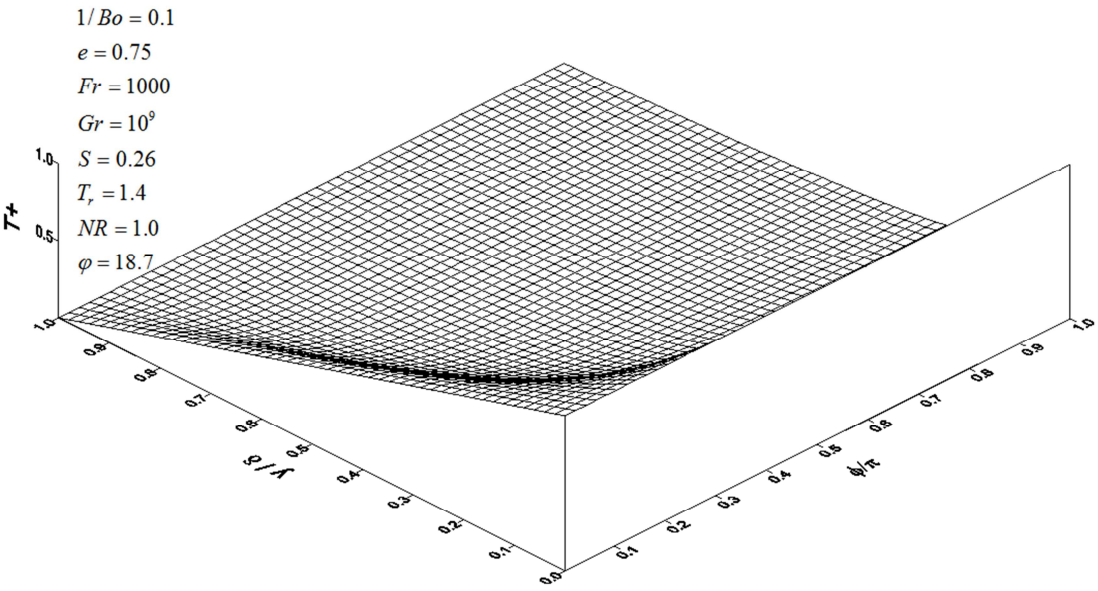

Figure 3. Local temperature distribution in vapor film.

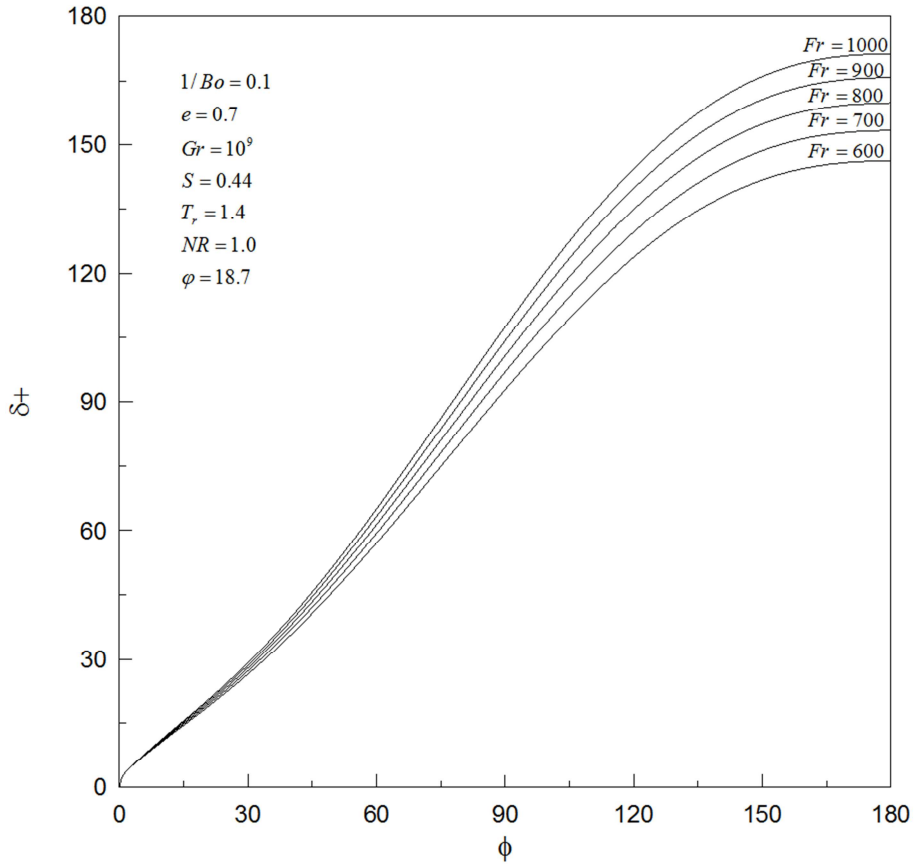

Figure 4. Dimensionless film thickness on tube surface.

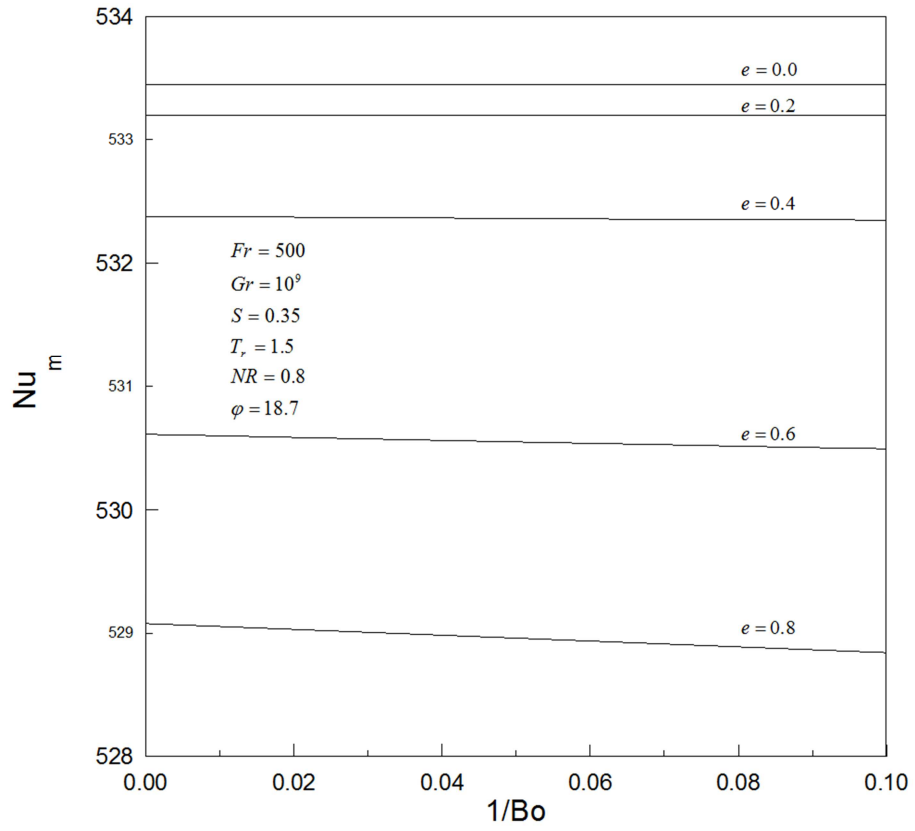

Figure 5. Effects of the surface tension on mean Nusselt number.

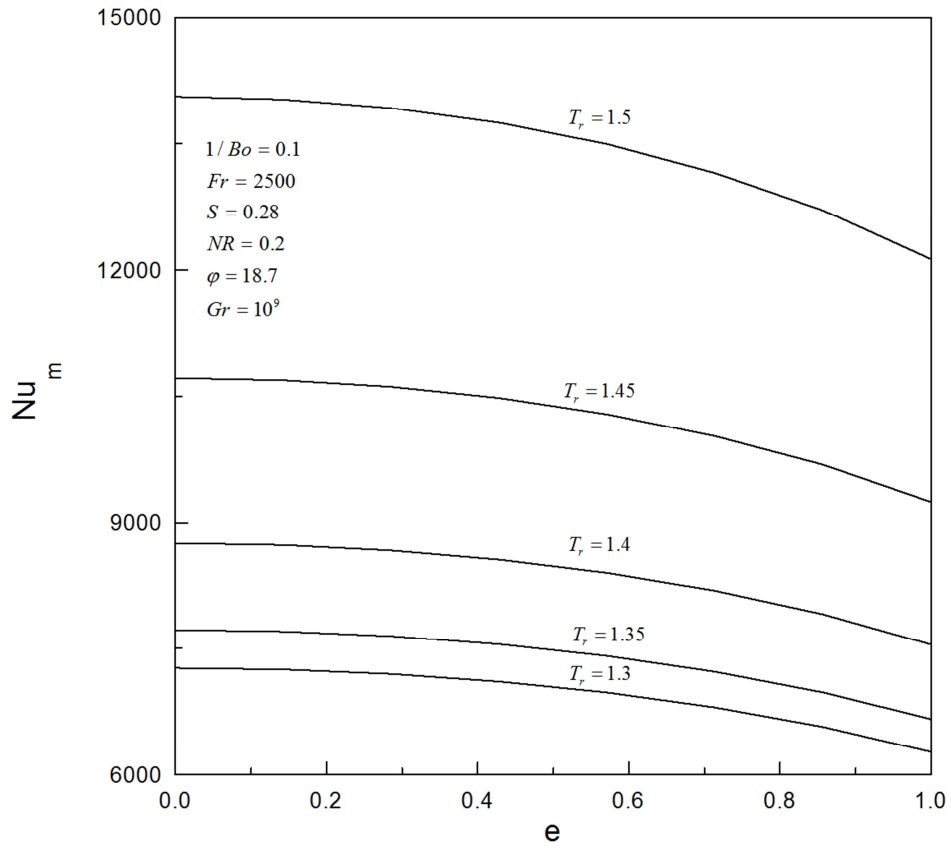

Figure 6. Effects of the eccentricity parameter on mean Nusselt number.

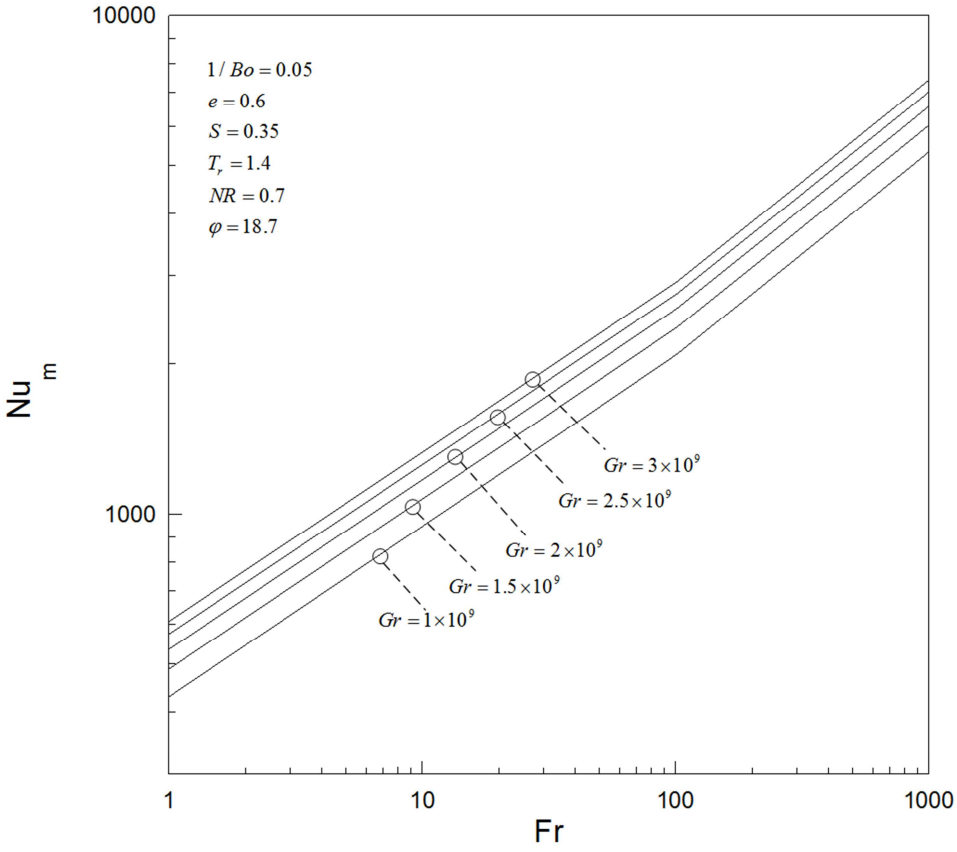

Figure 7. Effects of the Froude number on mean Nusselt number.

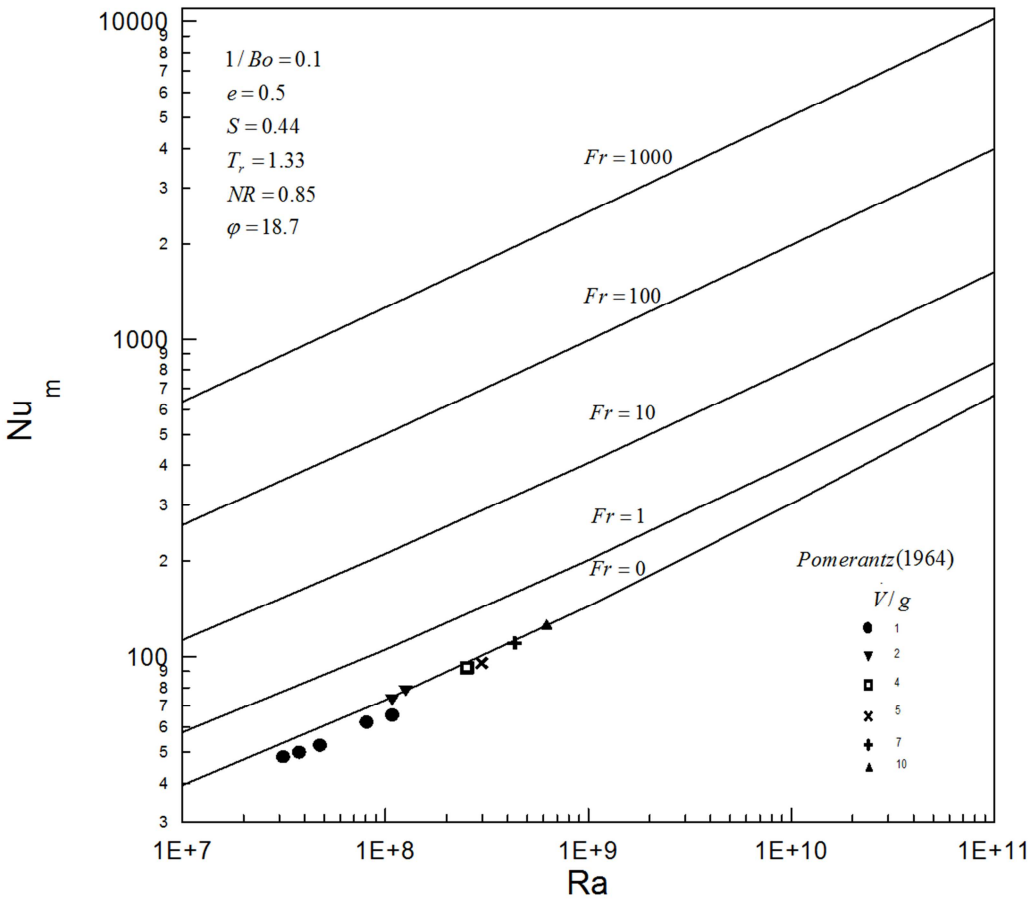

Figure 8. Effects of the Ra on mean Nusselt number.

5. Concluding Remarks

The following conclusions can be drawn from the results of the present theoretical study:

I. The surface tension effects increase in the forced convection boiling system under high velocities of flowing liquid, the mean Nusselt will very slightly decrease. For example, under the case of e=0.8, when the 1/Bo increase from 0.0 to 0.1, the mean Nusselt will decrease from 529.077 to 528.743.

II. The increase in the eccentricity of elliptical tube will decrease in the liquid velocity and lead to a lower mean Nusselt number. Besides, turbulent film boiling under the high velocity of external flowing liquid, the increase in the Froude number and Grashof number will bring out an increase in the mean Nusselt number.

III. Under the special case of free convection film boiling on a circular tube, it shows a good agreement between the present results and the previous experimental studies. Besides, the present model includes interfacial shear, Bond number, radiation effects, temperature ratio, eddy diffusivity and thermal properties of temperature dependent. The present model can exactly predicts the turbulent film boiling with external flowing liquid.

Acknowledgements

The authors gratefully acknowledge the support provided to this projects by the Ministry of Science and Technology of the Taiwan under Contract Number MOST 106-2221-E-019-062.

Nomenclature

a, b	semimajor, semiminior axis of ellipse
Bo	Bond number, $(\rho_l - \rho_s)g(D_e/2)^2/\sigma_{Bo}$
C_p	specific heat capacity, (J/KgK)
D_e	equivalent circular diameter of elliptical tube (m)
D_e^+	shear Reynolds, $D_e u^*/\nu_s$
D_e^*	wall shear parameter, $D_e^+/Gr^{1/3}$
e	eccentricity of ellipse, $\sqrt{1-(b/a)^2}$

Fr	Froude number, $\frac{u_\infty^2}{g(D_e/2)}$
Gr	modified Grashof number, $\frac{gD_e^3}{\nu_s^2}\frac{\rho_l-\rho_s}{\rho_s}$
g	acceleration due to gravity (m/s^2)
h	heat transfer coefficient, $w/(m^2k)$
h_{fg}	latent heat (J/kg)
k	thermal conductivity (W/m.k)
k^+	dimensionless thermal conductivity, $k(T)/k(T_s)$
k_w^+	$k(T)\big\|_{y^+=0}/k(T_s)$
Nu	Local Nusselt number, $h(D_e/2)/k_s$
Nu_m	mean Nusselt number
NR	radiation parameter, $\varepsilon\sigma T_s^3(D_e/2)/k_s$
Pr	Prandtl number, $C_p\mu/k$
Ra	modified Rayleigh number, $Gr\,\mathrm{Pr}\left[0.5+1/S(T_r-1)\right]$
Re_v	Reynolds number of vapor, $\frac{u_\infty D_e}{\nu_v}$
R_e^*	half of wall shear parameter, $D_e^*/2$
S	heat capacity parameter, $C_pT_s/(h_{fg}\,\mathrm{Pr})$
St	Stanton number, $Nu/(\mathrm{Re}\,\mathrm{Pr})$
T	temperature (K)
T_r	temperature ratio, T_w/T_s
T^+	dimensionless temperature, $(T-T_s)/(T_w-T_s)$
u	vapor velocity in x-direction (m/s)
u^*	shear velocity, $\sqrt{\tau_w/\rho_s}$
u^+	dimensionless velocity, u/u^*
v	velocity normal to the direction of flow (m/s)
$\dot{V}$	acceleration due to graviton force (m/s^2)
x	peripheral coordinate (m)
y	coordinate measured distance normal to tube surface (m)
y^+	dimensionless distance, yu^*/ν_s

Greek Symbols

δ	vapor film thickness (m)
δ^+	dimensionless film thickness, $\delta u^*/\nu_s$
μ	absolute viscosity (kg/ms)
μ^+	dimensionless absolute viscosity, μ/μ_s
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ	shear stress (N/m^2)
θ	angle measured from bottom of tube
φ	angle between the tangent to tube surface and the normal to direction of gravity
ϕ	interfacial shear parameter, $2^n\pi C\left(\frac{\nu_s}{\nu_l}\right)^{n-1}\left(\frac{\rho_l-\rho_s}{\rho_s}\right)^{\frac{1-n}{2}}Gr^{\frac{3n-1}{6}}$
ε_m	eddy diffusivity for momentum
ε	emissivity
σ	Stefan-boltzmann constant, w/m^2k^4

Subscripts

l	liquid
s	vapor at saturation temperature
v	vapor
w	tube wall
x	x-direction
δ	vapor-liquid interface

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