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Nonlinear Evolution of Gravity Waves on the Surface Deep-water Under the Action of Viscosity and Surfactant

Augustin Daïka[1, 2, *], Cesar Mbané Biouélé[1]

[1]Laboratory of Earth's Atmosphere Physics, Faculty of Science, University of Yaoundé 1, Yaoundé, Cameroon
[2]Department of Meteorology, Climatology, Hydrology and Pedology, National Advanced School of Engineering, University of Maroua, Maroua, Cameroon

Abstract

This paper presents a nonlinear evolution of gravity waves on the surface deep-water under the effects of viscosity and surfactant in terms of their space and time evolution, that is, their motion and also in terms of mechanical transformations that these systems may suffer in their dealings with other systems. We give a formal derivation of evolution equations, obtained from the modified nonlinear Schrödinger equation, for viscous capillary-gravity waves with surfactants in water of infinite depth. To fully simulate the non-linear evolution of the wave train in the presence of viscosity and surfactant, a new numerical model, based on the Bogning-Djeumen Tchaho-Kofane method (BDKm) and the Peregrine model, is developed. On the basis of these different approaches, the role of viscosity and surfactant on gravity waves in water of infinite depth is analyzed. The results show the effect of viscosity and surfactants on the nonlinear evolution of gravity waves on the surface deep-water. Naturally, they affect the remote images strongly in radar and lidar remote sensing of the sea surface.

Keywords

Gravity Waves, Surface Deep-water, Presence of Viscosity and Surfactant, Bdk Method and Peregrine Model

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1. Introduction

Descriptions of gravity waves on the sea surface for a short time have been considered as a part of marine folklore for a long time. A number of instrumental registrations have appeared recently making the community to pay more attention to this problem and to reconsider known observations of this gravity waves. However, when a layer of contaminant (surfactant) is present on the free surface, its concentration varies with the motion of the free surface, causing so a surface-tension gradient that must be balanced by a non-zero surface shear stress [1-3]. Moreover, the comprehension and prediction of the physical processes responsible of this phenomenon are not completely understood. Generally, mathematical models offer more formidable opportunities for understanding real phenomena whose physics is, at the current level of our knowledge, difficult to obtain. A mathematical model based on modified nonlinear Schrödinger equation coupled with assumptions derived from the literature on the nature to incorporate the effects of viscosity and surfactant of these waves is developed. In this work, we use a new method of construction of soliton solutions of modified nonlinear Schrödinger equation, named Bogning-Djeumen Tchaho-Kofané method (BDKm) [4-6] and Peregrine model [7, 8], to numerically simulate the equations that model the impact of viscosity and surfactant. The work presented in this paper is structured as follows: In Section 2, we describe the BDKm to

* Corresponding author
E-mail address: augustindaika@yahoo.fr (A. Daïka), cesar.mbane@yahoo.fr (C. M. Biouélé)

seek wave solutions of nonlinear evolution equations. In Section 3, we implement nonlinear evolution equation for the capillary-gravity wave with viscosity and surfactant. In section 4, we illustrate the methods in detail with the modified non-linear Schrodinger equation in deep water for the capillary-gravity wave with viscosity and surfactant. In Section 5, we investigate numerically nonlinear evolution equation for the capllary-gravity wave with viscosity and surfactant in water of infinite depth. In section 6, some conclusions are given.

2. Description of Bogning-Djeumen Tchaho-Kofane Method

This method has been adopted to facilitate the resolution of certain type of nonlinear partial differential equations where the nonlinear terms and dispersive terms coexist and intends to look for the solutions of certain categories of nonlinear partial differential equations on the form:

$$\gamma_i \sum_i \frac{\partial u}{\partial x_i} + b_i \sum_i \frac{\partial^2 u}{\partial x_i^2} + ... + c_i \sum_i \frac{\partial^l u}{\partial x_i^l} + d_i \sum_{m,n} \frac{\partial^n u \partial^m u}{\partial x_i^n \partial x_i^m} + f\left(u, |u|^2\right) = 0 \quad (1)$$

Where γ_i, b_i, c_i and d_i are the constants; i, j, l, $m$and n the positive integer; f a linear arbitrary function of u and $|u|^2$. u the function unknown to be determine and $|u|^2$ the magnitude of u.

Hence, considering equation (1), we propose to construct the solution under the form:

$$u = \sum_{ij} a_{ij} \frac{\sinh^i(\alpha x)}{\cosh^j(\alpha x)} \quad (2)$$

Where α is a fixed constant and a_{ij} the coefficients to determined.

We introduce equation (2) into (1) and we obtain the form:

$$\sum \frac{F(a_{ij})}{\cosh^n(\alpha x)} + \sum G(a_{ij}) \frac{\sinh(\alpha x)}{\cosh^n(\alpha x)} + \sum H(a_{ij}) \cosh^k(\alpha x) + \sum T(a_{ij}) \sinh(\alpha x) \cosh^l(\alpha x) + \sum W(a_{ij}) = 0 \quad (3)$$

where $F(a_{ij})$, $G(a_{ij})$, $H(a_{ij})$, $T(a_{ij})$ and $W(a_{ij})$ are functions of the coefficients a_{ij}.

Here the importance or priority makes reference to the range that permits to obtain good results or merely that which tends more to the exact value. Obtaining the coefficients through equation (3) boils down to solving the coefficient equation as follows:

The term in $\frac{1}{\cosh^n(\alpha x)}$, $\sum_{i,j} F(a_{ij}) = 0$ (4)

The term in $\frac{\sinh(\alpha x)}{\cosh^n(\alpha x)}$, $\sum_{i,j} G(a_{ij}) = 0$. (5)

The term in $\cosh^k(\alpha x)$, $\sum_{i,j} H(a_{ij}) = 0$ (6)

The term in $\sinh(\alpha x)\cosh^l(\alpha x)$, $\sum_{i,j} T(a_{ij}) = 0$ (7)

The ranges $W(a_{ij})$ is considered now as the one that brings no reliable information. It is important to mention that this method appears complicated in the case where the properties of transformations of hyperbolic functions are not mastered. A mastery of these transformations reduces the difficulties considerably as regard to the calculations.

It should be noted that the best solution depends on the shape of solution considered from the onset, the symmetry of the equation to solve as well as from its nonlinearity degree. In these conditions, one moves directly to the equations of lower powers until the good equation to solve is obtained.

3. Implementation of Equation for the Capillary-gravity Wave with Viscosity and Surfactant

Under the effects of gravity and dissipation, the wave moves through successive deformations of its surface. This situation is perfectly described by the modified Nonlinear Schrödinger Equation which can be obtained from the fully nonlinear potential theory by using the multi- scales method [9]. To investigate both viscosity and surfactants effects on the capillary-gravity wave, we have used the amplitude equation given by [10]:

$$ia_\tau + Pa_{\varsigma\varsigma} + Q|a|^2 a - Ra = 0 \quad (8)$$

Where

$$\varsigma = s\left(x - v_g t\right)\cos\theta + y\sin\theta\,,\ \tau = s^2 t$$

and

$$s = \frac{\varepsilon^{1/4}}{\lambda}$$

With

$$P = -\frac{1}{8}\left(4T^2 - 8T + 1\right)\left(2 - 3\cos\theta\right), Q = -\frac{9T^2 - 15T + 8}{4\left(1 - 3T\right)}$$

and

$$R = -\frac{i\lambda^2}{2\sqrt{2}} \frac{\kappa\left[\kappa + i\left(\sqrt{2} - \kappa\right)\right]}{\kappa^2 - \sqrt{2}\kappa + 1}$$

a is the wave envelope, p, q and r are three coefficients that depend on the weber number κ and the surface tension coefficient T, of the carrier wave. λ is a proportionality constant. θ is an angle to the wave direction and the transformation.

The free-surface elevation $\eta(x, y, t)$ is given by the relation:

$$\eta(x, y, t) = a(x, y, t)\exp\left[i\left(k_0 x - \omega\left(\vec{k}_0\right)\right)t\right] \quad (9)$$

This equation (8) called the modified nonlinear Schrodinger equation which can be obtained from the fully nonlinear potential theory. The just mentioned numerical results are all based on the amplitude equation for the capillary-gravity wave with viscosity and surfactant.

The dissipation term proportional to a in (8) is attributed to the surfactant, whereas for a clean surface the dissipation is of higher order, as shown in the previous section where different scaling was required. Therefore, if either λ or κ is zero, the dissipation term in (8) is absent, and we recover the amplitude equation for capillary-gravity waves in inviscid flow. The linear term of (8) has a complex coefficient; the imaginary part is related to the decay rate, and the real part corresponds to a frequency change due to the surfactant. This last is characterized by a surface-dilatational modulus, which measures the resistance to the compression/expansion type of surface deformation. Changes in surfactant concentration cause the surface tension to vary in time and space. The flow is caused by initial surface disturbances with small but finite amplitude, and is irrotational everywhere except in the boundary layer beneath the free surface, since the viscosity is considered to be small.

When the surfactant is insoluble and non-diffusive, the mass in a surface material element is conserved.

4. Illustration of Methods

4.1. Solitary Wave Solutions in the BDK Method for the Capillary-gravity Wave with Viscosity and Surfactant

We obtain an exact analytical solution of equation (8) for the capillary-gravity wave with viscosity and surfactant:

$$a(\varsigma, \tau) = a_0 \operatorname{sec}h\left(a_0\sqrt{\frac{Q}{2P}}\varsigma\right)\exp\left(iQa_0^2\tau\right).\exp\left(-iR\tau\right) \quad (10)$$

4.2. Homoclinic Solution for the Capillary-gravity Wave with Viscosity and Surfactant

We define the nonlinear solution of the amplitude under the analytical shape given as follows:

$$a(\varsigma, \tau) = \frac{\Gamma(\varsigma, \tau)}{\Lambda(\varsigma, \tau)} \quad (11)$$

Where $\Gamma(\varsigma, \tau)$ and $\Lambda(\varsigma, \tau)$ are both arbitrary functions we assume that $\Lambda(\varsigma, \tau)$ is real function.

The substitution of this relation (11) in (8), we obtain finally the exact analytical solution of the equation for the capillary-gravity wave with viscosity and surfactant in a homoclinic orbit to the fixed point:

$$a(\varsigma, \tau) = a_0 \exp\left(ia_0^2 Q\tau\right) \times \frac{1 + 2\cosh\left(\aleph\varsigma\right)\exp\left(\Omega_0 P\tau + 2i\beta + \gamma\right) + \operatorname{sec}h^2\left(\beta\right)\exp\left(2\Omega_0 P\tau + 4i\beta + 2\gamma\right)}{1 + 2\cosh\left(\aleph\varsigma\right)\exp\left(\Omega_0 P\tau + \gamma\right) + \operatorname{sec}h^2\left(\beta\right)\exp\left(2\Omega_0 P\tau + 2\gamma\right)} \times \exp\left(-iR\tau\right) \quad (12)$$

With $\aleph = a_0\sqrt{\frac{2Q}{P}}\sinh(\beta)$ and $\Omega_0 = \pm\aleph\sqrt{2a_0^2\frac{Q}{P} - \aleph^2}$

Where β is an arbitrary constant and γ is an arbitrary phase.

4.3. Peregrine Solution of the NLS Equation for the Capillary-gravity Wave with Viscosity and Surfactant

This peregrine solution has the peculiarity of being not periodic in time and in space. It has been recently reproduced experimentally in wave tank laboratories [11], in optical fibers [1] and in plasmas [12].

The exact analytical solution of the equation (8) for the capillary-gravity wave with viscosity and surfactant in a Peregrine breather type is given by:

$$a(\varsigma,\tau) = a_0 \exp\left(iQa_0^2\tau\right).\left[1 - \frac{4\left(1+i2Qa_0^2\tau\right)}{1+4Q^2a_0^4\tau^2+\dfrac{2Q}{P}a_0^2\varsigma^2}\right]\exp\left(-iR\tau\right) \quad (13)$$

5. Numerical Investigation and Discussion

The main objective of this section is to confirm the correctness of the analytical approach used in section 4. Despite the relevant phenomena describe by these solutions, they were obtained after some approximations and assumptions (or inputs). Thus, a set of numerical experiment is done in order to check the analytical solutions obtained from the modified nonlinear Schrodinger equation (8) by verifying the analytical predictions; checking the validity of the analytical solutions by comparing them with direct numerical integrations of the original equation of motion such that Euler and Navier-Stokes equations allowing a mathematical formulation of the gravity wave motion. The abovementioned numerical experiments are done because an analytical solution leads to wrong results if the initial condition used in the numerical integration is not close to the exact solution.

The upsurges of gravity wave on the sea surface are due to complex physical mechanisms. Figures presented in this manuscript provide evidence of surfactants and viscosity effects on a wave's packets propagating in deep water. Figures 1 (a)-(c), give a 3D representation of surface elevation as a function of space and time of the equation (10). In figure 1, the effects of viscosity (λ) and surfactant (T) are examined with the Weber number $\kappa=\sqrt{2}$: a – T = 0,02; $\lambda=0,25$; b- T = 0,02; $\lambda=0,125$ and c- T = 0,1; $\lambda=0,25$. One can see exactly the green color representing a calm ocean surface, gradually gains space.

Figures 2 (a)-(b) give the Simulations of geometry of oceans' surface wave with the presence of viscosity and surfactant shown on Figure 1: a- T = 0,1; $\lambda=0,5$ and b- T = 0,1; $\lambda=1$.

According to these figures, oceans' surface wave influence by the viscosity and surfactant.

Figures 3 (a)-(b) describe Evolution of oceans' surface wave in presence of viscosity and surfactant in the homoclinic orbit model: a – T = 0,1; $\lambda=0,125$ and b- T = 0,3; $\lambda=0,125$.

The simulations of geometry of oceans' surface wave with the presence of viscosity and surfactant shown on Figure 3b is represented in the figure 4. In the Figures 5 (a)-(b), we see the evolution of the slowly varying Water elevation with small viscosity: a – T = 0,1; $\lambda=0,125$ and b- T = 1; $\lambda=0,125$. Here, the 3D-representations of Peregrine model is simulated with $\kappa=\sqrt{2}$.

6. Conclusion

In this manuscript, the modified nonlinear Schrodinger equation has been used in a formal derivation of evolution equations for nonlinear viscous water waves. The fully nonlinear evolution of the wave packet in the presence of viscosity and surfactant was resolved using the BDKm and Peregrine model. This research work is an important tool for acquiring information on the scientifically conceivable reasons for the data acquisition in radar and lidar remote sensing of the sea surface and surfactants affects these remote images strongly. According to pertinent results obtained of this study, viscosity and surfactant determine the validity of the linear dissipation terms added to the amplitude equations of studies. Adding an insoluble surfactant with a linear dependence of surface-tension coefficient on concentration again allows only linear dissipation, consistent with many experimental observations. Soluble or diffusive surfactants generally exhibit smaller surface dilatational modulus, but we have determined that these modifications would also have dissipative terms.

Appendix

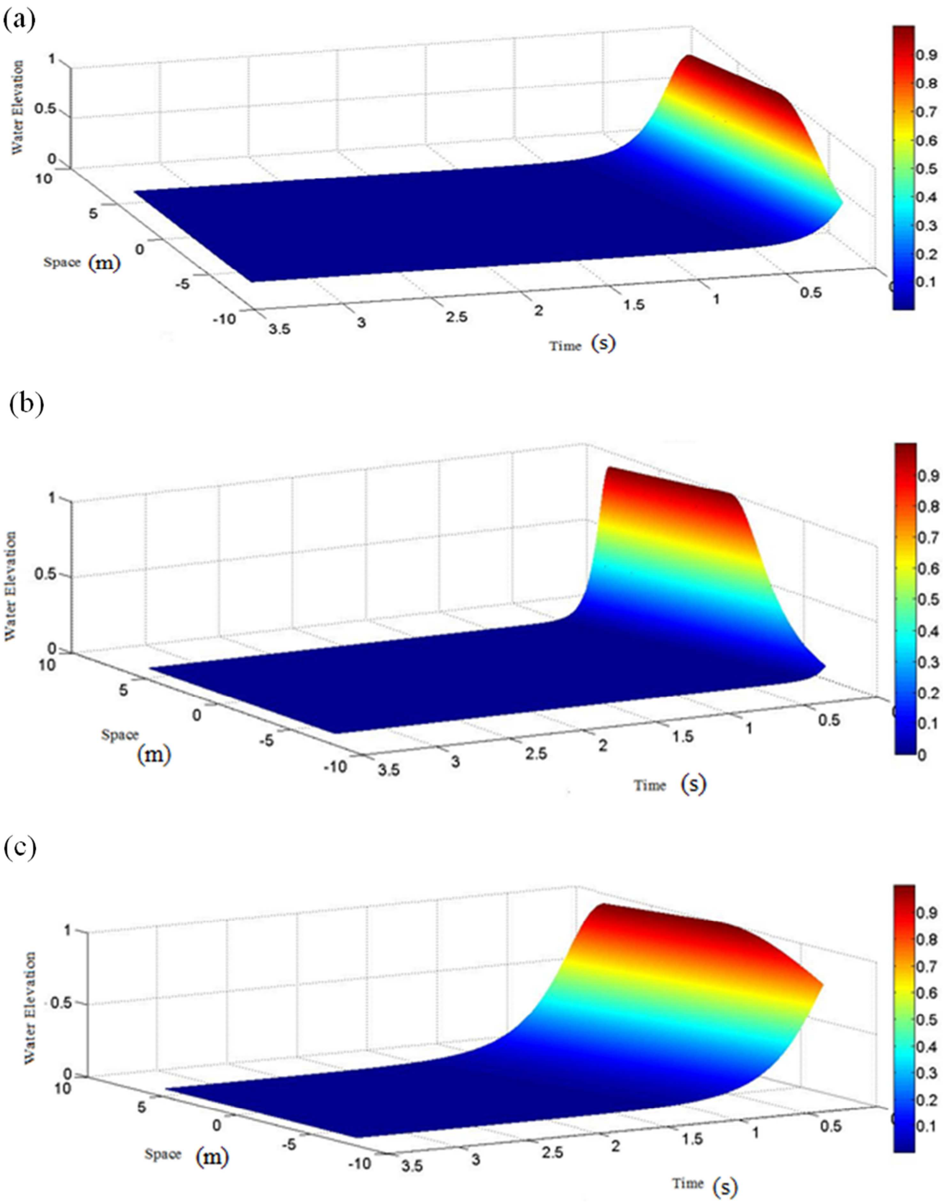
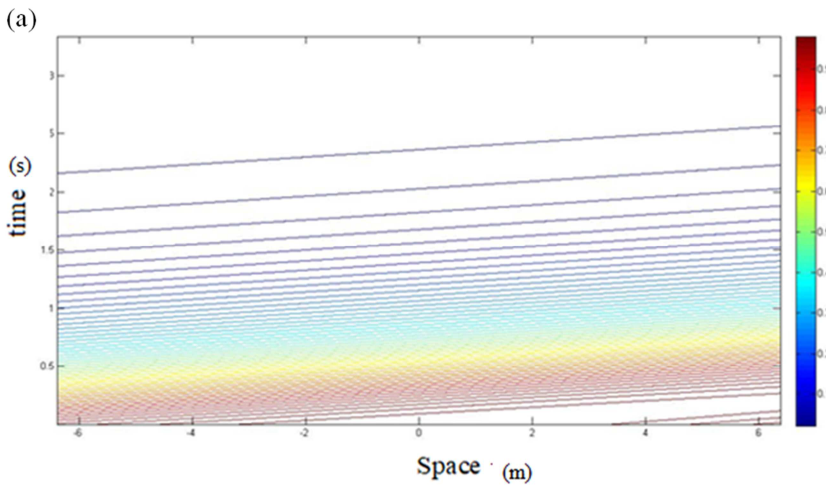


Figure 1. Evolution of Water elevation under the effects of viscosity and surfactant: a – T = 0,02; $\lambda = 0{,}25$; b- T = 0,02; $\lambda = 0{,}125$ and c- T = 0,1; $\lambda = 0{,}25$.



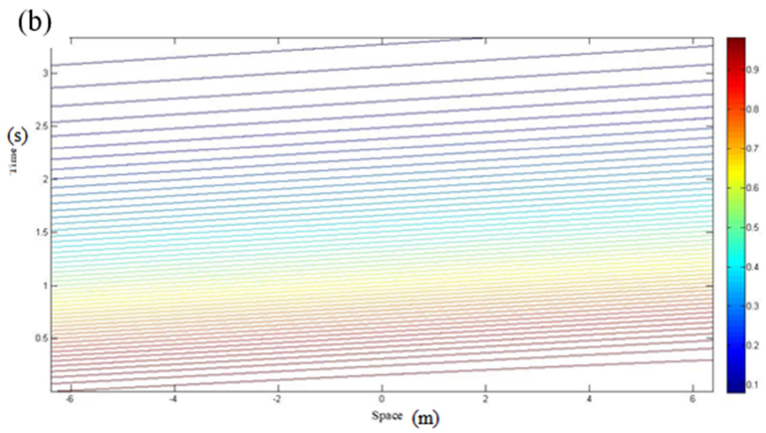


Figure 2. Simulations of geometry of oceans' surface wave with the presence of viscosity and surfactant shown on Figure 1: a- T = 0,1; $\lambda = 0{,}5$ and b- T = 0,1; $\lambda = 1$.

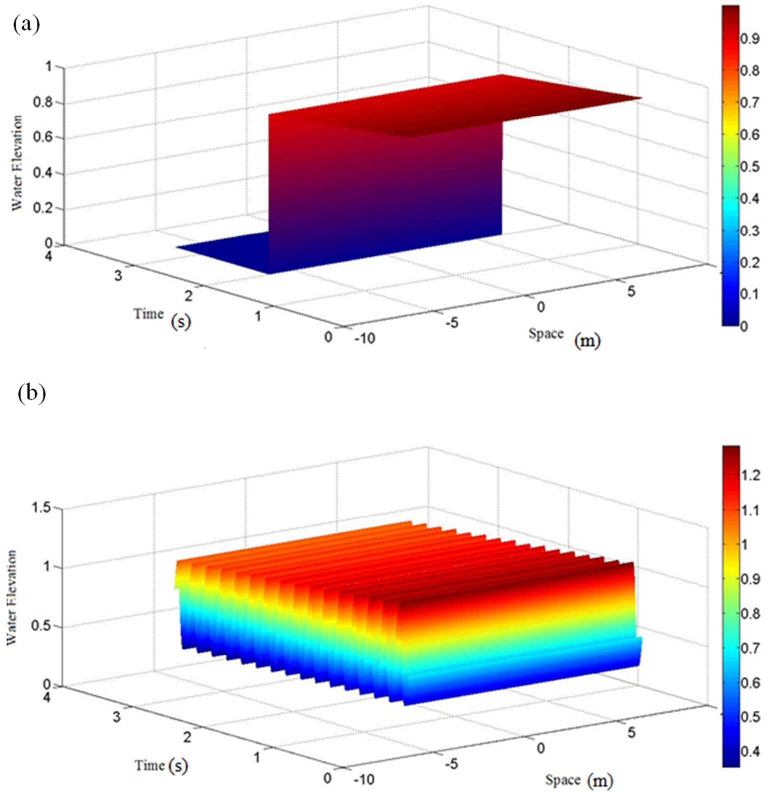


Figure 3. Evolution of Water elevation under the effects of viscosity and surfactant ((3D-representations of homoclinic orbit model): a – T = 0,1; $\lambda = 0{,}125$ and b- T = 0,3; $\lambda = 0{,}125$.

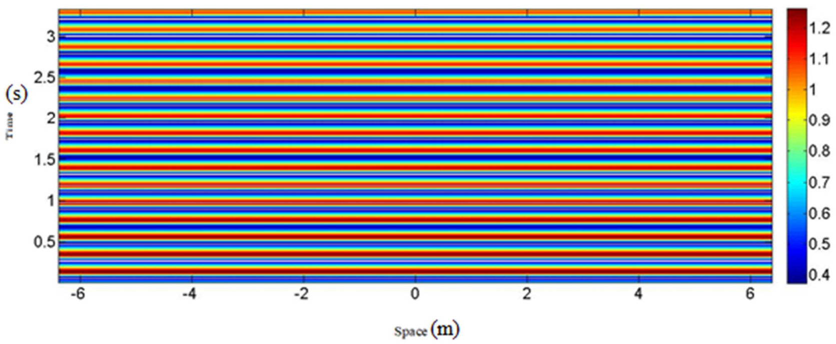


Figure 4. Simulations of geometry of oceans' surface wave with the presence of viscosity and surfactant shown on Figure 3b.

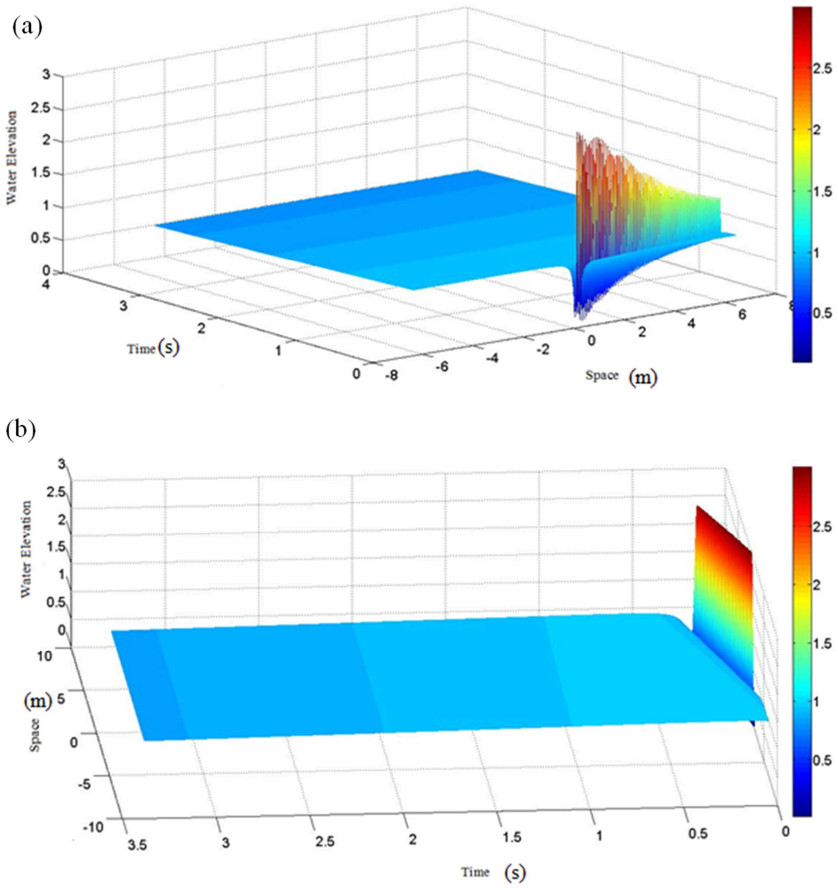


Figure 5. Evolution of Water elevation under the effects of viscosity and surfactant ((3D-representations of Peregrine model): a – T = 0,1; $\lambda = 0{,}125$ and b- T = 1; $\lambda = 0{,}125$.

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