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Malthus Meets Solow in an Endogenous Growth Model

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Abstract

This article investigates first, in how far disease such that HIV/AIDS, the 2008 food crisis consequences and poverty reduction in Africa may validate Malthus population growth theory. Second, if the observed high growth rates in emerging countries may validate Solow growth model consequences that fail to be proved empirically until now. Using an endogenous growth model, we find that, Malthus may be validated if sustainable actions are not conducted in poor countries in order to ensure a balanced population growth rate. Solow is validated specifically in emerging countries as shown by data observations i.e their growth rates are higher than in Western countries. Moreover, emerging countries exports have oriented monetary policies and contributed to the 2008 financial crisis in developed world. The intercept of the both thoughts yields the necessity for technology to be improved through research and development for sustainable development to be reached.

Keywords

Population Growth, HIV/AIDS, Marginal Capital Returns Productivity, Food, Relative Labor Efficiency, Growth, Development, R&D

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1. Introduction

The aim of this article is *first* to investigate in how far the observed poverty reduction in developing world in response to the 2015 Millennium Development goal of United Nation on poverty reduction target, the 2008 food crisis consequences and HIV/AIDS negative impact on population growth in Africa, may support Robert Malthus [1] population theory i.e sooner or later, population gets checked by famine and disease offsetting population growth because food increase is too low compare to fecundity rate evolution. *Second* if the observed fast and high growth rates in emerging countries despite of financial crisis in developed world in 2008, may support Robert Solow [2] neoclassical growth analysis consequences, where developing countries grow faster than developed countries because marginal product of capital is decreasing through the time. Thus, the question asked by the analysis is: can we find some empirical evidence of Solow and Malthus theories?

After the 2008 food crisis in developing world and at the same time, financial crisis in developed world added with HIV/AIDS crisis since the 1980s specifically in Africa as well as the emergence of growth acceleration in new developed countries mostly located in Asia, despite of the fact that Africa accelerated his GDP too caused by some cooperation with countries like China, development remains unsustainable caused by the difficulties to purchase food as well as drugs[1] to fully finance HIV/AIDS pandemic which still leading to many deaths despite of the World Organizations intervention, thus population growth matters there.

Moreover, using the measure of people living on $1.25 a day

1 See D. Loubaki et al (2015)

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or less, the World Bank's poverty measurement team, led Martin Ravallion, estimates that the percentage of poor Africans fell from 58 percent in 1999 to 47.5 percent in 2008. In the past, even when the poverty rate fell, he typically found that the absolute number of poor people rose because of rapid population growth. Between 2005 and 2008, for the first time, the absolute number of poor people also declined, from 395 million to 386 million. Nine million Africans, equivalent to the entire population of Benin, escaped poverty during those three years. In a survey of 18 relatively fast-growing African countries, Andy Mc Kay finds that poverty fell in almost all of them. Ghana and Uganda showed significant declines in poverty, helped by the fact that, inequality also declined (World Bank Weekly update, January 3, 2013). In parallel, most of the world's 34 million people living with HIV/AIDS reside in developing countries and face food shortages and the pandemic accuracy, thus are unable to work or to produce food. In 2011, 2.5 million people became newly infected with HIV, and 1.7 million died of HIV-related illnesses. Sub-Saharan Africa accounted for 69% of all new infections and nearly half of all HIV-related deaths globally. While 8 million people worldwide can access treatment, nearly 7 million additional people in need of access did not have it as of 2011 thus are led to death. Despite of the World Organizations intervention to treat the pandemic, the situation remains blocked by cultural inheritance like the difficulty to use condoms during sexual contact and transactional sex due to poverty in Sub-Saharan Africa.

In parallel, after the financial crisis of 2007-2008 in developed world also called the "subprime", growth in high-income countries remains weak, with their GDP expanding only 1.3 percent in 2012 and remains low at an identical 1.3 percent in 2013. In the Euro Area, growth returned positive in 2014 only, with GDP contraction by 0.1 percent in 2013, before edging up to 0.9 percent in 2014 and 1.4 percent in 2015. According to some experts, those results are due to rapid evolvement of the role of the governments around the world in response to the dramatic changes related to technological progress, emerging countries' sudden and high growth rates as well as real and financial economic integration. National strategic policies designed to drive the country's economic development got less and less effective caused by the larger variety of options available to companies and to households as well as massive interrelations among different geographical areas. Emerging countries exports have oriented monetary policies and contributed to the global imbalances that triggered the financial crisis since their money is used somewhere else. The regulations enacted by developed countries had facilitated the transmission of the crisis to the real economy, thus drives to unemployment rate increase almost everywhere in Western countries where the countries located in the South suffered more because when a country accumulates foreign reserves for strategic reasons, its primary objective is not their immediate use for the defense of its currency, the use of foreign reserves, partly locked up, and managed according to profit maximizing criteria by the sovereign wealth funds (SWF) which seems to indicate the prevalence of the strategic motivation behind the recent wave of foreign reserves accumulation. From the perspective of financial accounts, persisting trade surpluses are reflected into persisting financial surpluses: Citizens of Emerging countries became net providers of funds abroad because the monetary policy of reserve accumulation, however, requires the prior issue of local currency to be immediately converted into foreign currency. The resulting increases in the monetary base of the country led to inflationary pressures. Monetary policy, therefore, through the inflation channel triggers a displacement of resources from the private to the public sector, by reducing private savings and transferring it to the public sector. American households' savings have turned negative in recent years the imbalance has gradually extended up to support a portion of this private debt which is largely financed, as already noted, by developing countries. The United States has thus experienced a strong growth in private consumption, which can be argued to be largely financed by the large Chinese savings and not absorbed by the local economy. Unfortunately, the interaction between financial markets yields crisis transmission both in other countries and in real economy.

The scientific contribution of this article holds *first* on the geographical area of application of the growth analysis i.e in Poorest countries located in Africa and in Emerging countries mostly located in Asia and in Latin America for some of them, *second* the model tests theories in order to find empirical support through data observation on the actual economic development evolution, *third,* the standard growth literature failed to prove empirically Solow convergence notion until now, that it shown by data observation of emerging countries growth rates in this model.

Thus this article aim is to investigate in how far it can be considered that *HIV/AIDS in increasing death rate and decreasing life expectancy, is making population growth decrease?, The increasing per-capita income in past poor countries now classified as Middle income countries by the World Bank, is that international cooperation among those countries explain the improvements observed in poverty reduction and emerging countries high and fast growth rate compare to more developed countries?*.

The article follows the literatures of economic growth (technology and population) and food in economics as well as examines Malthus and Solow works consequences today.

For the *first literature,* the evolution of FS approaches has basically followed three phases. The first took place as a dominant theoretical explanatory framework for food crises since the time of Malthus (the late eighteenth century) until the year 80, what Amartya Sen [3] called the FAD approach, Food Availability Decline. This approach conceived famine as shortages of food supplies per capita, motivated by natural factors; e.g., drought, floods and other calamities that undermine crops or demographic factors i.e. vegetative growth that goes beyond the supply [4]. The second approach claimed that hunger and famine don't necessarily evolve from lack of food supplies in the market, but lack of resources in sectors to produce or purchase them. This approach focuses on the family rather than the country, but shortly after introducing the term individual, focuses not only on the availability but on the access to food as determined by vulnerable socio-economic degrees ([5]; [6]; [7]). Schneider et al [8] examine global food production development until 2030 with a partial equilibrium model of agriculture and forestry including population growth, technical change, and two alternative deforestation policies. They found that food prices, per capita consumption of food, and the ratio between plant and animal food change relatively little across scenarios. The third approach focuses on individual perspective. A person enjoys food security when his consumption is always greater than his needs thus defines physiological needs [9]. For the *second literature category*, since the 1950s, population growth rate is decreasing and it is projected to decrease during the next six decades. This decrease is particularly relevant in the group of developed countries but is also observable on a global scale. The decrease in the rate of growth is predominantly due to the aging of the population in developed countries and a dramatic increase in the number of deaths in poor countries due to calamities such that infections and food needs. From 2030 to 2050, the world population is projected to grow more slowly than ever before in its history. Since the 1980s the pandemic called HIV/AIDS appeared first in developed countries went to Africa where deaths occurrences increased re-opening Malthus population growth theory. The Essay on Population published in 1798 by Rev. Thomas Malthus postulates that population naturally tends to increase faster than nature can provide subsistence. Malthus based his theory on the growth of the North American colonies and concludes showing that population naturally tends to double every twenty-five years. Thus, population would increase at a geometrical ratio. Meanwhile, subsistence from land, under the most favorable circumstances, could not possibly increase faster than in an arithmetical ratio. Malthus relies to decreasing returns notion in economy initiated by Ricardo and Smith in the concern of land production capacity scarcity. Indeed, whereas Malthus sees population as constrained by food production, with population growth being the endogenous factor, Boserup [10] effectively reversed the picture. She assumes that "population growth" is independent variable which in turn is a major factor determining "agricultural developments". Instead of distinguishing between two different ways of raising agricultural output, bringing in more land or using the land more intensive, emphasis is placed on the frequency of cropping. This break, with the basis of the classical theory of rent, lays the foundation on which Boserup builds her theory. Lee [11] examines the implications of those theories. His point of departure is that Malthus and Boserup both share the assumption of diminishing returns to labor for a fixed technological level, but they use different endogenous variable. For Malthus, population growth is endogenous, while for Boserup, it is technological change which is endogenous. If seen from this perspective, the two theories can be combined to yield a theory of the interaction between population and technological change. From Malthus, we have that, if the population is sparse relative to the technology, which implies higher welfare, then population will grow. On the other hand, if population is dense relative to the technology, and welfare is low, therefore the population will decline. Kremer [12] tests the previous finding. However, Kremer sets out not to test Lee's model, but to examine the implications of some models of endogenous technological change. In these models, with technology being non rival and a constant share of resources devoted to research, a larger population implies faster technological progress. Assuming that, each person's research productivity is independent of the population, Kremer finds that the growth rate of technology is proportional to the size of the population. If one also makes the Malthusian assumption that the population is limited by the available technology, which is equivalent to the growth rate of population being proportional to the growth rate of technology, the model implies that the growth rate of population is proportional to the level of population. Testing this empirically, Kremer finds support for this hypothesis. However, throughout history, human populations have experienced deficiencies in food production and today's resource scarcity remain true in developing world where it must be purchased, no significant distribution policy exist, specifically since the end of Communism i.e the end of the use of planning models where food was provided to population by the system. Liberalism in order to achieve economic integration, rules out this possibility available before. Three arguments may illustrate the global dimension of this threat. *First*, the total use of resources for food production over all countries has reached substantial proportions (FAOSTAT, 2007). Rosegrant et al [13] differ substantially and range from minus 17% to plus 228%. This variation is due to methodological and data

differences. *The second argument* supporting a global dimension of food production challenges is that although some regions experience more problems than others, today's societies are increasingly connected. Globalization has opened the door to more international trade. Thus, regional commodity supply shortage or surplus can be transferred to and mitigated by world markets. Furthermore, globalization has also influenced governmental regulations. National land use related policies are increasingly embedded in international policies. *A third argument* is that, the cumulative impacts of local land use decisions may cause significant global environmental feedback, foremost through climate change ([14]). There are both positive and negative agricultural impacts which influence the availability and fertility of land, the length of the growing season, fresh water endowments, pest occurrences, CO_2 fertilization, and the frequency of extreme events related to draughts, flooding, fire, and frost. Galor and Weil [15] develop a unified growth model that captures the historical evolution of population, technology, and output. It encompasses the endogenous transition between three regimes that have characterized economic development. The economy evolves from a Malthusian regime, where technological progress is slow and population growth prevents any sustained rise in income per capita, into a Post-Malthusian regime, where technological progress rises and population growth absorbs only part of output growth. Ultimately, a demographic transition reverses the positive relationship between income and population growth, and the economy enters a Modern Growth regime with reduced population growth and sustained income growth. In macroeconomic theories on population, where HIV/AIDS has an influence like Young [16], the effect of the disease such that HIV/AIDS on rapid population growth in Africa, thereby increases per-capita GDP and per-capita income. He emphasizes two competing effects. On the one hand, the epidemic is likely to have a negative impact on the human capital accumulation of orphaned children. On the other hand, widespread community infection lowers fertility, both directly, through a reduction in the willingness to engage in unprotected sexual activity, and indirectly, by increasing the scarcity of labor and the value of a woman's time. He finds that, even with the most pessimistic assumptions concerning reductions in educational attainment, the fertility effect dominates.

Finally, the neoclassical model of economic growth of Solow assumes that labor force grows at a constant rate $n > 0$ implying that the labor force grows exponentially for any initial given level. That the marginal productivity of capital is decreasing thus, makes developing countries having more potentiality to drain growth than more developed countries in the literature of growth. Solow paper then raised a body of literature which contrasts his finding through the concept of convergence which may be β or σ i.e two concepts of convergence appear in discussions of economic growth across countries or regions. In one view ([17]; [18]; [19], [20], [21]), convergence applies if a poor economy tends to grow faster than a rich one, so that the poor country tends to catch up to the rich one in terms of levels of per capita income or product. This property corresponds to the concept of *β convergence*. The second concept concerns cross-sectional dispersion ([22]). In this context, convergence occurs if the dispersion—measured, for example, by the standard deviation of the logarithm of per capita income or product across a group of countries or regions—declines over time. We call this process *σ convergence*. Convergence of the first kind (poor countries tending to grow faster than rich ones) tends to generate convergence of the second kind (reduced dispersion of per capita income or product), but this process is offset by new disturbances that tend to increase dispersion. The difficulty to prove Solow empirically gave birth to the modern growth literature or endogenous growth literature which began with Romer [23], who explained Solow residual through knowledge which can't be kept secret and Lucas [24] explained Solow through human capital accumulation initiated by Becker [25]. The literature expansion introduces Schumpeterian growth models initiated by Aghion and Howitt [26] where innovations are endowed with the destruction and creation property such that when a new innovation appear, the previous become obsolete and left out to unskilled labor mainly. Indeed, the intercept of human capital and knowledge or technological change introduced previously by Romer, [27] without the creative destructive concept of innovations, explain also growth ([28]). Modern growth pursues its development in including almost all the body of interest in economics such that demography ([29]), environment ([30]; [31]), Health ([32]; [33]) and so on. Baldwin et al [34] consider an endogenous-growth model with heterogeneous firms and fixed costs of operation and of exporting. Grossman and Helpman [35] present the literature which investigates how globalization affects economic growth. Therefore, in the first decade of this 21th century data observation tends to validate the previous convergence concepts so that, this article aim is to test the basic model i.e Solow in the context of Emerging countries existence and experience known through data observation.

The model is organized like follows, section 2 establishes the model, section 3 presents the results and section 4 presents the discussion and section 5 presents the interaction between Solow and Malthus concepts for development, finally the article concludes on technological increase for negative

mechanisms on growth, not to play.

2. The Model

Consider a closed developing country economy where agents live for two periods. There exist stocks of HIV/AIDS infected agents denoted by N_t^S at time t who hardly live for two periods of time and of non HIV/AIDS infected agents denoted by N_t^H who fully live for two periods. There thus exist the constant growth rate μ^S and population growth rate is due to healthy agents' stock, μ^H then the sick and healthy agents' stocks move respectively such that $N_{t+1}^S = (1-\mu^S)N_t^S$ where μ^S is getting closer to 1 as the other time is approaching and $N_{t+1}^H = (1+\mu^H)N_t^H$. At the initial time, HIV/AIDS is assumed not to exist and thus $N_0^S = N_0^H = N_0 > 0$ which yield $N_{t+1}^S = (1-\mu^S)^{t+1}N_0$ and $N_{t+1}^H = (1+\mu^H)^{t+1}N_0$ but at $t \geq 1$ the infection exist. For the simplicity of the dynamic of the model, agents are controlled on HIV/AIDS prevalence at the beginning of each period only so that those who are not infected at that time are assumed to remain healthy until the end of the period of time. There is a lack of information on inter time periods infection occurrences. That hypothesis means that some agents have HIV/AIDS when they are born and some others do not have it at their born date.

2.1. Population Growth Rate

Total population at time t is $N_t = N_t^S + N_t^H$ and by definition, population growth rate is g^N such that $g^N = \left[\frac{N_{t+1}^H}{N_t^H} - 1\right] + \left[\frac{N_{t+1}^S}{N_t^S} - 1\right]$ which yields (1) i.e

$$g^N = \mu^H - \mu^S \tag{1}$$

Equation (1) means that HIV/AIDS decreases population growth rate

2.2. The Firms

The production function of the firm is $Y_t = F(K_t, uN_t)$ where $u = u^H + u^S$, $u^H > u^S$ and uN_t is labor stock in efficiency units. The production function is of Cobb Douglas i.e

$$Y_t = AK_t^{\alpha}\left(u^S N_t^S\right)^{\delta}\left(u^H N_t^H\right)^{1-\alpha-\delta} \tag{2}$$

Where α and δ are positive parameters, A is the productivity of good production sector. Profit maximization yields the infected and the healthy workers per-capita income and the interest rate such that:

$$w_t^S = \delta A k_t^{\alpha} n_t^{\delta-1} \tag{3}$$

$$w_t^H = (1-\alpha-\delta)A k_t^{\alpha} n_t^{\delta} \tag{4}$$

$$r_t = \alpha A k_t^{\alpha-1} n_t^{\delta} \tag{5}$$

Where $n_t = \frac{u^S N_t^S}{u^H N_t^H}$ and $k_t = \frac{K_t}{u^H N_t^H}$ are respectively labour efficiency gaps and *per-capita* capital stock

2.3. The Consumers

The two kinds of agents can be indexed by j=S,H and both the first and the second period food consumption are denoted by c_t^j and c_{t+1}^j and b_t^j is HIV/AIDS which hurt the agent utility function, the saving rate is stj and the interest rate is $R_t = 1 + r_t$. Assuming that the agent spends a fraction of his income, $x_t^j = \gamma_j b_t^j$ in HIV/ADS treatment only during the first period when young (we explicitly assume in this model that the disease comes from sexual contact when young or transmission during the pregnancy of the infected mother or breath feeding from an infected mother in developing countries), where $0 < \gamma < 1$ is a parameter. The agent consumes his saving at the second period when old and retired, therefore the first and the second period budget constraints can be written such that $w_t^j = s_t^j + x_t^j + c_t^j$ and $R_t s_t^j = c_{t+1}^j$. Indeed the intertemporal budget constraint which is found in eliminating the saving rate in the both previous equations can be written such that:

$$c_t^j + \frac{c_{t+1}^j}{1+r_t} = w_t^j - \gamma^j b_t^j \tag{6}$$

Therefore, the agents' equilibrium problem consists on maximizing the utility function over food consumption demands and the spending on the disease or healthcare, b_t when it is positive otherwise it is HIV/AIDS prevalence. Therefore, the expression of the way the disease hurt the utility function of the agent can be written such that

$$U_t^j = \ln\left(c_t^j\right) + \beta^j \ln\left(c_{t+1}^j\right) - \theta^j \ln\left(b_t\right) \tag{7}$$

The Lagrangian can be written such that

$$L_t^j = \left[\ln\left(c_t^j\right) + \beta^j \ln\left(c_{t+1}^j\right) - \theta^j \ln\left(b_t^j\right)\right] + \lambda\left[-c_t^j - \frac{c_{t+1}^j}{1+r_t} + w_t^j - \gamma^j b_t^j\right] \tag{8}$$

The first order conditions of the maximization problem yield the first and second consumption periods as well as HIV/AIDS equilibrium prevalence i.e

$$c_t^j = \left[\frac{1}{1+\beta^j-\theta^j}\right] w_t^j \quad (9)$$

$$\frac{c_{t+1}^j}{1+r_t} = \left[\frac{\beta^j}{1+\beta^j-\theta^j}\right] w_t^j \quad (10)$$

$$x_t^j = \gamma^j b_t^j = -\left[\frac{\theta^j}{1+\beta^j-\theta^j}\right] w_t^j \quad (11)$$

Indeed the saving rates can be expressed such that

$$s_t^{\ j} = \left[\frac{\beta^j}{1+\beta^j-\theta^j}\right] w_t^j \quad (12)$$

Where j=S,H

According to equation (5), the infected agent faces $\gamma^S > 0$, then $b_t^S > 0$ whereas the healthy agent faces $b_t^H = 0$ indeed, $s_t^{\ S} = \left[\frac{\beta^S}{1+\beta^S-\theta^S}\right] w_t^S < s_t^{\ H} = \left[\frac{\beta^H}{1+\beta^H}\right] w_t^H$ because the healthy agent faces $\gamma^H = 0$, indeed, $x_t^S = x_t$ and $x_t^H = 0$

Proposition 1: *HIV/AIDS prevalence is defined such that equation (13) i.e*

$$b_t = -\frac{\delta}{\gamma}\left[\frac{\theta^j}{1+\beta^j-\theta^j}\right] A k_t^{\alpha} n_t^{\delta-1} \quad (13)$$

Proof: Equation (12) results added to equation (11), yield expression (13). Thus, disease evolution affects both per-capita income and productivity gaps. The equation also means that sustainability prospects using capital funds may eradicate the pandemic and establish a balanced in population growth terms. The increase of the funds devoted to aids eradicates the pandemic.

2.4. The Capital Dynamics

Because capital doesn't depreciate over time, *per-capita* capital dynamics definition is given by $K_{t+1} = u^H N_{t+1}^H k_{t+1} = N_t^H s_t^H$ therefore $k_{t+1} = \frac{1}{u^H\left(1+\mu^H\right)} s_t^H$

leading to equation (14) expression i.e

$$k_{t+1} = \left[\frac{\beta^H\left(1-\alpha-\delta\right)}{\left(1+\beta^H\right)\left(1+\mu^H\right)u^H}\right] A k_t^{\alpha} n_t^{\delta} \quad (14)$$

The equation (14) stipulates that disease absence as well as the marginal propensity to save increase per-capita income through investment potentiality increase.

3. Other Results

Proposition 2: the economic growth rate defined by $g = \frac{k_{t+1}}{k_t} - 1$ *is expressed such that equation* (15) i.e

$$g = \left[\frac{\beta^H\left(1-\alpha-\delta\right)}{\left(1+\beta^H\right)\left(1+\mu^H\right)u^H}\right] A k_t^{\alpha-1} n_t^{\delta} - 1 \quad (15)$$

Proof: Replacing the equation (14) in the definition of the economic growth rate leads to equation (15) which increases in healthy agents' marginal propensity to save and elasticity in good production sector.

3.1. Examining Solow

Proposition 3: per capita income and population labor efficiency gaps can be expressed such that (16) *and* (17) i.e:

$$k^* = \left[\frac{A}{(1+e)}\frac{\beta^H}{\left(1+\beta^H\right)\left(1+\mu^H\right)u^H}\right]^{(1-\delta)\Delta}\left[\frac{A}{e}\left(\frac{\delta}{\gamma}\right)\frac{\theta}{1+\beta^S-\theta}\right]^{\delta\Delta} \quad (16)$$

$$n^* = \left[\frac{A}{(1+e)}\frac{\beta^H}{\left(1+\beta^H\right)\left(1+\mu^H\right)u^H}\right]^{\alpha\Delta}\left[\frac{A}{e}\left(\frac{\delta}{\gamma}\right)\frac{\theta}{1+\beta^S-\theta}\right]^{(1-\alpha)\Delta} \quad (17)$$

Where $\Delta = \frac{1}{\alpha\delta+(1-\alpha)(1-\delta)}$

PROOF: assuming *per-capita* income growth rate and disease to be equal i.e *-b=g=* $\frac{k_{t+1}}{k_t} - 1$ *and taking the logarithm i.e* $ln\left(\frac{k_{t+1}}{k_t}-1\right) = ln\left(\frac{k_{t+1}}{k_t}\right) + ln\left(1-\frac{k_t}{k_{t+1}}\right) \approx ln(k_{t+1}) - ln(k_t) + 1$

indeed in the long run we have $ln\left(\frac{k_{t+1}}{k_t}-1\right) \approx 1$. *We can take the exponential and normalize their evolution such that -b=g=e* i.e $\frac{\delta}{\gamma}\left[\frac{\theta}{1+\beta^S-\theta}\right] A k_t^{\alpha} n_t^{\delta-1} = \left[\frac{\beta^H\left(1-\alpha-\delta\right)}{\left(1+\beta^H\right)\left(1+\mu^H\right)u^H}\right] A k_t^{\alpha-1} n_t^{\delta} = e$. We obtain 2 equations

with 2 unknown variables which are population growth rate in efficiency terms n and per-capita income k. solving that system those variables can be written such that announced in the head of the proposition.

Proposition 4: from the previous proposition, endogenous marginal product of capital is expressed by equation (18) i.e

$$r^* = \alpha A\left[\frac{\left(1+\beta^H\right)\left(1+\mu^H\right)(1+e)u^H}{A\beta^H}\right]^{1-\alpha-\delta} \quad (18)$$

Proof: Setting the marginal productivity rate of investment to equal $m=\left[\frac{\beta^H}{1+\beta^H}\right]$ and because $\partial r^*/\partial m<0$ emerging countries or developing countries grow faster than rich countries as proved by the data.

3.2. Examining Malthus

The balanced growth rate, equation (1) moves such that population growth rate, equation (15) i.e $g=g^N$. Solving this equality leads to a relationship between *per-capita* income and productivity gaps or population growth expressed like following:

$$k^{**}=\left[\frac{\beta^H\left(1-\alpha-\delta\right)A}{\left(1+\beta^H\right)\left(1+\mu^H\right)\left(\mu^H-\mu^S\right)u^H}\right]^{1/1-\alpha} n^{**(1-\delta)/(1-\alpha)} \quad (19)$$

Indeed, we have $\partial k^{**}/\partial n^{**}>0$ and $\partial k^{**}/\partial \mu S>0$ which means that population productivity gaps due to disease and death rate increase per-capita capital and death rate do the same thing to per-capita capital or income. Therefore, Malthus tends to be validated i.e diseases and hunger lead to death and to income increase for the remaining population. But Malthus is offset by sustainability policy with the affectation of the funds no more to investment but to eradicate the pandemic viewed through the marginal propensity to save affectation to medical care.

4. Discussions

- Malthus represented by disease and death rates are not included in the final economic dynamic equation of capital marginal return, thus the model gives some credibility to his thesis on wealth creation relationship to accumulation against poverty and disease as well as high population growth. Malthus tends to be validated if there is a drama such that the extinction of the great part of the population if sustainable actions are not highly conducted in poorest countries. The mechanics are productivity gap increase between healthy and non healthy and death outcomes produces income increase. In conclusion, *first* HIV/AIDS disease reduces a part of the population but not at the significant way i.e in large scale such that there are species extinctions in the long run because there exists a part of healthy agents which is stronger and increasing and country's wealth creator. There are also actions conducted in favor of sustainable development in financing the eradication of the pandemic by developed countries as well as international organizations such that the World Bank and the World Health Organization in cooperation with poor countries governments. The study highlights the fact that, disease can be fought through more funds or an appropriate economic policy which are presently done as just said above. *Second* because population is increasing on the one side and decreasing on the other side, there exist a stable equilibrium in health state ensuring stabilities in terms of population growth and thus, in *per-capita* income and economic growth terms displayed in figure 1 and this level can be achieved through more fights not only against poverty but also looking for sustainable development which is not achieved yet in poorest developing countries and still expect to come.
- Solow validation by the model corresponds more to emerging countries growth rate data than Sub-Saharan Africa data. Why do Solow is right only now? Because emerging countries exports have oriented monetary policies and contributed to the global imbalances that triggered the financial crisis. From the perspective of financial accounts, persisting trade surpluses are reflected into persisting financial surpluses: Citizens of developing countries are net providers of funds abroad. The monetary policy of reserve accumulation, however, requires the prior issue of local currency, to be immediately converted into foreign currency. The resulting increases in the monetary base of the country lead to inflationary pressures (or alternatively, under sterilization policies, increases in domestic interest rates, which depress investment). Monetary policy therefore, through the inflation channel (or through its effects on interest rates under sterilization policies) triggers a displacement of resources from the private to the public sector by reducing private savings and transferring it to the public sector. The combination of low interest rates and significant debts incurred by the private sector, especially by American households, with emerging countries generated the conditions for the outbreak of the crisis. Indeed, the availability of abundant liquidity at a low price in the United States has triggered a significant use of leverage on the part of household, but also on credit institutions. They have used these resources thus accumulated for loans (so-called subprime) also to households that would be unable to provide adequate guarantees, thereby getting exposed to significant risk of

insolvency in accordance with the well known transmission mechanism involving financial institutions. A decline in collaterals prices, i.e. housing, has provided an incentive for borrowers to default on their loan (as the cost of redeeming the mortgage was higher than the value of the underlying house). The resulting string of defaults has filled the banks' assets with low-value houses, thereby leading to devaluations that deteriorated their balance sheet. In addition, the difficulty in valuing the complex mortgage-backed securities exacerbated the issue of asymmetric information, and, through the channel of negative expectations spreading out, contaminated even solid institutions that did not carry "toxic" assets. As a result of this turmoil, some of the financial institutions around the world were ultimately bailed out by their governments; others collapsed. Mario Deaglio [36] estimates the costs of bankruptcy in 2 million jobs lost only in the United Kingdom. The crisis quickly spread out to the real economy and in the whole globalized world markets. This, resulted in a significant drop in the growth rate, that turned negative in 2008 in almost all developed countries, and decreased although remaining positive in the developing countries [37]. According to preliminary estimates by the International Monetary Fund, the 2010 bounce has been less pronounced for Italy (1%) than it has been for the other major European countries (France, 1,6%, Germany, 3,3%), as well as for the United Kingdom (1,7%), the United States (2,6%) and Japan (2,8%). While data seem to converge in indicating that the worst point of the crisis is over, data on Italy confirm the Italian productivity stagnation. The financial crisis propagated from the financial to the real world roughly through two channels: the households credit on the one hand, and the bank credit on the other. Indeed, the financial crisis acts on the capital market by modifying credit supply and saving use, both directly invested by households, and those intermediated by credit intermediaries.

5. Malthus Meets Solow

The synthesis of Malthus and Solow in this article yields to the equalization of equations (18) and (19) i.e it is the locus on the space *endogenous marginal product of capital* expressed by equation (18) *and population growth* expressed by equation (19) are set equal i.e

$$r^{*}=\alpha A\left[\frac{\left(1+\beta^{H}\right)\left(1+\mu^{H}\right)\left(1+e\right)u^{H}}{A\beta^{H}}\right]^{1-\alpha-\delta}=k^{**}=\left[\frac{\beta^{H}\left(1-\alpha-\delta\right)A}{\left(1+\beta^{H}\right)\left(1+\mu^{H}\right)\left(\mu^{H}-\mu^{S}\right)u^{H}}\right]^{1/1-\alpha}n^{**(1-\delta)/(1-\alpha)}$$

Then yields

$$n^{**(1-\delta)/(1-\alpha)}=\left[\alpha A\left(1+e\right)u^{H}\right)^{1-\alpha-\beta}\left[\frac{\left(\mu^{H}-\mu^{S}\right)u^{H}}{1-\alpha-\delta}\right]^{1/1-\alpha}\left[\frac{\left(1+\beta^{H}\right)\left(1+\mu^{H}\right)}{\beta^{H}A}\right]^{\frac{1+(1-\alpha-\beta)(1-\alpha)}{1-\alpha}}\right] \tag{20}$$

The last equation expresses population growth rate in function of technology

This last result expresses that, technology and human birth are linked and the equilibrium on the both should be established, since technology in human being creation is medicine and improves over time, so that increases life expectancy, better health not only food availability plays a crucial role on human being, but technology too which also plays in food system. Consequently, sustainable development consists on research and development like in pandemic action both on health and food.

6. Conclusion

The model aim is to investigate in how far Malthus population growth is validated and Solow verified in developing countries. We found that, food shortages and disease make the sick agents' productivity decrease and per-capita income of the country gets increase and show the necessity for a sustainable economic policy to be conducted in poor countries though technology or research development that we find at the end, which will be devoted to food improvement system, health remedies and so on. Malthus predictions i.e death occurrences and population growth reduction is valid since nothing is done. The model had established those results at the equilibrium in population terms and shows that poorest countries growth potentiality could have been higher than the richer countries if disease matters were not too great or inexistent, in empirical terms poverty reduction observed doesn't explain sustainable development achievement yet. The increase of calamities and social security development absence needs funds to be drained from industrial needs to healthcare needs or to use potential savings from technological industry to medicine i.e pharmacy, and medicine staffs in Africa. The model also agrees with Solow implication in the observed empirical data concerning

high growth rates in emerging countries of Asia and poverty reduction in poorest countries of Africa. Consequently, the model shows advancement in economics thought and modeling but shows also that, at the time of Solow, the economic implications of the theory didn't have to be necessarily relied to empirical evidence for the theory to hold.

According to equation (1), the growth rate, g is an increasing function of μ^H whereas, g is an increasing function of of μ^H according to equation (15), therefore, the equilibrium exist and it is highlighted by figure 1.

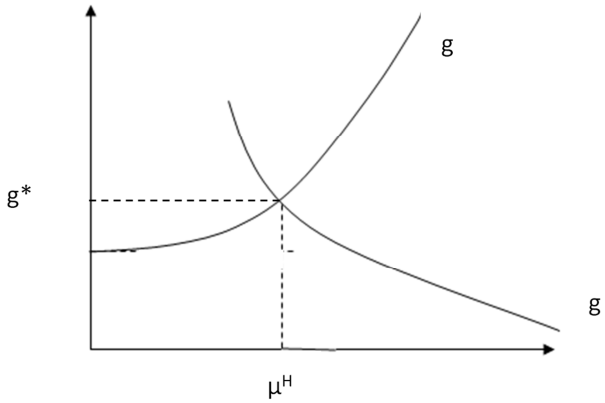

Figure 1. Displays the equilibrium of population growth rate in function of the economic growth rate.

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