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Relative Performance of Laminated Composite Doubly Curved Shell Roofs with Cutout

Sarmila Sahoo*

Department of Civil Engineering, Heritage Institute of Technology, Kolkata, India

Abstract

Performance characteristics of stiffened composite doubly curved shells with cutout are analyzed in terms of natural frequency. A finite element code is developed for the purpose by combining an eight noded curved shell element with a three noded curved beam element. The code is validated by solving benchmark problems available in the literature and comparing the results. The size of the cutout is varied for different edge constraints of cross-ply and angle-ply laminated composite shells. The results furnished here may be readily used by practicing engineers dealing with stiffened composite conoids, hyperbolic paraboloids and elliptic paraboloids with cutouts.

Keywords

Laminated Composite, Doubly Curved Shells, Cutout, Fundamental Frequency

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1. Introduction

Laminated composite structures have gained extensive importance in various fields of aerospace, marine and civil engineering. Shell roof structures can be conveniently built with composite materials that have many advantages, besides high specific strength and stiffness. Among the different shell panels which are commonly used as roofing units in civil engineering practice, the conoidal, hyperbolic paraboloidal among the anticlastic, and the elliptic paraboloidal among the synclastic shells are common as roofing units to cover large column-free areas. Conoidal shells being ruled surfaces provide ease of casting and allow north light to come in. Thus these are preferred in many situations. The hyperbolic paraboloidal shells are aesthetically appealing. But they are less stiff than other doubly curved shells. The elliptic paraboloidal shells, on the other hand, are architecturally acceptable as well as structurally stiff due to their surface geometry.

Research on conoidal shell was started long back with [1], who analysed static characteristics of conoidal shells using the variational method. Then it was continued and improved by researchers like Brebbia and Hadid [2], Choi [3], Ghosh and Bandyopadhyay [4, 5], Dey et al. [6] and Das and Bandyopadhyay [7]. Dey et al. [6] considered static analysis of conoidal shell while Chakravorty and Bandyopadhyay [8] applied the finite element technique to explore the free vibration characteristics of shallow conoids and also observed the effects of excluding some of the inertia terms from the mass matrix on the first four natural frequencies. Chakravorty et al. [9-11] published a series of papers where they reported on free and forced vibration characteristics of graphite-epoxy composite conoidal shells with regular boundary conditions. Later, Nayak and Bandyopadhyay [12-15] reported free vibration of stiffened isotropic and composite conoidal shells. Das Chakravorty [16, 17] considered bending and free vibration characteristics of un-punctured and un-stiffened composite conoids. Hota and Chakravorty [18] studied isotropic punctured conoidal shells with complicated boundary conditions along the four edges but no such study about composite conoidal shells is available in the literature. Number of researchers has worked on different behavioral

* Corresponding author
E-mail address: sarmila.sahoo@gmail.com

aspects of laminated doubly curved shells. Pradyumna and Bandyopadhyay [19, 20] reported static, dynamic and instability behaviour of laminated doubly curved shells. Application of doubly curved shells in structures often necessitates provision of cutouts for the passage of light, service lines and also sometimes for alteration of the resonant frequency. The free vibration of composite as well as isotropic plate with cutout was studied by different researchers from time to time. Shells of double curvature have the ability to span over relatively large distances without the need of intermediate supports in comparison with flat plates and cylindrical panels of the same general proportions. This aspect in particular attracts the designers to use such shell forms in places of large column free areas. Qatu et al. [21] reviewed the work done on the vibration aspects of composite shells between 2000-2009 and observed that most of the researchers dealt with closed cylindrical shells. Other shell geometries like conical shells and shallow shells on different planforms are receiving considerable attention. Recently, Kumar et al. [22-25] considered finite element formulation for shell analysis based on higher order zigzag theory. Vibration analysis of spherical shells and panels both shallow and deep has also been reported for different boundary conditions [26-29]. A complete and general view on mathematical modeling of laminated composite shells using higher order formulations has been provided in recent literature [30-32].

The review of the existing literature clearly reveals that the information available for vibration behavior of all common forms of both anticlastic and synclastic shells with cutout is far from complete. The present work is therefore aimed at investigating the relative performance in terms of free vibration characteristics of several laminated composite shell forms like conoids (Fig. 1), and hyperbolic paraboloids (Fig. 2) and elliptic paraboloids (Fig. 3) in presence of cutout by employing the finite element method based on an eight-noded isoparametric doubly curved shell element.

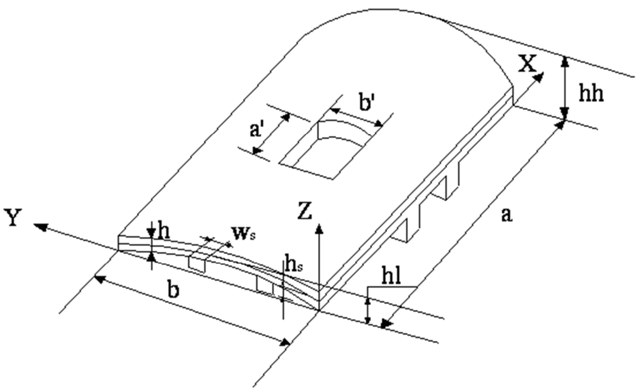

Fig. 1. Conoidal shell with a concentric cutout stiffened along the margins.

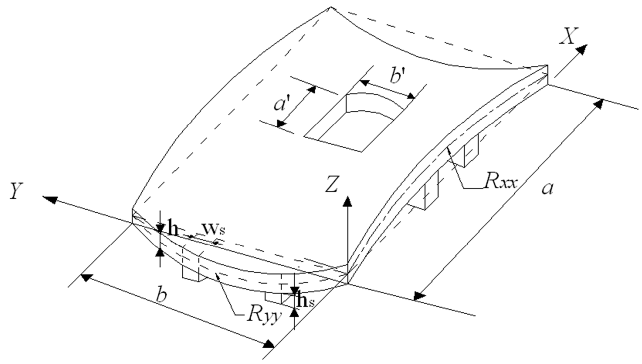

Fig. 2. Hyperbolic paraboloid shell with a concentric cutout stiffened along the margins.

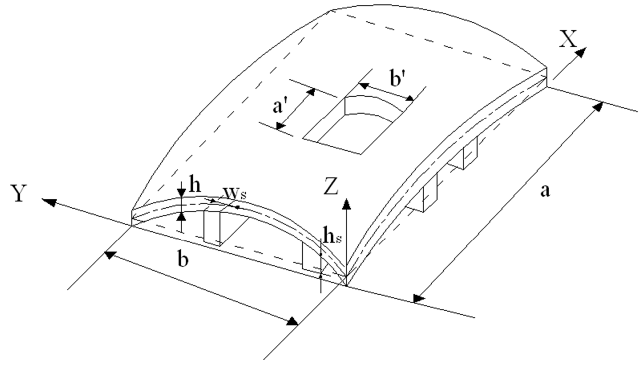

Fig. 3. Elliptic paraboloidal shell with a concentric cutout stiffened along the margins.

2. Formulation

A laminated composite shell of uniform thickness h is considered. Keeping the total thickness same, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the X-axis of the co-ordinate system. The constitutive equations for the shell are given by

$$\{F\}=[E]\{\varepsilon\} \tag{1}$$

where, $\{F\} = \{N_x, \ N_y, \ N_{xy}, \ M_x, \ M_y, \ M_{xy}, \ Q_x, \ Q_y\}^T$,

$$[E] = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [S] \end{bmatrix},$$

$$\{\varepsilon\} = \{\varepsilon_x^0, \ \varepsilon_y^0, \ \gamma_{xy}^0, \ k_x, \ k_y, \ k_{xy}, \ \gamma_{xz}^0, \ \gamma_{yz}^0\}^T.$$

The force and moment resultants are expressed as

$$\{N_x, \ N_y, \ N_{xy}, \ M_x, \ M_y, \ M_{xy}, \ Q_x, \ Q_y\}^T = \int_{-h/2}^{h/2} \{\sigma_x, \ \sigma_y, \ \tau_{xy}, \ \sigma_x.z, \ \sigma_y.z, \ \tau_{xy}.z, \ \tau_{xz}, \ \tau_{yz}\}^T dz \tag{2}$$

The submatrices [A], [B], [D] and [S] of the elasticity matrix [E] are functions of Young's moduli, shear moduli and the Poisson's ratio of the laminates. They also depend on the angle which the individual lamina of a laminate makes with the global X-axis. The detailed expressions of the elements of the elasticity matrix are available in several references. The strain-displacement relations on the basis of improved first order approximation theory for thin shell are established as

$$\begin{aligned}&\left\{\varepsilon_x, \ \varepsilon_y, \ \gamma_{xy}, \ \gamma_{xz}, \ \gamma_{yz}\right\}^T \\ &= \left\{\varepsilon_x^0, \ \varepsilon_y^0, \ \gamma_{xy}^0, \ \gamma_{xz}^0, \ \gamma_{yz}^0\right\}^T \\ &+ z\left\{k_x, \ k_y, \ k_{xy}, \ k_{xz}, \ k_{yz}\right\}^T\end{aligned} \tag{3}$$

where, the first vector is the mid-surface strain for a shell and the second vector is the curvature.

An eight-noded curved quadratic isoparametric finite element is used for shell analysis. The five degrees of freedom taken into consideration at each node are u, v, w, α, β. The following expressions establish the relations between the displacement at any point with respect to the co-ordinates ξ and η and the nodal degrees of freedom.

$$u = \sum_{i=1}^{8} N_i u_i \quad v = \sum_{i=1}^{8} N_i v_i \quad w = \sum_{i=1}^{8} N_i w_i \quad \alpha = \sum_{i=1}^{8} N_i \alpha_i$$
$$\beta = \sum_{i=1}^{8} N_i \beta_i \tag{4}$$

where the shape functions derived from a cubic interpolation polynomial are:

$N_i = (1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)/4$, for i=1,2,3,4

$N_i = (1+\xi\xi_i)(1-\eta^2)/2$, for i=5,7

$N_i = (1+\eta\eta_i)(1-\xi^2)/2$, for i=6,8

The generalized displacement vector of an element is expressed in terms of the shape functions and nodal degrees of freedom as:

$$\left[u\right] = \left[N\right]\left\{d_e\right\} \tag{5}$$

i.e., $$\left\{u\right\} = \begin{Bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} N_i & & & & \\ & N_i & & & \\ & & N_i & & \\ & & & N_i & \\ & & & & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix}$$

The strain-displacement relation is given by

$$\left\{\varepsilon\right\} = \left[B\right]\left\{d_e\right\}, \tag{6}$$

where $$\left[B\right] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & -\dfrac{N_i}{R_y} & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix}$$

The element stiffness matrix is

$$\left[K_e\right] = \iint \left[B\right]^T \left[E\right]\left[B\right] dxdy \tag{7}$$

The element mass matrix is obtained from the integral

$$\left[M_e\right] = \iint \left[N\right]^T \left[P\right]\left[N\right] dxdy\,, \tag{8}$$

where,

$$\left[N\right] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix},$$

$$\left[P\right] = \sum_{i=1}^{8} \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

in which

$$P = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} \rho dz \quad \text{and} \quad I = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} z\rho dz$$

Three noded curved isoparametric beam elements are used to model the stiffeners, which are taken to run only along the boundaries of the shell elements. In the stiffener element, each node has four degrees of freedom i.e. $u_{sx}, w_{sx}, \alpha_{sx}$ and β_{sx} for X-stiffener and $v_{sy}, w_{sy}, \alpha_{sy}$ and β_{sy} for Y-stiffener. The generalized force-displacement relation of stiffeners can be expressed as:

X-stiffener: $$\left\{F_{sx}\right\} = \left[D_{sx}\right]\left\{\varepsilon_{sx}\right\} = \left[D_{sx}\right]\left[B_{sx}\right]\left\{\delta_{sxi}\right\}; \tag{9}$$

Y-stiffener: $$\left\{F_{sy}\right\} = \left[D_{sy}\right]\left\{\varepsilon_{sy}\right\} = \left[D_{sy}\right]\left[B_{sy}\right]\left\{\delta_{syi}\right\} \tag{10}$$

where, $\{F_{sx}\} = [N_{sxx} \quad M_{sxx} \quad T_{sxx} \quad Q_{sxxz}]^T$;

$$\{\varepsilon_{sx}\} = [u_{sx.x} \quad \alpha_{sx.x} \quad \beta_{sx.x} \quad (\alpha_{sx} + w_{sx.x})]^T$$

And $\{F_{sy}\} = [N_{syy} \quad M_{syy} \quad T_{syy} \quad Q_{syyz}]^T$;

$$\{\varepsilon_{sy}\} = [v_{sy.y} \quad \beta_{sy.y} \quad \alpha_{sy.y} \quad (\beta_{sy} + w_{sy.y})]^T$$

The generalized displacements of the *y*-stiffener and the shell are related by the transformation matrix $\{\delta_{syi}\} = [T]\{\delta\}$ where

$$[T] = \begin{bmatrix} 1+\dfrac{e}{R_y} & symmetric & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This transformation is required due to curvature of *y*-stiffener and $\{\delta\}$ is the appropriate portion of the displacement vector of the shell excluding the displacement component along the x-axis.

Elasticity matrices are as follows:

$$[D_{sx}] = \begin{bmatrix} A_{11}b_{sx} & B'_{11}b_{sx} & B'_{12}b_{sx} & 0 \\ B'_{11}b_{sx} & D'_{11}b_{sx} & D'_{12}b_{sx} & 0 \\ B'_{12}b_{sx} & D'_{12}b_{sx} & \frac{1}{6}(Q_{44}+Q_{66})d_{sx}b_{sx}^3 & 0 \\ 0 & 0 & 0 & b_{sx}S_{11} \end{bmatrix}$$

$$[D_{sy}] = \begin{bmatrix} A_{22}b_{sy} & B'_{22}b_{sy} & B'_{12}b_{sy} & 0 \\ B'_{22}b_{sy} & \frac{1}{6}(Q_{44}+Q_{66})b_{sy} & D'_{12}b_{sy} & 0 \\ B'_{12}b_{sy} & D'_{12}b_{sy} & D'_{11}d_{sy}b_{sy}^3 & 0 \\ 0 & 0 & 0 & b_{sy}S_{22} \end{bmatrix}.$$

where,

$$D'_{ij} = D_{ij} + 2eB_{ij} + e^2A_{ij}\,; \quad B'_{ij} = B_{ij} + eA_{ij}\,,$$

and A_{ij}, B_{ij}, D_{ij} and S_{ij} are explained in literature.

Here the shear correction factor is taken as 5/6. The sectional parameters are calculated with respect to the mid-surface of the shell by which the effect of eccentricities of stiffeners is automatically included. The element stiffness matrices are of the following forms.

for X-stiffener: $[K_{xe}] = \int [B_{sx}]^T [D_{sx}][B_{sx}]dx$;

for Y-stiffener: $[K_{ye}] = \int [B_{sy}]^T [D_{sy}][B_{sy}]dy$

The integrals are converted to isoparametric coordinates and are carried out by 2-point Gauss quadrature. Finally, the element stiffness matrix of the stiffened shell is obtained by appropriate matching of the nodes of the stiffener and shell elements through the connectivity matrix and is given as:

$$[K_e] = [K_{she}] + [K_{xe}] + [K_{ye}]. \tag{11}$$

The element stiffness matrices are assembled to get the global matrices.

The element mass matrix for shell is obtained from the integral

$$[M_e] = \iint [N]^T [P][N]dxdy\,, \tag{12}$$

where,

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix},$$

$$[P] = \sum_{i=1}^{8} \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

in which

$$P = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} \rho dz \quad \text{and} \quad I = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} z\rho dz$$

Element mass matrix for stiffener element

$$[M_{sx}] = \iint [N]^T [P][N]dx \quad \textit{for X stiffener}$$

and $[M_{sy}] = \iint [N]^T [P][N]dy$ *for Y stiffener*

Here, $[N]$ is a 3x3 diagonal matrix.

$$[P] = \sum_{i=1}^{3} \begin{bmatrix} \rho.b_{sx}d_{sx} & 0 & 0 & 0 \\ 0 & \rho.b_{sx}d_{sx} & 0 & 0 \\ 0 & 0 & \rho.b_{sx}d_{sx}^2/12 & 0 \\ 0 & 0 & 0 & \rho(b_{sx}.d_{sx}^3 + b_{sx}^3.d_{sx})/12 \end{bmatrix}$$

for X-stiffener

$$[P] = \sum_{i=1}^{3} \begin{bmatrix} \rho.b_{sy}d_{sy} & 0 & 0 & 0 \\ 0 & \rho.b_{sy}d_{sy} & 0 & 0 \\ 0 & 0 & \rho.b_{sy}d_{sy}^2/12 & 0 \\ 0 & 0 & 0 & \rho(b_{sy}.d_{sy}^3 + b_{sy}^3.d_{sy})/12 \end{bmatrix}$$

for Y-stiffener

The mass matrix of the stiffened shell element is the sum of the matrices of the shell and the stiffeners matched at the appropriate nodes.

$$\left[M_e\right]=\left[M_{she}\right]+\left[M_{xe}\right]+\left[M_{ye}\right]. \quad (13)$$

The element mass matrices are assembled to get the global matrices.

The code developed can take the position and size of cutout as input. The program is capable of generating non uniform finite element mesh all over the shell surface. So the element size is gradually decreased near the cutout margins.

The free vibration analysis involves determination of natural frequencies from the condition

$$\left|\left[K\right]-\omega^2\left[M\right]\right|=0 \quad (14)$$

This is a generalized eigen value problem and is solved by the subspace iteration algorithm.

3. Results and Discussion

The validity of the present approach is checked through solution of benchmark problems. It is evident from Table 1 that the present results agree with those of Chakravorty et al. [11] and the fact that the cutouts are properly modeled in the present formulation is thus established.

Table 1. Non-dimensional fundamental frequencies ($\bar{\omega}$) for laminated composite conoidal shell with cutout.

a'/a	Corner point supported		Simply supported		Clamped	
	Chakravorty et al. [11]	Present model	Chakravorty et al. [11]	Present model	Chakravorty et al. [11]	Present model
0.0	23.863	23.494	75.450	74.892	124.736	123.306
0.1	23.554	23.872	75.098	75.278	123.811	123.987
0.2	23.746	23.485	73.668	73.324	122.074	120.588
0.3	23.510	23.768	69.979	69.763	120.515	119.101
0.4	23.205	23.101	61.824	61.524	116.924	115.924

a/b=1, a/h=100, a'/b'=1, a/hh=2.5, hl/hh=0.25

Numerical results are then obtained for several doubly curved shells with 0/90/0/90 and +45/-45/+45/-45 laminations by varying boundary constraints, cutout size and position. The shell thickness h is taken to be constant in all cases and lamina properties taken are: E_{11}/E_{22}= 25, G_{23} = $0.2E_{22}$, G_{13}= G_{12}= $0.5E_{22}$, ν_{12} =ν_{21} =0.25.Tables 2 and 3 contains the results of non-dimensional frequency [$\bar{\omega} = \omega a^2\left(\rho / E_{22}h^2\right)^{1/2}$] of 0/90/0/90 and +45/-45/+45/-45 stiffened shells with cutouts. The cutouts are also taken to be square in plan. The sizes of the concentric cutouts are varied from 0 to 0.4 and boundary conditions are varied along the four edges. The stiffeners, along the cutout periphery are extended up to the edge of the shell. It is seen that when a cutout is introduced to a stiffened shell the fundamental frequencies increases. The increase in frequency may be explained by the fact that when a cutout is introduced to an un-punctured surface the number of stiffeners increases from two to four in the present study. When the cutout size is further increased the number and dimensions of the stiffeners do not change, but the shell surface undergoes loss of both mass and stiffness. As the cutout grows in size the loss of mass is more significant than that of stiffness, and hence the frequency increases. But, for some angle ply shells with further increase in the size of the cutout, the loss of stiffness gradually becomes more important than that of mass, resulting in decrease in fundamental frequency. This leads to the engineering conclusion that cutouts with stiffened margins may always safely be provided on shell surfaces for functional requirements. This behavior is similar in all the shells. However, frequency varies depending on the boundary condition. Frequency is the maximum for clamped one and minimum for corner point supports while it is intermediate in case of simply supported one. Also the frequencies are dependent on the lamination schemes whether cross ply or angle ply. In general cross ply shells have lower frequencies compared to angle ply ones and the behavioral pattern is similar for all types of doubly curved shells. For angle ply shells, elliptic paraboloid geometry has got the maximum frequency followed by conoids and hyperbolic paraboloids in case of un-punctured shells, but the situation changes when the shell gets punctured. The situation gets more complicated in case of cross ply shells due to introduction of cutout.

Table 2. Non-dimensional Fundamental Frequencies ($\bar{\omega}$) for laminated composite (0/90/0/90) stiffened shell for different sizes of the central square cutout and different boundary conditions.

Shell Type	Boundary conditions	Cutout size (a'/a)				
		0	**0.1**	**0.2**	**0.3**	**0.4**
Conoid	Clamped	105.76	118.91	124.38	127.47	124.49
	Simply supported	54.77	56.67	59.27	62.36	65.21
	Corner Point supported	21.18	21.68	22.61	24.23	25.53
Hyperbolic Paraboloid	Clamped	103.94	118.21	142.85	155.42	157.23
	Simply supported	66.21	68.82	88.25	97.8	96.83
	Corner Point supported	28.56	33.64	40.85	49.17	57.67
Elliptic Paraboloid	Clamped	139.71	119.13	133.82	152.26	152.99
	Simply supported	66.29	72.77	76.75	80.43	84.75
	Corner Point supported	35.65	38.04	40.62	43.15	42.99

$a/b=1$, $a/h=100$, $a'/b'=1$, $E_{11}/E_{22}=25$, $G_{23}=0.2E_{22}$, $G_{13}=G_{12}=0.5E_{22}$, $\nu_{12}=\nu_{21}=0.25$.

Table 3. Non-dimensional Fundamental Frequencies ($\bar{\omega}$) for laminated composite (+45/-45/+45/-45) stiffened shell for different sizes of the central square cutout and different boundary conditions.

Shell Type	Boundary conditions	Cutout size (a'/a)				
		0	**0.1**	**0.2**	**0.3**	**0.4**
Conoid	Clamped	137.5363	149.748	156.932	154.6134	147.0345
	Simply supported	89.8159	91.6626	94.3912	89.284	83.3021
	Corner Point supported	26.4022	26.8267	27.3666	28.542	30.1836
Hyperbolic Paraboloid	Clamped	101.36	119.09	123.64	126.15	129.41
	Simply supported	73.24	90.05	96.76	99.26	99.61
	Corner Point supported	36.17	42.04	49.86	59.38	61.29
Elliptic Paraboloid	Clamped	143.55	156.08	165.22	166.77	157.25
	Simply supported	96.82	106.74	109.73	112.61	114.3
	Corner Point supported	32.82	35.56	38.02	41.03	42.2

$a/b=1$, $a/h=100$, $a'/b'=1$, $E_{11}/E_{22}=25$, $G_{23}=0.2E_{22}$, $G_{13}=G_{12}=0.5E_{22}$, $\nu_{12}=\nu_{21}=0.25$.

4. Conclusions

The present approach produces results in close agreement with those of the benchmark problems and the finite element code used here is suitable for analyzing free vibration problems of stiffened doubly curved roof panels with cutouts. The present study reveals that cutouts with stiffened margins may always safely be provided on shell surfaces for functional requirements. The arrangement of boundary constraints along the four edges is important so far the free vibration is concerned. The relative free vibration performances of doubly curved shells for different combinations of edge conditions along the four sides are expected to be very useful in decision-making for practicing engineers. The information regarding the behavior of stiffened shells with a wide spectrum of boundary conditions for cross ply and angle ply shells may also be used as design aids for structural engineers.

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