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# Application of Linear Differential Equation in an Analysis Transient and Steady Response for Second Order RLC Closed Series Circuit

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### Abstract

In this Paper, this work investigates the application of RLC diagrams in the catena study of linear RLC closed series electric circuits. The Relevant second order ordinary differential equations were solved by Kirchhoff's Voltage law. This solution obtained was employed to procedure RLC diagram simulated by MATLAB and Mathematica 9.0. A circuit containing an inductance L or capacitor C and resistor R with current and voltage variable given by differential equation. The general solution of differential equation has two parts complementary function (C. F) and particular integral (P. I) in which C. F. represents transient response and P. I. represents steady response. The general solution of differential equation represents the complete response of network. In this connection, this paper includes RLC circuit and ordinary differential equation of second order and its solution.

### Keywords

Circuit Analysis, RLC Circuit, Ordinary Differential Equation, Transient Response, Steady Response, Kirchhoff's Law

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## 1. Introduction

Nonlinear dynamic study of systems governed by equations in which a small change in one variable can induce a large systematic change is known as chaos. Dislike a linear system, in which a small change in one variable produces a small and easily quantifiable systematic change, a nonlinear system exhibits a sensitive dependence on initial conditions. Chaotic behaviour has been observed in nonlinear electric circuits, oscillating system, chemical reaction, magneto-mechanical devices, weather and climate etc. Ajide et al., (2011) [1]. RLC Circuit is extensively used in a diversity of applications such as filters in communications systems, surmise systems in automobiles, defibrillator circuits in biomedical applications etc. The RLC circuit has much portly and attractive response than RC or RL circuit. An equation which involves differential coefficient is called differential equation [2-5]. A differential equation involving derivatives with respect to single independent variable is called ordinary differential equation and involving partial derivatives with respect to more than one independent variable is called partial differential equation [6]. The inter-connection of simple electric device in which there is at least one closed path for current to flow is called electric circuit. The circuit is switch from one condition to another by change in the applied source or a change in the circuit elements there is a transition period during which the branch current and voltage changes from their former values to new ones. This period is

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called transient [7-10]. After the transient has passed the circuit is said to be steady state. The linear differential equation that describes the circuit will have two parts to its solution the complementary function corresponds to the transient and the particular solution corresponds to steady state. The *v-i* relation for an inductor or capacitor is a differential [11-13]. A circuit containing an inductance L or a capacitor C and resistor R with current and voltage variable given by differential equation of the same form. It is a linear second order differential equation with constant coefficient when the value of R, L, C are constant. L and C are storage elements. Circuit has two storage elements like one L and one C are referred to as second order circuit. Therefore, the series or parallel combination of R and L or R and C are first order circuit and LRC in series or parallel are second order circuit. The circuit changes are assumed to occur at time t=0 and represented by a switch [14-16]. The switch may be supposed to closed (on) and open (off) at t=0. The order of differential equation represent derivatives involve and is equal to the number of energy storing elements and differential equation considered as ordinary. The differential equation that formed for transient analysis will be linear ordinary differential equation with constant coefficient. The value of voltage and current during the transient period are known as transient response. The C. F. of differential equation represents the transient response. The value of voltage and current after the transient has died out are known as steady state response [17-20]. The P. I. of differential equation represents the steady state response. The complete or total response of network is the sum of the transient response and steady state response which is represented by general solution of differential equation. The value of voltage and current that result from initial conditions when input function is zero are called zero input response [21]. The value of voltage and current for the input function which is applied when all initial condition are zero called zero state response. In this Study, we sketch the entire figure by using $MATHLAB$ and $Mathematica$ 9.0.

## 2. Data and Methods

The formation of differential equation for an electric circuit depends upon the following laws.

The voltage drop across a resistor is given by

$$V_R = Ri \tag{1}$$

The voltage drop across an inductor is given by

$$V_L = L\frac{di}{dt} \tag{2}$$

The voltage drop across a capacitor is given by

$$V_C = \frac{1}{C}q \tag{3}$$

Where $R, L, C$ a constant of proportionality is called the resistance, inductance, capacitance, and $i$ is the current. Since $i = \frac{dq}{dt}$, this often can written

$$V_C = \frac{1}{C}\int i\,dt \tag{4}$$

**Table 1.** Elements symbol and units of measurements.

| S. No | Element | Symbol | Unit |
|---|---|---|---|
| 1. | Voltage | V | volt |
| 2. | Current | I | ampere |
| 3. | Charge | Q | coulomb |
| 4. | Resistance | R | ohm |
| 5. | Inductance | L | henry |
| 6. | Capacitance | C | farad |

The fundamental law in the study of electric circuits is the following:

### 2.1. Kirchhoff's Voltage Law (Form 1)

The algebraic sum of the instantaneous voltage drops around a close circuit in a specific direction is zero.

Since voltage drops across resistors, inductors and capacitors have the opposite sign from voltages arising from electromotive force, we may state this law in the following alternative from.

### 2.2. Kirchhoff's Voltage Law (Form 2)

The sum of the voltage drops across resistors, indicators and capacitors is equal to the total electromotive force in a closed circuit.

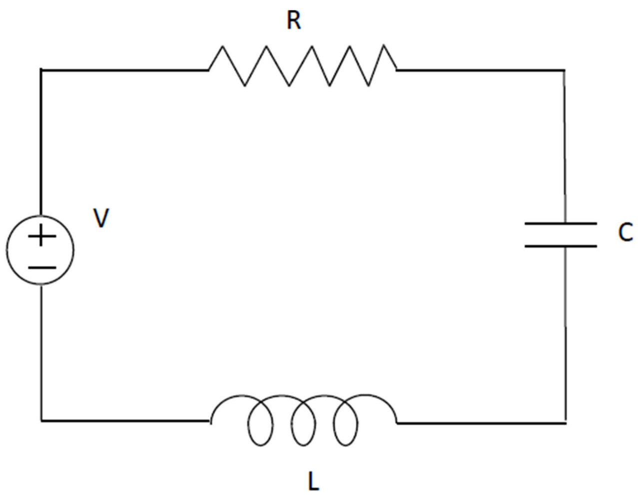


**Figure 1.** Initial RLC Circuit Diagram.

Applying by Kirchhoff's law to the circuit of Figure 1. Letting $E$ denoted the electromotive force and using the law (1), (2) & (3) for voltage drops that were given above, then it becomes the equation of the form is

$$L\frac{di}{dt} + Ri + \frac{1}{c}q = V \quad (5)$$

This equation contains two dependent variable $i\ and\ q$. However, it recall that these two variables are related to each other by the equation

$$i = \frac{dq}{dt} \quad (6)$$

Using equation (6) may eliminate $i$ from equation (5) and write it in the from

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}q = V \quad (7)$$

Equation (7) is a second-order linear differential equation in the single dependent variable $q$. On the other hand, if it differentiates equation (5) and write

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{c}i = \frac{dK}{dt} \quad (8)$$

This is a second-order linear differential equation in the single dependent variable $i$. Thus it have the two second-order linear differential equations (7) and (8) for the charge $q$ and current $i$ respectively. Further observe that in two very simple cases the problem reduces to first-order liner differential equation. If the circuit contains no capacitor, equation (5) itself reduce directly to RL series circuit.

$$L\frac{di}{dt} + Ri = V \quad (9)$$

If it solves this equation (9) then getting the solution is of the form

$$i = \frac{V}{R}(1 - e^{(-\frac{R}{L})t} \quad (10)$$

Here the graph of the equation (10) is

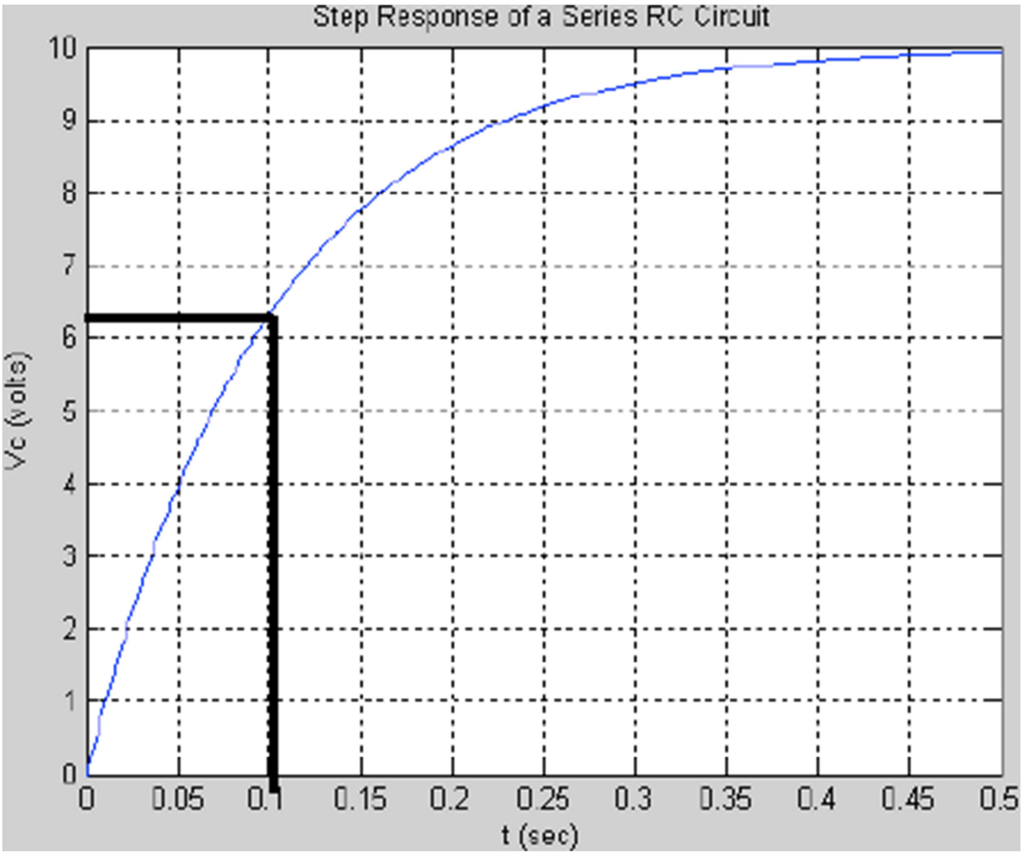


**Figure 2.** The graph of $i = \frac{V}{R}(1 - e^{(-\frac{R}{L})t}$.

The plot shows the transition period during which the current adjusts from its initial value of zero to final value $\frac{V}{R}$, which is steady state.

And while if no inductor is present, Equation (7) reduce to RC series circuit.

$$R\frac{dq}{dt} + \frac{1}{c}q = V$$

$$=> Ri + \frac{1}{c}\int idt = V \quad (11)$$

Differentiating equation (11) with respect to $t$ then it will be obtain

$$R\frac{di}{dt}+\frac{i}{C}=0 \quad (12)$$

Solving this equation then the equation is

$$i=\frac{V}{R}e^{-\frac{t}{RC}} \quad (13)$$

The function has an exponential decay shape as shown in the graph. The current stops flowing as the capacitor becomes fully charged.

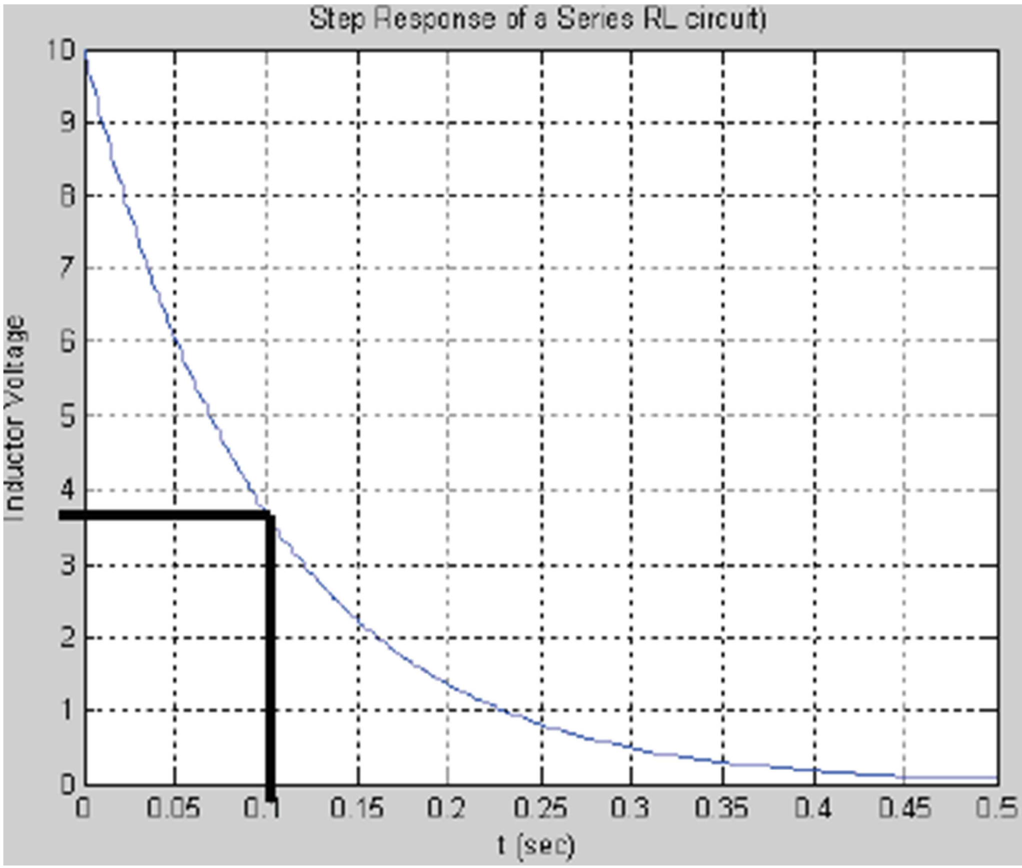


**Figure 3.** The graph of $i=\frac{V}{R}e^{-\frac{t}{RC}}$, an exponential decay curve.

Before considering example, we observe an interesting and useful analogy. The differential equation (7) for the charge is exactly the same as the differential equation (8).

## 3. An Example

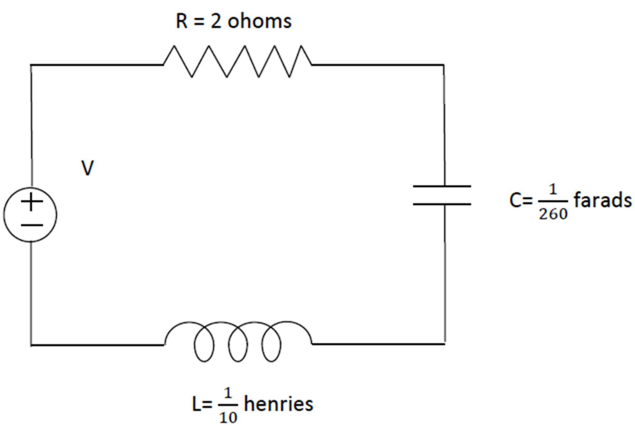


**Figure 4.** RLC Circuit Diagram.

The second order RLC circuits drew using linear technology spice software as shown in figure 4. The resistor value is selected at 2 ohm, capacitor at $\frac{1}{260}$, inductor at 0.1 Henry and electromotive force $100\sin 60t\ V$. If the initial current and initial charge on the capacitor is both zero then find the charge on the capacitor at any time $t>0$.

Formulation 1: By directly applying Kirchhoff's law; Let $i$ denoted the current and $q$ the charge on the capacitor at time t. the total electromotive force is$100\ sin\ 60t\ V$. Using the voltage drop laws (1), (2) and (3) then the equation is of the form:

$$\frac{1}{10}\frac{di}{dt}+2i+260q=100\sin 60t$$

since $q=\frac{di}{dt}$, this reduce to

$$\frac{1}{10}\frac{d^2q}{dt^2}+2\frac{dq}{dt}+260q=100\sin 60t \quad (14)$$

Formulation 2: Applying equation (7) for the charge it becomes $L = \frac{1}{10}, R = 2, C = \frac{1}{260}$ *and* $V = 100\sin 60t.$ substituting these values directly into equation (7) it again obtain equation (14) at once. Multiplying equation (14) through by 10, now considering the differential equation of the form is

$$\frac{d^2q}{dt^2} + 20\frac{dq}{dt} + 2600q = 1000\sin 60t \qquad (15)$$

Since the charge $q$ is initially zero, so the first initial condition is

$$q(0) = 0 \qquad (16)$$

Since the current $i$ is also initially zero and $i = \frac{dq}{dt}$, it takes the second initial condition in the form

$$q'(0) = 0 \qquad (17)$$

The homogeneous equation corresponding to (15) has the auxiliary equation

$$m^2 + 20m + 2600 = 0$$

The roots of this equation are -10 $\pm 50i$ and so the complementary function of equation (15) is

$$q_c = e^{-10t}(C_1 \sin 50t + C_2 \cos 50t) \qquad (18)$$

Employing the method of undermined coefficients to find a particular integral of (15), then it becomes

$$q_p = (A\sin 60t + B\cos 60t) \qquad (19)$$

Differentiating twice and substituting into equation (15), this gives

$$A = \frac{-25}{61} \text{ and } B = \frac{-30}{61}$$

And so the general solution of equation (15) is

$$q = e^{-10t}(C_1 \sin 50t + C_2 \cos 50t) - \frac{25}{61}\sin 60t - \frac{30}{61}\cos 60t \qquad (20)$$

$$\left.\begin{array}{l}\frac{dq}{dt} = e^{-10t}[(-10C_1 - 50C_2)\sin 50t + (50C_1 - 10C_2)\cos 50t] \\ \frac{-1500}{61}\cos 60t + \frac{1800}{61}\sin 60t\end{array}\right\} \qquad (21)$$

Applying condition (16) to equation (18) and condition (17) to equation (19), this gives

$$C_1 = \frac{36}{61} \text{ and } C_2 = \frac{30}{61}$$

Thus the solution of the problem is

$$q = \frac{6e^{-10t}}{61}(6\sin 50t + 5\cos 50t) - \frac{5}{61}(5\sin 60t + 6\cos 60t) \qquad (22)$$

Or,

$$q = \frac{6\sqrt{61}}{61}e^{-10t}Cos\,(50t - \emptyset) - \frac{5\sqrt{61}}{61}\cos(60t - \theta) \qquad (23)$$

Where $\cos\emptyset = \frac{5}{\sqrt{61}}, \sin\emptyset = \frac{6}{\sqrt{61}}$ *and* $\cos\theta = \frac{6}{\sqrt{61}}, \sin\theta = \frac{5}{\sqrt{61}}$ . From these equations we determine $\emptyset \approx 0.88\ (radians)$ *and* $\theta \approx 0.69\ (radians)$. Thus our solution is given approximately by

$$q = 0.77e^{-10t}\cos(50t - 0.88) - 0.64\cos(60t - 0.69) \qquad (24)$$

## 4. Result and Discussion

The first term in the above solution clearly becomes negligible after a relatively short time, it is the transient term. The transient term is equation (18), its indicates the Figure 5. After a sufficient time essentially all that remains is the periodic second term; this is the steady-state term. Observe that its period $\pi/20$ is the same as that of electromotive force. However, the phase angle $\emptyset \approx 0.88\ rad$ indicates that the electromotive force leads the steady state current by approximately$q$ *of equation* (24). This term mention equation (19), and its indicates the Figure 6. The Figure 6 and Figure 7 are represent actual and approximation sketch of steady part, and finally we see that the two graphs are both matching. The graphs of these two components and that of their sum (the complete solution) are shown in figure bellow:

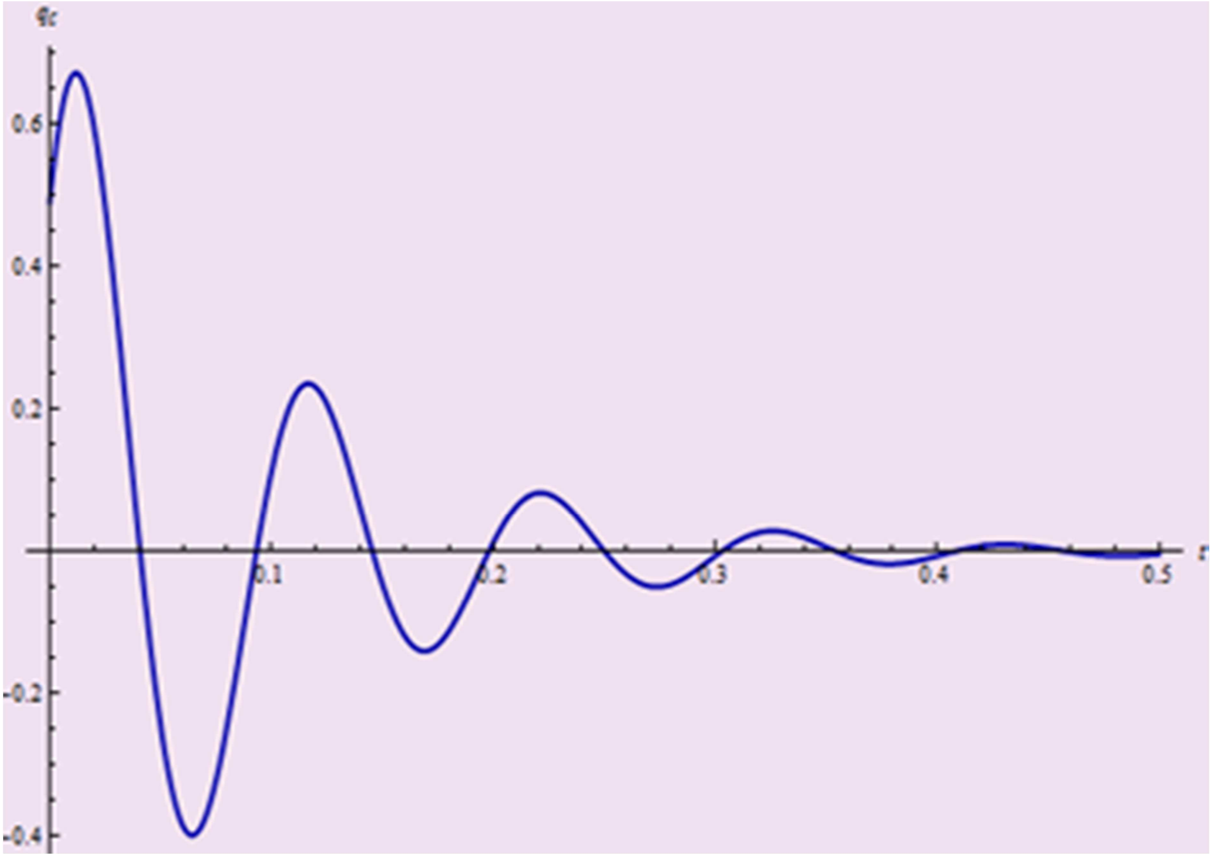


**Figure 5.** The charge $q_c$ when time t=0 and t=0.5.

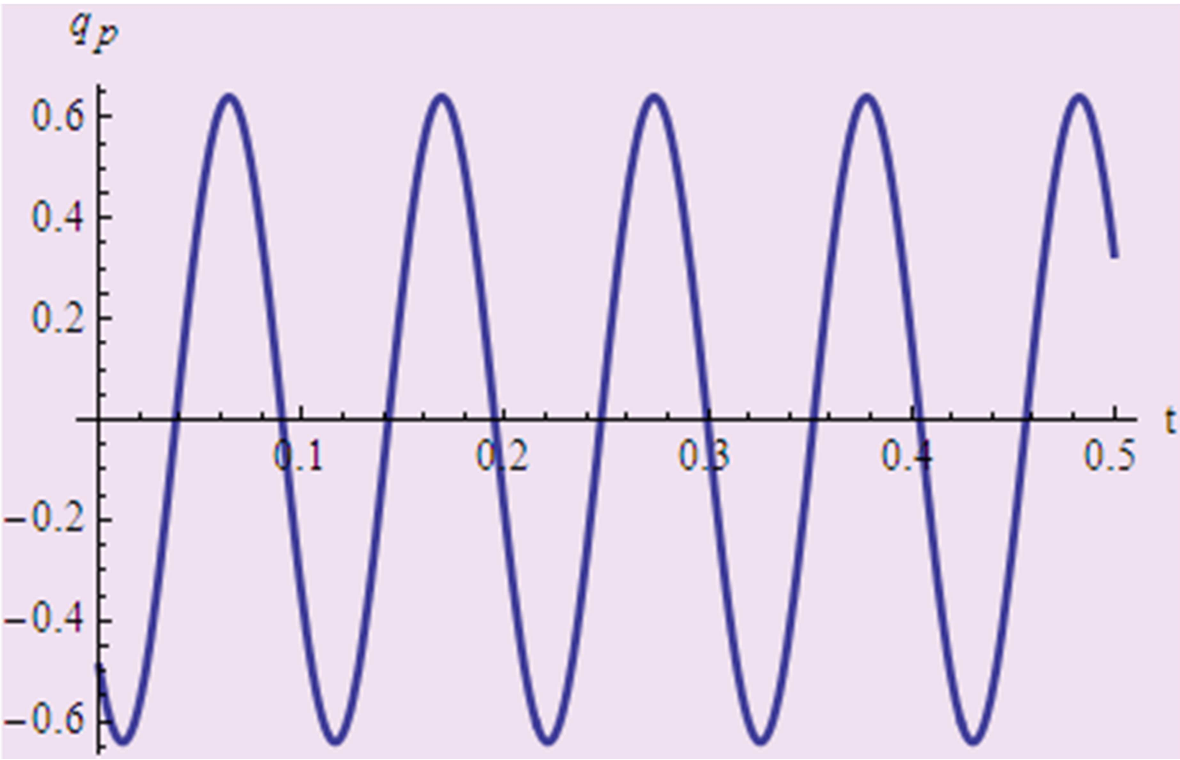


**Figure 6.** The charge $q_p$ when time t=0 and t=0.5.

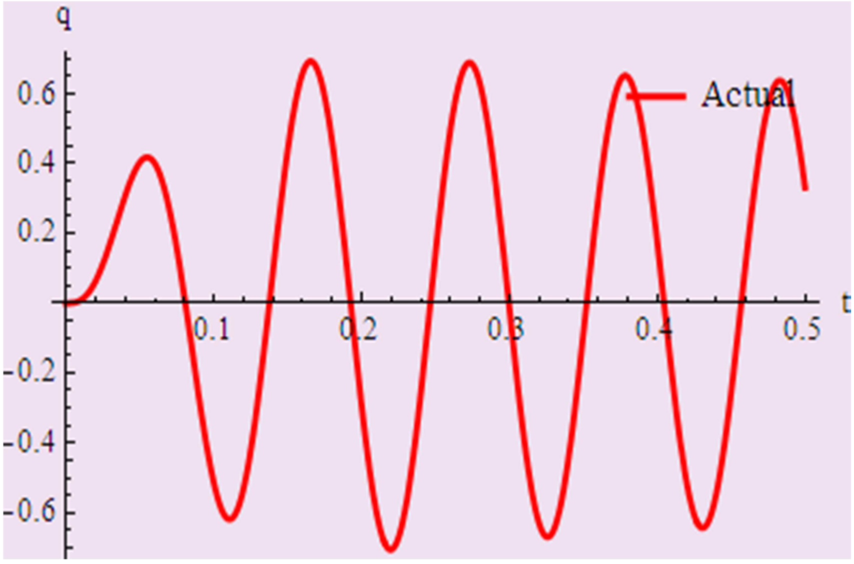


**Figure 7.** The charge $q$ actual result when time t=0 and t=0.5.

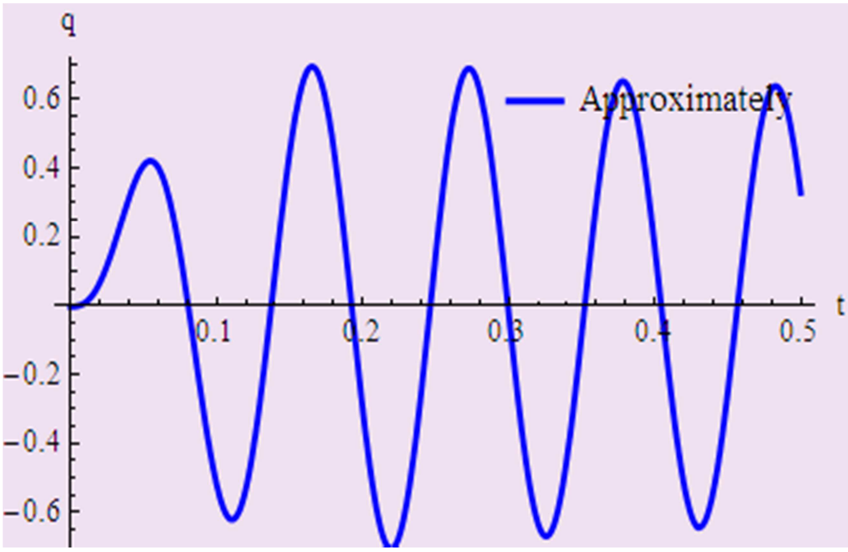


**Figure 8.** The charge $q$ approximate result when time t=0 and t=-0.5.

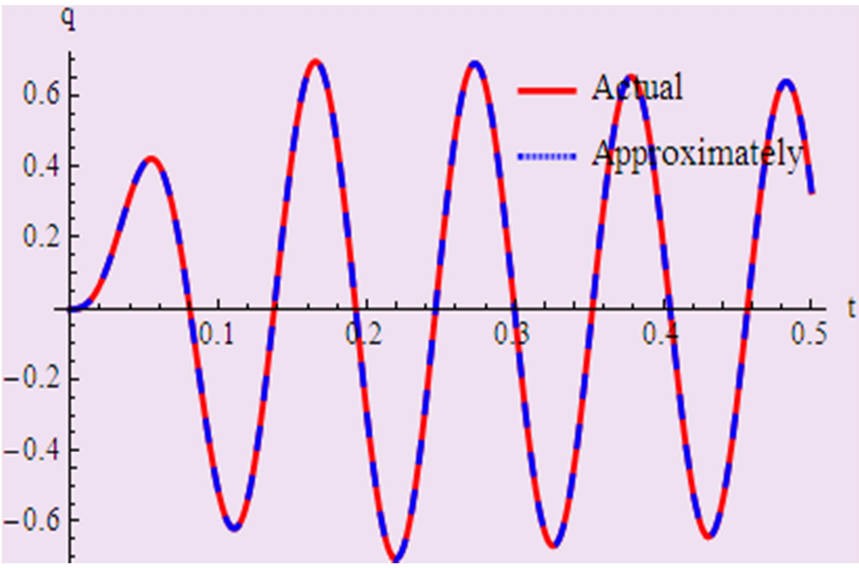


**Figure 9.** The charge $q$ Comparison between actual and approximate result when time t=0 & t=0.5.

## 5. Conclusion

In this paper, we have successfully applied the Kirchhoff's law modified into second order linear differential equation to series circuits containing an electromotive force, resistor, inductor and capacitor. We assume that the reader is somewhat familiar with these items. We have been simply recalled that the electromotive force produces a flow of current in a closed circuit and the current produces a so-called voltage drop across each resistor, inductor, and capacitor.

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