

Applications of Fourier Series in Electric Circuit and Digital Multimedia Visualization Signal Process of Communication System

Ahammodullah Hasan^{1, *}, Mohammad Al-Amin Meia²,
Mohammad Owaziuddin Mostofa³

¹Department of Mathematics, Faculty of Applied Science and Technology, Islamic University, Kushtia, Bangladesh

²Department of Mathematics, Government Adamjeenagar Merchant Worker (M. W) College, Narayanganj, Bangladesh

³Department of Civil Engineering, Southern University Bangladesh, Chattogram, Bangladesh

Abstract

The Fourier series, the founding principle behind the field of Fourier analysis, is an infinite expansion of function in terms sine's and cosines. Fourier transform provides a continuous complex frequency of a function. It is useful in the study of frequency response of a filter, solution PDE, Discrete Fourier transform and FFT in the signals analysis. The advent of Fourier transformation method has greatly extended our ability to implement Fourier methods on digital multimedia visualization system. It covers the mathematical foundations of Digital signal processing (DSP), classical sound, synthesis algorithms and multi-time frequency domain analysis associated musical sound. In this paper, we are analysis of square wave in terms of Fourier component, may occur in electric circuits designed to handle sharply rising pulses and how to convert analog to digital system by using Fourier transform and its applications in electronics and digital multimedia visualization signal process of communication system.

Keywords

Fourier Transform (FT), Fourier Sine and Cosine Transform, Inverse Fourier Transform, Inverse Fast Fourier Transform, Electric Circuit, Frequency, Multimedia Visualization, Digital Signal Process

Received: September 13, 2018 / Accepted: November 29, 2018 / Published online: January 16, 2019

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1. Introduction

The Fourier transformation is standard system analysis tool for viewing the spectral content of signal or sequence. The Fourier transform of sequence, commonly referred to as the discrete time Fourier transform or DTFT is not suitable for real-time implementation. The DTFT takes a sequence as input, but produces a continuous function of frequency as output. A close relative to the DTFT is discrete Fourier transform or DFT [1]. The DFT takes a finite length sequence as input and produces a finite length sequence as output. When the DFT is implemented as an efficient algorithm it is

called the Fast Fourier transformation (FFT). J. W. Cooley and J. W. Tukey are given credit for bringing the FFT to the world in their paper "An algorithm for the machine calculation of complex Fourier Series", Mathematics Computation, Vol. 19, 1965, pp. 297-301. In retrospect, others had discovered the technique many years before. For instance, the great German mathematician Karl Friedrich Gauss (1777-1855) had used the method more than a century earlier. This early work was largely forgotten because it lacked tool to make it practical: the digital computer [2-10]. Cooley and Tukey are honoured because they discovered the FFT at the right time. Mathematics is everywhere in every

* Corresponding author

E-mail address: ahammodullah.iu@gmail.com (A. Hasan)

phenomena, technology, observation, experiment etc. All we need to do is to understand the logic hidden behind. In this article we are focusing on application of Fourier series in electric circuit and communication system [11-13]. FT is name in the *honour* of Joseph Fourier (1749-1829), one of the greatest name in the history of mathematics and physics. Mathematically speaking, the Fourier transform is a linear operator that maps a functional space to another function space and decomposes a function into another function of its frequency components. In particular, the fields of electronics, quantum mechanics, and electrodynamics all mark heavy use to Fourier series. Additionally, other methods based on the Fourier series, such that as the FFT (Fast Fourier Transform *a from* Discrete Fourier Transform [DFT]), are particularly useful for the fields of Digital signal Processing (DSP) and Multi-channel sound analysis [14-22]. In this Study, we sketch the entire figure by using *Mathematica* 9.0.

2. Methodology

Let $f(t)$ is a real-valued function of period 2π and it's a finite sum of harmonically related sinusoids, mathematically the expression for a Fourier series is

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \text{ where, } -\pi \leq t \leq \pi \quad (1)$$

Where $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt dt$$

Fourier series finding principle behind the field of Fourier analysis is an infinite expansion in terms of sine and cosine or imaginary exponentials. The series is define in its imaginary exponential form is

$$f(t) = \sum_{n=-\infty}^{\infty} A_n e^{inx} \quad (2)$$

where the A_n 's are given by the expression

$$A_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx \quad (3)$$

3. Fourier Transform (F.T)

The Fourier series is only capable of analysing the frequency component of certain, discrete frequency (in -tigers) of a given function. In order to study the case where the frequency components of the sine and cosine terms are continuous, the concept of Fourier transform must be introduced. If $h(t)$ is piece-wise continuous and absolutely

integrable in $(-\infty, \infty)$, then the F.T of $h(t)$ is denoted by $\xi(\phi)$ and is define by

$$\xi[h(t)] = \xi(\phi) = \int_{-\infty}^{\infty} h(t) e^{i\phi t} dt \quad (4)$$

And inverse transform

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \xi(\phi) e^{-i\phi t} d\phi \quad (5)$$

Here, the $\xi(\phi)$ fulfills the role of the A_n 's in equations (2) and (3); it gives an indicator of "how much" a particular frequency oscillation damping contributes to the function $f(t)$.

Assume that it has a equidistance, finite set data $h_k = h(t_k)$, $t_k = k\Delta$, and it only interested in N equidistance, discrete frequencies in the range $-\phi_c$ to ϕ_c . It is wish to examine the frequencies $\phi_n = \frac{2\pi n}{N\Delta}$, for $N = \frac{-N}{2}, \dots, \frac{N}{2}$, where Δ be the same Δ be which defines is t_k 's. It is provided an approximation to the Fourier transformation in this range by *Reimann* sum as follows:

$$\begin{aligned} \xi(\phi_n) &= \int_{-\infty}^{\infty} h(t) e^{i\phi t} dt \\ &\approx \sum_{k=0}^{N-1} h_k e^{i\phi_n t_k \Delta} \end{aligned}$$

Using the definition of t_k , this expression reduce to

$$\xi_n \approx \sum_{k=0}^{N-1} h_k e^{ik \frac{2\pi}{N}} \quad (6)$$

This equation called Discrete Fourier transformation (DFT) of the function $h(t)$. If it is denote ξ_n as

$$\xi_n = \sum_{k=0}^{N-1} h_k e^{ik \frac{2\pi n}{N}} \quad (7)$$

The Fourier transform, $\xi(\phi)$, may then be approximated using the expression

$$\xi(\phi) \approx \Delta \xi_n \quad (8)$$

Comparing equation (6) with the Fourier series given equation (2), it is clear that this is a form of Fourier series with non-integer frequency component. Currently, the most common and efficient method of numerically calculating the DFT is using a class of algorithms called "Fast Fourier Transforms (FFTs)". The first known discovery of the FFT was by Gauss in 1805; however, the first modern "rediscovery" of the FFT was done in 1942 by Danielson and Lanczos. They were able to show one may divide any DFT into a sum of two DFT's which each correspond to $\frac{N}{2} - 1$ points. The proof of Danielson and Lanczos's assertion is the following complex number

$$W = e^{\frac{2\pi i}{N}} \quad (9)$$

Equation (6) may written as,

$$\xi_n = \sum_{k=0}^{N-1} W^{nk} h_k \quad (10)$$

Any DFT may then be written as following

$$\begin{aligned} \xi_n &= \sum_{k=0}^{N-1} e^{2\pi i n k / N} \\ &= \sum_{k=0}^{N/2-1} e^{2\pi i (2k) n / N} h_{2k} + \sum_{k=0}^{N/2-1} e^{2\pi i (2k+1) n / N} h_{2k+1} \\ &= \sum_{k=0}^{N/2-1} e^{2\pi i (2k) n / (\frac{N}{2})} h_{2k} + W^k \sum_{k=0}^{N/2-1} e^{2\pi i (2k) n / (\frac{N}{2})} h_{2k+1} \\ &= \xi_n^e + W^k \xi_n^o \end{aligned}$$

Here, ξ_n^e denotes the even terms of the sum (the one corresponding to the index $2k$) and ξ_n^o denotes the odd terms (the ones corresponding to the index $(2k + 1)$). The most useful part of this formula is that it can be used independently expand using the same algorithm, each time reducing the number of calculations by a factor of 2. In fact, this class of FFT algorithm shrinks the computation time from $O(N^2)$ operations to the much more manageable $O(N \log_2 N)$ operations. There are many different FFT algorithms; the one presented here is simply the most common time by 20 or 30 percent (so-called base-4 FFTs or base-8 FFTs). Most importantly, both classes of FFT algorithm are fast enough to embed into modern digital oscilloscopes, multimedia visualization, and electronics equipment. Thus, FFTs have many modern applications, such as Spectrum analyzers, digital signal processors (DSPs), multimedia visualization, digital to analog signal process, and telecommunications systems.

4. The Spectrum Analyzer

In modern technology, an important instrument to any experimentalist is the spectrum Analyzer. This instrument reads a digital signal (usually a voltage) and provides the operator with the Fourier coefficients which correspond to each of the sine and cosine terms of the Fourier series expansion of digital signal. Suppose an instrument takes a time-domain signal, such as the amplitude of output voltage of an instrument. Let us call this digital signal $V(t)$. Then the DFT of $V(t)$ is

$$\xi_n = \sum_{k=0}^{N-1} v_k e^{2\pi i n k / N} \quad (11)$$

We see that this equation is of the same form of equation (7), which means that the previously described methods of the FFT apply to the function. Thus, any digital oscilloscope that is sufficiently fast and equipped with a FFT algorithm is capable of providing the user with the frequency components

of the source signal. Oscilloscopes which are equipped with the ability to FFT their inputs are termed “Digital Spectral Analyzers”. Although they were once a separate piece of equipment for experimentalists, improvements in digital electronics has made it practical to merge the role of oscilloscopes. Spectrum Analyzer have many uses in the laboratory, but one of the most common uses in for signals gives the amplitudes of the various oscillatory components of the input. After normalization, this allows for the experimentalists to determine what frequencies dominate their signals. For example, if we have a DC signals, we would be expect the FFT to show only very low frequency oscillations (i.e., the largest amplitudes should correspond to ≈ 0). However, if we see a sharp peak of amplitudes around 60 HZ, we would know that something is feeding noise into our signal with a frequency of 60 HZ (for example, an AC leakage from our power source)

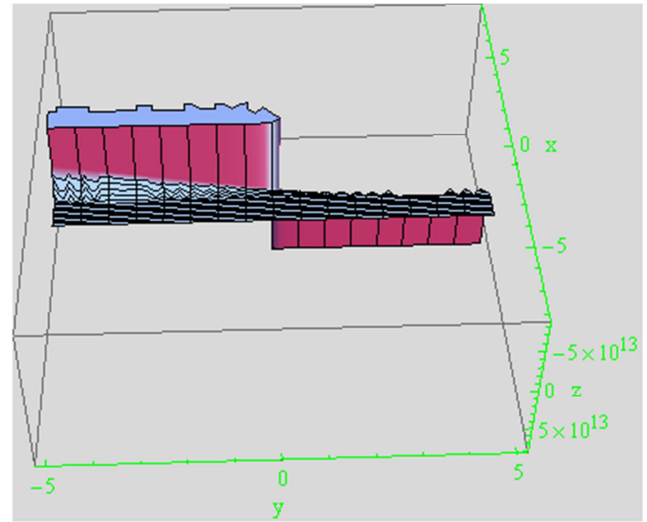


Figure 1. The shape of 3D figure DFT equation (10) when $V_k = -5$ & $V_k = 5$, $N = 6$ and $ik = -8$ and $ik = 8$ for spectrum analyzer.

5. Digital Signal Processing

Already seen how to Fourier series allows experimentalists to identify sources of noise. It may also be used to eliminate sources of noise by introducing the idea of the Inverse Fast Fourier Transform (IFFT). In general goal of an IFF is to take the equation (3) the ones that appear in equation (5) and use them to reconstruct the original function, $f(t)$ of equation (2). Analytically, this is done by multiplying each equation (2) by $e^{2\pi i k \frac{n}{N}}$ then taking the sum over n . However, this is an inefficient algorithm to use when the calculation must do numerically. Just as there is a fast numerical algorithm for approximating the Fourier coefficients, there is another efficient algorithm is called the IFFT which capable of calculating the IFT much faster than the brute-force method. In 1988, it shown by Duhamel, Piron, and Etcheto that the IFFT is simply

$$F^{-1}(x) = F(ix^*)^* \quad (12)$$

In other words, it can calculate the IFFT directly from the FFT; we are simply flipping the real and imaginary parts of the coefficients calculated by the original FFT. Thus, the IFFT algorithms are essentially the same as the FFT algorithm; all one must do is flip the numbers around at the beginning of the calculation. Since the IFFT inherits all of the speed benefits of the FFT, it is quite practical to use it in real time in the laboratory.

One of the most common applications of the IFFT in laboratory is to provide Digital signal processing (DSP). In general, the idea of DSP is to use configurable digital electronics to clean up, transform, or amplify a signal by FFT'ing the signal, removing, shifting or Damping the unwanted frequency components, and then transforming the signal back using the IFFT on the filtered signals.

There are many advantages to doing DSP as opposed to doing analog signal processing. To begin with, practically speaking, we can have a much more complicated filtering function (the function that transforms the coefficients of the DFT) with DSP than analog signal processing. While it is fairly easy to make a single band pass, low pass, or high pass filter with capacitors, resistors, and inductors, it is relatively difficult and time consuming to implement anything more complicated than these three simple filters. Furthermore, even if a more complicated filter was implemented with analog electronics, it is difficult to make even small modification to the filter (there are exceptions to this, such as FPGA's, but those are also more difficult to implement than sample software solution). DSP is not limited by either of those effects since the processing is (usually) done in software, which can be programmed to do whatever the user desires. Probably the most important advantage that DSP has over analog signal processing is the fact that the processing may be done after the signal has been taken.

In modern-day experiments, raw data is often recorded during the experiment and corrected for noise in software during the analysis step. If one filters the signal beforehand (with analog signal processing), it is possible that the later in the experiment, the experimenter could find that they filtered out good signals. The only option in this case is for experiment to be return. On the other hand, if the signal processing was done digitally, all that the experimenter has to do is edit their analysis code and rerun the analysis; this could save both time and money.

6. Preliminary

1. Communication: Fourier transform is essential to understand how to signals behaves when it passes through

filter, amplifiers and communication multi-channel.

2. Data analysis: Fourier transform can be used as high-pass, low-pass, and band- pass filters and it can also be applied to signals and noise estimation by encoding the time series.
3. Cell phone: Every mobile device net-book, notebook, tablet and phone have been built in high speed cellular data connection, just like Fourier transform because sound may be represented by a complex combination of their waves. Human very easily perform Fourier transform mechanically every day. For example when we are in room with great deal of noise and you selectively hear our name above a noise, we have just performed Fourier series.
4. Input transducer: The device that converts a physical signal from source to an electrical, mechanical, electronic signal more suitable for communicating.
5. Transmitter: The device that sends the transducer signal
6. Transmission channel: The physical medium on which the signals is carried.
7. Receiver: The device that recover the transmitted signal from the channel.
8. Output transducer: The device that converts the received signal back into useful quantity.

7. Pairs of Frequency Used in Dual-tone Multi-frequency Digital Signal

In above discussion, consider a list of set 12 channel frequency data is,

Frequency={ {697,1209}, {697,1336}, {697,1477}, {770,1209}, {770,1336}, {770,1477}, {852,1209}, {852,1336}, {852,1477}, {941,1209}, {941,1336}, {941,1477} }. Now calculate and simulation this data and figure by using *mathematica 9.0*, it has the following Digital signal processing figure bellow:

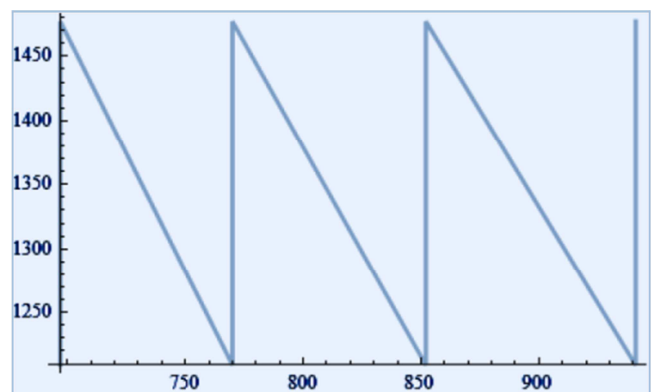


Figure 2. In frame of frequency set of data.

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

Figure 3. Create pairs of low and high frequencies for each key.

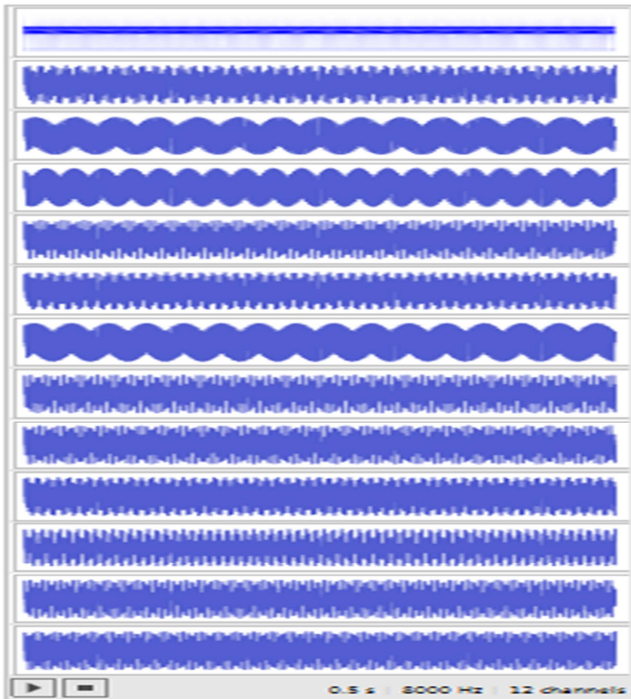


Figure 4. The physical medium on which the 12 channels digital signals.

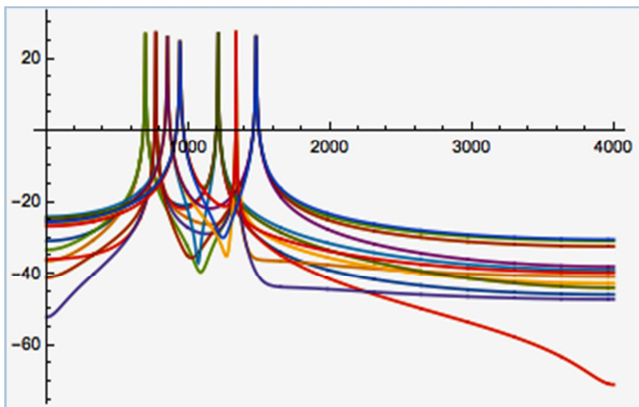


Figure 5. Visualize the periodogram of the create sound for 12 channel.

Now a days, in modern technology multimedia knowledge based over the internet in a user transparent way. The

distributed knowledge based can be used to archive reasonable libraries of the multimedia objects. These multimedia objects can be retrieved in real time over the internet in user transparent manner, and applied for many task. The task could be display of animated cartoon, movies, simulation and visualization of complex objects transmission of complex VLSI design layouts, detail geographical maps. Multimedia objects and their motion are displayed at client end a sequence of multimedia frames integrating synchronized digitalize video and sound at fast enough rate. Huge amount of data needs to be transmitted to display consecutive frames at such a rate, the size of the window, frame rate, the quality of the picture, the transmission bandwidth. Mathematical graph can be used the component level schematics of complex objects such as combustion device. The graph provided the ability to logically group the object and display the complex multiple digital visualization signals of 12-channel.

8. Example of Square Wave Rectifier

An example of the above discussion, consider a square wave by the function $f(t)$ such that

$$f(t) = \begin{cases} -0, & -\pi < t < 0 \\ h, & 0 < t < \pi \end{cases} \quad (13)$$

Now, comparing equation (13) with equation (1) then it becomes-

$$a_0 = \frac{1}{2\pi} \int_0^{\pi} h \, dt = \frac{h}{2} [t] = \frac{h}{2}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^{\pi} h \cos nt \, dt \\ &= \frac{h}{n\pi} [\sin nt]_0^{\pi} \\ &= \frac{h}{n\pi} [\sin n\pi - \sin 0] = 0 \end{aligned}$$

and

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{\pi} h \sin nt \, dt \\ &= -\frac{h}{n\pi} [\cos nt]_0^{\pi} \\ &= -\frac{h}{n\pi} [\cos n\pi - \cos 0] \\ &= \frac{h}{n\pi} [1 - \cos n\pi] \end{aligned}$$

$$= \frac{h}{n\pi} [1 - (-1)^n], \text{ where } n \text{ is odd.}$$

Now putting the value of a_0, a_n and b_n in (1), then the required Fourier series is

$$f(t) = \frac{h}{2} + \sum_{n=1}^{\infty} \left[0 + \frac{h}{n\pi} \{1 - (-1)^n\} \sin nt \right]$$

$$= \left[\frac{h}{2} + \frac{2h}{\pi} \left[\frac{\sin t}{1} + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \dots \dots \dots \right] \right]$$

Showing that the square wave rectifier does a fairly good job of approximating the Alternative current (AC).

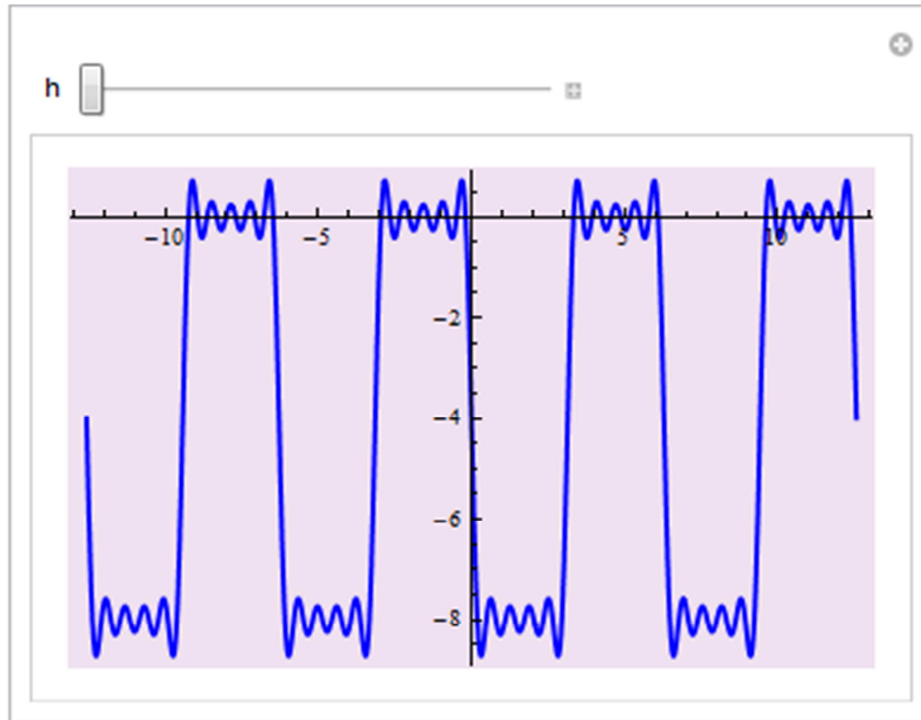


Figure 6. Upper part of square wave rectifier.

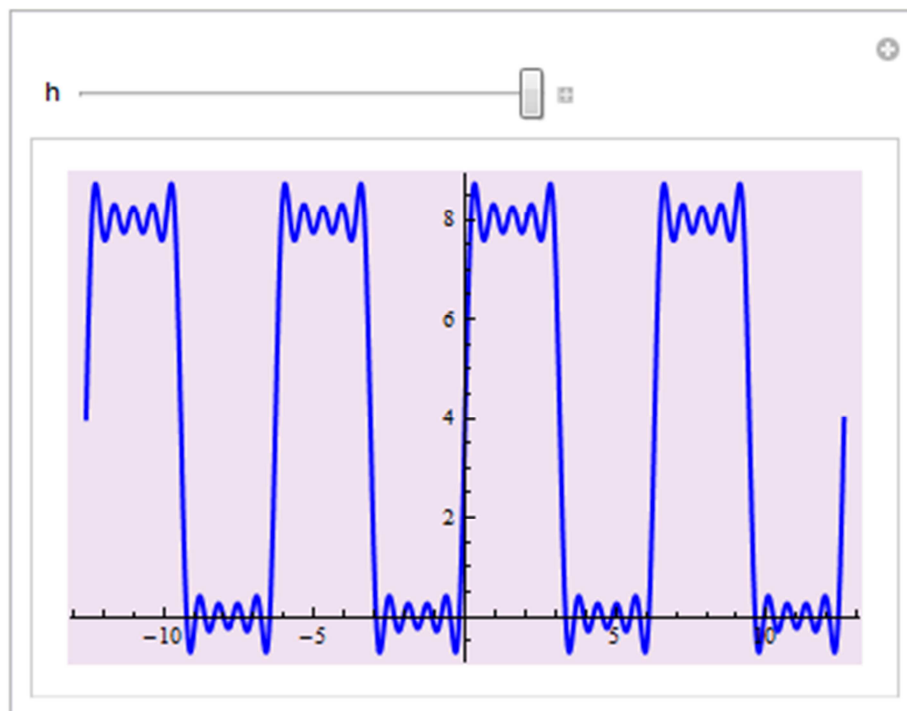


Figure 7. Lower part of square wave rectifier.

9. Example of Analog to Digital Conversion by Using Fourier Series

Consider the Fourier series of the following periodic function is

$$f(t) = \begin{cases} -\frac{h}{2}, & 0 < t < \pi \\ \frac{h}{2}, & \pi < t < 2\pi \end{cases} \quad (14)$$

Comparing equation (1) with equation (14), we obtain

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_0^{2\pi} f(t) dt \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} f(t) dt + \int_{\pi}^{2\pi} f(t) dt \right] \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} -\frac{h}{2} dt - \int_{\pi}^{2\pi} \frac{h}{2} dt \right] \\ &= \frac{1}{2\pi} \left[-\frac{\pi h}{2} - \frac{\pi h}{2} \right] = 0 \\ a_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \cos nt dt \end{aligned}$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} f(t) \cos nt dt + \int_{\pi}^{2\pi} f(t) \cos nt dt \right]$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} -\frac{h}{2} \cos nt dt - \int_{\pi}^{2\pi} \frac{h}{2} \cos nt dt \right]$$

$$= \frac{1}{\pi} \left[\frac{h}{2} \frac{\sin nt}{n} \right]_0^{\pi} - \left[\frac{h}{2} \frac{\sin nt}{n} \right]_{\pi}^{2\pi} = 0$$

$$\text{and } b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin nt dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} f(t) \sin nt dt + \int_{\pi}^{2\pi} f(t) \sin nt dt \right]$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} -\frac{h}{2} \sin nt dt - \int_{\pi}^{2\pi} \frac{h}{2} \sin nt dt \right]$$

$$= \frac{1}{\pi} \left[\frac{h}{2} \frac{\cos nt}{n} \right]_0^{\pi} - \left[\frac{h}{2} \frac{\cos nt}{n} \right]_{\pi}^{2\pi}$$

$$= \frac{-h(\cos[n\pi] - \cos[2n\pi] - 2\sin[\frac{n\pi}{2}]^2)}{2n\pi}$$

Now putting the value of a_0, a_n and b_n in (1), obtain the required Fourier series is

$$f(t) = \left[\frac{2h \sin t}{\pi} + \frac{2h \sin 3t}{3\pi} + \frac{2h \sin 5t}{5\pi} + \frac{2h \sin 7t}{7\pi} + \dots \right]$$

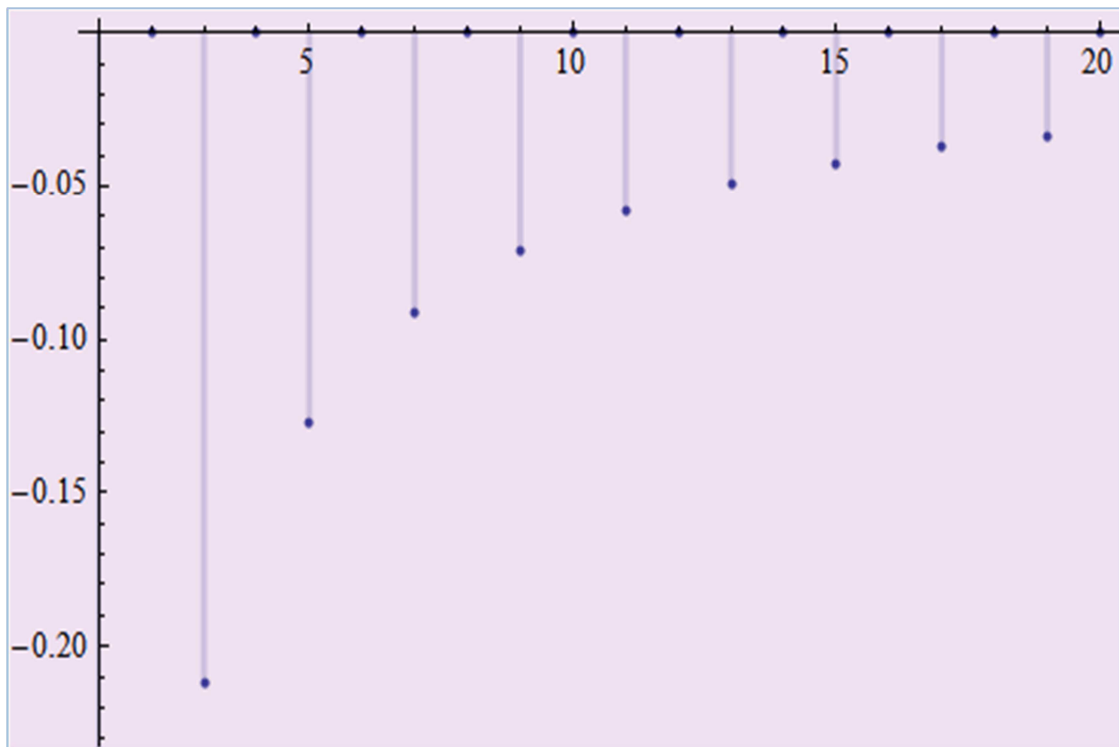


Figure 8. Discrete sketch of analog to Digital signal process.

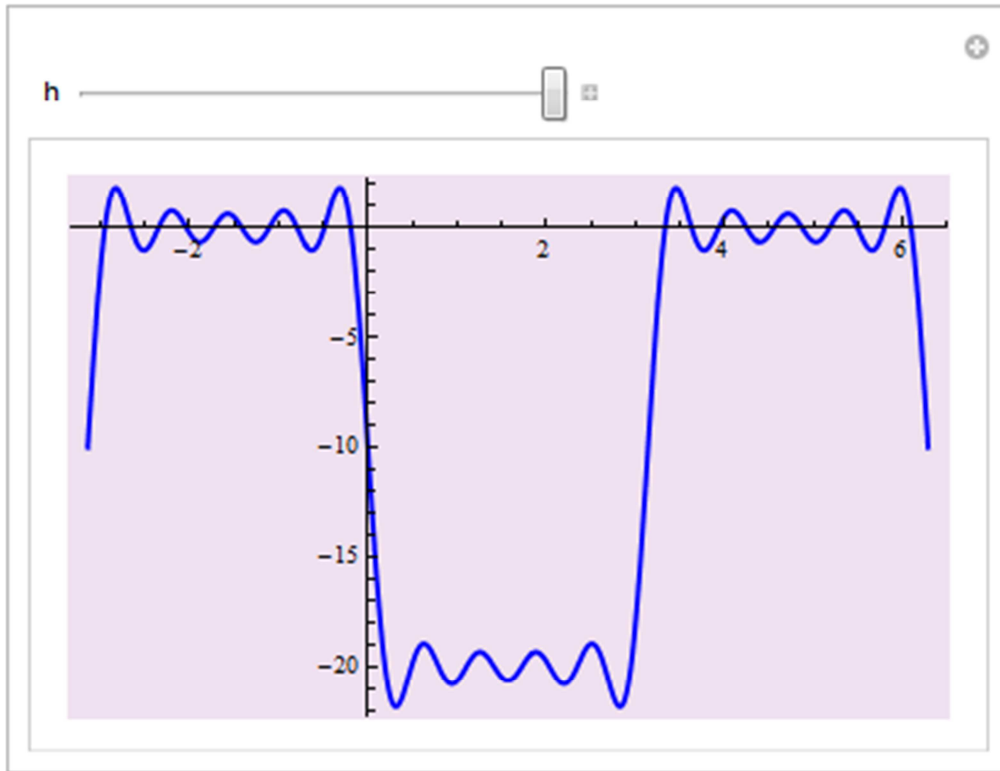


Figure 9. Upper part of square wave rectifier.

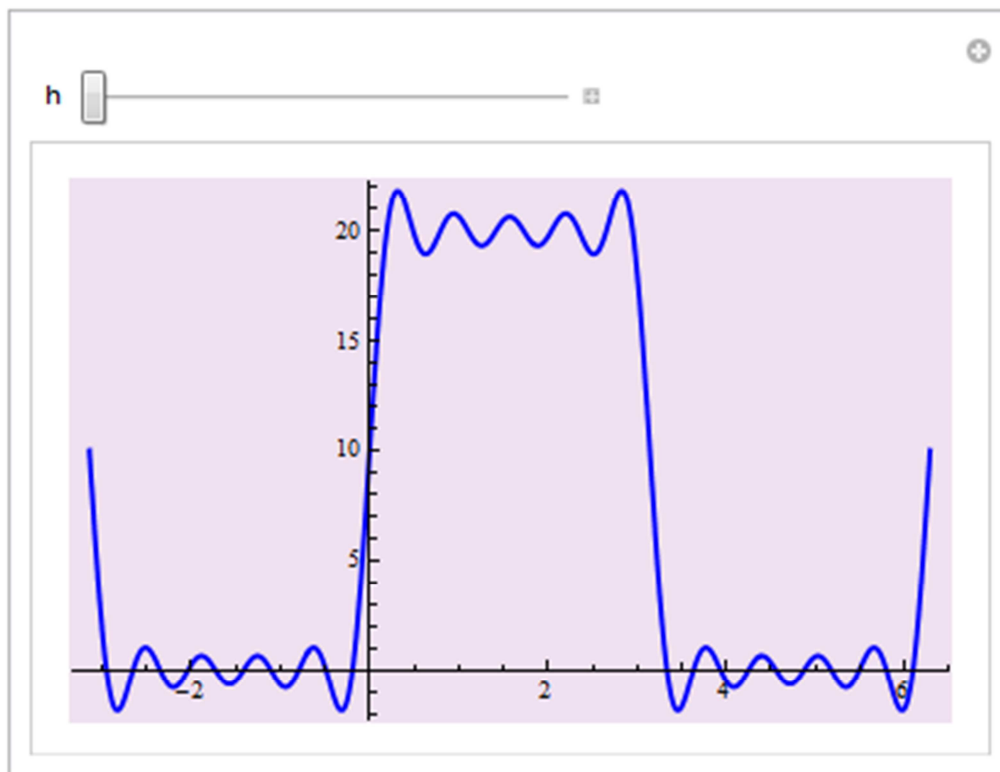


Figure 10. Lower part of square wave rectifier.

Thus the graph indicate that the series is convergent and has sum of $f(t)$.

10. Conclusion

Fourier series is useful in many applications ranging from

experimental instruments to rigorous mathematical analysis techniques. Thanks to modern developments in digital electronics and telecommunication. In this paper a brief overview Applications of Fourier transform. The primary use of Fourier transform of converting a time domain function into its frequency domain with digital communication signals. And we have seen Fourier converts signals from analog to digital signal, and how to square wave rectifier use in electricity. Fourier methods are commonly used for signal analysis and system design in modern electronics, The Spectrum Analyzer, information and communication technology. Like as cell phone, radio, television and vibration analysis. The Figure 1 represents the shape of 3D figure DFT equation (10) when $V_k = -5$ & $V_k = 5$ & $N = 6$ and $ik = -8$ & $ik = 8$ for spectrum analyzer it use in many laboratories. The Figure 2, Figure 3, Figure 4 represent pair wise frequency used in dual-tone multidimensional digital signal. The Figure 5 indicates the visualization periodogram creating the channel 12 frequency. The Figure 6, Figure 7 and Figure 9 and Figure 10 are square wave rectifier upper and lower part, the square wave rectifier does a fairly good job of approximating the Alternative current (AC). And Figure 8 discrete Fourier component analog to digital signal process.

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