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Construction of Solitary Wave Solutions of Higher-Order Nonlinear Partial Differential Equations Modeled in a Modified Nonlinear Noguchi Electrical Line

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Abstract

Nowadays, a soliton is considered as a future wave because it is a robust, a stable and a non-dissipative solitary wave. If one can use a soliton as a transmission signal in electrical lines, this will have advantages in the economic, technology and educative domains. Since the propagation of soliton is due to the interaction between nonlinearity and dispersion, it necessitates that the transmission medium should be nonlinear and dispersive. The physical system that one has chosen for this study is a Noguchi electrical line due to the fact that it is cheaper and very easy to manufacture than other transmission lines; then one find out analytically the variation that must undergo the charge of capacitors of that electrical line so that its transmission medium accepts the propagation of solitary wave of the wished type. The objective of this research is to define the analytical expression of the charge of capacitors in the line, to model higher-order nonlinear partial differential equations which govern the dynamics of those solitary waves in the line and to find out some exact solutions of solitary waves type of those equations. To attain our objective, one apply Kirchhoff laws to the circuit of a modified nonlinear Noguchi electrical line to model the higher-order nonlinear partial differential equations which describe the dynamics of those solitons. Then, one apply the direct and effective Bogning-Djeumen Tchaho-Kofane method based on the identification of basic hyperbolic function coefficients to construct some exact soliton solutions of modeled equations. The results obtained are supposed to permit: The amelioration of signals that will propagate in those line, the reduction of amplification stations of those lines, the facilitation of the choice of the type of the line relative to the type of signal one wishes to transmit, to augment the mathematical field knowledge, the manufacturing of new capacitors and new electrical lines susceptible of propagating those solitary waves.

Keywords

Noguchi Electrical Line, Construction, Model, Soliton Solution, Solitary Wave, Nonlinear Partial Differential Equation, Kink, Pulse

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1. Introduction

During the last decades, Solitary waves have evolved from a simple water wave to the propagation of Pulse solitons in optical fibers [1]. From the definition of solitary wave which is a wave capable to move on a long distance maintaining its shape and its velocity; it has come to mind that if such a signal is used in engineering of information through Noguchi electrical line, it will resist best on diverged dissipation

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factors. One has in this regard decided to come up with two definitions of nonlinear charge of capacitors constituting networks of modified Noguchi electrical line and apply them to model new higher-order nonlinear partial differential equations which describe the dynamics of solitary waves in the given line. The construction of solitary wave solutions of each modeled nonlinear partial differential equation by mathematical methods presented in [2-15] and new Bogning-Djeumen Tchaho-Kofane method presented in [16-21] has enabled to obtain solitary wave solutions of type Kink and Solitary wave solution of type pulse. The work presented in this paper is partitioned as follows: In part 2, one carries out the general presentation of Bogning-Djeumen Tchaho-Kofane methods, in part 3, one presents the general modeling of a modified Noguchi electrical line. In part 4, one constructs the solitary wave solution of type Kink of the obtained differential equation, in part 5, one constructs the solitary wave solution of type Pulse of the obtained differential equation. Finally, the conclusion is presented in part 6.

2. General Presentation of Bogning-Djeumen Tchaho-Kofane Method

Bogning, Djeumen and Kofane have developed an analytical method for obtaining solution of shape sech^n in certain class of nonlinear partial differential equations. This method is focused on the construction of solitary wave solution and has been adopted to facilitate the resolution of certain type of nonlinear partial differential equations where the nonlinear terms and dispersive terms coexist. This method of construction of the solitary wave solutions intends to look for the solutions of certain categories of nonlinear partial differential equations on the form

$$\gamma_i \sum_i \frac{\partial u}{\partial x_i} + b_i \sum_i \frac{\partial^2 u}{\partial x_i^2} + \ldots + c_i \sum_i \frac{\partial^l u}{\partial x_i^l} + d_i \sum_{m,n} \frac{\partial^n u \partial^m u}{\partial x_i^n \partial x_i^m} + f\left(u, |u|^2\right) = 0 . \tag{1}$$

Where γ_i, b_i, c_i, d_i are the constants; i, j, l, m, n the positive integer; f a linear arbitrary function of u and $|u|^2$; u the unknown to be determine and $|u|^2$ the magnitude of u. One looks for solution of equation (1) under the shape of a linear combination of the hyperbolic functions as follows

$$u = \sum_{ij} a_{ij} \frac{\sinh^i(\alpha x)}{\cosh^j(\alpha x)} . \tag{2}$$

Where α is a constant which depends on the system parameters which model the nonlinear partial differential equation and a_{ij}, the constants to be determined. Thus the combination of equation (1) and equation (2) permits to have an equation under the shape

$$\begin{aligned} &\sum \frac{F(a_{ij})}{\cosh^n(\alpha x)} + \sum G(a_{ij}) \frac{\sinh(\alpha x)}{\cosh^n(\alpha x)} + \sum H(a_{ij}) \cosh^k(\alpha x) \\ &+ \sum T(a_{ij}) \sinh(\alpha x) \cosh^l(\alpha x) + \sum W(a_{ij}) = 0. \end{aligned} \tag{3}$$

This equation presents five ranges of equations of coefficients a_{ij} which are: the range of the coefficients $F(a_{ij})$ of power $\frac{1}{\cosh^n(\alpha x)}$, the range of coefficients $G(a_{ij})$ of power $\frac{\sinh(\alpha x)}{\cosh^n(\alpha x)}$, the range of coefficients $H(a_{ij})$ of power $\cosh^k(\alpha x)$, the range of coefficients $T(a_{ij})$ of power $\sinh(\alpha x)\cosh^l(\alpha x)$ and the range of coefficients $w(a_{ij})$ of power 1. In the ranges of coefficients of $\frac{1}{\cosh^n(\alpha x)}$ and $\frac{\sinh(\alpha x)}{\cosh^n(\alpha x)}$, the equations that best seeks the solutions are those raised to the must elevated powers. In the ranges of coefficients of $\cosh^k(\alpha x)$ and $\sinh(\alpha x)\cosh^l(\alpha x)$ priority is given to the equations of coefficients of low powers. The last range of equations of coefficients $w(a_{ij})$ is not very important because it is considered like a confused domain for the correct solutions obtainable. One can classify these equations of coefficients in order of decreasing priority $F(a_{ij})$, $G(a_{ij})$, $H(a_{ij})$, $T(a_{ij})$ and $w(a_{ij})$. Here the importance or priority makes reference to the range that permits to obtain good results or merely that which tends more to the exact value. While identifying the coefficients of equation (1) to zero, one gets the first range

$$\sum_{ij} F\left(a_{ij}\right)=0 \tag{4}$$

and

$$\sum_{ij} G\left(a_{ij}\right)=0 . \tag{5}$$

In the set of the above equations, priority is given to equations which are to the power $\frac{1}{\cosh^{n}(\alpha x)}$ and $\frac{\sinh(\alpha x)}{\cosh^{n}(\alpha x)}$ from most elevated. But care must be taken because it is not the most elevated power which gives the best solution directly; it depends on the shape of solution considered from the onset, the symmetry of the equation to solve as well as from its nonlinearity degree. In these conditions, one moves directly to the equations of lower powers until the good equation to solve is obtained. In the case where the first two set of equations (4) and (5) don't give a satisfactory solution, one moves to the set of equations of the following range

$$\sum_{ij} H\left(a_{ij}\right)=0 \tag{6}$$

and

$$\sum_{ij} T\left(a_{ij}\right)=0 . \tag{7}$$

In the set of equation (6) and (7), priority is given to the equations of low powers of $\cosh^{k}(\alpha x)$ and $\sinh(\alpha x)\cosh^{l}(\alpha x)$. In general, the first two set of equations (4) and (5) permit to find the solution of the problem. In the case where that they don't give satisfactory solution, it would be cautious to change the shape of the solution or merely the form of solution we want to construct or simply the method. The ranges $w\left(a_{ij}\right)$ is considered now as the one that brings no reliable information. It is important to mention that this method appears complicated in the case where the properties of transformations of hyperbolic functions are not mastered. A mastery of these transformations reduces the difficulties considerably as regard to the calculations.

3. General Modeling of a Modified Nonlinear Noguchi Electrical Line

Let us consider in figure 1, a modified Noguchi electrical line constituting various identical networks numbered by positive integer numbers n. In network n, we have: an inductor with a constant inductance L connected in parallel with a capacitor having a constant capacitor C; i_n is the current flowing through the set; another capacitor whose charge varies in nonlinear manner relative to the voltage u_n across it.

It is to be noted that the electrical line presented in figure 1 is that of Noguchi which was modified by the fact that, the inductor with constant inductance that used to be connected in parallel with nonlinear capacitor was disconnected or has an elevated capacitance and constituting an open branch.

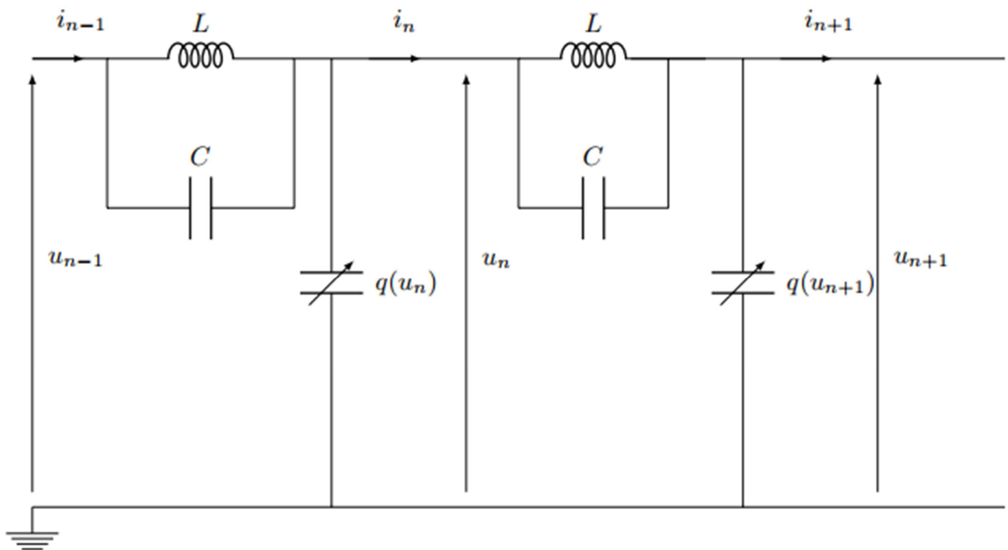

Figure 1. Presentation of a modified nonlinear Noguchi electrical line.

Applying Kirchhoff laws to the circuit of figure 1, one obtains the following equations

$$\frac{\partial i_{n-1}}{\partial t}=\frac{\partial i_n}{\partial t}+\frac{\partial^2 q_n}{\partial t^2}, \quad (8)$$

and

$$u_{n-1}=L\frac{\partial i_{n-1}}{\partial t}-LC\frac{\partial^2\left(u_{n-1}-u_n\right)}{\partial t^2}+u_n. \quad (9)$$

Substituting $\frac{\partial i_{n-1}}{\partial t}$ of (8) in (9) one obtains the equation below

$$u_{n-1}=L\frac{\partial i_n}{\partial t}+L\frac{\partial^2 q_n}{\partial t^2}-LC\frac{\partial^2\left(u_{n-1}-u_n\right)}{\partial t^2}+u_n. \quad (10)$$

Replacing $L\frac{\partial i_n}{\partial t}$ of (9) evaluated on the next order in (10), the following equation is obtained

$$u_{n+1}-2u_n+u_{n-1}+LC\frac{\partial^2\left(u_{n+1}-2u_n+u_{n-1}\right)}{\partial t^2}=L\frac{\partial^2 q_n}{\partial t^2}. \quad (11)$$

To obtain the continuum model, the left hand member of equation (11) must be approximated to a spatial partial derivative relative to $x=nh$ which is the distance measured from the beginning of the line. h indicates the distance that separates two consecutives nodes and is equivalent to the spatial sampling derivative period. As such we obtain the spatial partial derivative by using Taylor expansion of u_{n+1} and u_{n-1} closely to u_n considering the terms tills the fourth order in the following manner

$$u_{n+1}=u_n+\frac{h}{1!}\frac{\partial u_n}{\partial x}+\frac{h^2}{2!}\frac{\partial^2 u_n}{\partial x^2}+\frac{h^3}{3!}\frac{\partial^3 u_n}{\partial x^3}+\frac{h^4}{4!}\frac{\partial^4 u_n}{\partial x^4}. \quad (12)$$

and

$$u_{n-1}=u_n-\frac{h}{1!}\frac{\partial u_n}{\partial x}+\frac{h^2}{2!}\frac{\partial^2 u_n}{\partial x^2}-\frac{h^3}{3!}\frac{\partial^3 u_n}{\partial x^3}+\frac{h^4}{4!}\frac{\partial^4 u_n}{\partial x^4}. \quad (13)$$

Equations (12) and (13) permit us to have

$$u_{n+1}-2u_n+u_{n-1}=h^2\frac{\partial^2 u_n}{\partial x^2}+\frac{h^4}{12}\frac{\partial^4 u_n}{\partial x^4}. \quad (14)$$

Using equation (14), equation (11) can be rewritten as follows

$$h^2\frac{\partial^2 u_n}{\partial x^2}+\frac{h^4}{12}\frac{\partial^4 u_n}{\partial x^4}+LCh^2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 u_n}{\partial x^2}\right)+\frac{LCh^4}{12}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^4 u_n}{\partial x^4}\right)-L\frac{\partial^2 q_n}{\partial t^2}=0. \quad (15)$$

We can therefore present the general continuum model of a modified nonlinear Noguchi electrical line shown in figure 1 by the partial differential equation written as follows

$$\frac{h^4}{12}\frac{\partial^4 u(x,t)}{\partial x^4}+h^2\frac{\partial^2 u(x,t)}{\partial x^2}+\frac{LCh^4}{12}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^4 u(x,t)}{\partial x^4}\right)+LCh^2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)-L\frac{\partial^2 q(u(x,t))}{\partial t^2}=0. \quad (16)$$

4. Construction of Solitary Wave Solutions of Type Kink Relative to the Partial Differential Equation (16)

We define the nonlinear charge of the capacitor under the analytical shape given as follows:

$$q(u(x,t))=A_1u(x,t)+A_2u^3(x,t)+A_3u^5(x,t)+A_4\ln\left(\frac{u(x,t)+A_0}{u(x,t)-A_0}\right). \quad (17)$$

With $|u(x,t)|>|A_0|$. A_1; A_2; A_3 and A_4 are non-nil real numbers whose conditions of choice will be established. Replacing $q(u(x,t))$ of (17) in differential equation (16) we obtain higher order nonlinear partial differential equation given below

$$
\begin{aligned}
&\left(2LCh^4A_0^2u^2(x,t)-LCh^4u^4(x,t)-LCh^4A_0^4\right)\frac{\partial^6 u(x,t)}{\partial x^4\partial t^2}+\begin{pmatrix}-12LCh^2u^4(x,t)\\+24LCh^2A_0^2u^2(x,t)\\-12LCh^2A_0^4\end{pmatrix}\frac{\partial^4 u(x,t)}{\partial x^2\partial t^2}\\
&+\begin{pmatrix}\left(36LA_2A_0^4-24LA_1A_0^2-24LA_4A_0\right)u^2(x,t)\\+\left(12LA_1-72LA_2A_0^2+60LA_3A_0^4\right)u^4(x,t)\\+\left(-120LA_3A_0^2+36LA_2\right)u^6(x,t)+60LA_3u^8(x,t)\\+12LA_1A_0^4+24LA_4A_0^3\end{pmatrix}\frac{\partial^2 u(x,t)}{\partial t^2}\\
&+\left(-h^2u^4(x,t)+2h^4A_0^2u^2(x,t)-h^4A_0^4\right)\frac{\partial^4 u(x,t)}{\partial x^4}\\
&+\left(-12h^2u^4(x,t)+24h^2A_0^2u^2(x,t)-12h^2A_0^4\right)\frac{\partial^2 u(x,t)}{\partial x^2}\\
&+\begin{pmatrix}\left(72LA_2A_0^4+48LA_4A_0\right)u(x,t)\\+\left(240LA_3A_0^4-144LA_2A_0^2\right)u^3(x,t)\\+\left(-480LA_3A_0^2+72LA_2\right)u^5(x,t)+240LA_3u^7(x,t)\end{pmatrix}\left(\frac{\partial u(x,t)}{\partial t}\right)^2=0.
\end{aligned}
\tag{18}
$$

Let us find out the solution of differential equation (18) under the analytical shape written as follow

$$u(x,t)=a\tanh(kx-vt). \tag{19}$$

Where a, k and v are non-nil real numbers that will be determined in terms of Noguchi electrical line parameters. Replacing $u(x,t)$ of (19) in equation (18), one obtains the following:

$$
\begin{aligned}
&\begin{pmatrix}8h^4a^5k^4-240LA_3a^7v^2+96LCh^2a^5k^2v^2\\-120La^3a^9v^2+24h^2a^5k^2+48LA_4a^3A_0v^2\\-16h^4a^3A_0^2k^4+24h^2A_0^4ak^2-24LA_1a^5v^2\\-72LA_2a^3v^2A_0^4+32LCh^4A_0^4ak^2v^2\\-192LCh^2a^3A_0^2k^2v^2+72LA_2a^7v^2\\+96LCh^2aA_0^4k^2v^2+32LCh^4a^5k^4v^2\\+144LA_2a^5v^2A_0^2-24LA_1av^2A_0^4\\+48LA_1a^3v^2A_0^2+8h^4A_0^4ak^4\\-64LCh^4a^3A_0^2k^4v^2-48LA_4av^2A_0^3\\-48h^2A_0^2a^3k^2-120LA_3a^5A_0^4v^2\end{pmatrix}\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)}+\begin{pmatrix}768LCh^2a^3A_0^2k^2v^2-48h^2a^5k^2\\-480LCh^2a^5k^2v^2-48LA_1a^3v^2A_0^2\\-40h^4a^5k^4-1200LA_3a^7A_0^2v^2\\-480LCh^4aA_0^4k^4v^2+480LA_3a^5A_0^4v^2\\+64h^4A_0^2a^3k^4+720LA_3a^9v^2\\-288LCh^2aA_0^4k^2v^2+288LA_2a^7v^2\\-24h^4A_0^4ak^4+144LA_2a^3v^2A_0^4\\-432LA_2a^5v^2A_0^2-544LCh^4a^5k^4v^2\\+48LA_1a^5v^2+48h^2a^3A_0^2k^2+1024LCh^4a^3k^4v^2\end{pmatrix}\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)}\\
&+\begin{pmatrix}1680LA_3a^7A_0^2v^2+24h^2a^5k^2\\-572LCh^4a^3A_0^2k^2v^2+56c+1440LA_3a^9v^2\\-48h^4A_0^2a^3k^4-24LA_1a^5v^2-360LA_3a^5A_0^4v^2\\+672LCh^2a^5k^2v^2+720LCh^4aA_0^4k^4v^2\\+288LA_2a^5A_0^2v^2+1712LCh^4a^5k^2v^2\\-2400LCh^4a^3A_0^2k^4v^2-360LA_2a^7v^2\end{pmatrix}\frac{\sinh(kx-vt)}{\cosh^7(kx-vt)}+\begin{pmatrix}1200LA_3a^9v^2-288LCh^2a^5k^2v^2\\-720LA_3a^7A_0^2v^2-24h^4a^5k^4+144LA_2a^7v^2\\-1920LCh^4a^5k^4v^2+1440LCh^4a^3k^4v^2\end{pmatrix}\frac{\sinh(kx-vt)}{\cosh^9(kx-vt)}\\
&+\left(-360LA_3a^9v^2+720LCh^4a^5k^4v^2\right)\frac{\sinh(kx-vt)}{\cosh^{11}(kx-vt)}=0.
\end{aligned}
\tag{20}
$$

Equation (20) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permit to obtain the set of equations presented below

$$\begin{cases}
\begin{pmatrix}
8h^4a^5k^4-240LA_3a^7v^2+96LCh^2a^5k^2v^2-120La^3a^9v^2+24h^2a^5k^2\\
+48LA_4a^3A_0v^2-16h^4a^3A_0^2k^4+24h^2A_0^4ak^2-24LA_1a^5v^2-72LA_2a^3v^2A_0^4\\
+32LCh^4A_0^4ak^2v^2-192LCh^2a^3A_0^2k^2v^2+72LA_2a^7v^2\\
+96LCh^2aA_0^4k^2v^2+32LCh^4a^5k^4v^2+144LA_2a^5v^2A_0^2\\
-24LA_1av^2A_0^4+48LA_1a^3v^2A_0^2+8h^4A_0^4ak^4\\
-64LCh^4a^3A_0^2k^4v^2-48LA_4av^2A_0^3-48h^2A_0^2a^3k^2-120LA_3a^5A_0^4v^2
\end{pmatrix}=0\\
\begin{pmatrix}
768LCh^2a^3A_0^2k^2v^2-48h^2a^5k^2-480LCh^2a^5k^2v^2-48LA_1a^3v^2A_0^2\\
-40h^4a^5k^4-1200LA_3a^7A_0^2v^2-480LCh^4aA_0^4k^4v^2+480LA_3a^5A_0^4v^2\\
+64h^4A_0^2a^3k^4+720LA_3a^9v^2-288LCh^2aA_0^4k^2v^2+288LA_2a^7v^2\\
-24h^4A_0^4ak^4+144LA_2a^3v^2A_0^4-432LA_2a^5v^2A_0^2-544LCh^4a^5k^4v^2\\
+48LA_1a^5v^2+48h^2a^3A_0^2k^2+1024LCh^4a^3k^4v^2
\end{pmatrix}=0\\
\begin{pmatrix}
1680LA_3a^7A_0^2v^2+24h^2a^5k^2-572LCh^4a^3A_0^2k^2v^2+56c+1440LA_3a^9v^2\\
-48h^4A_0^2a^3k^4-24LA_1a^5v^2-360LA_3a^5A_0^4v^2+672LCh^2a^5k^2v^2+720LCh^4aA_0^4k^4v^2\\
+288LA_2a^5A_0^2v^2+1712LCh^4a^5k^2v^2-2400LCh^4a^3A_0^2k^4v^2-360LA_2a^7v^2
\end{pmatrix}=0\\
\begin{pmatrix}
1200LA_3a^9v^2-288LCh^2a^5k^2v^2-720LA_3a^7A_0^2v^2-24h^4a^5k^4+144LA_2a^7v^2\\
-1920LCh^4a^5k^4v^2+1440LCh^4a^3k^4v^2
\end{pmatrix}=0\\
\left(-360LA_3a^9v^2+720LCh^4a^5k^4v^2\right)=0.
\end{cases} \tag{21}$$

Solving the set of equation (21), one obtains the solution and the conditions given in (22) of higher-order nonlinear partial differential equation (18) which govern the dynamics of solitary wave of type Kink in the modeled Noguchi electrical line

$$a=A_0\,;\;k=\pm\frac{A_0}{h}\left(\frac{A_3}{2C}\right)^{\frac{1}{4}};\;v=\pm\frac{A_0\sqrt{A_3}}{2\sqrt{LC\left(3A_2-3\sqrt{2CA_3}+5A_3A_0^2\right)}}\,;$$

$$A_1=-\frac{-6A_2\sqrt{2C}+12C\sqrt{A_3}-10A_3A_0^2\sqrt{2C}+A_2A_0^2\sqrt{A_3}+A_0^4A_3^{\frac{3}{2}}}{\sqrt{A_3}}$$

$$A_4=\frac{A_0\left(-6A_2\sqrt{2C}+12C\sqrt{A_3}-10A_3A_0^2\sqrt{2C}+A_2A_0^2\sqrt{A_3}+A_0^4A_3^{\frac{3}{2}}\right)}{2\sqrt{A_3}}\,;\;A_3>0\;\;3A_2+5A_3A_0^2>3\sqrt{2CA_3}\,;$$

$$u(x,t)=A_0\tanh\left(\pm\frac{A_0}{h}\left(\frac{A_3}{2C}\right)^{\frac{1}{4}}x\pm\frac{A_0\sqrt{A_3}}{2\sqrt{LC\left(3A_2-3\sqrt{2CA_3}+5A_3A_0^2\right)}}t\right). \tag{22}$$

5. Construction of Solitary Wave Solutions of Type Pulse Relative to the Partial Differential Equation (16)

We define the nonlinear charge of the capacitor under the analytical shape given below

$$q(u(x,t))=A_1u(x,t)+A_2u^3(x,t)+A_3u^5(x,t). \tag{23}$$

Where A_1; A_2; A_3 and A_4 are non-zeros real numbers whose conditions of choice will be established. Replacing $q(u(x,t))$ of (23) in differential equation (16) one obtains the nonlinear partial differential equation given as follow

$$
\begin{aligned}
&\frac{h^4}{12}\frac{\partial^4 u(x,t)}{\partial x^4}+h^2\frac{\partial^2 u(x,t)}{\partial x^2}+\frac{LCh^4}{12}\frac{\partial^6 u(x,t)}{\partial x^4\partial t^2}+LCh^2\frac{\partial^4 u(x,t)}{\partial x^2\partial t^2}\\
&+\left(-LA_1-3LA_2u^2(x,t)-5LA_3u^4(x,t)\right)\frac{\partial^2 u(x,t)}{\partial t^2}\\
&+\left(-6LA_2u(x,t)-20LA_3u^3(x,t)\right)\left(\frac{\partial u(x,t)}{\partial t}\right)^2=0.
\end{aligned}
\tag{24}
$$

Let us find the solution of equation (24) under the analytical shape

$$
u(x,t)=a\,\mathrm{sech}\left(kx-vt\right). \tag{25}
$$

Where a, k and v are non-nil real numbers that will be determined in terms of Noguchi electrical line parameters. Replacing $u(x,t)$ of (25) in equation (24) yields following equation

$$
\begin{aligned}
&\left(12LA_2a^3v^2+24LCh^2av^2k^2-25LA_3a^5v^2+70LCh^4av^2k^4+2h^4ak^4\right)\frac{1}{\cosh^5\left(kx-vt\right)}\\
&+\left(-LA_1av^2+h^2ak^2+\frac{1}{12}h^4ak^4+LCh^2av^2k^2+\frac{1}{12}LCh^4av^2k^4\right)\frac{1}{\cosh\left(kx-vt\right)}\\
&+\left(-2h^2ak^2-\frac{91}{6}LCh^4av^2k^4-9LA_2a^3v^2-20LCh^2av^2k^2+2LA_1av^2-\frac{5}{3}h^4ak^4\right)\frac{1}{\cosh^3\left(kx-vt\right)}\\
&+\left(-60LCh^4av^2k^4+30LA_3a^5v^2\right)\frac{1}{\cosh^7\left(kx-vt\right)}=0.
\end{aligned}
\tag{26}
$$

Equation (26) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permit us to obtain the set of four equations written below

$$
\begin{cases}
12LA_2a^3v^2+24LCh^2av^2k^2-25LA_3a^5v^2+70LCh^4av^2k^4+2h^4ak^4=0,\\
-LA_1av^2+h^2ak^2+\frac{1}{12}h^4ak^4+LCh^2av^2k^2+\frac{1}{12}LCh^4av^2k^4=0,\\
-2h^2ak^2-\frac{91}{6}LCh^4av^2k^4-9LA_2a^3v^2-20LCh^2av^2k^2+2LA_1av^2-\frac{5}{3}h^4ak^4=0,\\
-60LCh^4av^2k^4+30LA_3a^5v^2=0.
\end{cases}
\tag{27}
$$

Solving the set of nonlinear equations (27), one obtains the solutions and the conditions presented in (28) of nonlinear partial differential equation (24) which governs the dynamics of solitary waves of type Pulse in the modeled Noguchi electrical line:

$$
v=\pm\frac{1}{2L}\left(\frac{-\left(\begin{aligned}&120A_3^{\frac{7}{2}}C^{\frac{7}{2}}+54\sqrt{2}A_3^3C^3A_2L+20A_1A_3^{\frac{7}{2}}C^{\frac{5}{2}}L-6LA_3^{\frac{5}{2}}C^{\frac{5}{2}}A_2^2\\&-6\left(\begin{aligned}&256A_3^7C^7L^2+224\sqrt{2}A_3^{\frac{13}{2}}L^2A_2-12A_3^7C^6L^2A_1+66A_3^6C^2L^2A_2^2\\&-12\sqrt{2}A_3^{\frac{13}{2}}C^{\frac{11}{2}}A_2L^2A_1-14\sqrt{2}A_3^{\frac{11}{2}}C^{\frac{11}{2}}A_2^3L^2-6A_1A_3^6C^5L^2A_2^2+L^2A_3^5C^5A_2^4\end{aligned}\right)^{\frac{1}{2}}\end{aligned}\right)}{-3A_3^{\frac{5}{2}}C^{\frac{7}{2}}A_2^2+24\sqrt{2}A_2C^4A_3^3+54A_3^{\frac{7}{2}}C^{\frac{9}{2}}+50A_1A_3^{\frac{7}{2}}C^{\frac{7}{2}}}\right)^{\frac{1}{2}};
$$

$$a=\frac{2v\sqrt{-3A_3LC^{\frac{3}{2}}\left(10Lv^2C+1\right)\left(A_2C+\sqrt{2A_3}\right)}}{A_3\left(10Lv^2C+1\right)};\ k=\pm\frac{a}{h}\left(\frac{A_3}{2C}\right)^{\frac{1}{4}};\ u(x,t)=a\,\mathrm{sech}\left(kx-vt\right). \quad (28)$$

Let's recall that a; v; and K must be evaluated with real numbers A_1 ; A_2 and A_3 chosen in sort a way that the expressions in the roots should be positive.

6. Conclusion

At the end of this work where one has modeled a modified Noguchi electrical line by two different higher-order nonlinear partial differential equations which have permitted us to construct solitary wave solutions; it is necessary to point out that the results obtained will first of all enable in physics and engineering of telecommunication, the manufacturing of two new transmission lines notably those of Noguchi electrical line presented in figure 1 whose charge of nonlinear capacitors of its networks varies for one in nonlinear manner defined in (17) and the other in nonlinear manner defined in (24). In addition, these results will permit an amelioration of the quality of signals that will be propagated in the new lines. In fact, those signals are solitary waves of type Kink obtained in (22) and type Pulse obtained in (28) which by their definition are propagated on a long distance without changing their shape and velocity by resisting best on different dissipation factors. Finally, in a typical mathematical domain, the results obtained has permitted to define in (18) and in (24) two new higher-order nonlinear partial differential equations which have respectively exact solitary wave solutions (22) and (28); this by increasing the field of mathematical knowledge. In order to bring new ideas on the stability of the obtained solitary waves; it is necessary for us to study next their modulational instability before carrying out a practical study where we will experiment the applicability and the perfection of those two new lines.

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