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Construction of Solutions in the Shape (Pulse; Pulse) and (Kink; Kink) of a Set of Two Equations Modeled in a Nonlinear Inductive Electrical Line with Crosslink Capacitor

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Abstract

In the course of the last years, a soliton has been the must soliciting solitary wave for the propagation of energy or information through transmission media. This is due to its stable properties and it does not dissipate energy in the course of its propagation. Solitons are encountered in the analysis of water wave, plasmas, fiber optics, shock compression and nonlinear transmission line. The physical system that had been studied in this paper is a nonlinear inductive electrical line with crosslink capacitor reason being that it is cheapest, easy to manufacture and therefore more accessible than any other transmission lines. One apply Kirchhoff laws to the circuit of a nonlinear inductive electrical line with crosslink capacitor to model new set of two partial differential equations with higher-order of nonlinearity which govern the dynamics of the set of two solitary waves on the given line. Many authors look for numerical solutions of those nonlinear equations whereas exact analytical solutions lead best to the information of the electrical line. Therefore the construction of the coupled solitary wave solutions of these equations by the direct and effective Bogning-Djeumen Tchaho-Kofane method which is based on the identification of basic hyperbolic function coefficients has permitted one to realize that solitary wave of type (Kink; Kink) and type (Pulse; pulse) are easily propagated through the line when certain conditions we have presented are respected. The results obtained are supposed to permit the amelioration of signals that will propagate in those lines and the reduction of amplification station of those lines. The inductive electrical line with crosslink capacitor that we are studying is advantageous for the fact that it permits simultaneously the propagation of a set of two solitary waves contrary to a non-coupled inductive electrical line which only enables the propagation of one solitary wave when the signal considered is the current; the more we will multiply the crosslink in the line, the more we will multiply the simultaneous propagation of solitary wave in the line.

Keywords

Inductive Electrical Line, Crosslink Capacitor, Modeling, Construction, Soliton Solution, Coupled Solution, Coupled Solitary Wave, Solitary Wave, Nonlinear Partial Differential Equation, Kink, Pulse

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1. Introduction

Solitary waves, have evolved from simple water waves to the propagation of solitons in optical fibers [1]. Since a solitary wave is defined as a wave capable of propagating on longer distances without changing its shape and its velocity; it has come in mind the fact that if one of such signals is used in engineering of information through an inductive electrical line with crosslink capacitor, it will resist best on different

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dissipation factors. In this effect, one has decided to render definitions of nonlinear magnetic flux linkage of inductors constituting the two nonlinear parts linked through the capacitors in the said line. Then, one has applied them to model new set of two nonlinear partial differential equations, which govern the dynamics of a coupled solitary wave in the given line. In order to construct exact coupled solitary wave solutions of every set of two nonlinear partial differential equations obtained, we rely first on methods presented in [2-15]. Furthermore, one has decided to adopt the new Bogning-Djeumem Tchaho-Kofane method [16-21] reason being that it facilitates the construction of a solitary wave solution by identification of the basic hyperbolic function coefficients of nonlinear partial differential equations in a direct and effective manner. Having solved the equations, we have come up with solitary wave solutions of type (Kink; Kink) and type (Pulse; Pulse). The work presented in this paper is partitioned as follows: In part 2, we present a general modeling of a nonlinear inductive electrical line with crosslink capacitor; In part 3, we construct a coupled solitary wave solutions of type (Kink; Kink); In part 4, we construct a coupled solitary wave solutions of type (Pulse; Pulse) and we present at the end the conclusion in part 5.

2. General Modeling of a Nonlinear Inductive Electrical Line with Crosslink Capacitor

Let us consider a nonlinear inductive electrical line shown in figure 1. The line is constituted by a good number of identical networks numbered by the positive integer n. The network number n is constituted by a capacitor with capacitance C_0 which link the two nonlinear inductive parts; two inductors in which each of the magnetic flux linkage ϕ_1^n and ϕ_2^n changes respectively in nonlinear manner in terms of the currents i_1^n and i_2^n which flow through them; u_1^n and u_2^n are respectively the voltage across each capacitor with capacitance C_1 and C_2.

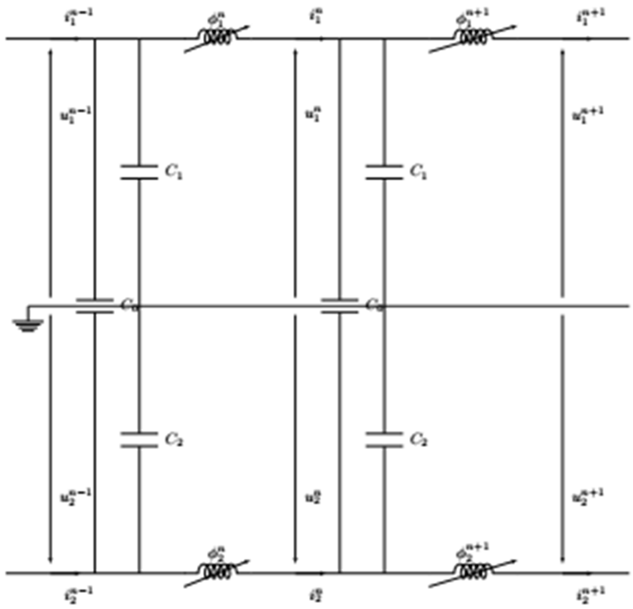

Figure 1. Presentation of a nonlinear inductive electrical line with crosslink capacitor.

Applying Kirchhoff's laws to the circuit shown in figure 1, one obtain the following equations:

$$u_1^n - u_1^{n-1} = -\frac{\partial \phi_1^n}{\partial t} \tag{1}$$

$$u_2^n - u_2^{n-1} = -\frac{\partial \phi_2^n}{\partial t} \tag{2}$$

$$i_1^{n-1} - i_1^n = C_0 \frac{\partial\left(u_1^{n-1} - u_2^{n-1}\right)}{\partial t} + C_1 \frac{\partial u_1^{n-1}}{\partial t} \tag{3}$$

$$i_2^{n-1} - i_2^n = -C_0 \frac{\partial\left(u_1^{n-1} - u_2^{n-1}\right)}{\partial t} + C_2 \frac{\partial u_2^{n-1}}{\partial t} \tag{4}$$

Relations (1), (2), (3) and (4) permit one to obtain the following system:

$$\begin{cases} i_1^{n-1} - i_1^n = C_0 \dfrac{\partial^2\left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_1 \dfrac{\partial^2 \phi_1^n}{\partial t^2} + C_0 \dfrac{\partial\left(u_1^n - u_2^n\right)}{\partial t} + C_1 \dfrac{\partial u_1^n}{\partial t} \\ i_2^{n-1} - i_2^n = -C_0 \dfrac{\partial^2\left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_2 \dfrac{\partial^2 \phi_2^n}{\partial t^2} - C_0 \dfrac{\partial\left(u_1^n - u_2^n\right)}{\partial t} + C_2 \dfrac{\partial u_2^n}{\partial t} \end{cases} \tag{5}$$

Considering (3) and (4) in next order, one rewrites a set of two partial differential equations (5) in the form below:

$$\begin{cases} i_1^{n+1} - 2i_1^n + i_1^{n-1} = C_0 \dfrac{\partial^2 \left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_1 \dfrac{\partial^2 \phi_1^n}{\partial t^2} \\ i_2^{n+1} - 2i_2^n + i_2^{n-1} = -C_0 \dfrac{\partial^2 \left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_2 \dfrac{\partial^2 \phi_2^n}{\partial t^2} \end{cases} \quad (6)$$

To obtain the continuum model, the left hand side of each equation in (6) has to be approximated to a spatial partial derivative with respect to $x = nh$ which represents the distance measured from the beginning of the line. h represents the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivative period. One obtains as such spatial partial derivatives using Taylor expansion of i_1^{n+1} and i_1^{n-1} closely to i_1^n by considering the terms till fourth order in the following manner:

$$i_1^{n+1} = i_1^n + \frac{h}{1!}\frac{\partial i_1^n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_1^n}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 i_1^n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_1^n}{\partial x^4} \quad (7)$$

$$i_1^{n-1} = i_1^n - \frac{h}{1!}\frac{\partial i_1^n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_1^n}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 i_1^n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_1^n}{\partial x^4} \quad (8)$$

$$i_1^{n+1} - 2i_1^n + i_1^{n-1} = \frac{h^2}{1}\frac{\partial^2 i_1^n}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4 i_1^n}{\partial x^4} \quad (9)$$

In the same light using Taylor expansion of i_2^{n+1} and i_2^{n-1} closely to i_2^n by considering the terms till fourth order one obtains the equation below:

$$i_2^{n+1} - 2i_2^n + i_2^{n-1} = \frac{h^2}{1}\frac{\partial^2 i_2^n}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4 i_2^n}{\partial x^4} \quad (10)$$

Equation (9) and (10) permits to rewrite the set of two differential equations (6) as follows:

$$\begin{cases} \dfrac{h^4}{12}\dfrac{\partial^4 i_1^n}{\partial x^4} + h^2 \dfrac{\partial^2 i_1^n}{\partial x^2} = C_0 \dfrac{\partial^2 \left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_1 \dfrac{\partial^2 \phi_1^n}{\partial t^2} \\ \dfrac{h^4}{12}\dfrac{\partial^4 i_2^n}{\partial x^4} + h^2 \dfrac{\partial^2 i_2^n}{\partial x^2} = -C_0 \dfrac{\partial^2 \left(\phi_1^n - \phi_2^n\right)}{\partial t^2} + C_2 \dfrac{\partial^2 \phi_2^n}{\partial t^2} \end{cases} \quad (11)$$

Finally, one obtains the continuum model of the nonlinear inductive electrical line with crosslink capacitor presented in figure 1 by the set of two nonlinear partial differential equation below:

$$\begin{cases} \dfrac{h^4}{12}\dfrac{\partial^4 i_1(x,t)}{\partial x^4} + h^2 \dfrac{\partial^2 i_1(x,t)}{\partial x^2} = C_0 \dfrac{\partial^2 \left(\phi_1\left(i_1(x,t)\right) - \phi_2\left(i_2(x,t)\right)\right)}{\partial t^2} + C_1 \dfrac{\partial^2 \phi_1\left(i_1(x,t)\right)}{\partial t^2} \\ \dfrac{h^4}{12}\dfrac{\partial^4 i_2(x,t)}{\partial x^4} + h^2 \dfrac{\partial^2 i_2(x,t)}{\partial x^2} = -C_0 \dfrac{\partial^2 \left(\phi_1\left(i_1(x,t)\right) - \phi_2\left(i_2(x,t)\right)\right)}{\partial t^2} + C_2 \dfrac{\partial^2 \phi_1\left(i_2(x,t)\right)}{\partial t^2} \end{cases} \quad (12)$$

3. Construction of Solitary Wave Solution in the Shape (Kink; Kink) of the Set of Two Partial Differential Equations (12)

Let us define each nonlinear magnetic flux linkage of the two inductors constituting each of the two linked parts under the analytical shape as follows:

$$\begin{cases} \phi_1\left(i_1(x,t)\right) = A_1 i_1(x,t) + A_2 i_1^3(x,t) + A_3 \ln\left(i_1(x,t) + A_0\right) + A_4 \ln\left(i_1(x,t) - A_0\right) \\ \phi_2\left(i_2(x,t)\right) = B_1 i_2(x,t) + B_2 i_2^3(x,t) + B_3 \ln\left(i_2(x,t) + B_0\right) + B_4 \ln\left(i_2(x,t) - B_0\right) \end{cases} \quad (13)$$

With $\left|i_1(x,t)\right| > \left|A_0\right|$; $\left|i_2(x,t)\right| > \left|B_0\right|$. A_1 ; A_2 ; A_3 ; A_4 ; B_1 ; B_2 ; B_3 and B_4 are non-zero real numbers which will be chosen conveniently. By substituting each of the nonlinear flux linkage $\phi_1\left(i_1(x,t)\right)$ and $\phi_2\left(i_2(x,t)\right)$ of (13) in (12) we obtain the set of two nonlinear partial differential equations written as:

$$\begin{cases} \frac{h^4}{12}\frac{\partial^4 i_1(x,t)}{\partial x^4}+h^2\frac{\partial^2 i_1(x,t)}{\partial x^2}+\frac{C_0h^4}{12(C_0+C_2)}\frac{\partial^4 i_2(x,t)}{\partial x^4}+\frac{C_0h^2}{(C_0+C_2)}\frac{\partial^2 i_2(x,t)}{\partial x^2} \\ +\left(\frac{C_0^2}{C_0+C_2}-C_0-C_1\right)\left(A_1+3A_2i_1^2(x,t)+\frac{A_3}{i_1(x,t)+A_0}+\frac{A_4}{i_1(x,t)-A_0}\right)\frac{\partial^2 i_1(x,t)}{\partial t^2} \\ +\left(\frac{C_0^2}{C_0+C_2}-C_0-C_1\right)\left(6A_2i_1(x,t)-\frac{A_3}{\left(i_1(x,t)+A_0\right)^2}-\frac{A_4}{\left(i_1(x,t)-A_0\right)^2}\right)\left(\frac{\partial i_1(x,t)}{\partial t}\right)^2=0 \\ \frac{h^4}{12}\frac{\partial^4 i_2(x,t)}{\partial x^4}+h^2\frac{\partial^2 i_2(x,t)}{\partial x^2}+\frac{C_0h^4}{12(C_0+C_1)}\frac{\partial^4 i_1(x,t)}{\partial x^4}+\frac{C_0h^2}{(C_0+C_1)}\frac{\partial^2 i_1(x,t)}{\partial x^2} \\ +\left(\frac{C_0^2}{C_0+C_1}-C_0-C_2\right)\left(B_1+3B_2i_2^2(x,t)+\frac{B_3}{i_2(x,t)+B_0}+\frac{B_4}{i_2(x,t)-B_0}\right)\frac{\partial^2 i_2(x,t)}{\partial t^2} \\ +\left(\frac{C_0^2}{C_0+C_1}-C_0-C_2\right)\left(6B_2i_2(x,t)-\frac{B_3}{\left(i_2(x,t)+B_0\right)^2}-\frac{B_4}{\left(i_2(x,t)-B_0\right)^2}\right)\left(\frac{\partial i_2(x,t)}{\partial t}\right)^2=0 \end{cases} \quad (14)$$

Let us use Bogning-Djeumen Tchaho-Kofane method [16-21] to come out with the solution of (14) under the analytical shape below:

$$\begin{pmatrix} i_1(x,t)=a\tanh(kx-vt) \\ i_2(x,t)=b\tanh(kx-vt) \end{pmatrix} \quad (15)$$

Where a; b; k and v are non-zero real numbers to be determined in terms of modeled line parameters. Replacing $i_1(x,t)$ and $i_2(x,t)$ given by (15) in (14) we yield the following set of two equations which are written in a simplified form when $a=A_0$ and $b=B_0$:

$$\begin{cases} \begin{pmatrix} 6C_0h^4bk^4A_0^4-36A_0^7v^2A_2C_0C_2-36A_0^7v^2A_2C_1C_2 \\ +6h^4A_0^5k^4C_2+6h^4A_0^5k^4C_0-36A_0^7v^2A_2C_0C_1 \end{pmatrix}\sinh(kx-vt) \\ +\begin{pmatrix} -6h^2A_0^5k^2C_0-2h^4A_0^5k^4C_2+18A_0^7v^2A_2C_0C_2+18A_0^7v^2A_2C_1C_2 \\ +18A_0^7v^2A_2C_0C_1-2C_0h^4bk^4A_0^4+6A_0^5v^2A_1C_0C_1-2h^4A_0^5k^4C_0 \\ +6A_0^5v^2A_1C_0C_2-6h^2A_0^5k^2C_2+6A_0^5v^2A_1C_1C_2-6C_0h^2bk^2A_0^4 \end{pmatrix}\sinh(kx-vt)\cosh^2(kx-vt) \\ +\begin{pmatrix} 3A_0^4v^2A_3C_0C_2+3A_0^4v^2A_3C_1C_0+3A_0^4v^2A_3C_1C_2 \\ +3A_0^4v^2A_4C_0C_2+3A_0^4v^2A_4C_0C_1+3A_0^4v^2A_4C_1C_2 \end{pmatrix}\cosh^3(kx-vt)=0 \\ \begin{pmatrix} 6C_0h^4ak^4B_0^4-36B_0^7v^2B_2C_0C_1-36B_0^7v^2B_2C_1C_2 \\ +6h^4B_0^5k^4C_1+6h^4B_0^5k^4C_0-36B_0^7v^2B_2C_0C_2 \end{pmatrix}\sinh(kx-vt) \\ +\begin{pmatrix} -6h^2B_0^5k^2C_0-2h^4B_0^5k^4C_1+18B_0^7v^2B_2C_0C_1+18B_0^7v^2B_2C_1C_2 \\ +18B_0^7v^2B_2C_0C_2-2C_0h^4ak^4B_0^4+6B_0^5v^2B_1C_0C_2-2h^4B_0^5k^4C_0 \\ +6B_0^5v^2B_1C_0C_1-6h^2B_0^5k^2C_1+6B_0^5v^2B_1C_1C_2-6C_0h^2ak^2B_0^4 \end{pmatrix}\sinh(kx-vt)\cosh^2(kx-vt) \\ +\begin{pmatrix} 3B_0^4v^2B_3C_0C_1+3B_0^4v^2B_3C_2C_0+3B_0^4v^2B_3C_1C_2 \\ +3B_0^4v^2B_4C_0C_1+3B_0^4v^2B_4C_0C_2+3B_0^4v^2B_4C_1C_2 \end{pmatrix}\cosh^3(kx-vt)=0 \end{cases} \quad (16)$$

Equation (16) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permits to obtain the following set of fourteen equations:

$$\left\{\begin{array}{l}6C_0h^4bk^4A_0^4-36A_0^7v^2A_2C_0C_2-36A_0^7v^2A_2C_1C_2\\+6h^4A_0^5k^4C_2+6h^4A_0^5k^4C_0-36A_0^7v^2A_2C_0C_1=0\\-6h^2A_0^5k^2C_0-2h^4A_0^5k^4C_2+18A_0^7v^2A_2C_0C_2+18A_0^7v^2A_2C_1C_2\\+18A_0^7v^2A_2C_0C_1-2C_0h^4bk^4A_0^4+6A_0^5v^2A_1C_0C_1-2h^4A_0^5k^4C_0\\+6A_0^5v^2A_1C_0C_2-6h^2A_0^5k^2C_2+6A_0^5v^2A_1C_1C_2-6C_0h^2bk^2A_0^4=0\\3A_0^4v^2A_3C_0C_2+3A_0^4v^2A_3C_1C_0+3A_0^4v^2A_3C_1C_2\\+3A_0^4v^2A_4C_0C_2+3A_0^4v^2A_4C_0C_1+3A_0^4v^2A_4C_1C_2=0\\6C_0h^4ak^4B_0^4-36B_0^7v^2B_2C_0C_1-36B_0^7v^2B_2C_1C_2\\+6h^4B_0^5k^4C_1+6h^4B_0^5k^4C_0-36B_0^7v^2B_2C_0C_2=0\\-6h^2B_0^5k^2C_0-2h^4B_0^5k^4C_1+18B_0^7v^2B_2C_0C_1+18B_0^7v^2B_2C_1C_2\\+18B_0^7v^2B_2C_0C_2-2C_0h^4ak^4B_0^4+6B_0^5v^2B_1C_0C_2-2h^4B_0^5k^4C_0\\+6B_0^5v^2B_1C_0C_1-6h^2B_0^5k^2C_1+6B_0^5v^2B_1C_1C_2-6C_0h^2ak^2B_0^4=0\\3B_0^4v^2B_3C_0C_1+3B_0^4v^2B_3C_2C_0+3B_0^4v^2B_3C_1C_2\\+3B_0^4v^2B_4C_0C_1+3B_0^4v^2B_4C_0C_2+3B_0^4v^2B_4C_1C_2=0\end{array}\right. \quad (17)$$

Haven solved the set of equation (17), it has permitted to present in (18) the coupled solution with conditions of the set of two nonlinear partial differential equations obtained in (14) which model the dynamics of solitary wave of type (Kink; Kink):

$$a=A_0;\ b=B_0;\ v=\pm\sqrt{\frac{6A_0A_2\left(C_0A_0+C_0B_0+C_2A_0\right)}{\left(C_1C_0+C_2C_0+C_1C_2\right)\left(A_0^2A_2+A_1\right)^2}};\ k=\pm\frac{A_0}{h}\sqrt{\frac{6A_2}{A_0^2A_2+A_1}};\ \frac{6A_0A_2\left(C_0A_0+C_0B_0+C_2A_0\right)}{\left(C_1C_0+C_2C_0+C_1C_2\right)\left(A_0^2A_2+A_1\right)^2}>0;$$

$$\frac{6A_2}{A_0^2A_2+A_1}>0;\ B_2=\frac{C_0h^4k^4+C_1h^4k^4}{6B_0^2v^2\left(C_1C_0+C_2C_0+C_1C_2\right)}+\frac{C_0A_0h^4k^4}{6B_0^3v^2\left(C_1C_0+C_2C_0+C_1C_2\right)};$$

$$B_1=\frac{6C_0h^2k^2+6C_1h^2k^2-C_0h^4k^4-C_1h^4k^4}{6v^2\left(C_1C_0+C_2C_0+C_1C_2\right)}+\frac{6C_0A_0h^2k^2-C_0A_0h^4k^4}{6B_0v^2\left(C_1C_0+C_2C_0+C_1C_2\right)};$$

$$B_3=\frac{C_0B_0h^4k^4+C_1B_0h^4k^4+C_0A_0h^4k^4-6C_0B_0h^2k^2-6C_1B_0h^2k^2-6C_0A_0h^2k^2}{12v^2\left(C_1C_0+C_2C_0+C_1C_2\right)};$$

$$A_3=\frac{C_0A_0h^4k^4+C_2A_0h^4k^4+C_0B_0h^4k^4-6C_0A_0h^2k^2-6C_2A_0h^2k^2-6C_0B_0h^2k^2}{12v^2\left(C_1C_0+C_2C_0+C_1C_2\right)};\ A_4=-A_3;\ B_4=-B_3;$$

$$\left(\begin{array}{l}i_1(x,t)=A_0\tanh\left(\pm\frac{A_0}{h}\sqrt{\frac{6A_2}{A_0^2A_2+A_1}}x\pm\sqrt{\frac{6A_0A_2\left(C_0A_0+C_0B_0+C_2A_0\right)}{\left(C_1C_0+C_2C_0+C_1C_2\right)\left(A_0^2A_2+A_1\right)^2}}t\right)\\ i_2(x,t)=B_0\tanh\left(\pm\frac{A_0}{h}\sqrt{\frac{6A_2}{A_0^2A_2+A_1}}x\pm\sqrt{\frac{6A_0A_2\left(C_0A_0+C_0B_0+C_2A_0\right)}{\left(C_1C_0+C_2C_0+C_1C_2\right)\left(A_0^2A_2+A_1\right)^2}}t\right)\end{array}\right) \quad (18)$$

4. Construction of Solitary Wave Solution in the Shape (Pulse; Pulse) of the Set of Two Partial Differential Equations (12).

Let us define each nonlinear magnetic flux linkage of the two inductors constituting each of the two linked parts under the analytical shape as follows:

$$\begin{cases} \phi_1\left(i_1\left(x,t\right)\right) = A_1 i_1\left(x,t\right) + A_2 i_1^3\left(x,t\right) \\ \phi_2\left(i_2\left(x,t\right)\right) = B_1 i_2\left(x,t\right) + B_2 i_2^3\left(x,t\right) \end{cases} \tag{19}$$

With A_1; A_2; B_1 and B_2 are non-zero real numbers which will be chosen conveniently. By substituting each of the nonlinear magnetic flux linkage $\phi_1\left(i_1\left(x,t\right)\right)$ and $\phi_2\left(i_2\left(x,t\right)\right)$ of (19) in (12) one obtains the set of two nonlinear partial differential equations written as:

$$\begin{cases} \frac{h^4}{12}\frac{\partial^4 i_1(x,t)}{\partial x^4} + h^2\frac{\partial^2 i_1(x,t)}{\partial x^2} + \frac{C_0 h^4}{12(C_0+C_2)}\frac{\partial^4 i_2(x,t)}{\partial x^4} + \frac{C_0 h^2}{(C_0+C_2)}\frac{\partial^2 i_2(x,t)}{\partial x^2} \\ +\left(\frac{C_0^2}{C_0+C_2} - C_0 - C_1\right)\left(A_1 + 3A_2 i_1^2(x,t)\right)\frac{\partial^2 i_1(x,t)}{\partial t^2} \\ +6A_2\left(\frac{C_0^2}{C_0+C_2} - C_0 - C_1\right) i_1(x,t)\left(\frac{\partial i_1(x,t)}{\partial t}\right)^2 = 0 \\ \frac{h^4}{12}\frac{\partial^4 i_2(x,t)}{\partial x^4} + h^2\frac{\partial^2 i_2(x,t)}{\partial x^2} + \frac{C_0 h^4}{12(C_0+C_1)}\frac{\partial^4 i_1(x,t)}{\partial x^4} + \frac{C_0 h^2}{(C_0+C_1)}\frac{\partial^2 i_1(x,t)}{\partial x^2} \\ +\left(\frac{C_0^2}{C_0+C_1} - C_0 - C_2\right)\left(B_1 + 3B_2 i_2^2(x,t)\right)\frac{\partial^2 i_2(x,t)}{\partial t^2} \\ +6B_2\left(\frac{C_0^2}{C_0+C_1} - C_0 - C_2\right) i_2(x,t)\left(\frac{\partial i_2(x,t)}{\partial t}\right)^2 = 0 \end{cases} \tag{20}$$

Let us use Bogning-Djeumen Tchaho-Kofane method [16-21] to come out with the solution of (20) under the analytical shape below:

$$\begin{pmatrix} i_1(x,t) = a\,\mathrm{sech}(kx - vt) \\ i_2(x,t) = b\,\mathrm{sech}(kx - vt) \end{pmatrix} \tag{21}$$

Where a; b; k and v are non-zero real numbers to be determined in terms of modeled line parameters. Replacing $i_1(x,t)$ and $i_2(x,t)$ given by (21) in (20) we yield the following set of two equations which are written in a simplified form:

$$\begin{cases} 24h^4ak^4C_0 + 24h^4ak^4C_2 + 24h^4bk^4C_0 + 144A_2a^3v^2C_1C_2 + 144A_2a^3v^2C_1C_0 + 144A_2a^3v^2C_0C_2 \\ +\begin{pmatrix} h^4ak^4C_0 - 12av^2A_1C_0C_2 + 12h^2ak^2C_0 + 12h^2ak^2C_2 + 12h^2bk^2C_0 \\ +h^4bk^4C_0 - 12av^2A_1C_1C_2 + h^4ak^4C_2 - 12av^2A_1C_0C_1 \end{pmatrix}\cosh^4(kx - vt) \\ +\begin{pmatrix} -24h^2ak^2C_0 - 24h^2ak^2C_2 - 20h^4ak^4C_2 - 20h^4ak^4C_0 \\ -20h^4bk^4C_0 + 24av^2A_1C_1C_0 + 24av^2A_1C_0C_2 + 24av^2A_1C_1C_2 \\ -108A_2a^3v^2C_0C_2 - 108A_2a^3v^2C_1C_0 - 108A_2a^3v^2C_1C_2 - 24h^2bk^2C_0 \end{pmatrix}\cosh^2(kx - vt) = 0 \\ 24h^4bk^4C_0 + 24h^4bk^4C_1 + 24h^4ak^4C_0 + 144B_2b^3v^2C_1C_2 + 144B_2b^3v^2C_2C_0 + 144B_2b^3v^2C_0C_1 \\ +\begin{pmatrix} h^4bk^4C_0 - 12bv^2B_1C_0C_1 + 12h^2bk^2C_0 + 12h^2bk^2C_1 + 12h^2ak^2C_0 \\ +h^4ak^4C_0 - 12bv^2B_1C_1C_2 + h^4bk^4C_1 - 12bv^2B_1C_0C_2 \end{pmatrix}\cosh^4(kx - vt) \\ +\begin{pmatrix} -24h^2bk^2C_0 - 24h^2bk^2C_1 - 20h^4bk^4C_1 - 20h^4bk^4C_0 \\ -20h^4ak^4C_0 + 24bv^2B_1C_2C_0 + 24bv^2B_1C_0C_1 + 24bv^2B_1C_1C_2 \\ -108B_2b^3v^2C_0C_1 - 108B_2b^3v^2C_2C_0 - 108B_2b^3v^2C_1C_2 - 24h^2ak^2C_0 \end{pmatrix}\cosh^2(kx - vt) = 0 \end{cases} \tag{22}$$

Equation (22) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permits to obtain the following set of twelve equations:

$$\begin{cases} 24h^4ak^4C_0+24h^4ak^4C_2+24h^4bk^4C_0+144A_2a^3v^2C_1C_2+144A_2a^3v^2C_1C_0+144A_2a^3v^2C_0C_2=0 \\ h^4ak^4C_0-12av^2A_1C_0C_2+12h^2ak^2C_0+12h^2ak^2C_2+12h^2bk^2C_0 \\ +h^4bk^4C_0-12av^2A_1C_1C_2+h^4ak^4C_2-12av^2A_1C_0C_1=0 \\ -24h^2ak^2C_0-24h^2ak^2C_2-20h^4ak^4C_2-20h^4ak^4C_0 \\ -20h^4bk^4C_0+24av^2A_1C_1C_0+24av^2A_1C_0C_2+24av^2A_1C_1C_2 \\ -108A_2a^3v^2C_0C_2-108A_2a^3v^2C_1C_0-108A_2a^3v^2C_1C_2-24h^2bk^2C_0=0 \\ 24h^4bk^4C_0+24h^4bk^4C_1+24h^4ak^4C_0+144B_2b^3v^2C_1C_2+144B_2b^3v^2C_2C_0+144B_2b^3v^2C_0C_1=0 \\ h^4bk^4C_0-12bv^2B_1C_0C_1+12h^2bk^2C_0+12h^2bk^2C_1+12h^2ak^2C_0 \\ +h^4ak^4C_0-12bv^2B_1C_1C_2+h^4bk^4C_1-12bv^2B_1C_0C_2=0 \\ -24h^2bk^2C_0-24h^2bk^2C_1-20h^4bk^4C_1-20h^4bk^4C_0 \\ -20h^4ak^4C_0+24bv^2B_1C_2C_0+24bv^2B_1C_0C_1+24bv^2B_1C_1C_2 \\ -108B_2b^3v^2C_0C_1-108B_2b^3v^2C_2C_0-108B_2b^3v^2C_1C_2-24h^2ak^2C_0=0 \end{cases} \quad (23)$$

Haven solved the set of equations (23), it has permitted to present in (24) the coupled solution with conditions of the set of two nonlinear partial differential equations obtained in (20) which model the dynamics of solitary waves of type (Pulse; Pulse):

$$k=\pm\frac{1}{h}\left(-6+\left(\begin{matrix}36+6B_1v^2C_0+6v^2A_1C_1+6C_0v^2A_1+6B_1v^2C_2\\ +6v^2\left(\begin{matrix}B_1^2C_0^2-2C_0B_1A_1C_1+2B_1C_0^2A_1+2B_1^2C_0C_2+A_1^2C_1^2\\ +2A_1^2C_1C_0-2B_1A_1C_2C_1+C_0^2A_1^2-2C_0B_1A_1C_2+B_1^2C_2^2\end{matrix}\right)^{\frac{1}{2}}\end{matrix}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}};\ a=\pm hk\sqrt{\frac{-2A_1}{A_2\left(12+h^2k^2\right)}};$$

$$b=-\frac{a\left(6A_2v^2a^2C_0C_2+6A_2v^2a^2C_1C_0+6A_2v^2a^2C_1C_2+k^4h^4C_0+k^4h^4C_2\right)}{k^4h^4C_0};\ v=\pm\frac{k^2h^2}{B_2b^2}\sqrt{\frac{-B_2b\left(C_0b+C_1b+C_0a\right)}{6\left(C_0C_1+C_0C_2+C_1C_2\right)}};$$

$$B_1>0\,;\ A_2<0\,;\ B_2<0\,;\ A_1>0\,;\ \begin{pmatrix} i_1(x,t)=a\,\mathrm{sech}(kx-vt)\\ i_2(x,t)=b\,\mathrm{sech}(kx-vt)\end{pmatrix} \quad (24)$$

5. Conclusion

One has modeled and constructed a coupled solitary wave solution of two different set of nonlinear partial derivative equations of an inductive electrical line with crosslink capacitor. It is therefore important to point out that the results obtained will first of all enable us in the domain of physics and telecommunication of engineering, the manufacturing of new transmission lines as inductive electrical lines whose magnetic flux linkage of inductors in which, one varies in a nonlinear shape defined in (13) and the other one varies in a nonlinear shape defined in (19). In addition, these results will permit to ameliorate the quality of signals that will be propagated in those new lines. In fact, those signals are solitary waves of type (Pulse; Pulse) obtained in (24) and type (Kink; Kink) obtained in (18) which by their definitions, propagate on a very long distance by maintaining their shape; their speed and resist best on different dissipation factors. Finally, in a typical mathematics domain, the results obtained have permitted to define in (14) and (20) two new set of nonlinear partial derivative equations which have respectively for exact solutions the coupled solitary wave (18) and (24); this augments the field of mathematical knowledge. It is necessary to recall that the inductive electrical line with crosslink capacitor that we have studied is advantageous for the fact that it permits simultaneously the propagation of a set of two solitary waves contrary to a non-coupled inductive electrical line which only enables the propagation of one solitary wave when the signal considered is the current; the more we will multiply the crosslink in the line, the more we will multiply the simultaneous propagation of solitary waves in the line. In order to inquire ideas concerning the stability of obtained solitary waves, it seems for us to study later their modulational instability before carrying out the practical survey where we will experiment the applicability and the perfection of these new inductive electrical lines with crosslink capacitor.

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