American Journal of Circuits, Systems and Signal Processing
Vol. 4, No. 2, 2018, pp. 15-22
http://www.aiscience.org/journal/ajcssp
ISSN: 2381-7194 (Print); ISSN: 2381-7208 (Online)

Solitary Wave Solutions of Modeled Equations in a Nonlinear Capacitive Electrical Line

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Abstract

Solitary wave is the most used signal for the transmission of information in the different transmission media for the fact that it is robust, localized, and stable. It is particularly that property of non-loss of energy by solitary wave during its movement that has motivated us to find out in which measure one can propagate it in transmission media like electrical line. The choice of electrical line for our study is due to the fact that they are low cost transmission supports and very easy to manufacture. In this article, we define analytically nonlinear properties that must obey the electrical line components so that their media accept to propagate solitary waves. We use the analytical definition of nonlinear charge of capacitors and we apply Kirchhoff laws to the circuit of nonlinear capacitive electrical line to model new higher-order nonlinear partial differential equations which govern the dynamics of solitary waves in the line. The application of Bogning-Djeumen Tchaho-Kofane method that facilitate the construction of solitary wave solutions of nonlinear partial differential equations by the identification of basic hyperbolic function coefficients in a direct and effective manner has permitted to obtain exact solutions of the modeled nonlinear equations. These solution are solitary waves of type Kink and type Pulse that are susceptible to propagate in the line when some conditions we have established are respected. The results obtained in this paper notably the analytical definitions that must undergo nonlinear charges of capacitors in the line, the nonlinear partial differential equations and their exact soliton solutions will permit: The amelioration of the quality of signal susceptible of propagating in the line, to facilitate the choice of the type of the line relative to the choice of the type of signal to transmit, to economize by reducing amplification stations, to increase the mathematical knowledge, to enable the manufacturing of new nonlinear capacitors.

Keywords

Capacitive Electrical Line, Construction, Model, Soliton Solution, Solitary Wave, Nonlinear Partial Differential Equation, Kink, Pulse

Received: July 17, 2018 / Accepted: August 1, 2018 / Published online: August 31, 2018

1. Introduction

Since the discovery of soliton wave by the Scottish engineer John Scott Russel, the theory of solitary wave has greatly evolved. The theory has gone through steps from the elaboration of the first equations which model its dynamics, from the determination of analytical form (solitary wave types) [1] to the stage where one uses to explain certain natural phenomena such as Tsunami, hurricane and certain forms of earth quakes. If a good number of researchers continue to develop this theory, it is because the solitary wave given its characteristic properties is presented as the wave of the future. If the wave can propagate on longer distances without changing its form, this simple means that if such a wave is sent in a transmission line, it will be less vulnerable to dissipation and instability phenomena [2, 3], therefore its usage in engineering of telecommunication will

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enforce the quality of signal, the gap zone, the reduction of amplification stations. Electrical transmission lines are very convenient tools to study the wave propagation in one-dimensional nonlinear dispersive media [4, 5] and investigation of nonlinear lumped elements transmission line has long been carried out to understand the principle of soliton generation [6, 7]. This is why we have decided to study a nonlinear capacitive electrical line where only capacitive components are nonlinear [8]. In case where the technology of telecommunication will be based on the propagation of solitary waves, there will be the need to construct a type of solitary wave able to propagate in a given transmission line in terms of its characteristics and its properties. We wish to mention by this the necessity of constructing solutions and even the forced ones that can be used as signals in certain transmission lines. Mathematically, it is a very difficult task since the formation of soliton requires a combination of the nonlinearity effect and the dispersion effect of the transmission line [9-11] but few solutions are found by using certain appropriate methods [12-23] to solve certain nonlinear partial differential equations. Being part of this dynamic research, we model in this article the nonlinear partial derivative equations which describe the dynamics of waves in a capacitive electrical line whose charge of capacitors is nonlinear. We therefore use the Bogning-Djeumen Tchaho-Kofane method [12-17] to construct solitary wave solutions susceptible of propagating in that electrical line. The work presented in this paper is partitioned as follows: In part 2, we present a general modeling of a nonlinear capacitive electrical line; In part 3, we construct solitary wave solutions of type Kink; In part 4, we construct solitary wave solutions of type Pulse, In part 5, we construct solitary wave solutions in the form $u(x,t) = a\dfrac{\sinh^{3m}(kx-vt)}{\cosh^{3n}(kx-vt)}$ and we present at the end the conclusion in part 6.

2. General Modeling of a Nonlinear Capacitive Electrical Line

Let us consider an electrical line constituting a good number of identical networks shown in figure 1 where R is the resistance of the resistor and G the conductance of another resistor, connected on two parallel lines with a capacitor whose charge $q(u_n)$ changes in nonlinear manner in terms of the voltage u_n across that capacitor.

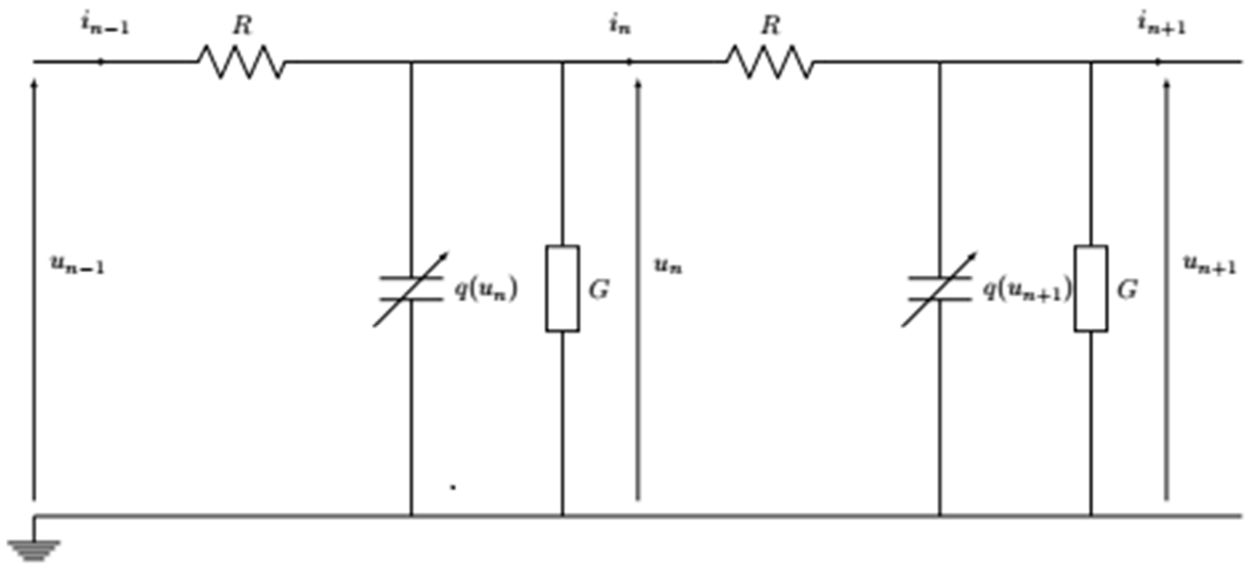

Figure 1. Presentation of a nonlinear capacitive electrical line.

By applying Kirchhoff laws to the circuit shown in figure 1, we obtain the following equations

$$u_n - u_{n+1} = Ri_n, \tag{1}$$

$$i_{n-1} - i_n = Gu_n + \frac{\partial q_n}{\partial t}. \tag{2}$$

Where n is a positive integer that numbers each network of the line, u_n and u_{n+1} indicate respectively the voltage across capacitor of the network order n and the voltage across capacitor of the network order $n+1$. i_{n-1} and i_n indicate respectively the current that flows through the resistor network order $n-1$ and the resistor network order n. q_n represents the nonlinear charge of the capacitor. Considering equation (2), equation (1) become

$$u_n - u_{n+1} = Ri_{n-1} - RGu_n - R\frac{\partial q_n}{\partial t}. \quad (3)$$

The substitution of $Ri_{n-1} = u_{n-1} - u_n$ of equation (1) obtained during the previous order in equation (3), one obtains the differential equation below

$$u_{n+1} - 2u_n + u_{n-1} = R\frac{\partial q_n}{\partial t} + RGu_n. \quad (4)$$

To obtain the continuum model, the left hand side of equation (4) has to be approximated to a spatial partial derivative with respect to $x = nh$ which represents the distance measured from the beginning of the line. h represents the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivatives period. We obtain as such spatial partial derivatives using Taylor expansion of u_{n+1} and u_{n-1} closely to u_n by considering the terms till fourth order in the following manner:

$$u_{n+1} = u_n + \frac{h}{1!}\frac{\partial u_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u_n}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 u_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 u_n}{\partial x^4}, \quad (5)$$

$$u_{n-1} = u_n - \frac{h}{1!}\frac{\partial u_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u_n}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 u_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 u_n}{\partial x^4}, \quad (6)$$

$$u_{n+1} - 2u_n + u_{n-1} = h^2\frac{\partial^2 u_n}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4 u_n}{\partial x^4}. \quad (7)$$

Equation (7) and (4) permits us to derive the result as follows:

$$-h^2\frac{\partial^2 u_n}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 u_n}{\partial x^4} + R\frac{\partial q_n}{\partial t} + RGu_n = 0. \quad (8)$$

Finally, we obtain the continuum model of the nonlinear capacitive electrical line presented in figure 1 by the nonlinear partial differential equation below:

$$-h^2\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 u(x,t)}{\partial x^4} + R\frac{\partial q(u(x,t))}{\partial t} + RGu(x,t) = 0. \quad (9)$$

Let's find out the solitary wave solutions of equation (9).

3. Construction of Solitary Wave Solution of Type Kink of Partial Differential Equation (9)

We define the nonlinear charge of the capacitor on the analytical shape as follows

$$q(u(x,t)) = A_1 u^4(x,t) + A_2 u^2(x,t) + A_3 \ln(u^2(x,t) - A_0^2). \quad (10)$$

With $|u(x,t)| > |A_0|$. A_1; A_2 and A_3 are non-zero real numbers which will be chosen conveniently. By substituting the charge $q(u(x,t))$ of (10) in equation (9) we obtain the nonlinear partial differential equation written as

$$\begin{aligned}&\frac{A_0^2 h^4}{12}\frac{\partial^4 u(x,t)}{\partial x^4} - \frac{h^4}{12}u^2(x,t)\frac{\partial^4 u(x,t)}{\partial x^4} + A_0^2 h^2\frac{\partial^2 u(x,t)}{\partial x^2} - h^2 u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2}\\ &+(2RA_3 - 2A_0^2 RA_2)u(x,t)\frac{\partial u(x,t)}{\partial t} + (2RA_2 - 4A_0^2 RA_1)u^3(x,t)\frac{\partial u(x,t)}{\partial t}\\ &+4RA_1 u^5(x,t)\frac{\partial u(x,t)}{\partial t} - A_0^2 RGu(x,t) + RGu^3(x,t) = 0.\end{aligned} \quad (11)$$

Considering $n_1 = \frac{A_0^2 h^4}{12}$, $n_2 = -\frac{h^4}{12}$, $n_3 = A_0^2 h^2$, $n_4 = -h^2$, $n_5 = 2RA_3 - 2A_0^2 RA_2$, $n_6 = 2RA_2 - 4A_0^2 RA_1$, $n_7 = 4RA_1$, $n_8 = -A_0^2 RG$, $n_9 = RG$, equation (11) takes the following shape

$$\begin{aligned}&n_1\frac{\partial^4 u(x,t)}{\partial x^4} + n_2 u^2(x,t)\frac{\partial^4 u(x,t)}{\partial x^4} + n_3\frac{\partial^2 u(x,t)}{\partial x^2} + n_4 u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2} + n_5 u(x,t)\frac{\partial u(x,t)}{\partial t}\\ &+n_6 u^3(x,t)\frac{\partial u(x,t)}{\partial t} + n_7 u^5(x,t)\frac{\partial u(x,t)}{\partial t} + n_8 u(x,t) + n_9 u^3(x,t) = 0.\end{aligned} \quad (12)$$

Let us use Bogning-Djeumen Tchaho-Kofane method [12-17] to come out with the solution of equation (12) under the analytical shape below

$$u(x,t) = a\tanh(kx - vt). \quad (13)$$

Where a; k and v are non-zero real numbers to be determined. Replacing $u(x,t)$ given by (13) in equation (12) we yield the following equation:

$$\begin{aligned}&(-24n_2a^3k^4-n_7a^6v)\frac{\sinh(kx-vt)}{\cosh^7(kx-vt)}+(2n_7a^6v+24n_1ak^4+n_6a^4v+32n_2a^3k^4+2n_4a^3k^2)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)}+\\&(-n_7a^6v-8n_1ak^4-2n_3ak^2-8n_2a^3k^4-n_9a^3-n_5a^2v-n_6a^4v-2n_4a^3k^2)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)}+\\&(n_8a+n_9a^3)\frac{\sinh(kx-vt)}{\cosh(kx-vt)}=0.\end{aligned} \tag{14}$$

Equation (14) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permits us to obtain the following set of four equations

$$\begin{cases}-24n_2a^3k^4-n_7a^6v=0,\\2n_7a^6v+24n_1ak^4+n_6a^4v+32n_2a^3k^4+2n_4a^3k^2=0,\\-n_7a^6v-8n_1ak^4-2n_3ak^2-8n_2a^3k^4-n_9a^3-n_5a^2v-n_6a^4v-2n_4a^3k^2=0,\\n_8a+n_9a^3=0.\end{cases} \tag{15}$$

Solving the set of equation (15) has permitted us to obtain the following results

$$a=\pm\sqrt{\frac{-n_8}{n_9}},\ k=\pm\frac{1}{2}\sqrt{\frac{n_4n_8n_7}{2n_2n_8n_7+3n_1n_9n_7-3n_6n_2n_9}},\ v=\frac{3n_2n_4^2n_7\left(-n_8n_9\right)^{\frac{3}{2}}}{2n_8\left(2n_2n_8n_7+3n_1n_9n_7-3n_6n_2n_9\right)^2},$$

$$\begin{pmatrix}n_8n_1^2n_9^2n_7^2-2n_8n_1n_9^2-n_7n_6n_2+n_8n_6^2n_2^2n_9^2+\frac{4}{3}n_2n_8^2n_7^2n_1n_9\\-\frac{1}{6}n_8n_9n_7^2n_1n_3n_4-\frac{4}{3}n_2^2n_8^2n_7n_6n_9-\frac{1}{6}n_8n_5n_2n_4^2n_7n_9+\frac{1}{6}n_8n_9n_7n_2n_4n_3n_6\\+\frac{4}{9}n_2^2n_8^3n_7^2+\frac{1}{9}n_4^2n_8^2n_7^2n_1-\frac{1}{9}n_8^2n_7^2n_3n_4n_2=0\end{pmatrix},\ n_8n_9<0. \tag{16}$$

Replacing n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8 and n_9 by their different expressions in (16), we obtain the solution of the nonlinear partial differential equation (11) which models the dynamics of solitary waves of type Kink in the capacitive line as follows

$$a=A_0,\ k=\pm A_0\left(\frac{-RGA_1}{h^4A_3}\right)^{\frac{1}{4}},\ v=\frac{-GA_0}{2A_3},\ A_2=-\frac{4}{3}A_1A_0^2-2\sqrt{\frac{-A_1A_3}{RG}},\ A_1A_3<0$$

$$u(x,t)=A_0\tanh\left(\pm A_0\left(\frac{-RGA_1}{h^4A_3}\right)^{\frac{1}{4}}x+\frac{GA_0}{2A_3}t\right). \tag{17}$$

4. Construction of Solitary Wave Solution of Type Pulse Relative to Nonlinear Partial Differential Equation (9)

We define the nonlinear charge of the capacitor with analytical shape as given:

$$q(u(x,t))=A_1u(x,t)\sqrt{1-\left(\frac{u(x,t)}{A_0}\right)^2}+A_2u^3(x,t)\sqrt{1-\left(\frac{u(x,t)}{A_0}\right)^2}+A_3\arctan\left(\sqrt{\frac{u^2(x,t)}{A_0^2-u^2(x,t)}}\right). \tag{18}$$

With $\left|A_0\right| > \left|u(x,t)\right|$. A_1 ; A_2 and A_3 are non-zero real numbers whose conditions of choice will be established. By substituting $q(u(x,t))$ of (18) in differential equation (9) has permitted to obtain the nonlinear partial differential equation

$$\begin{aligned} &A_0\sqrt{A_0^2-u^2(x,t)}\left(-h^2\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{h^4}{12}\frac{\partial^4 u(x,t)}{\partial x^4}+RGu(x,t)\right)\\ &+R\left(-4A_2u^4(x,t)+\left(3A_0^2A_2-2A_1\right)u^2(x,t)+A_0^2A_1+A_0A_3\right)\frac{\partial u(x,t)}{\partial t}=0. \end{aligned} \tag{19}$$

Let us find out the result of equation (19) on the analytical shape

$$u(x,t)=a\,\mathrm{sech}\left(kx-vt\right). \tag{20}$$

Where a; k and v are non-zero real numbers to be determined. Substituting $u(x,t)$ of (20) in differential equation (19), we obtain the equation as follows:

$$\begin{aligned} &\left(3a^3vRA_0^2A_2-2a^3vRA_1\right)\frac{\sinh\left(kx-vt\right)}{\cosh^4\left(kx-vt\right)}+\left(avRA_0A_3+avRA_0^2A_1\right)\frac{\sinh\left(kx-vt\right)}{\cosh^2\left(kx-vt\right)}\\ &-4a^5vRA_2\frac{\sinh\left(kx-vt\right)}{\cosh^6\left(kx-vt\right)}+\left(A_0RGa-\frac{1}{12}A_0h^4ak^4-A_0h^2ak^2\right)\frac{\sqrt{A_0^2-\dfrac{a^2}{\cosh^2\left(kx-vt\right)}}}{\cosh\left(kx-vt\right)}\\ &+\left(2A_0h^2ak^2+\frac{5}{3}A_0h^4ak^4\right)\frac{\sqrt{A_0^2-\dfrac{a^2}{\cosh^2\left(kx-vt\right)}}}{\cosh^3\left(kx-vt\right)}-2A_0h^4ak^4\frac{\sqrt{A_0^2-\dfrac{a^2}{\cosh^2\left(kx-vt\right)}}}{\cosh^5\left(kx-vt\right)}=0 \end{aligned} \tag{21}$$

We realize that to be able to transform the hyperbolic functions of (21) to the basic hyperbolic functions as recommended by the new Bogning-Djeumen Tchaho-Kofane [12-17] we must consider $A_0=a$ such that:

$$\sqrt{A_0^2-\frac{a^2}{\cosh^2\left(kx-vt\right)}}=a\tanh\left(kx-vt\right) \tag{22}$$

The right-hand side of equation (22) has enabled us to rearrange (21) as

$$\begin{aligned} &\left(3a^3vRA_0^2A_2+2A_0a^2h^2k^2+\frac{5}{3}A_0a^2h^4k^4-2a^3vRA_1\right)\frac{\sinh\left(kx-vt\right)}{\cosh^4\left(kx-vt\right)}\\ &+\left(A_0a^2RG+avRA_0A_3-A_0a^2h^2k^2-\frac{1}{12}A_0a^2h^4k^4+avRA_0^2A_1\right)\frac{\sinh\left(kx-vt\right)}{\cosh^2\left(kx-vt\right)}\\ &+\left(-2A_0a^2h^4k^4-4a^5vRA_2\right)\frac{\sinh\left(kx-vt\right)}{\cosh^6\left(kx-vt\right)}=0. \end{aligned} \tag{23}$$

Equation (23) is valid if each coefficient of its basic hyperbolic function is equal to zero. This enables to obtain the set of three equations as follows

$$\begin{cases} 3a^3vRA_0^2A_2+2A_0a^2h^2k^2+\dfrac{5}{3}A_0a^2h^4k^4-2a^3vRA_1=0,\\ A_0a^2RG+avRA_0A_3-A_0a^2h^2k^2-\dfrac{1}{12}A_0a^2h^4k^4+avRA_0^2A_1=0,\\ -2A_0a^2h^4k^4-4a^5vRA_2=0. \end{cases} \tag{24}$$

The result of the set of nonlinear equation (24) enables us to realize that solitary waves of type Pulse are easily displaced in the nonlinear capacitive line with analytical shape given below

$$a = A_0\,,\ v = \frac{-GA_0}{A_3}\,,\ k = \pm\left(\frac{2A_0^3 A_2 RG}{h^4 A_3}\right)^{\frac{1}{4}},\ A_3 = \frac{RG\left(A_2^2 A_0^4 + 12A_2 A_0^2 A_1 + 36A_1^2\right)}{72A_0 A_2}\,,$$

$$u(x,t) = A_0\,\mathrm{sech}\left(\pm\left(\frac{2A_0^3 A_2 RG}{h^4 A_3}\right)^{\frac{1}{4}} x + \frac{GA_0}{A_3}t\right). \tag{25}$$

5. Construction of Third Order Soliton Solution of a General Partial Differential Equation (9)

We define the nonlinear charge of capacitors on the following analytical form

$$q\left(u(x,t)\right) = A_1 u^2(x,t) + A_2\left(\frac{u(x,t)}{A_0}\right)^{\frac{2}{3}} + A_3\left(\frac{u(x,t)}{A_0}\right)^{\frac{4}{3}} + A_4 \ln\left(\left(\left(\frac{u(x,t)}{A_0}\right)^{\frac{1}{3}} - 1\right)\left(\left(\frac{u(x,t)}{A_0}\right)^{\frac{1}{3}} + 1\right)\right). \tag{26}$$

With $\frac{u(x,t)}{A_0} > 1$. A_1, A_2, A_3 and A_4 are real numbers whose conditions of choice will be elaborated. Substituting $q(u(x,t))$ of (26) in (9), we obtain the nonlinear partial differential equation bellow

$$\begin{aligned}
&\left(h^4 A_0^{\frac{8}{3}} u^{\frac{1}{3}}(x,t) - h^4 A_0^2 u(x,t)\right)\frac{\partial^4 u(x,t)}{\partial x^4} + \left(12h^2 A_0^{\frac{8}{3}} u^{\frac{1}{3}}(x,t) - 12h^2 A_0^2 u(x,t)\right)\frac{\partial^2 u(x,t)}{\partial x^2} \\
&+\left(\begin{aligned}&24RA_1 A_0^2 u^2(x,t) - 24RA_1 A_0^{\frac{8}{3}} u^{\frac{4}{3}}(x,t) + 8RA_2 A_0^{\frac{4}{3}} u^{\frac{2}{3}}(x,t)\\ &+16RA_3 A_0^{\frac{2}{3}} u^{\frac{4}{3}}(x,t) - 16RA_3 A_0^{\frac{4}{3}} u^{\frac{2}{3}}(x,t) - 8RA_2 A_0^2 + 8RA_4 A_0^2\end{aligned}\right)\frac{\partial u(x,t)}{\partial t} \\
&+12RGA_0^2 u^2(x,t) - 12RGA_0^{\frac{8}{3}} u^{\frac{4}{3}}(x,t) = 0.
\end{aligned} \tag{27}$$

Let us construct the solitary wave solution of the nonlinear partial differential equation (27) under the shape

$$u(x,t) = a\frac{\sinh^{3m}(kx - vt)}{\cosh^{3n}(kx - vt)}. \tag{28}$$

where a, k, v, m and n are non-zero real numbers which will be determined in terms of electrical line parameters. By substituting $u(x,t)$ of (28) in (27), we obtain an equation in the following form

$$f_1(a,k,v,m,n)\frac{\sinh^{5m-1}(kx-vt)}{\cosh^{5n-1}(kx-vt)}+f_2(a,k,v,m,n)\frac{\sinh^{5m+1}(kx-vt)}{\cosh^{5n+1}(kx-vt)}+f_3(a,k,v,m,n)\frac{\sinh^{3m-2}(kx-vt)}{\cosh^{3n-2}(kx-vt)}$$
$$+f_4(a,k,v,m,n)\frac{\sinh^{3m+2}(kx-vt)}{\cosh^{3n+2}(kx-vt)}+f_5(a,k,v,m,n)\frac{\sinh^{3m-4}(kx-vt)}{\cosh^{3n-4}(kx-vt)}+f_6(a,k,v,m,n)\frac{\sinh^{3m+4}(kx-vt)}{\cosh^{3n+4}(kx-vt)}$$
$$+f_7(a,k,v,m,n)\frac{\sinh^{6m+1}(kx-vt)}{\cosh^{6n+1}(kx-vt)}+f_8(a,k,v,m,n)\frac{\sinh^{6m-1}(kx-vt)}{\cosh^{6n-1}(kx-vt)}+f_9(a,k,v,m,n)\frac{\sinh^{4m}(kx-vt)}{\cosh^{4n}(kx-vt)}$$
$$+f_{10}(a,k,v,m,n)\frac{\sinh^{6m+2}(kx-vt)}{\cosh^{6n+2}(kx-vt)}+f_{11}(a,k,v,m,n)\frac{\sinh^{6m-2}(kx-vt)}{\cosh^{6n-2}(kx-vt)}+f_{12}(a,k,v,m,n)\frac{\sinh^{3m-1}(kx-vt)}{\cosh^{3n-1}(kx-vt)} \quad (29)$$
$$+f_{13}(a,k,v,m,n)\frac{\sinh^{6m+4}(kx-vt)}{\cosh^{6n+4}(kx-vt)}+f_{14}(a,k,v,m,n)\frac{\sinh^{6m-4}(kx-vt)}{\cosh^{6n-4}(kx-vt)}+f_{15}(a,k,v,m,n)\frac{\sinh^{3m+1}(kx-vt)}{\cosh^{3n+1}(kx-vt)}$$
$$+f_{16}(a,k,v,m,n)\frac{\sinh^{9m+1}(kx-vt)}{\cosh^{9n+1}(kx-vt)}+f_{17}(a,k,v,m,n)\frac{\sinh^{9m-1}(kx-vt)}{\cosh^{9n-1}(kx-vt)}+f_{18}(a,k,v,m,n)\frac{\sinh^{6m}(kx-vt)}{\cosh^{6n}(kx-vt)}=0.$$

Where $f_i(a,k,v,m,n)$, $i=(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18)$ are analytical functions of real parameters a, k, v, m and n. We remark that when $5m-1=6m-1$, $5m-1=4m$, $5m+1=6m-1$, $5m-1=6m-4$, $3m+4=4m$ and $5n-1=6n-1$, $5n-1=4n$, $5n+1=6n-1$, $5n-1=6n-4$, $3n+4=4n$ meaning $m=0,1,2,3,4$ and $n=0,1,2,3,4$ certain terms of (29) merge. As such considering a case where $m=1$, $n=1$ and $a=A_0$ equation (29) is written on the simplified form as follows

$$\begin{aligned}&\left(6RvA_1A_0^2+2RvA_2-2h^4A_0k^4-6h^2A_0k^2+4RvA_3+RGA_0\right)\cosh^4(kx-vt)\\&+\left(12h^2A_0k^2+22h^4A_0k^4-4RvA_3-12RvA_1A_0^2\right)\cosh^2(kx-vt)\\&+\left(-RGA_0-2RvA_4\right)\cosh^6(kx-vt)-30h^4A_0k^4+6RvA_1A_0^2=0.\end{aligned} \quad (30)$$

Equation (30) is verified if and only if each coefficient of its basics hyperbolic functions is zero. This permit to obtain the set of four equations below

$$\begin{cases}6RvA_1A_0^2+2RvA_2-2h^4A_0k^4-6h^2A_0k^2+4RvA_3+RGA_0=0,\\12h^2A_0k^2+22h^4A_0k^4-4RvA_3-12RvA_1A_0^2=0,\\-RGA_0-2RvA_4=0,\\-30h^4A_0k^4+6RvA_1A_0^2=0.\end{cases} \quad (31)$$

Solving the set of four equations (31), we obtain the following results

$$a=A_0,\ v=-\frac{A_0G}{2A_4},\ k=\pm\frac{\left(-A_1RA_0^2GA_4^3\right)^{\frac{1}{4}}}{10^{\frac{1}{4}}A_4h},\ A_2=A_4+A_1A_0^2+\frac{3\sqrt{-10A_1RA_0^2GA_4^3}}{5A_4RG},\ A_1A_4<0,$$

$$A_3=-\frac{19A_1A_0^2}{10}-\frac{3\sqrt{-10A_1RA_0^2GA_4^3}}{5A_4RG},\ u(x,t)=A_0\frac{\sinh^3\left(\pm\frac{\left(-A_1RA_0^2GA_4^3\right)^{\frac{1}{4}}}{10^{\frac{1}{4}}A_4h}x+\frac{A_0G}{2A_4}\right)}{\cosh^3\left(\pm\frac{\left(-A_1RA_0^2GA_4^3\right)^{\frac{1}{4}}}{10^{\frac{1}{4}}A_4h}x+\frac{A_0G}{2A_4}\right)}. \quad (32)$$

Equation (32) represents the analytical expression of the exact solitary wave solution of nonlinear partial differential equation (27) which govern its dynamics in the capacitive electrical line.

6. Conclusion

In this paper, we have modeled nonlinear partial derivatives equations which govern the dynamics of waves in an electrical line made of capacitors whose charge is nonlinear. Having obtained those nonlinear partial derivative equations, we have apply the Bogning-Djeumen Tchaho-Kofane method [12-17] to propose solitary wave solutions susceptible of being used as signals of propagation in those electrical lines. We have shown that it is possible to obtain solitary wave solutions of type Kink or Pulse. In our next studies, we will verify the propagation of these obtained solutions.

Acknowledgements

We are grateful to all those who have been a source of inspiration in our studies; to the University of Yaoundé 1 and the ministry of higher education of Cameroon whom through their support has enabled us to achieve this work.

References

[1] J. R. Bogning, A. S. Tchakoutio Nguetcho, T. C. Kofané, Gap solitons coexisting with bright soliton in nonlinear fiber arrays. International Journal of nonlinear science and numerical simulation, Vol. 6 (4), (2005) 371-385.

[2] N. J. Zabusky and M. D. Kruskal, "Interaction of "Solitons" in a collisionless plasma and the recurrence of initial states," *Phys. Rev. Lett.*, 15, pp. 240-243 (1965).

[3] B. A. P. Taylor, "What is a solitary wave?" *Am. J. Phys.*, vol. 47, pp. 847 (1979).

[4] D. C. Hutchings and J. M. Arnold, J. Opt. Soc. Am. B 16, 513 (1999).

[5] A. R. Djordjevic, T. K. Sarkar, and R. F. Harrington, IEEE Trans. Microwave Theory Tech. MTT-34, 660 (1986).

[6] H. Nagashima and Y. Amagishi, "Experiment on the Toda lattice using nonlinear transmission lines," *J. Phys. Soc. Japan*, vol. 45, pp. 690-688 (1978).

[7] T. Kuusela, J. Hietarinta, K. Kokko, and R. Laiho, "Soliton experiments in a nonlinear electrical transmission line," *Eur. J. Phys.*, vol. 8, pp. 27-33 (1987).

[8] N. S. Kuek, A. C. Liew, E. Schamiloglu and J. O. Rossi, "Pulsed RF oscillations on a nonlinear capacitive transmission line," *IEEE Transactions on Dielectrics and Electrical Insulation*, vol. 20, no. 4, pp. 1129-1135, Aug. 2013.

[9] N. Seddon, C. R. Spikings, and J. E. Dolan, "RF pulse formation in nonlinear transmission lines," *Proc. of 16th IEEE Int. Pulsed Power Conf.,* (Albuquerque, NM, 2007), pp. 678-681.

[10] E. Afshari and A. Hajimiri, "Nonlinear transmission lines for pulse shaping in silicon," *Solid-State Circuits, IEEE Journal of*, vol. 40, no. 3, pp. 744-752, March 2005.

[11] J. Gaudet, E. Schamiloglu, J. O. Rossi, C. J. Buchenauer, and C. Frost, "Nonlinear transmission lines for high power microwave applications – A survey," *Proc. of 28th IEEE Int. Power Modulator Symposium and 2008 High Voltage Workshop* (Las Vegas, NV, 2008), pp. 131-138.

[12] J. R. Bogning, C. T. Djeumen Tchaho and T. C. Kofané, "Construction of the soliton solutions of the Ginzburg-Landau equations by the new Bogning-Djeumen Tchaho-Kofané method", Phys. Scr, Vol. 85, (2012), 025013-025017.

[13] J. R. Bogning, C. T. Djeumen Tchaho and T. C. Kofané, "Generalization of the Bogning- Djeumen Tchaho-Kofane Method for the construction of the solitary waves and the survey of the instabilities", Far East J. Dyn. Sys, Vol. 20, No. 2, (2012), 101-119.

[14] C. T. Djeumen Tchaho, J. R. Bogning, and T. C. Kofané, "Modulated Soliton Solution of the Modified Kuramoto-Sivashinsky's Equation", American Journal of Computational and Applied Mathematics", Vol. 2, No. 5, (2012), 218-224.

[15] C. T. Djeumen Tchaho, J. R. Bogning and T. C. Kofane, "Multi-Soliton solutions of the modified Kuramoto-Sivashinsky's equation by the BDK method", Far East J. Dyn. Sys. Vol. 15, No. 2, (2011), 83-98.

[16] C. T. Djeumen Tchaho, J. R. Bogning and T. C. Kofane, "Construction of the analytical solitary wave solutions of modified Kuramoto-Sivashinsky equation by the method of identification of coefficients of the hyperbolic functions", Far East J. Dyn. Sys. Vol. 14, No. 1, (2010), 14-17.

[17] J. R. Bogning, "Pulse Soliton Solutions of the Modified KdV and Born-Infeld Equations" International Journal of Modern Nonlinear Theory and Application, 2, (2013), 135-140.

[18] A. M. Wazwaz, A reliable treatment of the physical structure for the nonlinear equation K(m,n), Appl. Math. Comput, 163, (2005) 1081-1095.

[19] A. M. Wazwaz, Explicit traveling waves solutions of variants of the K(n,n) and ZK(n,n) equations with compact and non compact structures, Appl. Math. Comput, 173, (2006) 213-220.

[20] M. L. Wang, Exact solutions for a compound Käv- Burgers equation, Phys. Lett. A. 213, (1996) 279-287.

[21] E. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A, 277, (2000) 212-218.

[22] E. Fan, J. Zhang, A note on the homogeneous balance method, Phys. Lett. A, 305, (2002) 383-392.

[23] Y. B. Zhou, M. L. Wang, Y. M. Wang, Periodic wave solutions to a coupled KdV equations with variable coefficients, Phys. Lett. A, 308, (2003) 31-37.