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Researches on Threshold Selection Rules of Wavelet Denoising

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Abstract

A comparative analysis is made on four common threshold selection methods of wavelet denoising in this paper. First, analyze the theoretical definitions of the four typical threshold selection rules. Next, design corresponding MATLAB simulation experiments [1] to verify. Finally, we can safely draw the conclusion that for one dimensional voltage signal, adaptive threshold has the best effect in reducing the noise caused by large Gaussian noise, while fixed threshold plays an important role in weak noise. As for the picture, adaptive threshold and mixed threshold denoising reduced the noise obviously.

Keywords

Wavelet Denoising, Threshold Selection Rules, MATLAB Simulation

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1. Introduction

The signals of sound and picture can be influenced by noise in every aspects. And the noise is mainly thermal noise (Gaussian white noise) produced by the machine. According to its characteristic, wavelet decomposition and reconstruction method can be used to reduce the noise.

There are three mature wavelet denoising methods, the first one is based on Mallat algorithm with the method of the maximin wavelet series modulus [2], the second method was proposed by Xu chang-fa who used the spatial correlation of wavelet coefficients to denoising [3], and the third was came up by Stanford professor Donoho D L and Johnstone in 1994, which is the most common method of wavelet threshold denoising currently [4]. In the wavelet threshold denoising, selection of wavelet basis, selection of threshold value, determination of decomposition coefficient and selection of threshold function will influence the effect of denoising directly [5].

Nowadays, researches on wavelet threshold denoising are mainly concerntrate on the selection of threshold function [6]. Scholars built new functions which based on the hard threshold function and soft threshold function, so as to achieve better effect on denoising. Usually, fixed threshold rule is used in their experiments, while the influence of different threshold selection rules on denoising effect is ignored.

Therefore, we will discuss the effect of different threshold selection rules on denoising effect.

2. Methodology

2.1. Wavelet Transform

Traditional signal processing is based on Fourier transform, which is of great use in smooth and steady signals, however, most signals from the real life are not ideal, which are not completely smooth and steady. Thus, the limitations of Fourier transform in singular point will produce Gibbs effect.

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And wavelet transform can solve the problem effectively.

Wavelet transform is a signal decomposes into a series of wavelet function groups which is composed by the translation and expansion of mother wavelet. The time domain expression is as follows:

$$WT_x(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\phi^*(\frac{t-\tau}{a})dt \quad (1)$$

Where $\phi(t)$ is the mother wavelet, a is the scale factor, which stands for the transformation of mother wavelet, τ equal for the translation of mother wavelet, $\phi(\frac{t-\tau}{a})$ equals for the wavelet basis function which translate by the translation and expansion of mother wavelet. The wavelet transform process can be regarded as the inner product of signal x(t) and a group of wavelet basis. The frequency domain expression can also be obtained:

$$WT_x(a,w) = \frac{\sqrt{a}}{2\pi} \int_{-\infty}^{+\infty} X(w)\Psi^*(aw)e^{+jw\pi}dw \quad (2)$$

According to the time frequency analysis of wavelet transform, we can obtain the following features of wavelet transform [7]:

(1) When a gets small, the time domain of x(t) is narrowed, but the range of X(w) is widened in frequency observation, and the center frequency of observation is moved to a high frequency. While a gets large, the time domain of x(t) is widened, and the range of X(w) is narrowed in frequency, and the center frequency is moved to a low frequency.

(2) According to the invariable properties of the quality factor Q of wavelet transform, it can be seen that in high frequency, frequency resolution is low while time resolution is high. In low frequency, frequency resolution is high while time resolution is low.

In real life, high frequency components in the signal often correspond to the rapid changes in time domain, such as steep front, back edge, and sharp pulse. Analyze this kind of pulse signals, high resolution ratio in time domain is needed to meet the requirements of rapidly changes, while the resolution ratio in frequency is less demanded, and the analysis of center frequency should be moved to a high frequency. So as to low frequency signal in real life, low frequency signals are often change slowly, this kind of signals' analysis generally needs a better resolution ratio in frequency, while resolution of time can be relaxed, at the same time, the center frequency of analysis should be moved to a low frequency. Obviously, the characteristics of wavelet transform can automatically meet these objective practical needs.

2.2. Wavelet Threshold Denoising

The basic idea of the wavelet threshold denoising is to set a critical threshold value λ. If the wavelet coefficient is less than λ, the coefficient is considered to be caused by the noise, and the coefficient set to zero. If the coefficient of the wavelet coefficients is bigger than λ, the coefficient is considered to be caused by the signal, and the coefficient retain. Then, the wavelet coefficients of the processed wavelet coefficients can be changed to obtain the signal after denoising. The specific steps are as follows:

(1) define the signal with noise as f(t)=x(t)+n(t) (where x(t) is the original signal, n(t) is the Gaussian White noise with the variance of σ^2), then do wavelet transform, thus, a set of wavelet decomposition coefficients $w_{j,k}$ is obtained. The wavelet decomposition coefficients are composed by wavelet decomposition coefficients of noise $v_{j,k}$ and wavelet decomposition coefficients of the original signal $\mu_{j,k}$. Due to uneven distribution of space in original signal x(t), the scale wavelet coefficients $\mu_{j,k}$, in some specific location has great value, which corresponding to important information of the original signal x(t), while other places $\mu_{j,k}$ are small. As for white noise n(t) with Gaussian distribution, the corresponding wavelet coefficients $v_{j,k}$ are uniformly distributed on each scale.

(2) choose a proper threshold λ , quantitative threshold processing the wavelet decomposition coefficient $w_{j,k}$, set the wavelet coefficients $w_{j,k}$ which are lower than λ to zero(mainly caused by signal n(t)), and maintain the wavelet coefficients $w_{j,k}$ which are higher than λ (mainly caused by signal x(t)). Then, the estimated wavelet coefficients $T_{j,k}$ are

obtained, and make $\left\| T_{j,k} - \mu_{j,k} \right\|$ as small as possible.

(3) Using the estimated wavelet coefficients $T_{j,k}$ to do wavelet reconstruction, then the estimation signal $\hat{f}(t)$ is obtained, which is denoising signal.

Therefore, the selection of the appropriate threshold is of great importance. We will discuss the effect of four common threshold selection rules on denoising effect as follow.

2.3. Threshold Selection Rules

There are four common threshold selection rules:

(1) Fixed threshold (sqtwolog)

This method is based on in the independent Gaussian variables of N dimensional. With the increase of N, the probability of the maximum value less than λ is approaching to 1. The threshold expression is $\lambda = \sigma\sqrt{2*\log N}$, where σ stands for the deviation of the noise signal, and N is the length of the signal. Usually we choose $\sigma = \frac{\text{medium}(T_{ij})}{0.6745}$, where T_{ij} are wavelet coefficients.

From the formula, it is not difficult to find that this choice of threshold is associated with the size of the signal, the big signal leads to the big threshold value, which will make effective signal reset and then produce non-negligible error.

(2) Adaptive threshold (rigrsure)

Adaptive threshold is based on the idea that threshold change with the change of wavelet coefficients. This algorithm is based on the unbiased estimation principle of Stein, and determines the minimum risk amount of the corresponding the wavelet coefficients as the threshold [8]. The specific steps are as follows:

First, wavelet coefficients are squared and then arranged in the order of small to large, vector $P=[P_1, P_2, \dots, P_n]$ $P_1 \le P_2 \le \dots \le P_n$ is obtained. Then risk vector $R=[R_1, R_2, \dots, R_N]$ is calculated according to the vector P, where Ri satisfies: $R_i = [N-2*i+(N-i)*P_i+\sum_{k=1}^{i} P_k]/N$.

Finally, the threshold value is calculated by wavelet coefficients square corresponding to the risk value by the equation $\lambda = \sigma\sqrt{P_i}$. When the signal is in high frequency, the determination of this threshold can extract weak signal.

(3) Mixed threshold (heursure)

This threshold selection rule is based on fixed threshold and adaptive threshold, and the optimal results are selected.

First, the sum of squares of wavelet coefficients is obtained as $\sum_{i=1}^{N} P_i$, and then define $eta = \left[\sum_{i=1}^{N} P_i - N\right]/N$, $crit = (\frac{InN}{In2})^{1.5}$. The expression of the mixed threshold can be expressed as:

$$\lambda = \begin{cases} \lambda = \sigma\sqrt{2*\log N} & eta < crit \\ \min(\lambda = \sigma\sqrt{2*\log N}, \lambda = \sigma\sqrt{P_i}) & eta \ge crit \end{cases} \quad (3)$$

(4) Minimaxi

The principle of this method is to minimize the maximum risk of estimation.

$$\lambda = \begin{cases} 0.3936 + 0.1829 In(N-2) & N > 32 \\ 0 & N \le 32 \end{cases} \quad (4)$$

Different threshold selection rules inevitably lead to different denoising effect, for fixed threshold and minimax threshold, the threshold value of these two ways are fixed, which don't change with the change of wavelet coefficients, this would be easy to produce truncation phenomenon. This is bound to produce local jitter in signal reconstruction and will cause the denoising effect not ideal. As for adaptive threshold and mixed threshold, both are based on threshold changes along with the change of wavelet coefficients, this treatment can reduce truncation errors effectively. From the theoretical analysis, the threshold value with the change of wavelet coefficient is better than the fixed threshold. Therefore, it is necessary to further discussion about the comparison of the denoising effect of the four different threshold selection rules in the actual situation.

3. Experimental Evalutione

3.1. Index of Denoising Effect

In order to find out the experimental effect of different threshold denoising, signal-to-noise ratio and mean square error are used to evaluate the signal [9].

(1) signal-to-noise ratio (SNR)

This index is refers to the average power ratio of the signal and noise, which is usually described by dB. The larger the value is, the smaller the noise is, and the definition is as follows:

$$SNR = 10\log\left(\frac{\frac{1}{N}\sum_{n=1}^{N} f^2(n)}{\frac{1}{N}\sum_{n=1}^{N}\left[f(n)-\hat{f}(n)\right]^2}\right) \quad (5)$$

(2) mean square error (MSE)

This index reflects the difference between the estimator and the estimated quantity. The smaller the value is, the higher the accuracy is. The expression is as follows:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left[f(n)-\hat{f}(n)\right]^2 \quad (6)$$

Where $f(n)$ equals to the signal with noise and $\hat{f}(n)$ equals to denoised signal.

3.2. Experiment Simulation

(1) Leleccum

We used one-dimensional voltage signal (leleccum) to simulate the speech signal and select 1024 sample points. Then we get the original signal, as shown in Figure 1 (a). On the basis of the original signal, Gaussian white noise with large amplitude is added to simulate the noise interference in the process of signal acquisition and transmission, as shown in Figure 1 (b). Then, select fixed threshold, adaptive threshold, mixed threshold, and minimaxi threshold to reduce the noise respectively, and the signals after denoising are obtained, as shown in Figure 1 (c) - (f).

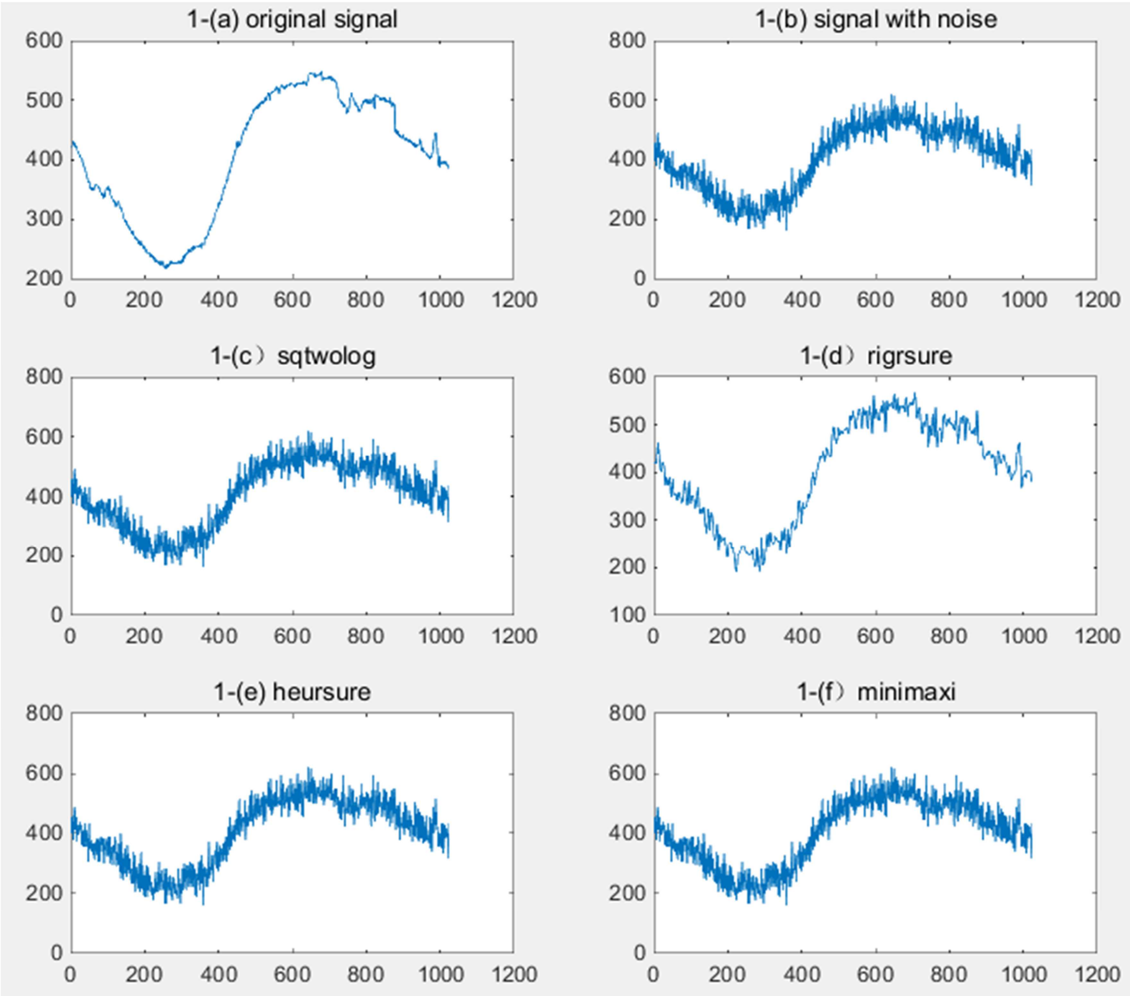

Figure 1. Different threshold in reducing signal with large noise.

As it depicted in the diagram, all of the four kinds of denoising methods reduced the noise to some degree, but except for the effect of adaptive threshold denoising is obvious, the

other three threshold denoising results are not ideal.

Through the calculation of signal-to-noise ratio and mean square error of the four threshold selection rules, the table 1 is as follows:

Table 1. SNR and MSE of the signal processed by different threshold.

signal	signal with noise	sqtwolog	rigrsure	heursure	maximini
SNR	18.3899	19.1353	27.8401	19.1353	18.8339
MSE	942.0917	793.5051	106.9222	793.5051	850.5367

According to the analysis of the SNR and MSE, it is known that adaptive threshold denoising is the most effectively when the one-dimensional voltage signal is interfered by large noise.

When the Gaussian noise signal is very small, the experimental processes are repeated. A new set of experimental results were obtained:

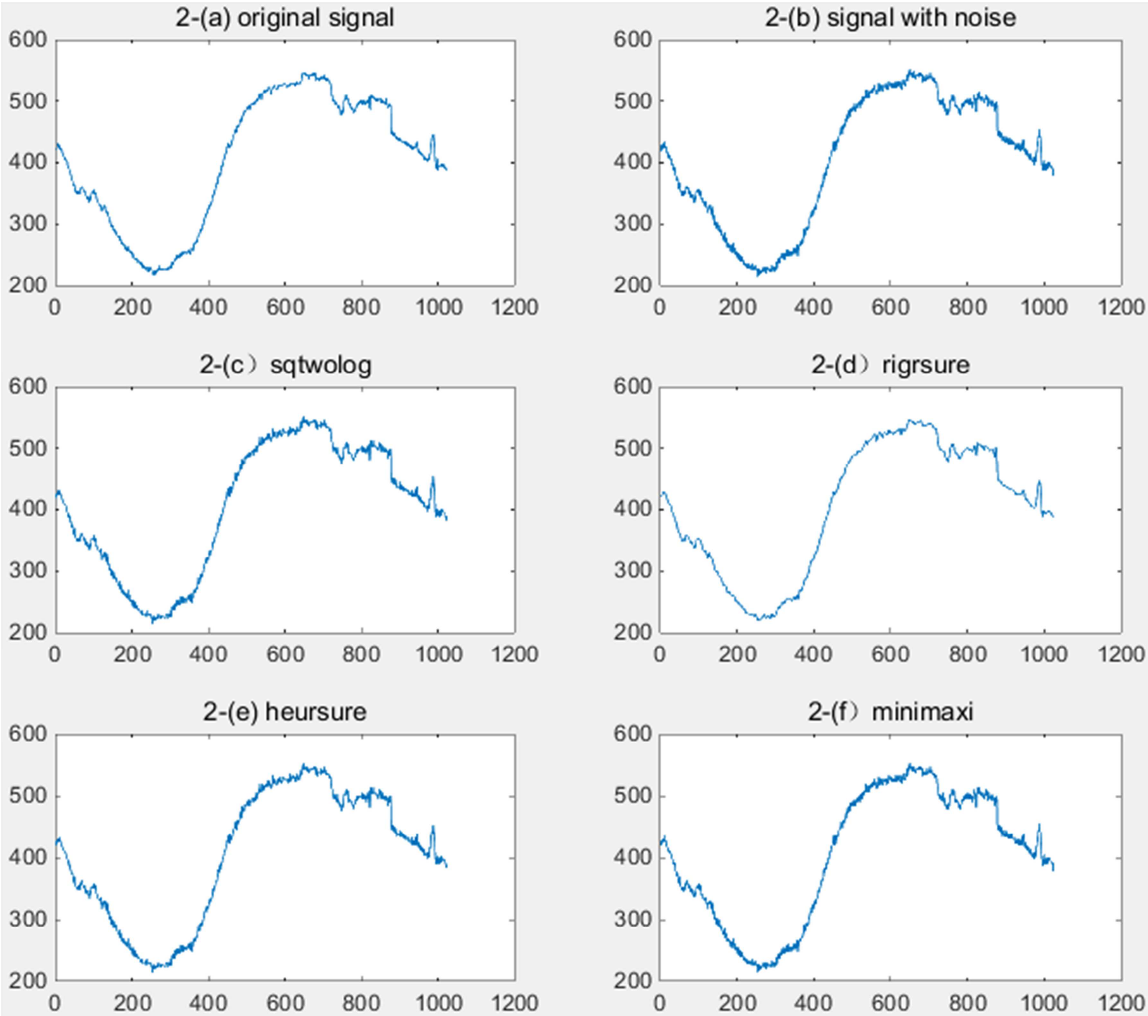

Figure 2. Different threshold in reducing signal with small noise.

It can be seen from Figure 2 (c) - (f), since the interference signal is relatively small, the signal-to-noise ratio of the noisy signal is large and mean square error is small.

Through the calculation of the signal-to-noise ratio and mean square error of the four threshold selection rules, the table 2 is as follows:

Table 2. SNR and MSE of the signal processed by different threshold.

signal	signal with noise	sqtwolog	rigrsure	heursure	maximini
SNR	38.3899	42.0162	38.8963	42.0162	41.3592
MSE	9.4209	4.0876	8.3839	4.0876	4.3592

By analyzing the signal-to-noise ratio and mean square error, it is known that fixed threshold and mixed threshold are the best for the one-dimensional voltage signal with large noise. The mixed threshold result is consistent with the fixed threshold value, which is in line with the mixed threshold algorithm.

(2) Picture [10]

Select a picture taken by MATLAB as the original signal, as shown in Figure 3 (a). Then add Gaussian white noise to simulate the noise in the process of image acquisition and transmission, as shown in Figure 3 (b). Next, fixed threshold, adaptive threshold and mixed threshold and maximini are respectively selected to reduce the noise, finally Figure 3 (c)-(f) are obtained.

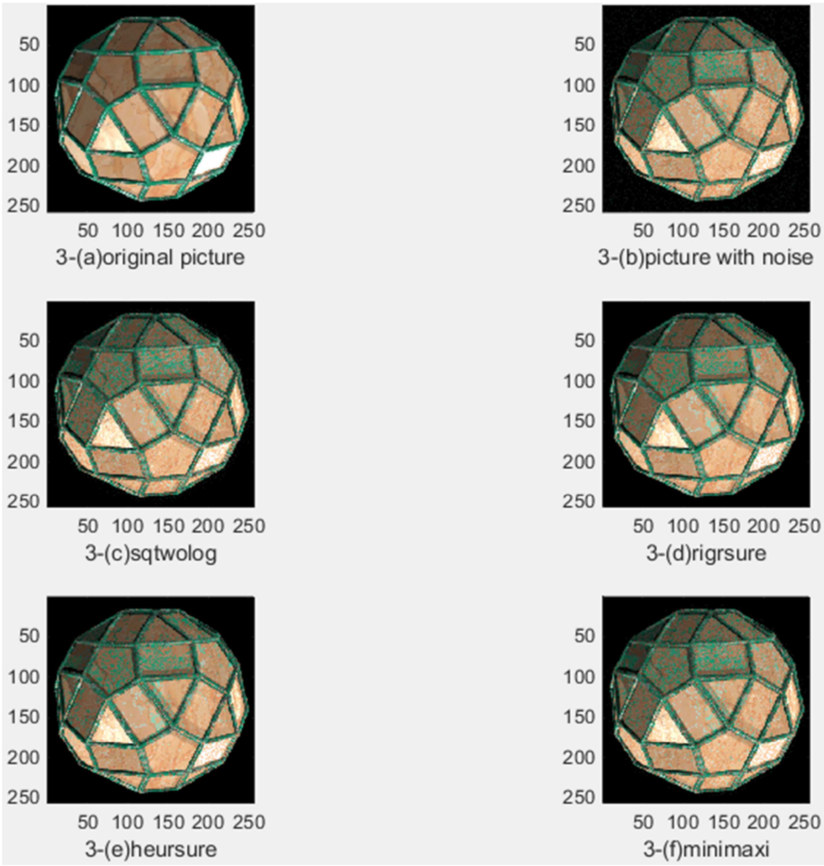

Figure 3. Different threshold in reducing picture with noise.

As it can be seen from the figures, the denoising of the four threshold values reduced the

noise of original image in varying degrees, but there are some deviations.

The table below is obtained by calculating the signal-to-noise ratio and mean square error of the image signals processed by the four threshold values (table 3).

Table 1. SNR and MSE of the picture processed by different threshold.

signal	Signal with noise	sqtwolog	rigrsure	heursure	maximini
SNR	8.1058	46.2750	122.6608	122.6608	49.8191
MSE	1.0058e+4	1.5331	3.5237e-08	3.5237e-08	0.6779

Analyze the results of the form, it can be seen that 4 kinds of threshold selection rules can reduce the noise in a certain degree, adaptive threshold and mixed threshold is worth to use due to its high SNR and low MSE. This illustrate the validity and reliability of the denoising results are great.

4. Conclusion

By introducing the principle of wavelet de-noising, this paper introduces four factors that influence the effect of denoising. Nowadays, the researches are mostly based on structuring a new threshold function while ignore the influence of different threshold selection rules. However, the selection rules of threshold value also play an important role in denoising effect. Further discussion is taken to analyze the theoretical effects of threshold selection rules through expressions. Then take the corresponding simulations analysis, the conclusion is as follows:

For one dimensional voltage signal, when the noise is big, adaptive method of threshold denoising is of great use. There is still a big error in signal, therefore secondary denoising with wavelet decomposition can be used. When the noise is weak, adaptive threshold denoising may remove the useful signal, so the choice of fixed threshold and mixed threshold algorithm is better.

For pictures denoising, due to fixed threshold method is highly affected by the size of picture. When picture is large, threshold will be very big, which will cause truncation errors and then result in inaccurate signal reconstruction. Adaptive threshold denoising obtain high PSNR and small MSE, and its denoising effect is the best of four.

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