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H-crystal as a Core Structure in Multilayer Weighted Networks

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Abstract

Extending the network h-core in single layer weighted networks, a method to extract a multilayer weighted network's core structure, called h-crystal, has been proposed and verified. By applying the algorithms of h-degree and h-strength to each individual layer, a network's h-core consisting of all the nodes having the h-degree above within edges and an h-subnet consisting of all the edges having the h-strength above with the nodes adjacent to the edges had been obtained, for each layer, at first. H-crystal is then identified by constructing layer-bridges between the layers' network h-cores and h-subnets. Via two empirical cases of information networks, it is found that the h-crystals of the networks exist, while their features and properties are revealed.

Keywords

H-degree, H-strength, Network H-core, H-subnet, H-crystal, Multilayer Networks, Weighted Network, Information Network, Heterogeneous Network

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1. Introduction

Stimulated by Watts & Strogatz [1] and Barabási & Albert [2] among others, the term complex networks became a household word among the 21th century scientists, leading to new developments in network (i.e. graph) theory, dynamics of networks, social networks, ecological networks, infrastructure related networks, molecular networks, spatial scientometrics, informetrics, webometrics as well as cognitive networks [3-9].

While most studies of classical complex networks focus on single-layer networks [6, 10], new research on complex networks often focuses on multilayer networks [11-12], multiplex network [13-15], and hierarchical and heterogeneous networks [7-9]. In the long review of Boccaletti et al. [12], the characteristics of a multilayer network had been defined, where a graph G with different layers G_α and G_β had common elements as interconnections. Then a multiplex network was defined as a special type of multilayer network.

Meanwhile, the classical homogeneous scholarly graph contains one type of nodes only, either author nodes, keyword nodes, or document nodes. The heterogeneous scholarly graph, on the other hand, characterizes the complex relations between different kinds of nodes, and various types of paths provide great potential to interconnect different research objects, i.e., publications, terms, authors, venues, etc. [16-20], which enriched the studies of multilayer heterogeneous networks.

Moreover, after h-index was introduced [21], its applications

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in homogeneous networks have been discussed [22]. In our early work, we have extended the h-index to h-degree, h-strength and h-subnets [23-25], to reveal network properties and core structures. In this paper, we continue extending the h-type terminology to multilayer (and heterogeneous) networks, which may apply to reveal the core structure of multilayer weighted networks, address the core documents and look potential applications in three-dimensional (3D) visualization.

In this research, we focus on the multilayer weighted networks and translate the h-type concepts and methods from single-layer networks to multilayer networks. In Particular, we proposed and studied a new multilayer core, h-crystal, for investigating core structure and context in the multilayer weighted networks. While similarity method was used to explore "core documents" [26-28] in single-layer (and homogeneous) networks, h-crystal can be a new method to identify another kind of core structures in multilayer (and heterogeneous) networks.

2. Methodology

It is well known that a network or graph consists of nodes and edges (links) [6, 29-30]. When nodes and edges represent information-related objects, we refer to such networks as information networks. Figure 1 shows some objects used to build information networks in scientific literature, and different objects and edges may belong to different types.

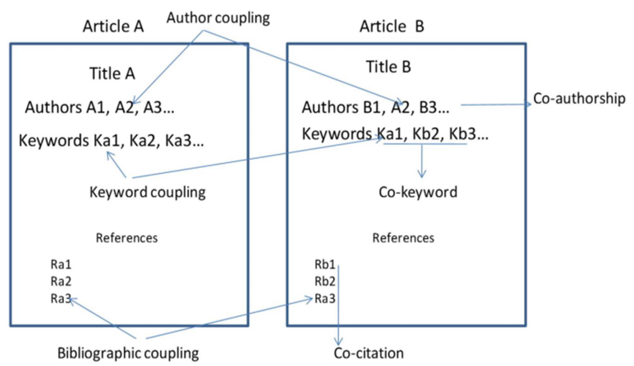

Fig. 1. Information network relations in document.

Co-authorship networks, co-citation networks, bibliographic coupling networks and similar networks are examples of (homogeneous) single-layer networks, which host only the same kind of information. Now, following Boccaletti et al. [12], we consider several aspects simultaneously, leading to a multilayered structure. For example, we may consider the three layers illustrated in Figure 2: a co-citation layer, a bibliographic coupling layer and a co-keyword layer, which can be integrated into a multilayered graph within more scholarly information, comparing with single-layer network. This construction is an example of a multilayer network in which constituents, i.e. nodes and edges, are consist of different layers. In these examples nodes are papers (layers 1 and 2) and terms (layer 3), while edges have different meanings as well, namely referring to co-citation, bibliographic coupling and being co-keywords.

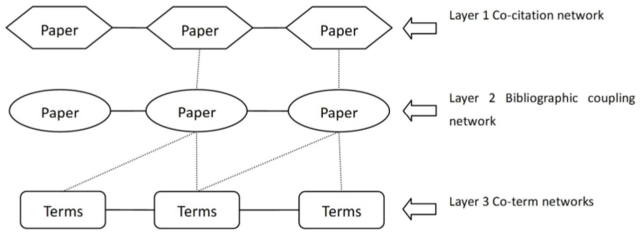

Fig. 2. An example of a multilayer information network.

In the following sections, along the h-type core structure in the single-layer networks, we extend h-type concepts from the single-layer networks to the multilayer networks.

2.1. Characterizing and Measuring the Core Structure of Single-Layer Network

Taking the lead from Hirsch's h-index [21] we introduced the notions of h-degree, for nodes, and h-strength, for edges [23-25]. These notions can be used to characterize weighted networks.

Definition 1 [23]. The h-degree (d_h) of node n in a weighted network is equal to d_h(n) if d_h(n) is largest natural number such that node n has at least d_h(n) links each with strength at least equal to d_h(n).

Using the notion of an h-degree leads to a network's h-core, a substructure of the complete network.

Definition 2. A network's h-core is set of nodes and their links, that all have an h-degree at least h.

It is important to point out that the h-degree (d_h) is a node-based measure, while a network's h-core is a set of the nodes and their links.

The h-strength is also introduced as follows [25]:

Definition 3. The h-strength (h_s) of a network is equal to h_s, if h_s is the largest natural number such that there are h_s links each with strength at least equal to h_s in the network.

The h-strength characterizes the core edges of a network in terms of link strengths. Dually to the notion of network's h-core we now define the h-subnet.

Definition 4. The h-subnet of a network is a sub-network consisting of all edges with strengths larger than or equal to the h-strength of the network and the nodes adjacent to these edges.

Note that h-strength (h_s) is a measure defined on edges and h-subnet includes all the edges and their nodes. The h-strength is an edge-based measure, while h-subnet is a set of the edges and their nodes. All these informetric indicators characterize the importance of the nodes within one layer.

A simple numbered example for identifying the network's h-core and the h-subnet in a single-layer weighted network can be explained as shown as Figure 3.

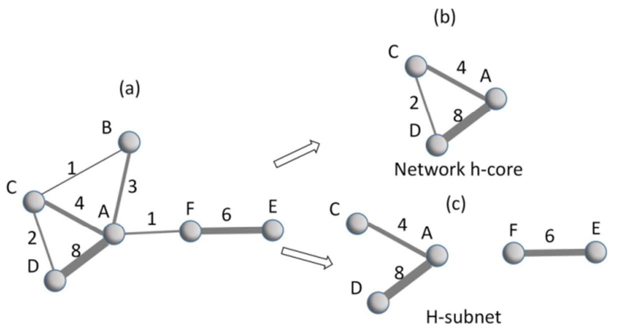

Fig. 3. A numbered example of network's h-core and h-subnet in single-layer network.

In Figure 3, the graph (a) is a weighted network within 6 nodes, where each node has its h-degree as A:3, B:1, C:2, D:2, E:1 and F:1 so that the subgraph (b) consisted by the nodes with h-degree ≥ 2 as well as their links yields the network's h-core, while the weights of 7 edges in the graph (a) rank as 8, 6, 4, 3, 2, 1, 1 and lead the h-strength equaling to 3 so that all edges with weights ≥ 3 generates h-subnet, i.e. subgraph (c).

2.2. Characterizing and Measuring the Core Structure of a Multilayer Network

It is necessary and important to find the core structure or sub-structure of a complex multilayer network [8], for judging the relatively important nodes and edges as well as their functions in the multilayer network. For the aim, we first introduce the notion of a layer bridge to connect the core structures of all single-layer networks so that we can approach the core structure of the multilayer network.

Definition 5. If two nodes in two layers of a multilayer network represent the same object, these nodes are artificially linked, which are called layer-bridges of the multilayer network.

When two layers in a multilayer network have layer-bridges they are connected, otherwise, they are not. The network's h-core and the h-subnet are sub-structures of single-layer weighted networks, which might not be connected. When all layers are connected through layer-bridges, this may lead to the complete connected graph of the multilayer network.

Combing h-degree, h-strength with layer-bridges, linking through multilayer network's h-cores and h-subnets, a core structure of whole multilayer weighted network, called h-crystal, is defined as follows.

Definition 6. The h-crystal is a core structure existing in a multilayer weighted network which consists of all network's h-cores and h-subnets in each layer of the network, connected by layer-bridges between two layers.

By definition, an h-crystal is a core structure, linking through network's h-cores and h-subnets in all layers via layer-bridges, where h-crystal must be connected. If there is only unconnected or broken structure, we say that h-crystal does not exist in the multilayer weighted network. Here, the layer-bridges between two layers link the really same nodes (same documents). As the h-core and h-subnet among single-layer networks are connected by the bridges, h-crystal will be connected and lead unique core structure in the multilayer weighted network so long as h-crystal exists.

A simple numbered example for identifying the h-crystal in a multilayer weighted network can be explained as shown as Figure 4.

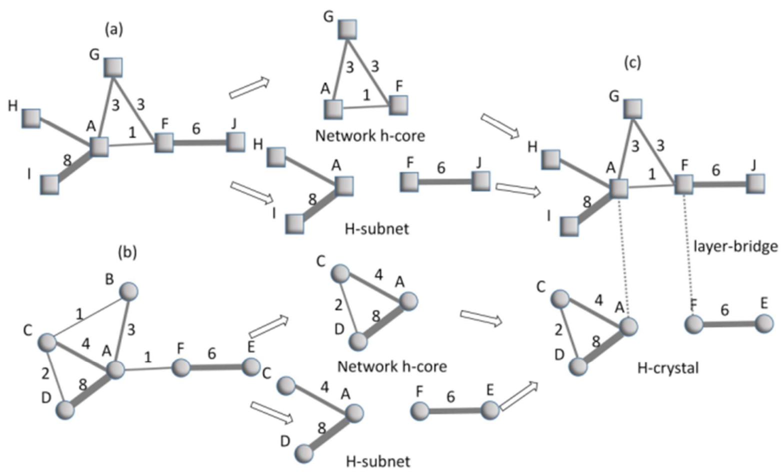

Fig. 4. A numbered example of h-crystal in multilayer network.

In Figure 4, both the graph (a) and graph (b) are weighted networks, respectively, within 6 nodes, which construct a two-layer network, where there are two same nodes (A and F). After computing and formulating the network's h-cores and h-subnets of (a)-layer and (b)-layer similarly to Figure 3, the h-crystal can be generated by connecting same A and F as layer-bridges.

It is noted that there are only few methods for extracting network core in a complex network. In homogeneous single-layer networks, we can mention k-core [31] and backbone [8]. However, the methods focus on node degree and cannot be applied to multilayer weighted networks. Here, we compute k-cores in each layer and put them together as a comparison. As both k-core and backbone need artificial parameters for approaching results, in which we cannot determine the changeable parameter α for backbone and we may set k=h for extracting k-core according to node degree, we compare with k-core only.

2.3. The Procedure of Identifying H-crystal

In next empirical studies, a multilayer network is constructed by a co-citation layer, a bibliographic coupling layer and a co-keyword layer, where the h-crystal of the multilayer network can be identified as three steps as illustrated in Figure 5, in the weighted network.

Step 1: Extracting the network's h-core using the algorithm of h-degree in each layer;

Step 2: Finding the h-subnet using the algorithm of h-strength in each layer;

Step 3: Constructing layer-bridges through linking the same nodes in two layers.

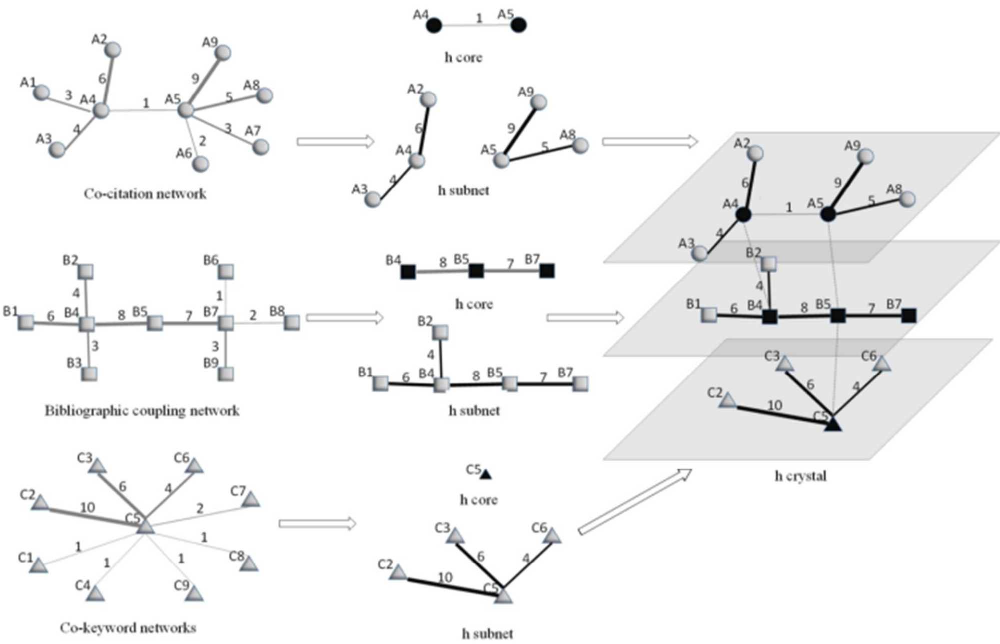

Fig. 5. Identifying the h-crystal in a multilayer network.

The dark black nodes and edges in Figure 5 mark the network's h-core, and the white nodes with edges form the h-subnet. The same paper nodes in Layer 1 and Layer 2 construct layer-bridges between Layer 1 and Layer 2, and the same keywords in different papers in Layer 2 and Layer 3 produce layer-bridges between Layer 2 and Layer 3. As the co-citation layer, bibliographic coupling layer and co-keyword layer are linked by layer-bridges, we got connected h-crystal.

Pseudocodes for identifying h-crystal in a multilayer network are provided in Appendix 1. While we draw 2D diagram with using the open-source software packages NetDraw (cf. https://sites.google.com/site/netdrawsoftware/home), we prefer to create 3D diagram with using Mage (cf. http://kinemage.biochem.duke.edu/software/mage.php), for clear crystal effects and potential 3D visualization developments.

3. Empirical Studies

Practical experiments have been run to test the above method of identifying h-crystal in a multilayer network.

3.1. Datasets and Experiments

Two sets of data were generated from the Web of Science (WoS) for the experiment:

(1) The document set referred to as the "h-set", which is retrieved from the Web of Science (WoS) by the following search strategy: TS= (h-index OR h-type ind* OR h-like ind* OR Hirsch index) OR TI="An index to quantify*" in the WoS for the publication period 2005-2012. Results were restricted to the two fields Information Science & Library Science and Multidisciplinary Sciences.

(2) The document set referred to as the "GR&SM-set", which is retrieved by the search strategy: TS= (General relativity) AND TS=(Standard model) in the database SCI-E for the publication period 1915-2012, without other restrictions.

The data sets were downloaded on August 1, 2014.

These two data sets represent two multilayer information networks with a combination of a co-citation network, a bibliographic coupling network and a co-keyword network (including the keywords in the ID and DS records). Tables 1 and 2 show the main features of these multilayer weighted networks.

Table 1. Multilayer weighted network parameters of the "h-set".

Type	Co-citation network	Bibliographic coupling network	Co-keyword network
number of nodes	6614	484	1237
number of edges	467,151	89,123	11,718
h-degree	15	12	8
number of nodes in the network's h core	20	23	14
h-strength	40	27	21
number of nodes in the network's h core and h subnet	27	38	20
number of edges in the network's h core and h subnet	199	263	103

Table 2. Multilayer weighted network parameters of the "GR&SM-set".

Type	Co-citation network	Bibliographic coupling network	Co-keyword network
number of nodes	22,434	625	2142
number of edges	1,684,501	22,972	20,563
h-degree	10	12	7
number of nodes in the network's h core	17	15	8
h-strength	20	35	15
number of nodes in the network's h core and h subnet	21	36	11
number of edges in the network's h core and h subnet	105	85	34

3.2. The H-crystal of the "H-set"

The h-crystal identified by the h-crystal procedure for the "h-set" is shown in Figure 6, where A refers to the co-citation layer, B to the bibliographic coupling layer and C to the co-keyword layer. Linked nodes between layers A and B represent the same papers, i.e. A270=B195, and so on.

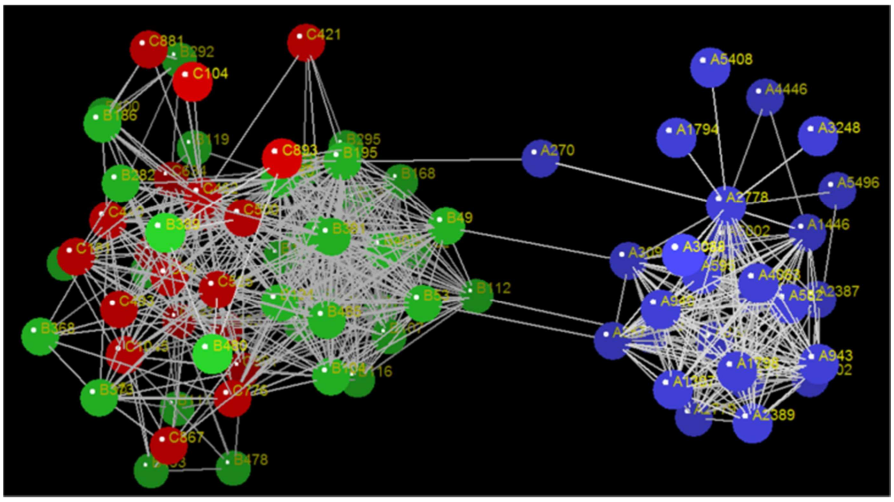

Fig. 6. H-crystal of the dataset "h-set".

In Figure 6, the A-type nodes express the core nodes of the co-citation network (A-core); B-type nodes mark the core nodes of the bibliographic coupling network (B-core), while C-type nodes refer to the core nodes of the co-keyword network (C-core). To illustrate the network structural information, we select the top five core nodes and rank according to their betweenness centralities, shown in Table 3, which form the core of the multilayer weighted network.

Table 3. Top 5 core nodes by betweenness centrality (denoted as bc) in the h-crystal of the "h-set".

Layer of co-citation network		Layer of bibliographic coupling network		Layer of co-keyword network	
Core node	**bc**	**Core node**	**bc**	**Core node**	**bc**
A4063	0.46	B296	0.1934	C462	0.5439
A2778	0.3569	B116	0.0713	C493	0.5234
A2387	0.2985	B53	0.043	C547	0.1345
A1802	0.0554	B120	0.0368	C440	0.0673
A1446	0.0554	B391	0.0345	C1045	0.0497

All these core nodes can be observed in Figure 6.

3.3. The H-crystal of "GR&SM-set"

Applying the h-crystal procedure to the "GR&SM-set", a similar h-crystal can be created and shown in Figure 7, where 3D-visual graphs are given, with similar A, B, C layers and symbols as in the previous example.

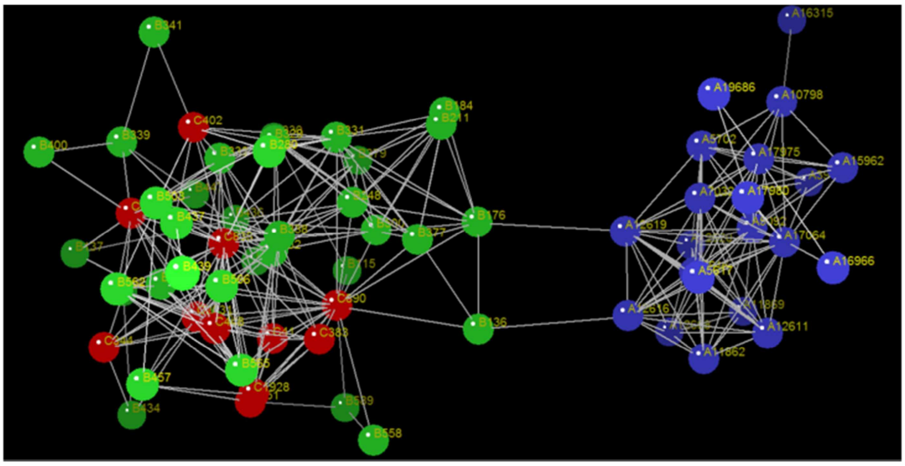

Fig. 7. H-crystal of the dataset "GR&SM-set".

Similarly, we list in Table 4 the top five core nodes according to their betweenness centrality. They form the core of the network core in the multilayer weighted network.

Table 4. Top 5 core nodes by betweenness centrality (bc) in the h-crystal of the "GR&SM-set".

Layer of co-citation network		Layer of bibliographic coupling network		Layer of co-keyword network	
Core node	**bc**	**Core node**	**bc**	**Core node**	**bc**
A17064	0.4363	B436	0.1914	C890	0.3667
A7032	0.1962	B330	0.1805	C448	0.2444
A17975	0.1224	B331	0.1227	C838	0.2
A5702	0.1221	B503	0.0426	C351	0.1778
A12620	0.1198	B377	0.0319	C2031	0.0667

Again, all these core nodes can easily be observed in the visual representations in Figure 7.

As a comparison, we also list the total numbers of nodes of both h-crystal and k-core in two datasets, as shown in Table 5, in which the k-core is calculated by k-core function in the software package Networkx (https://networkx.github.io), with setting k=h-degree in each layer then summing the nodes of all layers together. Here, the A, B, C three layers' k-cores of dataset 'h-set' contain nodes as 6222, 471, 924 when k=15, 12, 8 respectively, with same nodes between A and B as 288, so that the total nodes of k-core become 6222+471+924-288=7329. Similarly, the A, B, C three layers' k-cores of dataset 'GR&SM-set' contain nodes as 22345, 530, 1888 when k=10, 12, 7 respectively, with same nodes between A and B as 95, so that the total nodes of k-core are 22345+530+1888-95=24668.

Table 5. Total nodes of the h-crystal and k-core.

Dataset "h-set"		Dataset "GR&SM-set"	
Nodes of h-crystal	Nodes of k-core	Nodes of h-crystal	Nodes of k-core
81	7329	66	24668
Node percentage of h-crystal vs. k-core (%)		Node percentage of h-crystal vs. k-core (%)	
1.1		0.26	

Comparing the nodes in h-crystal and k-core, we find that all nodes of h-crystal are covered by the nodes of k-core (while k=h-degree), which means h-crystal is real core. However, h-crystal is much smaller (only about 1% nodes of k-core) and k-core may be unconnected, so that h-crystal may become the minimum connected core in a multilayer weighted network.

The above two empirical cases demonstrate that an 'h-crystal' can be identified in real-world multilayer information networks and h-crystal can form the core structure of a multilayer weighted network, which could lead to effective applications for extracting and judging important core. The documents of the core nodes of the A- and the B-layer are listed in Appendices 2 and 3. It is clear that those nodes provide the key information for the constructed multilayer networks. For example, the first ranked items at the A-layers are Hirsch's original article (A2778) introducing the h-index in "h-set" and Reiss et al.'s famous paper (A17975) on the accelerating universe that led to the 2011 Nobel Prize in physics, which are really the most important documents.

4. Analysis and Discussion

As the h-crystal consists of core nodes (nodes in the network's h-core), core edges (edges in the h-subnet) in each single-layer weighted networks, and layer-bridges in-between two layers, we can derive some of their theoretical properties following the ideas of an h-degree and h-strength [23-25]. Actually, the analytical properties of h-crystal can be obtained by merging all the layers' network's h-cores and h-subnets, where network's h-cores generated by nodes following h-degree algorithm and h-subnets by edges following h-strength algorithm. However, as there is complex mathematical structure in multilayer networks [11], a general result has not yet been reached.

4.1. The Nodes of the H-crystal

Consider a multilayer weighted network consisting of three layers A, B and C, with N_A, N_B and N_C nodes, and the three layers have the h-degree N_{Ah}, N_{Bh} and N_{Ch}, respectively. Suppose that the numbers of nodes that the node's h-degree equals N_{Ah}, N_{Bh} and N_{Ch} are N_{Ad}, N_{Bd} and N_{Cd} respectively. Let $N_{Ah\text{-}cs}$, $N_{Bh\text{-}cs}$ and $N_{Ch\text{-}cs}$ represent number of nodes in the network's h-core and h-subnet, $N_{Ah\text{-}s}$, $N_{Bh\text{-}s}$ and $N_{Ch\text{-}s}$ denote nodes in the h-subnet only (except nodes in the network's h core). If the number of same nodes of N_{Ah} and N_{Bh} is N_{AB} and that of N_{Bh} and N_{Ch} is N_{BC}, the total number of nodes in the h-crystal ($N_{h\text{-crystal}}$) can be calculated by following formulas

$$N_{h-crystal} = N_{Ah\text{-}cs} + N_{Bh-cs} + N_{Ch-cs} - N_{AB} - N_{BC} \quad (1)$$

$$N_{Ah\text{-}cs} = N_{Ah-c} + N_{Ah-s} = N_{Ah} + N_{Ad} - 1 + N_{Ah-s} \quad (2)$$

$$N_{Bh\text{-}cs} = N_{Bh-c} + N_{Bh-s} = N_{Bh} + N_{Bd} - 1 + N_{Bh-s} \quad (3)$$

$$N_{Ch\text{-}cs} = N_{Ch-c} + N_{Ch-s} = N_{Ch} + N_{Cd} - 1 + N_{Ch-s} \quad (4)$$

$$N_{Ad} \geq 1, N_{Bd} \geq 1, N_{Cd} \geq 1 \quad (5)$$

$$N_{Ah-s} \geq 0, N_{Bh-s} \geq 0, N_{Ch-s} \geq 0 \quad (6)$$

In our examples, we find that the $N_{h\text{-crystal}}$=27+38+20 -4-0=81 for the "h-set" and $N_{h\text{-crystal}}$=21+36+11 -2-0=66 for 'GR&SM-set', theoretically.

If the total number of nodes $N = N_A + N_B + N_C$ and $N_h = N_{Ah} + N_{Bh} + N_{Ch}$, in which node i has degree d_i, with h-degree h_i, we get

$$N_{h-crystal} \geq N_h \quad (7)$$

$$0 \leq N_h \leq N \leq \sum_{i=1}^{N} h_i \leq \sum_{i=1}^{N} d_i \quad (8)$$

$$N_h \leq \sqrt{\sum_{i=1}^{N} h_i} \leq \sqrt{\sum_{i=1}^{N} d_i} \quad (9)$$

This means that $N_{h\text{-crystal}}$ is restricted by N and N_h.

4.2. The Edges of the H-crystal

Suppose that a multilayer weighted network consists of three layers A, B and C, with number of edges L_A, L_B and L_C respectively, each with h-strength L_{Ah}, L_{Bh} and L_{Ch} and the numbers of edges for which the edge's h-strength equals L_{Ah}, L_{Bh} and L_{Ch} are L_{Ad}, L_{Bd} and L_{Cd} respectively. Let $L_{Ah\text{-}cs}$, $L_{Bh\text{-}cs}$ and $L_{Ch\text{-}cs}$ represent number of edges in the network's h-core and h-subnet. If the number of links of two layers A, B is L_{AB} and that of two layers B, C is L_{BC}, the total number of edges of the h-crystal ($L_{h\text{-crystal}}$) is:

$$L_{h-crystal} = L_{Ah-cs} + L_{Bh-cs} + L_{Ch-cs} + L_{AB} + L_{BC} \quad (10)$$

In our cases, for 'h-set', the $L_{h\text{-crystal}}$=199+263+103+0+207=772; for 'GR&SM-set', $L_{h\text{-crystal}}$=105+85 +34 +0+96=320.

If the total number of edges $L = L_A + L_B + L_C$ and $L_h = L_{Ah} + L_{Bh} + L_{Ch}$, with weights s_j in edge j and $L \leq N \cdot (N-1)$, we obtain

$$0 \leq L_h \leq L \leq \sum_{j=1}^{L} s_j \quad (11)$$

$$L_h \leq \sqrt{\sum_{j=1}^{L} s_j} = \sqrt{\frac{1}{2}\sum_{i=1}^{N} d_i} \quad (12)$$

where $\sum_{j=1}^{L} s_j$ is the sum of the total weight of all edges in the network and L indicates total edges.

It is worth to point out that our results show general ways to reach the core structure of both multilayer and heterogeneous networks, as the h-crystal covers heterogeneous structure (where co-keyword is different from co-citation and bibliographic coupling), though the three layers belong to homogeneous layer each respectively.

4.3. Potential Applications

The h-crystal revealed a core structure in a multilayer weighted network, which could be useful in three-dimensional (3D) visualization. In a complex network, it is impossible to show all nodes and edges in visualization, so that it is important to find and show the core nodes and edges. The h-crystal provides a potential way to approach the core structure in weighted networks, so that it may be meaningful in future studies.

Meanwhile, the h-crystal can tell the most significant core objects in a multilayer (and heterogeneous) graphical environment, so that it may reveal important core information.

4.4. Limitation of H-crystal

In this paper, we show only how h-crystal can be defined and constructed in a multilayer weighted network. As there are complex cases for different constitutes of h-crystals (such as different choice of layers), we have not attempted to provide a mathematical proof for the generalizability of the h-crystal. Also, although we tried various centralities, we only chose betweenness centrality for showing typical comparison and k-core for core comparison.

More researches will need to be done to validate and extend the h-crystal method to general multilayer (and heterogeneous) networks in future.

5. Conclusion

In this paper, a new method to find a core structure in a multilayer weighted network of information, called h-crystal, has been introduced. The core structure represents the most significant nodes and edges in the network thus this method can be applied to simplify multilayer weighted networks. Moreover, the h-crystal integrates core nodes and edges of the multilayer weighted network, for which important information of the multilayer weighted network can be obtained.

Other core structures may exist in multilayer and heterogeneous weighted networks, depending on the operationalization of the notion of a core. Yet, h-crystal can be considered a basic one and it has highly simplified efficiency. Like all the other h-type measures, h-crystal will be an efficient method for identifying and selecting objects from large information. A potential application of h-crystal, which we are currently working on, is a retrieval system with 3D visualization displays. The h-crystal-based system will show the retrieved document set in 3D with the core documents highlighted and with interactions to allow the user to interact with the 3D structure of the whole h-crystal.

Although multilayer weighted information networks are only one kind of multilayer weighted networks, the h-crystal methodology can be generalized to other types of weighted networks. Currently, our case study addresses only undirected multilayer weighted information networks, leaving directed multilayer as well as heterogeneous weighted networks for future investigations. Also, the dynamical issues leave for future works.

Acknowledgements

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Appendix 1: Pseudocodes of the H-crystal Algorithm

Algorithm1 generate bibliographic coupling network

Input: the record data of WoS

Output: bibliographic coupling network and the node information

Initialize the empty bibliographic coupling network

Initialize the sparse matrix M

for each record r in the record data of WoS do

  for each reference c in the record r do

    M←add the ID of the record r, the ID of the reference c and 1

    the node information←add the node information of the record

  end for

end for

M←M * M.T

for each nonzero element a_{ij} in M do

bibliographic coupling network←add i, j, a_{ij} into the Graph

end for

Return bibliographic coupling network, the node information

Algorithm2 generate co-citation network

Input: the record data of WoS

Output: co-citation network and the node information

Initialize the empty co-citation network

for each record in the record data of WoS do

for the reference c_j in the references of the record do

for the reference c_j in the references of the record do

if i < j then

if the reference c_i and the reference c_j in the co-citation network then

the weight of edge between the reference c_i and the reference c_j <-1

elseif then

co-citation network ←add the ID of reference c_i, the ID of the reference c_j and 1

the node information←add the node information of the reference

end if

end if

end for

end for

end for

Return co-citation network, the node information

Algorithm3 generate co-term network

Input: the content of ID and DE field in the record data of WoS

Output: co-term network and the node information

Initialize the empty co-term network

for each record in the record data of WoS do

for the term t_i in the content of ID and DE field of the record do

for the term t_j in the content of ID and DE field of the record do

if i < j then

if the term t_i and the term t_j in the co-term network then

the weight of edge about the term t_i and the term t_j <-1

elseif then

co-term network ←add the ID of term t_i, the ID of the term t_j and 1

the node information←add the node information of the term

end if

end if

end for

end for

end for

Return co-term network, the node information

Algorithm4 compute h core and h subnet of each layer network

Input：G(bibliographic coupling network/co-citation network/co-term network)

Output: h core and h subnet of the corresponding network

Initialize the empty h core and h subnet of G

for each node n in G do

h degree of the node ← n has at least h(n) links each with strength at least equal to h(n)

end for

h degree of the G← h index of h degree of every node in G

for each node n in G do

if h degree of node n >= h degree of the G then

h core of G←add node n into h core of G，meanwhile add the link if node n have a link with other nodes in h core of G

end if

end for

h strenth of the G←h index of the strenth of every edge in G

for each edge e in G do

if the strength of edge e >= h strenth of the G then

h subnet of G←add edge e into h subnet of G

end if

end for

h core and h subnet of G ← combine h core of G with h subnet of G

Return h core and h subnet of G

Algorithm5 construct bridge relations between bibliographic coupling network and co-citation network

Input: h core and h subnet of bibliographic coupling network, h core and h subnet of co-citation network

Output: bridge relations between bibliographic coupling network and co-citation network

for each node n_i in h core and h subnet of bibliographic coupling network do

for each node n_j in h core and h subnet of co-citation network do

if n_i equal n_j then

bridge relations ←construct a link between node n_i and node n_j

end if

end for

end for

Return bridge relations between bibliographic coupling network and co-citation network

Algorithm6 construct bridge relations between bibliographic coupling network and co-term network

Input: h core and h subnet of bibliographic coupling network, h core and h subnet of co-term network

Output: bridge relations between bibliographic coupling network and co-term network

for each node n_i in h core and h subnet of bibliographic coupling network do

for each node n_j in h core and h subnet of co-term network do

if n_i associated with n_j then

bridge relations ←construct a link between node n_i and node n_j

end if

end for

end for

Return bridge relations between bibliographic coupling network and co-term network

Algorithm7 construct h crystal of heterogeneous weighted networks

Input: h core and h subnet of co-citation network, h core and h subnet of bibliographic coupling network, h core and h subnet of co-term network, bridge relations between bibliographic coupling network and co-citation network, bridge relations between bibliographic coupling network and co-term network

Output: h crystal of heterogeneous weighted networks

Initialize the empty graph(h crystal of heterogeneous weighted networks)

h crystal of heterogeneous weighted networks ← combine(h core and h subnet of co-citation network ,h core and h subnet of bibliographic coupling network, h core and h subnet of co-term network, bridge relations between bibliographic coupling network and co-citation network, bridge relations between bibliographic coupling network and co-term network)

if h crystal of heterogeneous weighted networks is connected then

h crystal of heterogeneous weighted networks is existing

end if

Return h crystal of heterogeneous weighted networks

Appendix 2 A- and B- Core Nodes in H-crystal of "H-set", Ranked by Total Citations (TC)

ID	AU	TI	SO	PY	TC
A2778*	Hirsch, JE	An index to quantify an individual's scientific research output	PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES OF THE UNITED STATES OF AMERICA	2005	1828
A1802*	Egghe, L	Theory and practise of the g-index	SCIENTOMETRICS	2006	440
A4063*	Meho, LI; Yang, K	Impact of data sources on citation counts and rankings of LIS faculty: Web of science versus scopus and google scholar	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2007	260
A6002	Van Raan, AFJ	Comparison of the Hirsch-index with standard bibliometric indicators and with peer judgment for 147 chemistry research groups	SCIENTOMETRICS	2006	239
A2779	Hirsch, JE	Does the h index have predictive power?	PROCEEDINGS OF THE NATIONAL	2007	226

ID	AU	TI	SO	PY	TC
			ACADEMY OF SCIENCES OF THE UNITED STATES OF AMERICA		
A943	Braun, T; Glanzel, W; Schubert, A	A Hirsch-type index for journals	SCIENTOMETRICS	2006	198
A3092	Jin, BH; Liang, LM; Rousseau, R; Egghe, L	The R- and AR-indices: Complementing the h-index	CHINESE SCIENCE BULLETIN	2007	194
A582	Ball, P	Index aims for fair ranking of scientists	NATURE	2005	174
A671	Batista, PD; Campiteli, MG; Kinouchi, O; Martinez, AS	Is it possible to compare researchers with different scientific interests?	SCIENTOMETRICS	2006	173
A1801	Egghe, L; Rousseau, R	An informetric model for the Hirsch-index	SCIENTOMETRICS	2006	154
A857	Bornmann, L; Daniel, HD	What do we know about the h index?	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2007	156
A1446*	Cronin, B; Meho, L	Using the h-index to rank influential information scientists	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2006	141
A863	Bornmann, L; Mutz, R; Daniel, HD	Are there better indices for evaluation purposes than the h index? a comparison of nine different variants of the h index using data from biomedicine	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2008	141
A853	Bornmann, L; Daniel, HD	Does the h-index for ranking of scientists really work?	SCIENTOMETRICS	2005	135
A270	Alonso, S; Cabrerizo, FJ; Herrera-Viedma, E; Herrera, F	h-Index: A review focused in its variants, computation and standardization for different scientific fields	JOURNAL OF INFORMETRICS	2009	136
A3248	Kelly, CD; Jennions, MD	The h index and career assessment by numbers	TRENDS IN ECOLOGY & EVOLUTION	2006	124
A1397	Costas, R; Bordons, M	The h-index: Advantages, limitations and its relation with other bibliometric indicators at the micro level	JOURNAL OF INFORMETRICS	2007	112
A2389	Glanzel, W	On the h-index - A mathematical approach to a new measure of publication activity and citation impact	SCIENTOMETRICS	2006	101
A940	Braun, T; Glanzel, W; Schubert, A	A Hirsch-type index for journals	SCIENTIST	2005	97
A5408	Schubert, A; Glanzel, W	A systematic analysis of Hirsch-type indices for journals	JOURNAL OF INFORMETRICS	2007	92
A5496	Sidiropoulos, A; Katsaros, D; Manolopoulos, Y	Generalized Hirsch h-index for disclosing latent facts in citation networks	SCIENTOMETRICS	2007	98
A595	Banks, MG	An extension of the Hirsch index: Indexing scientific topics and compounds	SCIENTOMETRICS	2006	75
A4446	Oppenheim, C	Using the h-index to rank influential British researchers in information science and librarianship	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2007	58
A1794	Egghe L	Power Laws in the Information Production Process: Lotkaian Informetrics	LIBR INFORM SCI SER	2005	250 (Google scholar)
A1798	Egghe L	An improvement of the H-index: the G-index	ISSI newsletter	2006	267 (Google scholar)
A3088	Jin B.	Scientists designed a new indicator for themselves: h-index	SCI FOCUS	2006	54 (Google scholar)
A2387*	Glanzel W	A Discussion on the opportunities and limitations of h-index	SCI FOCUS	2006	15 (Google scholar)
B49	Jin, BH; Liang, LM; Rousseau, R; Egghe, L	The R- and AR-indices: Complementing the h-index	CHINESE SCIENCE BULLETIN	2007	194
B53*	Bornmann, L; Daniel, HD	What do we know about the h index?	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2007	156
B112	Bornmann, L; Mutz, R; Daniel, HD	Are there better indices for evaluation purposes than the h index? a comparison of nine different variants of the h index	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2008	141

ID	AU	TI	SO	PY	TC
		using data from biomedicine			
B195	Alonso, S; Cabrerizo, FJ; Herrera-Viedma, E; Herrera, F	h-Index: A review focused in its variants, computation and standardization for different scientific fields	JOURNAL OF INFORMETRICS	2009	136
B297	Egghe, L	The Hirsch Index and Related Impact Measures	ANNUAL REVIEW OF INFORMATION SCIENCE AND TECHNOLOGY	2010	88
B120*	Bar-Ilan, J	Informetrics at the beginning of the 21st century - A review	JOURNAL OF INFORMETRICS	2008	81
B402	Bornmann, L; Mutz, R; Hug, SE; Daniel, HD	A multilevel meta-analysis of studies reporting correlations between the h index and 37 different h index variants	JOURNAL OF INFORMETRICS	2011	52
B104	van Eck, NJ; Waltman, L	Generalizing the h- and g- indices	JOURNAL OF INFORMETRICS	2008	45
B118	Jacso, P	Testing the calculation of a realistic h-index in Google Scholar, Scopus, and Web of Science for F. W. Lancaster	LIBRARY TRENDS	2008	41
B194	Panaretos, J; Malesios, C	Assessing scientific research performance and impact with single indices	SCIENTOMETRICS	2009	43
B296*	Garcia-Perez, MA	Accuracy and Completeness of Publication and Citation Records in the Web of Science, PsycINFO, and Google Scholar: A Case Study for the Computation of h Indices in Psychology	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2010	33
B116*	Jacso, P	The pros and cons of computing the h-index using Web of Science	ONLINE INFORMATION REVIEW	2008	25
B186	Bontis, N; Serenko, A	A follow-up ranking of academic journals	JOURNAL OF KNOWLEDGE MANAGEMENT	2009	30
B119	Thelwall, M	Bibliometrics to webometrics	JOURNAL OF INFORMATION SCIENCE	2008	27
B107	Egghe, L; Rao, IKR	Study of different h-indices for groups of authors	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2008	19
B282	Franceschini, F; Maisano, D	The Hirsch spectrum: A novel tool for analyzing scientific journals	JOURNAL OF INFORMETRICS	2010	17
B295	Norris, M; Oppenheim, C	The h-index: a broad review of a new bibliometric indicator	JOURNAL OF DOCUMENTATION	2010	18
B182	Liu, YX; Rousseau, R	Properties of Hirsch-type indices: the case of library classification categories	SCIENTOMETRICS	2009	18
B480	Schreiber, M; Malesios, CC; Psarakis, S	Exploratory factor analysis for the Hirsch index, 17 h-type variants, and some traditional bibliometric indicators	JOURNAL OF INFORMETRICS	2012	17
B399	Kousha, K; Thelwall, M; Rezaie, S	Assessing the Citation Impact of Books: The Role of Google Books, Google Scholar, and Scopus	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2011	18
B403	Zhang, L; Thijs, B; Glanzel, W	The diffusion of H-related literature	JOURNAL OF INFORMETRICS	2011	14
B292	Serenko, A	The development of an AI journal ranking based on the revealed preference approach	JOURNAL OF INFORMETRICS	2010	13
B111	Egghe, L	The Influence of transformations on the h-index and the g-index	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2008	12
B400	Serenko, A; Dohan, M	Comparing the expert survey and citation impact journal ranking methods: Example from the field of Artificial Intelligence	JOURNAL OF INFORMETRICS	2011	11
B289	Franceschini, F; Maisano, D; Perotti, A; Proto, A	Analysis of the ch-index: an indicator to evaluate the diffusion of scientific research output by citers	SCIENTOMETRICS	2010	10
B168	Gagolewski, M; Grzegorzewski, P	A geometric approach to the construction of scientific impact indices	SCIENTOMETRICS	2009	10
B485	Liu, JS; Lu, LYY	An Integrated Approach for Main Path Analysis: Development of the Hirsch Index as an Example	JOURNAL OF THE AMERICAN SOCIETY FOR INFORMATION SCIENCE AND TECHNOLOGY	2012	11
B110	Bar-Ilan, J	The h-index of h-index and of other informetric topics	SCIENTOMETRICS	2008	8
B391*	Franceschini, F; Maisano, D	Bibliometric positioning of scientific manufacturing journals: a comparative	SCIENTOMETRICS	2011	8

ID	AU	TI	SO	PY	TC
		analysis			
B368	Jacso, P	The h-index, h-core citation rate and the bibliometric profile of the Scopus database	ONLINE INFORMATION REVIEW	2011	6
B493	Jacso, P	Grim tales about the impact factor and the h-index in the Web of Science and the Journal Citation Reports databases: reflections on Vanclay's criticism	SCIENTOMETRICS	2012	11
B457	Bornmann, L; Marx, W	HistCite analysis of papers constituting the h index research front	JOURNAL OF INFORMETRICS	2012	5
B117	Rodriguez, V; Janssens, F; Debackere, K; De Moor, B	On material transfer agreements and visibility of researchers in biotechnology	JOURNAL OF INFORMETRICS	2008	4
B389	Schreiber, M; Malesios, CC; Psarakis, S	Categorizing h-index variants	RESEARCH EVALUATION	2011	4
B478	Jacso, P	Using Google Scholar for journal impact factors and the h-index in nationwide publishing assessments in academia - siren songs and air-raid sirens	ONLINE INFORMATION REVIEW	2012	3
B55	Rodriguez, V; Janssens, F; Debackere, K; De Moor, B	On material transfer agreements and visibility of researchers in biotechnology	Proceedings of ISSI 2007: 11th International Conference of the International Society for Scientometrics and Informetrics, Vols I and II	2007	5
B373	Jacso, P	The h-index, h-core citation rate and the bibliometric profile of the Web of Science database in three configurations	ONLINE INFORMATION REVIEW	2011	1
B378	Kousha, K; Thelwall, M	Assessing the Citation Impact of Book-Based Disciplines: The Role of Google Books, Google Scholar and Scopus	PROCEEDINGS OF ISSI 2011: THE 13TH CONFERENCE OF THE INTERNATIONAL SOCIETY FOR SCIENTOMETRICS AND INFORMETRICS, VOLS 1 AND 2	2011	0

* asterisk signifies the top 5 core nodes.

Appendix 3 A- and B- Core Nodes in H-crystal of "GR&SM-set", ranked by Total Citations

ID	AU	TI	SO	PY	TC
A17975*	Riess, AG; Filippenko, AV; Challis, P; Clocchiatti, A; Diercks, A; Garnavich, PM; Gilliland, RL; Hogan, CJ; Jha, S; Kirshner, RP; Leibundgut, B; Phillips, MM; Reiss, D; Schmidt, BP; Schommer, RA; Smith, RC; Spyromilio, J; Stubbs, C; Suntzeff, NB; Tonry, J	Observational evidence from supernovae for an accelerating universe and a cosmological constant	ASTRONOMICAL JOURNAL	1998	7507
A17064*	Perlmutter, S; Aldering, G; Goldhaber, G; Knop, RA; Nugent, P; Castro, PG; Deustua, S; Fabbro, S; Goobar, A; Groom, DE; Hook, IM; Kim, AG; Kim, MY; Lee, JC; Nunes, NJ; Pain, R; Pennypacker, CR; Quimby, R; Lidman, C; Ellis, RS; Irwin, M; McMahon, RG; Ruiz-Lapuente, P; Walton, N; Schaefer, B; Boyle, BJ; Filippenko, AV; Matheson, T; Fruchter, AS; Panagia, N; Newberg, HJM; Couch, WJ	Measurements of Omega and Lambda from 42 high-redshift supernovae	ASTROPHYSICAL JOURNAL	1999	7022
A19686	Spergel, DN; Verde, L; Peiris, HV; Komatsu, E; Nolta, MR; Bennett, CL; Halpern, M; Hinshaw, G; Jarosik, N; Kogut, A; Limon, M; Meyer, SS; Page, L; Tucker, GS; Weiland, JL; Wollack, E; Wright, EL	First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters	ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES	2003	6776
A17980	Riess, AG; Strolger, LG; Tonry, J; Casertano, S; Ferguson, HC; Mobasher, B; Challis, P; Filippenko, AV; Jha, S; Li, WD; Chornock, R; Kirshner, RP; Leibundgut, B; Dickinson, M; Livio, M; Giavalisco, M; Steidel, CC; Benitez, T; Tsvetanov, Z	Type Ia supernova discoveries at z > 1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution	ASTROPHYSICAL JOURNAL	2004	2359
A5702*	Copeland, EJ; Sami, M; Tsujikawa, S	Dynamics of dark energy	INTERNATIONAL JOURNAL OF MODERN	2006	2040

ID	AU	TI	SO	PY	TC
			PHYSICS D		
A16966	Peebles, PJE; Ratra, B	The cosmological constant and dark energy	REVIEWS OF MODERN PHYSICS	2003	1962
A7032*	Dvali, G; Gabadadze, G; Porrati, M	4D gravity on a brane in 5D Minkowski space	PHYSICS LETTERS B	2000	1805
A5092	Carroll, SM; Duvvuri, V; Trodden, M; Turner, MS	Is cosmic speed-up due to new gravitational physics?	PHYSICAL REVIEW D	2004	1037
A5617	Colladay, D; Kostelecky, VA	Lorentz-violating extension of the standard model	PHYSICAL REVIEW D	1998	964
A15962	Nojiri, S; Odintsov, SD	Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration	PHYSICAL REVIEW D	2003	811
A5616	Colladay, D; Kostelecky, VA	CPT violation and the standard model	PHYSICAL REVIEW D	1997	774
A10798	Hu, W; Sawicki, I	Models of f(R) cosmic acceleration that evade solar system tests	PHYSICAL REVIEW D	2007	593
A11862	KOSTELECKY, VA; SAMUEL, S	SPONTANEOUS BREAKING OF LORENTZ SYMMETRY IN STRING THEORY	PHYSICAL REVIEW D	1989	569
A12619	Kostelecky, VA	Gravity, Lorentz violation, and the standard model	PHYSICAL REVIEW D	2004	423
A12616	Kostelecky, VA; Mewes, M	Signals for Lorentz violation in electrodynamics	PHYSICAL REVIEW D	2002	343
A12611	Kostelecky, VA; Lehnert, R	Stability, causality, and Lorentz and CPT violation	PHYSICAL REVIEW D	2001	313
A11869	KOSTELECKY, VA; POTTING, R	CPT, STRINGS, AND MESON FACTORIES	PHYSICAL REVIEW D	1995	276
A12608	Kostelecky, VA; Lane, CD	Constraints on Lorentz violation from clock-comparison experiments	PHYSICAL REVIEW D	1999	234
A3910	Buchert, T	On average properties of inhomogeneous fluids in general relativity: Dust cosmologies	GENERAL RELATIVITY AND GRAVITATION	2000	224
A16315	Oyaizu, H; Lima, M; Hu, W	Nonlinear evolution of f(R) cosmologies. II. Power spectrum	PHYSICAL REVIEW D	2008	126
A12620*	Kostelecky, VA; Mewes, M	Lorentz and CPT violation in the neutrino sector	PHYSICAL REVIEW D	2004	102
B176	Kostelecky, VA	Gravity, Lorentz violation, and the standard model	PHYSICAL REVIEW D	2004	423
B136	Kostelecky, VA; Mewes, M	Signals for Lorentz violation in electrodynamics	PHYSICAL REVIEW D	2002	343
B152	Brax, P; van de Bruck, C	Cosmology and brane worlds: a review	CLASSICAL AND QUANTUM GRAVITY	2003	178
B332	Buchert, T	Dark Energy from structure: a status report	GENERAL RELATIVITY AND GRAVITATION	2008	188
B434	Iocco, F; Mangano, G; Miele, G; Pisanti, O; Serpico, PD	Primordial nucleosynthesis: From precision cosmology to fundamental physics	PHYSICS REPORTS-REVIEW SECTION OF PHYSICS LETTERS	2009	165
B436*	Maartens, R; Koyama, K	Brane-World Gravity	LIVING REVIEWS IN RELATIVITY	2010	152
B282	Tsujikawa, S	Matter density perturbations and effective gravitational constant in modified gravity models of dark energy	PHYSICAL REVIEW D	2007	119
B377*	Kostelecky, VA; Mewes, M	Electrodynamics with Lorentz-violating operators of arbitrary dimension	PHYSICAL REVIEW D	2009	126
B248	Bailey, QG; Kostelecky, VA	Signals for Lorentz violation in post-Newtonian gravity	PHYSICAL REVIEW D	2006	116
B115	Thiemann, T	Gauge field theory coherent states (GCS): I. General properties	CLASSICAL AND QUANTUM GRAVITY	2001	98
B329	Tsagas, CG; Challinor, A; Maartens, R	Relativistic cosmology and large-scale structure	PHYSICS REPORTS-REVIEW SECTION OF PHYSICS LETTERS	2008	87
B437	Centrella, J; Baker, JG; Kelly, BJ; van Meter, JR	Black-hole binaries, gravitational waves, and numerical relativity	REVIEWS OF MODERN PHYSICS	2010	76
B184	Cane, F; Bear, D; Phillips, DF; Rosen, MS; Smallwood, CL; Stoner, RE; Walsworth, RL; Kostelecky, VA	Bound on Lorentz and CPT violating boost effects for the neutron	PHYSICAL REVIEW LETTERS	2004	74
B500	Kostelecky, VA; Tasson, JD	Matter-gravity couplings and Lorentz violation	PHYSICAL REVIEW D	2011	57
B331*	Turyshev, SG	Experimental Tests of General Relativity	ANNUAL REVIEW OF NUCLEAR AND PARTICLE SCIENCE	2008	52
B503*	Buchert, T	Toward physical cosmology: focus on inhomogeneous geometry and its non-	CLASSICAL AND QUANTUM GRAVITY	2011	45

ID	AU	TI	SO	PY	TC
		perturbative effects			
B211	Lane, CD	Probing Lorentz violation with Doppler-shift experiments	PHYSICAL REVIEW D	2005	42
B457	Wiegand, A; Buchert, T	Multiscale cosmology and structure-emerging dark energy: A plausibility analysis	PHYSICAL REVIEW D	2010	30
B339	Pun, CSJ; Kovacs, Z; Harko, T	Thin accretion disks onto brane world black holes	PHYSICAL REVIEW D	2008	31
B330*	Rovelli, C	Loop Quantum Gravity	LIVING REVIEWS IN RELATIVITY	2008	29
B338	Deruelle, N; Sasaki, M; Sendouda, Y	Detuned f(R) gravity and dark energy	PHYSICAL REVIEW D	2008	24
B280	Turyshev, SG; Israelsson, UE; Shao, M; Yu, N; Kusenko, A; Wright, EL; Everitt, CWF; Kasevich, M; Lipa, JA; Mester, JC; Reasenberg, RD; Walsworth, RL; Ashby, N; Gould, H; Paik, HJ	Space-based research in fundamental physics and quantum technologies	INTERNATIONAL JOURNAL OF MODERN PHYSICS D	2007	23
B565	Xu, C; Saridakis, EN; Leon, G	Phase-space analysis of teleparallel dark energy	JOURNAL OF COSMOLOGY AND ASTROPARTICLE PHYSICS	2012	30
B379	Bailey, QG	Time delay and Doppler tests of the Lorentz symmetry of gravity	PHYSICAL REVIEW D	2009	27
B380	Coc, A; Olive, KA; Uzan, JP; Vangioni, E	Nonuniversal scalar-tensor theories and big bang nucleosynthesis	PHYSICAL REVIEW D	2009	23
B562	Liu, D; Reboucas, MJ	Energy conditions bounds on f(T) gravity	PHYSICAL REVIEW D	2012	19
B341	Harko, T; Sabau, VS	Jacobi stability of the vacuum in the static spherically symmetric brane world models	PHYSICAL REVIEW D	2008	14
B566	Gonzalez, PA; Saridakis, EN; Vasquez, Y	Circularly symmetric solutions in three-dimensional teleparallel, f(T) and Maxwell-f(T) gravity	JOURNAL OF HIGH ENERGY PHYSICS	2012	16
B439	Centrella, J; Baker, JG; Kelly, BJ; van Meter, JR	The Final Merger of Black-Hole Binaries	ANNUAL REVIEW OF NUCLEAR AND PARTICLE SCIENCE, VOL 60	2010	13
B585	Wu, YP; Geng, CQ	Matter density perturbations in modified teleparallel theories	JOURNAL OF HIGH ENERGY PHYSICS	2012	9
B447	Bohmer, CG; De Risi, G; Harko, T; Lobo, FSN	Classical tests of general relativity in brane world models	CLASSICAL AND QUANTUM GRAVITY	2010	8
B137	Maartens, R	Brane-world cosmological perturbations - A covariant approach -	PROGRESS OF THEORETICAL PHYSICS SUPPLEMENT	2002	3
B589	Baccetti, V; Tate, K; Visser, M	Lorentz violating kinematics: threshold theorems	JOURNAL OF HIGH ENERGY PHYSICS	2012	3
B179	Sasaki, M	Brane-world cosmology and inflation	PRAMANA-JOURNAL OF PHYSICS	2004	1
B400	Harko, T	Matter Accretion by Brane-World Black Holes	JOURNAL OF THE KOREAN PHYSICAL SOCIETY	2009	0
B558	Baccetti, V; Tate, K; Visser, M	Inertial frames without the relativity principle	JOURNAL OF HIGH ENERGY PHYSICS	2012	0

* asterisk signifies the top 5 core nodes.

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