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A Study on Geometric Model of CCD Camera Based on Projective Geometry

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Abstract

Aiming at enhancing the spatial coordinates precision solved from 2D images, a new method for calculation of 3D model coordinate transformation between image side and object side space is proposed based on projective geometry. CCD camera imaging geometric principle is analyzed based on perspective transformation and cross-ratio invariant properties. Imaging geometric model based on linear feature is established using the relationship of parallel, perpendicular and intersecting between straight lines. With acquired photographs and camera parameters, the shape and size of the corresponding scene space is deduced and proved on the basis of the cross ratio invariability of collinear points with perspective projection. The coordinate transformation between image side and object side space of the 3D model is calculated. And perspective correspondence from image side to object side space is built. Experimental results prove that calculation accuracy of the spatial points coordinates can be enhanced using image geometric information. The method can be applied to enhance the precision of image feature extraction and location of geometric features such as circle, and to increase the measurement precisiong of the photogrammetry system and 3-D coordinate reconstruction.

Keywords

CCD Camera, Projective Geometry, Perspective Transformation, Cross Ratio, Imaging Model

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1. Introduction

According to the two-dimensional images acquired at different viewpoints to reconstruct the original three-dimensional model space objects is a hot research in photogrammetry and computer vision field. Transformation relationship between them from two-dimensional images to three-dimensional reconstruction is determined by the camera imaging geometry[1-3].Camera imaging geometry is the basis of three-dimensional reconstruction of data processing, mainly described camera image space and object space coordinate transformation relations[4-6]. Literature [7-8] were studied to explore space and precise positioning and testing of the circle. About camera lens distortion problems, literature[9] calculated camera parameters based cross-ratio invariance principle advantage of fitting functions; [10] using a nonlinear optimization method achieved a calibrated lens distortion parameters; [11] using the properties of the section plane conjugate established a geometric model of image distortion. In this paper, the establishment of CCD camera imaging geometry model are derived to calculate the image side perspective space, and the perspective correspondence of object space boundary contour and image boundary contour are analysized, depending on the camera parameters.

2. Camera Imaging Principle

The typical camera model is based on the principle of collinear, each point in the object space through a straight line through the center of the projection image projected onto the plane

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[12-13]. Using camera to shoot the object, getting a picture of an object, the corresponding relations between space objects and the objects in the image is projective geometry of the central projection, also known as perspective projection[14]. As shown in Figure 1, the perspective center is the center of photography S, namely the optical center of the camera. Optical axis for SS_i, perspective axis is the intersection l of a image plane (A'B'C'D') and objects plane (ABCD). Without loss of generality, assuming the angle between the image plane and the plane of the object is α. The following further studies the correspondence of perspective plane and object plane.

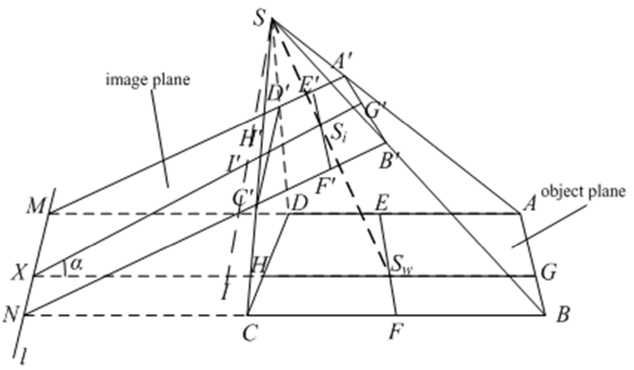


Fig. 1. Geometry model of camera imaging.

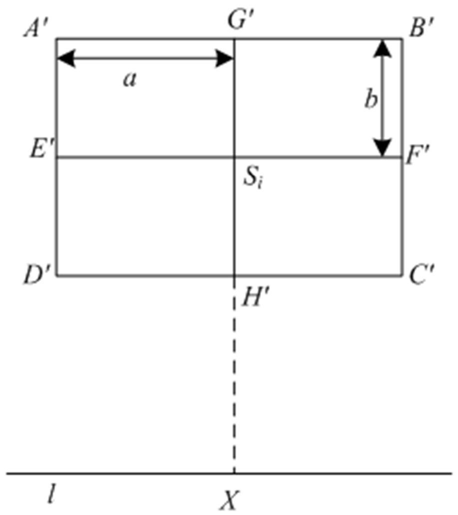


Fig. 2. The image plane.

3. Correspondence between the Image Space and Object Space

3.1. Information of Image Plane

Shoot a image, it is rectangular in shape, and its length and width is known. Setting image plane is a rectangle A'B'C'D', a length of 2a, width 2b. as the image center, from s_i to perspective axis doing vertical line, pedal of. Center of the image is s_i, from s_i perpendicular to do the perspective axis l, pedal for X.

On the image plane is known $A'B' = C'D' = E'F' = 2a$, $A'D' = B'C' = G'H' = 2b$, S_i as the center of rectangle $A'B'C'D'$, therefore, $G'S_i = S_iH' = b, E'S_i = S_iF' = a$.

3.2. Imaging Geometry Deduced

Established the plane geometry relations over the optical axis, perpendicular to perspective axis are shown in Figure 1. The camera optical center of S, camera focal length of f, then $ss_i = f$, the distance from optical center to object plane in optical axis is d, $d = ss_w$, the angle between image plane and object plane of α, $\angle S_iXS = \angle S_wSI = \angle S_iSI'$, camera pitching angle θ, we have $\angle S_iSH' = \angle S_iSG' = \theta$; $\tan\theta = b/f$. $\sin\theta = b/\sqrt{b^2+f^2}$, $\cos\theta = f/\sqrt{b^2+f^2}$

As shown in Figure 1, making perpendicular from the center of photography to the object space plane, doing $SI \perp GX$, intersection $G'X$ to point I'. In the $Rt\Delta SS_iI'$, easy to know:

$$S_i = f\tan\alpha, SI' = f/\cos\alpha \qquad (1)$$

In the $Rt\Delta SIS_w$, can be derived:

$$SI = d\cos\alpha, \quad S_wI = d\sin\alpha \qquad (2)$$

By the formula (1), the formula (2) can be derived:

$$I'I = d\cos\alpha - f/\cos\alpha \qquad (3)$$

In the $Rt\Delta XS_iS_w$, can be derived:

$$S_wX = (d-f)/\sin\alpha, S_iX = (d-f)\cot\alpha \qquad (4)$$

$Rt\Delta SIG$, because of $\tan\theta = b/f, \theta = \arctan(b/f)$, can be derived:

$$SH' = SG' = \sqrt{f^2+b^2} \qquad (5)$$

In the $Rt\Delta SIG$, because of $\angle GSI = \alpha+\theta$, by the formula (2), can be derived:

$$SG = SI/\cos(\alpha+\theta) = d\cos\alpha/\cos(\alpha+\theta) \qquad (6)$$

$$GI = SG\sin(\alpha+\theta) = d\cos\alpha\tan(\alpha+\theta) \qquad (7)$$

$$\begin{aligned} GS_w &= GI - S_wI = d(\cos\alpha\tan(\alpha+\theta) - \sin\alpha) \\ &= d((f\sin\alpha + b\cos\alpha)/(f - b\tan\alpha) - \sin\alpha) \end{aligned} \qquad (8)$$

4. Object Space Boundary Shape Analysis and Proof

The following discussion is on geometry features of the rectangular image obtained by the camera. Combination of the above analysis, and further proof is derived as follows: object along the optical axis of the plane distance d , then $SS_i = f, SS_w = d$. Perspective transformation perspective has the following properties: the axis parallel to the perspective of a set of parallel chords, still parallel to the axis of a rear perspective.

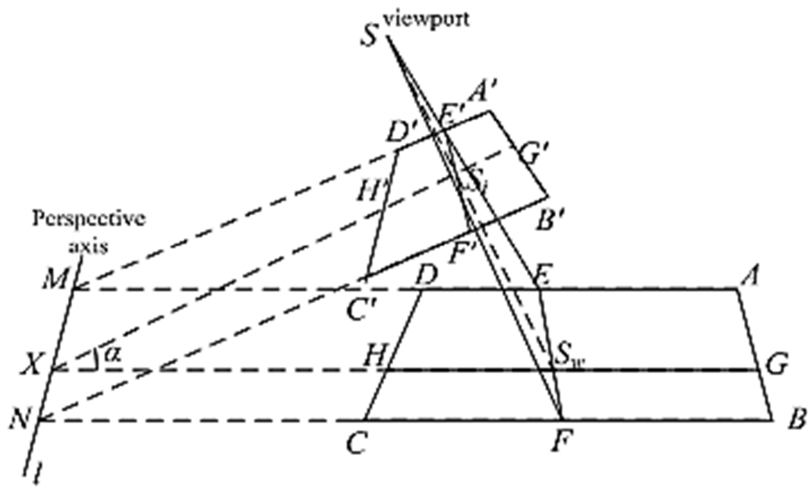
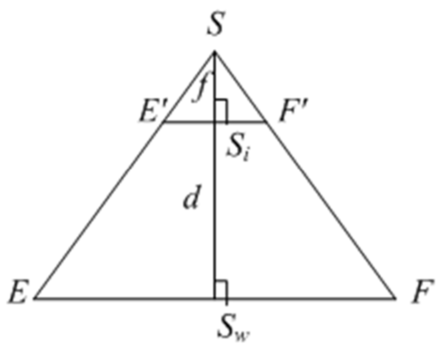


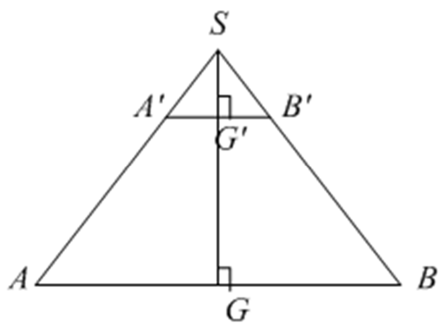
Fig. 3. Geometric relation of object space

Object side boundary contour analysis shown in Figure 4, in ΔESF and $\Delta E'SF'$ can be derived by the following formula:

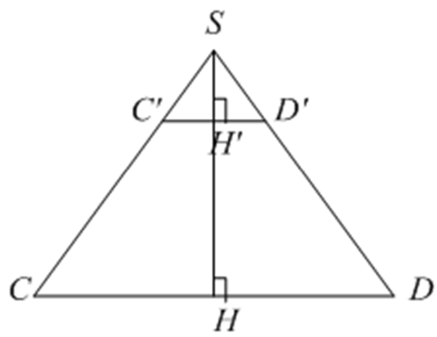
$$ES_w = S_w F = ad / f \tag{9}$$



(a) boundary 1



(b) boundary 2



(c) boundary 3

Fig. 4. Object space boundary contour analysis

As shown in Figure 4(a), with (9),you can know $E'F'/EF = f/d$,therefore:

$$EF = 2ad / f \tag{10}$$

In the same way in Figure 3, because of edge $A'B'//l, AB//l$,so $AB//l, CD//l$,therefore $AB//CD$.

As shown in Figure 4(b),in $\Delta SAB : BG = GA$.

Because of $\dfrac{A'B'}{AB} = \dfrac{SG'}{SG} = \dfrac{\sqrt{f^2+b^2}}{d\cos\alpha\cos(\alpha+\theta)}$, then:

$$AB = \frac{2ad}{f - b\tan\alpha} \tag{11}$$

As shown in Figure 4(c), in $\Delta SCD : DH = HC$.

$$\frac{C'D'}{CD} = \frac{SH'}{SH} = \frac{\sqrt{f^2+b^2}}{\sqrt{GH^2+SI^2}} \tag{12}$$

Integrated the above conclusions, we analyzed the object space contour shape. As shown in Figure 5, $l \perp \Delta SS_w X$, $\therefore GH \perp l, GH \perp AB$,and GH to split up and down, we can derive the quadrilateral is isosceles trapezoid. Getting to see, therefore, the rectangle $A'B'C'D'$ of image plane is corresponding with the isosceles trapezoid $ABCD$ of object plane.

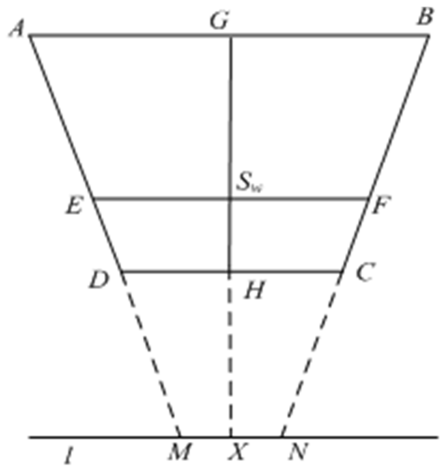


Fig. 5. Object space boundary contour.

From the above analysis, the rectangular image plane is perspective corresponding with isosceles trapezoid perspective object space, it also illustrates a phenomenon, two other long-term segment are parallel to the perspective axis, far away from the eyes of the piece looks shorter than the distance near this length.

5. The Determination of the Size of Perspective Corresponding Contour

In projective geometry, the center projection has the following property:

Property projective transformation keep collinear points and the nature of the cross ratio unchanged.

That is to say, if the four points are collinear, after projective transformation, the corresponding four points are still collinear, and keeping the nature of the cross ratio unchanged, namely, $cross(A,B;C,D) = cross(Ac,Bc;Cc,Dc)$.

According to above property, discussed the coplanar harness SG, SS_w, SH, SX cross ratio. We can be converted to calculate the four points cross ratio on $G'X$ and GX .

$$(SG', SS_i; SH', SX) = (G'S_i, H'X) = (GS_w, HX) \tag{13}$$

With (4), deprived (13):

$$(G'S_i, H'X) = \frac{G'H' \cdot S_i X}{S_i H' \cdot G'X} = \frac{2b\cdot(d-f)\cot\alpha}{b\cdot((d-f)\cot\alpha+b)} = \frac{2(d-f)\cot\alpha}{(d-f)\cot\alpha+b},$$

$$(GS_w, HX) = \frac{GH\cdot S_w X}{S_w H\cdot GX} = \frac{(GS_w+S_w H)\cdot S_w X}{S_w H\cdot(GS_w+S_w X)} = \frac{GS_w / S_w H+1}{GS_w / S_w X+1}$$

According to above (GS_w, HX) calculating formula, with formula (8), can obtain $S_w H$,having:

$$S_w H = \frac{b/f\cos\alpha\cos 2\alpha(d-f+b\tan\alpha)}{1-f/d+b^2/df\tan^2\alpha} \tag{14}$$

And according to $HX = S_w X - S_w H$, calculated HX .

Coming here, you can use formula (2) and (4) then simplifying formula, obtaining:

At this point, using equation (2), formula (14), the formula (12) can be simplified to give

$$CD = \frac{2a\sqrt{(d\sin a - S_w H)^2 + d^2\cos^2\alpha}}{\sqrt{f^2+b^2}} \tag{15}$$

Finally, as shown in Figure5, according to the size of the rectangular image, calculating a perspective view corresponding deduced trapezoidal space size, including the upper and lower end of the trapezoid, and the high and the position of a projection center in the trapezoid.

Based on the above analysis and derivation, you can get the following conclusions:

(1)With a rectangular image corresponding isosceles trapezoid perspective, perspective axis is parallel to the bottom of an isosceles trapezoid, small perspective axis near bottom, big bottom far.

(2) Given the size of the rectangular image, the camera focal length, angle between the plane and the distance between the camera and the object can be calculated isosceles trapezoid position corresponding to the size perspective.

(3) The line segment is bisected imaging center and is vertical to the perspective axis on the rectangle imaging, in space plane, its perspective line segment is vertical to the perspective axis, but imaging central perspective point is more close than the line segment center from the perspective axis.

6. Conclusion

In this paper, the discussion about the camera perspective imaging model and the correspondence between the above outline for the corresponding characteristics of pinhole camera's perspective, the nature of projective geometry based on perspective transformation, the image size and camera parameters are known, the shape of the object space is derived, location and size, the exact expression for the two-dimensional image and perspective correspondence between space objects provide a theoretical basis that can be used like a square space and object space coordinate conversion three-dimensional reconstruction, has some theoretical significance and application value. At the same time we see the need for future research in the camera distortion practical problems do further research and discussion.

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