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Analysis of Heart Rate Variability of Healthy Person Using Fractal Dimension

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Abstract

In this research, our aim is to calculate Fractal Dimension (FD) to analyse the heart rate variability (HRV) of healthy person from Electrocardiogram (ECG) signal. Some non-linear techniques are applied to different raw data (RR intervals of ECG) that are derived from the sample ECG records from MIT-BIH database. Electrocardiogram (ECG) signal gives significant information for the cardiologist to detect cardiac diseases. ECG signal is a self-similar object. So, fractal analysis can be implemented for proper utilization of the gathered information. A technique of nonlinear analysis- the fractal analysis is recently having its popularity to many researchers working on. In general, fractals can be any type of infinitely scaled and repeated pattern. A fractal is a natural phenomenon or a mathematical set that exhibits a repeating pattern that displays at every scale due to the self-similarity in the Heart's electrical conduction mechanism and self-affine behaviour of Heart Rate (HR). It is also known as expanding symmetry or evolving symmetry. Fractal analysis is measures complexity using the fractal dimension. Self-similarity dimension is one of the classifications of Fractal Dimension (FD). If the replication is exactly the same at every scale, it is called a self-similar pattern. So, fractal analysis can be implemented for proper utilization of the gathered information. It is expected that the proposed technique will provide a better result by comparison will others to calculate FD of ECG signal.

Keywords

Fractal Dimension (FD), Heart Rate Variability (HRV), Electrocardiogram (ECG), Instantaneous Heart Rate (IHR)

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1. Introduction

ECG signal of a human heart is a self-similar object, so it must have a fractal dimension that can be extracted using mathematical methods to identifying and distinguish specific states of heart pathological conditions [1]. Three non-linear methods for computing FD values will be investigated from ECG time series signals depending on fractal geometry in order to extract its main features.

1.1. Electrocardiogram (ECG)

An electrocardiogram (ECG) is a test which measures the electrical activity of heart [2]. With each heartbeat, an electrical signal spreads from the top of the heart to the bottom. In a healthy adult heart at rest, the SA node sends an electrical signal to begin a new heartbeat 60 to 100 times a minute [3]. The signal travels through the right and left atria from SA node. This electrical signal moving through the atria is recorded as the P wave on the ECG. When an electrical signal passes between the atria and ventricles it is called the atrioventricular (AV) node [4]. This process is the flat line between the end of the P wave and the beginning of the Q wave. The electrical signal

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then leaves the AV node and travels into the right and left bundle branches. As a result, blood pumps to the lungs and rest of the body. This process is recorded QRS waves on the ECG. When the ventricles recover their normal state, then it is shown as T wave on the ECG. Total heart's rhythm and activity then records on a moving strip of paper or a line on a screen.

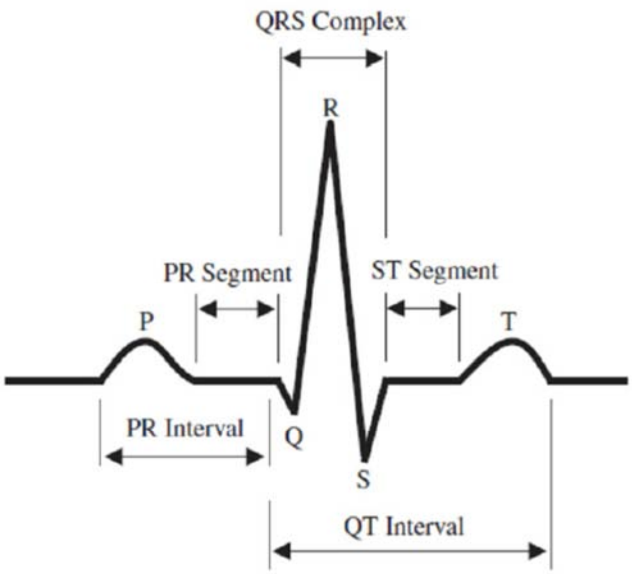

Figure 1. The components of an E.C.G signal.

1.2. Fractal Dimension

The term "fractals" is derived from the Latin word ‘Fractus’, the adjectival form of ‘Franger’, or "to break. Fractals have been used to explain objects and geometrical formations. Fractal dimension is a quantity, very often non-integer, often it is the only one measure of fractals [5]. Just a small group of fractals have one certain fractal dimension, which is scale invariant. These fractals are mono-Fractals. Fractal is strictly self-similar if it can be expressed as a union of sets, each of which is an exactly reduced copy (is geometrically similar to) of the full set. The most fractal looking in nature does not display this precise form. Natural objects are not union of exact reduced copies of whole. A magnified view of one part will not precisely reproduce the whole object, but it will have the same qualitative appearance [6] [7]. This property is called statistical self-similarity or semi-self-similarity. The most of natural fractals have different fractal dimensions depending on the scale. They are composed of many fractals with the different fractal dimension. They are called “multi-Fractals”. A compilation of mathematical procedures used to establish fractal dimension with the smallest error.

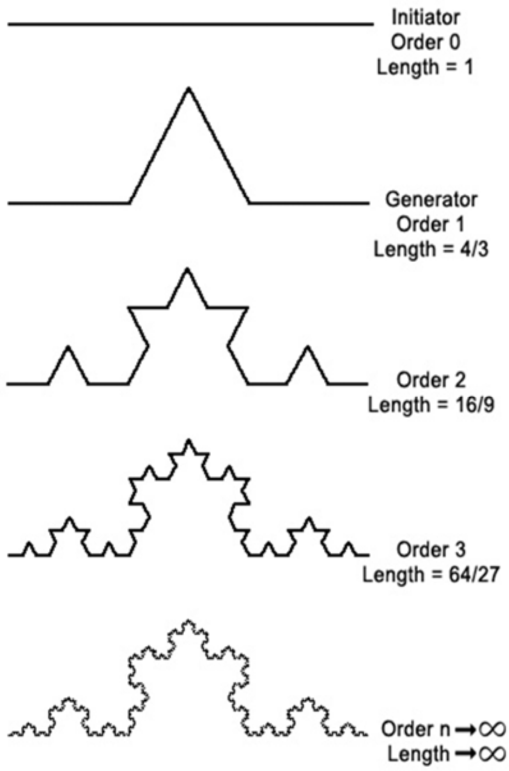

Figure 2. Fractal Dimension (FD) is according to the order and length.

2. Methodology

Fractal Dimension is a descriptive measure that has been proven useful in quantifying the complexity or self-similarity of biomedical signals. [8]. In general, a fractal is defined as a set having non-integer dimension. Consequently, the fractal dimension (FD) is introduced as a factor highly correlated with the human perception of object’s roughness. FD fills the gap between one- and two-dimensional objects. The more complex the contour of the curve, the more it fills the plane and the more its fractal dimension will be closer to 2. This section investigates three different methods for computing FD values from ECG time series signals (RR intervals) depending on fractal geometry in order to extract its main features [9].

A. Relative Dispersion (RD) Method

B. Power Spectral Density (PSD) Method

C. Rescaled Range Method

The heart rate data of healthy persons are loaded using MATLAB code. The code is simulated using appropriate data and the output Y is saved in ‘.mat’ format. Collected ‘115.mat’ or ‘117.mat’ or ‘127.mat’ or ‘230.mat’ provides ECG of healthy person. The plot of these matrices is shown in figure 3, 4, 5 and 6.

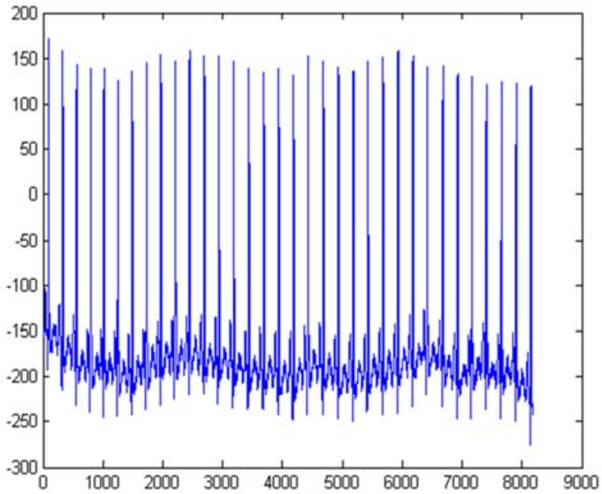

Figure 3. Heart rate of a healthy person (Data 122).

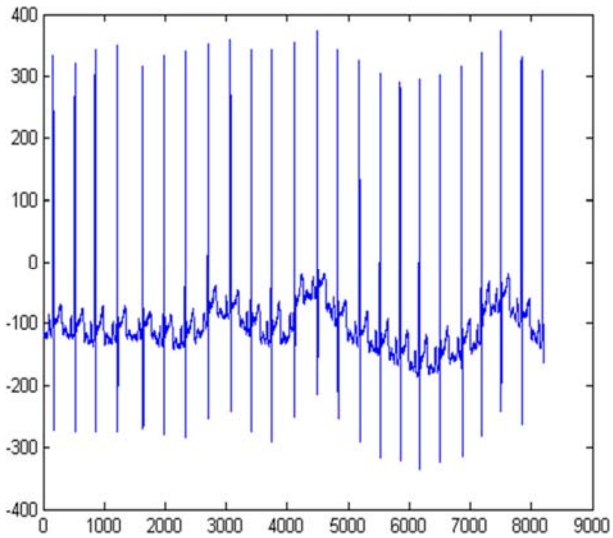

Figure 4. Heart rate of a healthy person (Data 115).

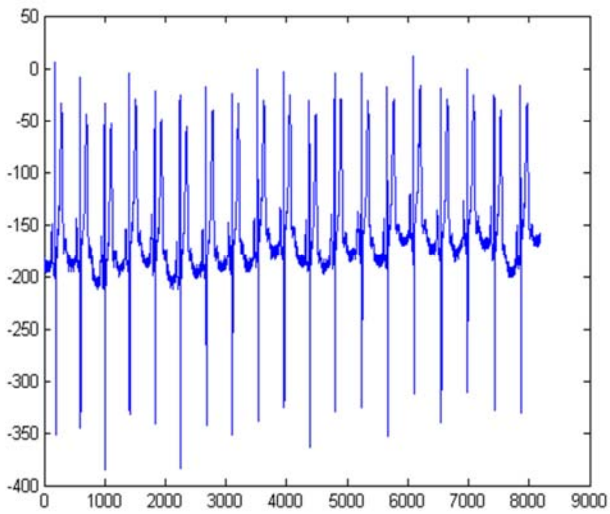

Figure 5. Heart rate of a healthy person (Data 117).

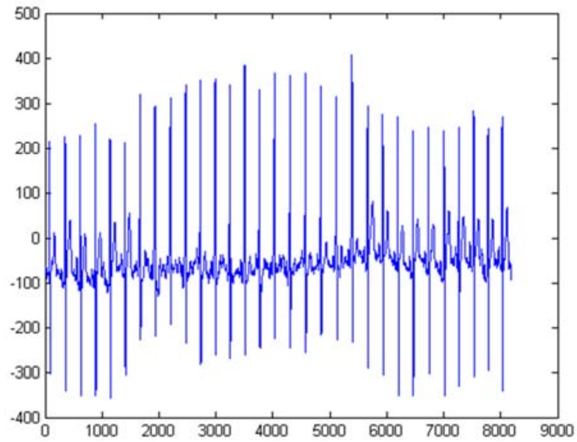

Figure 6. Heart rate of a healthy person (Data 230).

2.1. Relative Dispersion (RD) Method

One-dimensional approach that can be applied to an isotropic signal of any dimension [10]. Making estimates of the variance of the signal at each of several different levels of resolution form the basis of the technique; for fractal signals a plot of the log of the standard deviation versus the log of the measuring element size (the measure of resolution) gives a straight line with a slope of 1 - D, where D is the fractal dimension. The characterizing Hurst coefficient, H is a measure of irregularity; the irregularity or anti correlation in the signal is maximal at H near zero. White noise with zero correlation has H = 0.5. For one-dimensional series, H = 2 - D, where D is the fractal dimension, 1<D<2[9].

There is a strong relationship between the measure of variation (the coefficient of variation) and the resolution of measurement. This is a simple one dimensional spatial analysis which we level as RD analysis, where RD is the relative dispersion i.e. the standard deviation divided by the mean. Mathematically it can be expressed as: $\mathrm{RD} = \mathrm{SD}/\mathrm{Mean}$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{n}[x_i - \bar{x}]^2/n} \qquad (1)$$

Where, x_i = Random variable

$\bar{x}$= Mean of the variables

N = Number of Samples

H not equal to 0.5, the SD will be proportional to n^{H-1}, n being the bin size or time resolution interval. By calculating the RD ($\mathrm{RD} = {}^{\mathrm{SD}}/_{\mathrm{Mean}}$) for different bin sizes, n and fitting the square law function:

$$\mathrm{RD}=\mathrm{RD}_0(n^{H-1}/no) \qquad (2)$$

Where, RD_0 is the RD for some reference bin size no.

The exponent can be best estimated by a log-log transformation.

$$\log(RD)=\log(RD_0)+(H-1)\log(n/n_0) \quad (3)$$

Here H-1=slope, For Fractal Dimension, D=2-H. So, D=1-slope. So, fractal dimension from this equation can easily be estimated. The fit is little improved when longer signals are used. [9]

a) Analysis Procedure in Brief

1) At first specified data (RR intervals of ECG signal) is divided into different series of Bin size.
2) The data series is then segmented into intervals and individual Standard deviation and Mean of each segment are calculated.
3) Determined the arithmetic mean of data set by adding all of the individual values of the set together and dividing by the total number of values.
4) Then standard deviation is normalized by dividing with the arithmetic mean, which yields the relative dispersion RD of the data series.
5) The results are then plotted against $\log_2$ (n) versus $\log_2$ (RD), where n is the bin size or the time resolution interval.
6) The MATLAB function 'polyfit()' is used to fit a least square straight line over the $\log_2$ (n) versus $\log_2$ (RD) curve.

b) The given data numbers '115' or '117' or '122' provides ECG of healthy person

Table 1. RD method using 1024 simulated data for 'data-115'.

Bin size, n	Mean	SD	RD	$\log_2 n$	$\log_2(RD)$
512	-98.9805	1.8086	-0.0183	9	-5.7742
256	-98.9805	3.8381	-0.0388	8	-4.6887
128	-98.9805	13.6422	-0.1378	7	-2.8591
64	-98.9805	19.0539	-0.1925	6	-2.3771
32	-98.9805	25.3241	-0.2558	5	-1.9666
16	-98.9805	32.1144	-0.3245	4	-1.6239
8	-98.9805	51.2082	-0.5174	3	-0.9508
4	-98.9805	56.5814	-0.5716	2	-0.8068
2	-98.9805	58.704	-0.5931	1	-0.7537

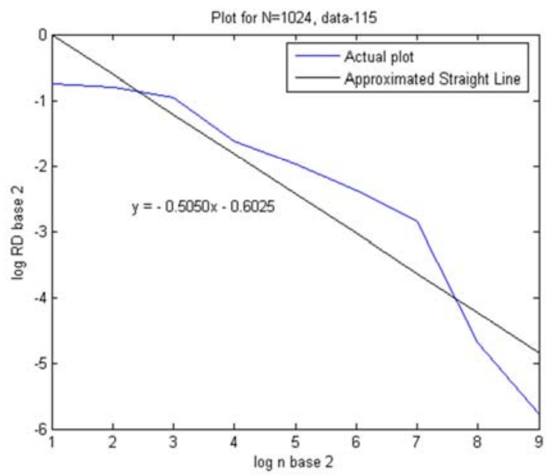

Figure 7. Actual and approximated straight line using N=1024 for 'data-115'.

Table 2. RD method using 2048 simulated data for 'data-115'.

Bin size, n	Mean	SD	RD	$\log_2 n$	$\log_2(RD)$
1024	-101.6660	2.6855	-0.0264	10	-5.2425
512	-101.6660	3.6850	-0.0362	9	-4.7860
256	-101.6660	5.7818	-0.0569	8	-4.1362
128	-101.6660	13.8885	-0.1366	7	-2.8719
64	-101.6660	19.6121	-0.1929	6	-2.3740
32	-101.6660	27.3504	-0.2690	5	-1.8942
16	-101.6660	37.8055	-0.3719	4	-1.4272
8	-101.6660	51.2683	-0.5043	3	-0.9877
4	-101.6660	56.7174	-0.5579	2	-0.8420
2	-101.6660	58.8620	-0.5790	1	-0.7884

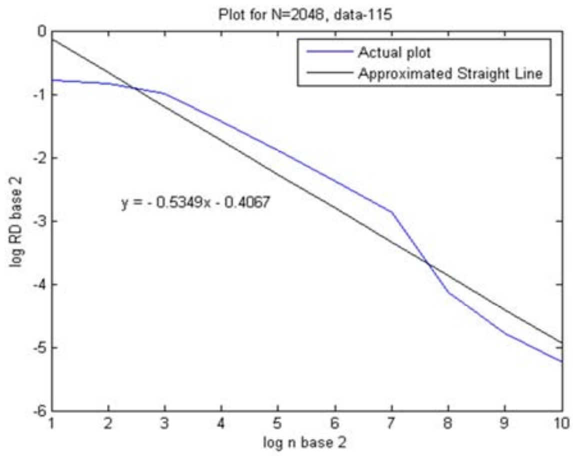

Figure 8. Actual and approximated straight line using N=2048 for 'data-115'.

Table 3. RD method using 4096 simulated data for 'data-115'.

Bin size, n	Mean	SD	RD	$\log_2 n$	$\log_2(RD)$
2048	-97.6292	4.0369	-0.0413	11	-4.5960
1024	-97.6292	4.8560	-0.0497	10	-4.3295
512	-97.6292	12.1435	-0.1244	9	-3.0071
256	-97.6292	13.3000	-0.1362	8	-2.8759
128	-97.6292	18.8721	-0.1933	7	-2.3711
64	-97.6292	23.2309	-0.2380	6	-2.0713
32	-97.6292	30.5692	-0.3131	5	-1.6752
16	-97.6292	40.7325	-0.4172	4	-1.2611
8	-97.6292	55.5996	-0.5183	3	-0.9482
4	-97.6292	56.1919	-0.5753	2	-0.7977
2	-97.6292	58.1929	-0.5961	1	-0.7464

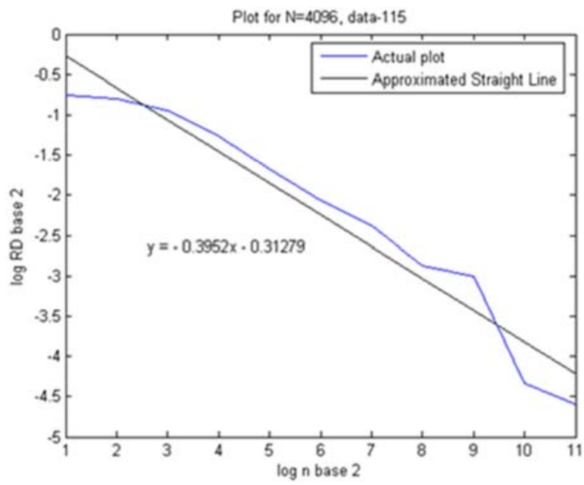

Figure 9. Actual and approximated straight line using N=4096 for 'data-115'.

Table 4. RD method using 8192 simulated data for ‘data-115’.

Bin size, n	Mean	SD	RD	log_2n	$log_2(RD)$
4096	-98.6744	1.0453	-0.0106	12	-6.5607
2048	-98.6744	4.3451	-0.0440	11	-4.5052
1024	-98.6744	23.6631	-0.2398	10	-2.0600
512	-98.6744	27.3933	-0.2776	9	-1.8489
256	-98.6744	28.5327	-0.2892	8	-1.7901
128	-98.6744	31.0265	-0.3144	7	-1.6692
64	-98.6744	33.7273	-0.3418	6	-1.5488
32	-98.6744	39.1148	-0.3964	5	-1.3350
16	-98.6744	47.6433	-0.4828	4	-1.0504
8	-98.6744	56.8525	-0.5762	3	-0.7955
4	-98.6744	62.5687	-0.6341	2	-0.6572
2	-98.6744	64.6028	-0.6547	1	-0.6111

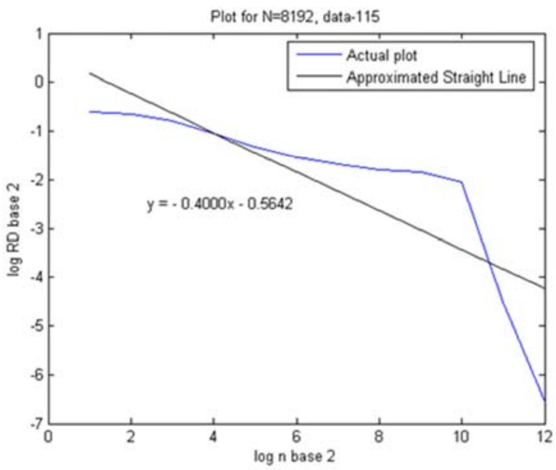

Figure 10. Actual and approximated straight line using N=8192 for ‘data-115’

Table 5. RD method using 1024 simulated data for ‘data-122’.

Bin size, n	Mean	SD	RD	log_2n	$log_2(RD)$
512	-152.5850	9.5088	-0.0623	9	-4.0042
256	-152.5850	11.1898	-0.0733	8	-3.7694
128	-152.5850	19.4844	-0.1277	7	-2.9692
64	-152.5850	31.1854	-0.2044	6	-2.2907
32	-152.5850	41.7397	-0.2736	5	-1.8701
16	-152.5850	60.2320	-0.3947	4	-1.3410
8	-152.5850	69.7918	-0.4574	3	-1.1285
4	-152.5850	72.7961	-0.4771	2	-1.0677
2	-152.5850	73.6464	-0.4827	1	-1.0509

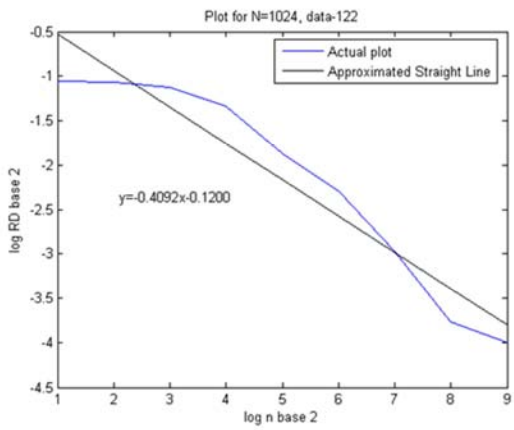

Figure 11. Actual and approximated straight using N=1024 for ‘data-122’.

Table 6. RD method using 2048 simulated data for ‘data-122’.

Bin size, n	Mean	SD	RD	log_2n	$log_2(RD)$
1024	-163.0415	10.4565	-0.0641	10	-3.9628
512	-163.0415	12.6741	-0.0777	9	-3.6853
256	-163.0415	13.7009	-0.0840	8	-3.5729
128	-163.0415	21.6276	-0.1327	7	-2.9143
64	-163.0415	30.8450	-0.1892	6	-2.4021
32	-163.0415	43.2240	-0.2651	5	-1.9153
16	-163.0415	57.2942	-0.3514	4	-1.5088
8	-163.0415	67.8111	-0.4159	3	-1.2656
4	-163.0415	70.7211	-0.4338	2	-1.2050
2	-163.0415	71.5694	-0.4390	1	-1.1878

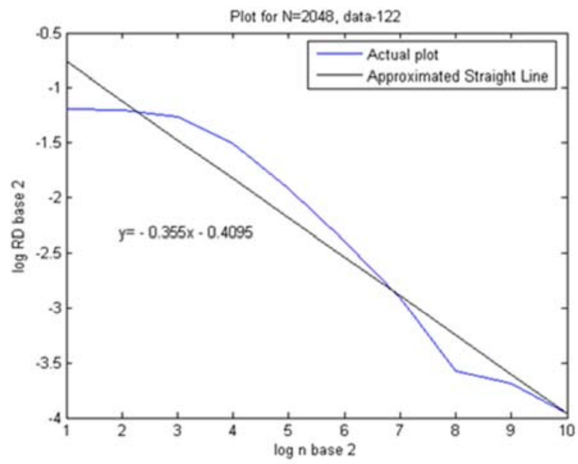

Figure 12. Actual and approximated straight using N=2048 for ‘data-122’.

Table 7. RD method using 4096 simulated data for ‘data-122’.

Bin size, n	Mean	SD	RD	log_2n	$log_2(RD)$
2048	-165.5513	2.5098	-0.0152	11	-6.0436
1024	-165.5513	9.2726	-0.0560	10	-4.1582
512	-165.5513	10.7165	-0.0647	9	-3.9494
256	-165.5513	11.4114	-0.0689	8	-3.8587
128	-165.5513	20.7424	-0.1253	7	-2.9966
64	-165.5513	30.1411	-0.1821	6	-2.4575
32	-165.5513	42.5256	-0.2569	5	-1.9609
16	-165.5513	56.1678	-0.3393	4	-1.5595
8	-165.5513	66.1902	-0.3998	3	-1.3226
4	-165.5513	68.8921	-0.4161	2	-1.2649
2	-165.5513	69.6806	-0.4209	1	-1.2484

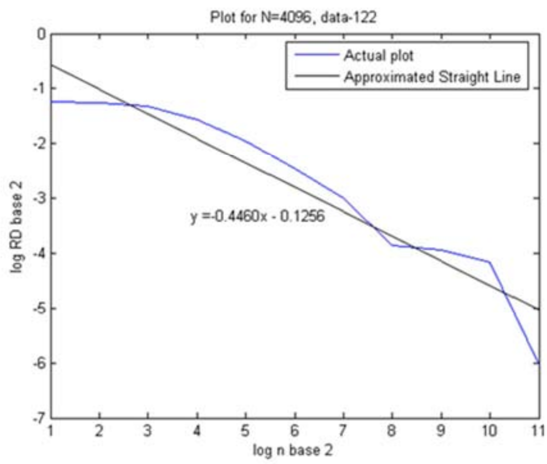

Figure 13. Actual and approximated straight using N=4096 for ‘data-122’

Table 8. RD method using 8192 simulated data for 'data-122'.

Bin size, n	Mean	SD	RD	log_2n	$log_2(RD)$
4096	-169.7992	4.2479	-0.0250	12	-5.3209
2048	-169.7992	4.6620	-0.0275	11	-5.1867
1024	-169.7992	8.7722	-0.0517	10	-4.2748
512	-169.7992	10.1694	-0.0599	9	-4.0615
256	-169.7992	10.9584	-0.0645	8	-3.9537
128	-169.7992	19.6442	-0.1157	7	-3.1117
64	-169.7992	29.3029	-0.1726	6	-2.5347
32	-169.7992	43.4798	-0.2561	5	-1.9654
16	-169.7992	56.5836	-0.3332	4	-1.5854
8	-169.7992	65.9676	-0.3885	3	-1.3640
4	-169.7992	68.7177	-0.4047	2	-1.3051
2	-169.7992	69.5263	-0.4095	1	-1.2882

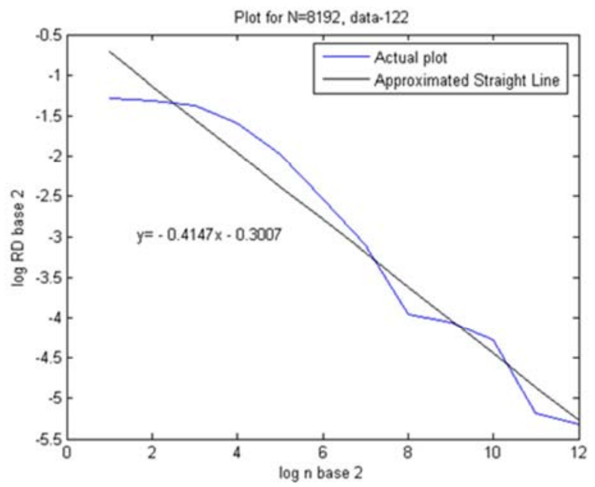

Figure 14. Actual and approximated straight using N=8192 for 'data-115'.

c) Analysis

The slope of the straight line is then used to calculate the fractal dimension (D=1-slope) (4)

Table 9. Range of Fractal Dimension of healthy persons obtained for different data Using Relative Dispersion (RD) Analysis.

Data Number	Data Length N	Fractal Dimension
'Data 115'	1024	1.5500
	2048	1.5349
	4096	1.3952
	8192	1.4000
'Data 122'	1024	1.4092
	2048	1.3550
	4096	1.4460
	8192	1.4147
'Data 117'	1024	1.3458
	2048	1.3562
	4096	1.5349
	8192	1.3192

The above calculation Standard Deviation is least when data length is 8192. In general, with the increase of data length, calculated fractal dimension in general gets closer to the actual result. So, Relative Dispersion (RD) Analysis is well suited for long signals.

From the above calculation, we can conclude that the range of fractal dimension for healthy person is 1.31-1.41.

2.2. Power Spectral Density Analysis

The power spectrum (the square of the amplitude from the Fourier transform) of an unpolluted fractional Brownian motion is known to be described by a power law function:

$$|A|^2 = 1/ f^{\beta} \quad (5)$$

Where |A| is the magnitude of the spectral density at frequency f, with an exponent equal to β = 2H + 1. In general, fractal signals always have such a very broad spectrum. When the derivative is taken from a fractal signal, β is reduced by two. Thus, for fractional Brownian noise, fBn, β is expected to be:

$$\beta=2H+1. \quad (6)$$

Here again, a straight line is fitted from a log-log plot, and H is calculated from the slope β. Power spectrum method applies the power law variation of time series. A strong relationship exists between fractal dimension and power law index of time series. In the frequency domain, fractal time series exhibit power law properties:

$$P(f) \sim f^{-\alpha} \quad (7)$$

Where P(f) is the power spectral density, f, and the exponent α is the so, called power-spectral index. For the values region between FD=1 and FD =2 the following relationship between FD and α is valid:

$$FD=(5-\alpha)/2, \text{ for } 1<FD<2 \quad (8)$$

In other words, the fractal dimension of a time series can be calculated directly from its power spectrum.

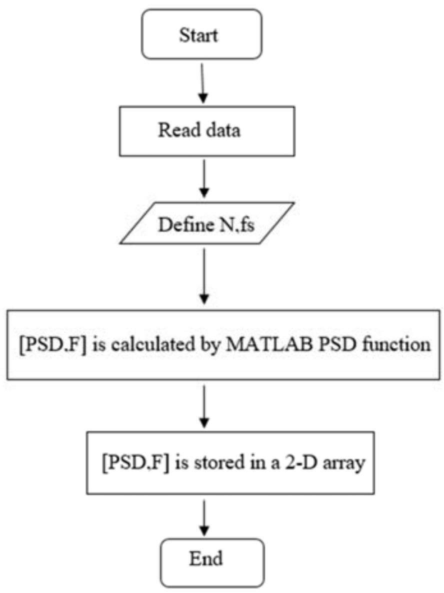

Figure 15. Flow Chart for Power Spectral Density Analysis.

a) Analysis Procedure in Brief

1) At first simulated data is loaded.
2) Power spectral density is estimated by using MATLAB function 'PSD' [10] [11].

Function Format: [Pxx,F]= psd(x,nfft,fs,window,noverlap)

It returns a vector of frequencies, the same size as Pxx at which the PSD is estimated. The PSD is plotted on log scale by using 'log log' command. The plots of the power spectral densities against the normalized frequency in the log scale are for normal data assuming sampling frequency.

3) Least-square straight line is then fitted over the PSD plotted.
4) The slope of the estimated straight line is used to calculate the fractal dimension of the simulated data. for different data series. Fractal dimension obtained for each data in then tabulated. Comparing these values the range of fractal dimension for healthy person is determined. Deviation from this range predicts the presence of abnormality.

b) The given data numbers '115' or '117' or '127' or '230' provides ECG of healthy person

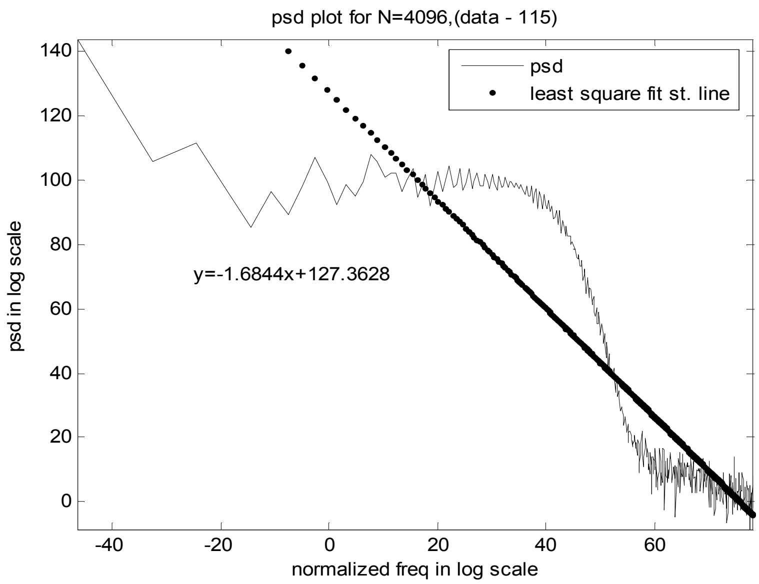

Figure 16. PSD and approximated straight line using N=4096 for 'data 115'.

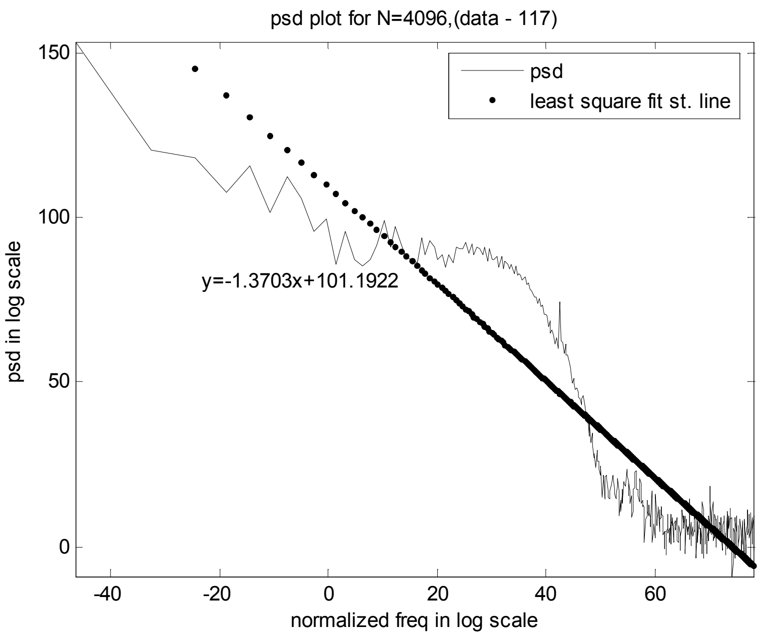

Figure 17. PSD and approximated straight line using N=4096 for 'data 117'.

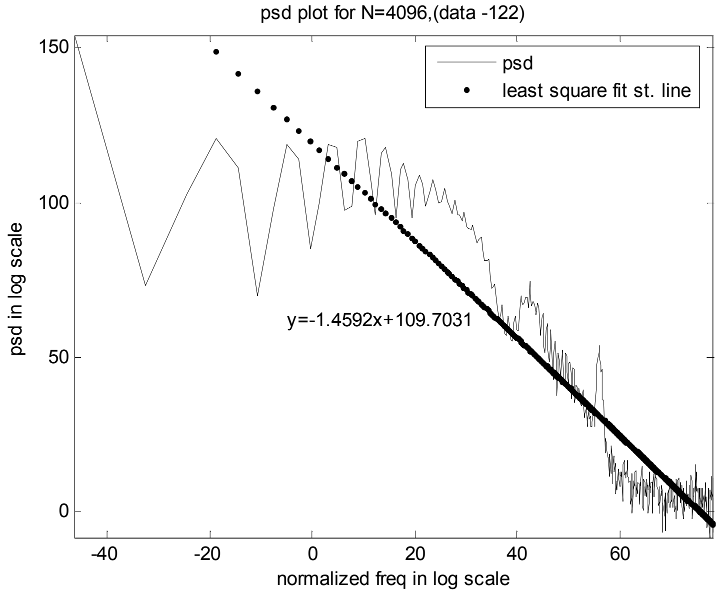

Figure 18. PSD and approximated straight line using N=4096 for 'data 122'.

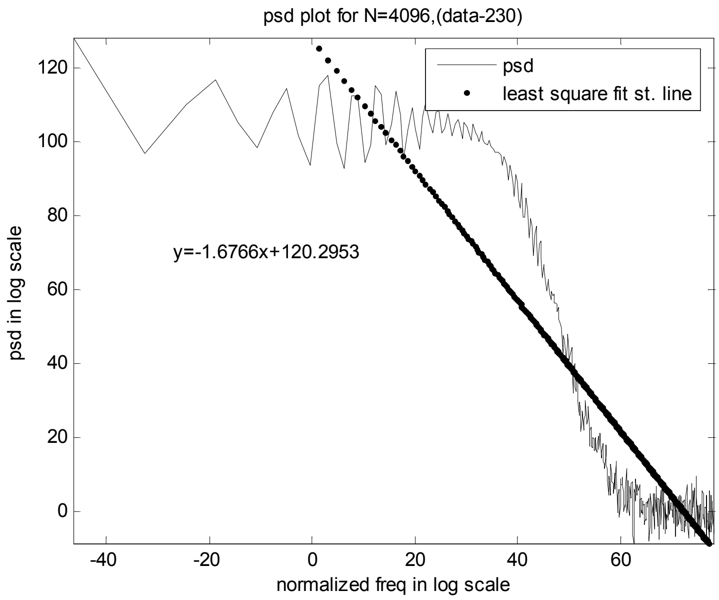

Figure 19. PSD and approximated straight line using N=4096 for 'data 230'.

c) Analysis

We know that for PSD method,

$$\text{Fractal dimension, } D = (5\text{-slope})/2 \quad (8)$$

As in the case of RD method, the best result is achieved with longer data series. From the analysis, it is obvious that PSD method shows least biased results. However, no single equation expression can be established at this stage to relate the nature of data series and the calculated fractal dimension.

From the above calculation, we can conclude that the range of fractal dimension for healthy person is 1.7494 - 1.7989.

Table 10. Range of Fractal Dimension of Healthy Person for different simulated data using Power spectral density analysis.

Data Number	Data Length N	Fractal Dimension
	1024	*1.7578*
'Data 115'	2048	*1.7655*
	4096	*1.7965*
	1024	*1.7647*
'Data 117'	2048	*1.7989*
	4096	*1.8148*
	1024	*1.7149*
'Data 122'	2048	*1.7494*
	4096	*1.7704*
	1024	*1.7470*
'Data 230'	2048	*1.7614*
	4096	*1.7617*

2.3. Rescaled Range Analysis

Hurst (1965) developed the rescaled range analysis, a statistical method is used to analyse long records of natural phenomena. There are two factors used in this analysis: firstly, the range R, this is the difference between the minimum and maximum 'accumulated' values or cumulative sum of X(t,τ) of the natural phenomenon at discrete integer-valued time t over a time span τ (taw), and secondly the standard deviation S, estimated from the observed values Xi(t). Hurst found that the ratio R/S is very well described for a large number of natural phenomena by the following empirical relation [12-18]:

$$\frac{R(\tau)}{S(\tau)} \propto \tau^H \quad (9)$$

where τ is the time span, and H the Hurst exponent. The coefficient c was taken equal to 0.5 by Hurst. R and S are

$$R(\tau) = Max(\mathrm{X}(\mathrm{t},\tau)) – \mathrm{Min}\,(\mathrm{X}(\mathrm{t},\tau)) \quad (10)$$

for the range $1 \leq t \leq \tau$

$$\text{and } S(\tau) = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}\{\xi(t) - (\xi)_\tau\}^2} \quad (11)$$

The Relation between Hurst exponent and the fractal dimension is simply D=2-H. We calculate the individual calculations for each interval length. A straight line is fitted in the log-log plot: Log[R(T)/S(T)] = c + H log(T). Where H = slope. So, Fractal dimension, D= 2-H. With the help of this equation we can easily evaluate fractal dimension in rescaled range analysis [19].

a) Analysis Procedure in Brief

1) At first specified data is divided into different series of *Bin size or* resolution time interval.
2) Then calculated standard deviation for different *Bin size* of data.
3) Mean of the data series is then calculated. Maximum and minimum of integral for each lag interval is then calculated to find out the range, R of each interval.
4) Finally, the R/S ratio is calculated from the average of each slot R/S ratio.
5) The results are then plotted against $\log_2$(t) vs $\log_2$(R/S), where t is the length of the time.
6) The MATLAB function 'polyfit()' is used to fit a least square straight line over the $\log_2$ (t) vs $\log_2$ (R/S) curve.

b) In the given '115.mat' or '117.mat' or '122.mat' provides ECG of healthy person

Table 11. R/S Analysis using 1024 simulated data for Data-115.

Slot size, T	$\log_2$(T)	$\log_2$(R/S)
512	9	12.6237
256	8	12.0999
128	7	11.9111
64	6	11.5401
32	5	11.1825

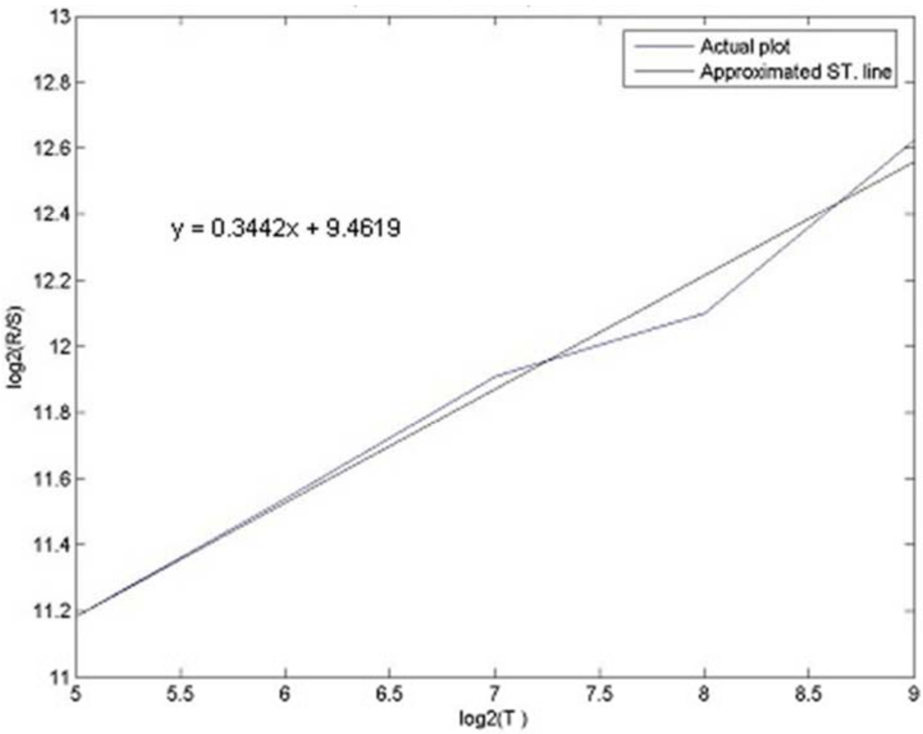

Figure 20. Actual and approximated straight line using N=1024 for *data115*.

Table 12. R/S Analysis using 2048 simulated data for Data-115.

Slot size, T	$\log_2(T)$	$\log_2(R/S)$
512	9	12.1189
256	8	11.999
128	7	11.1111
64	6	10.5401
32	5	10.1825

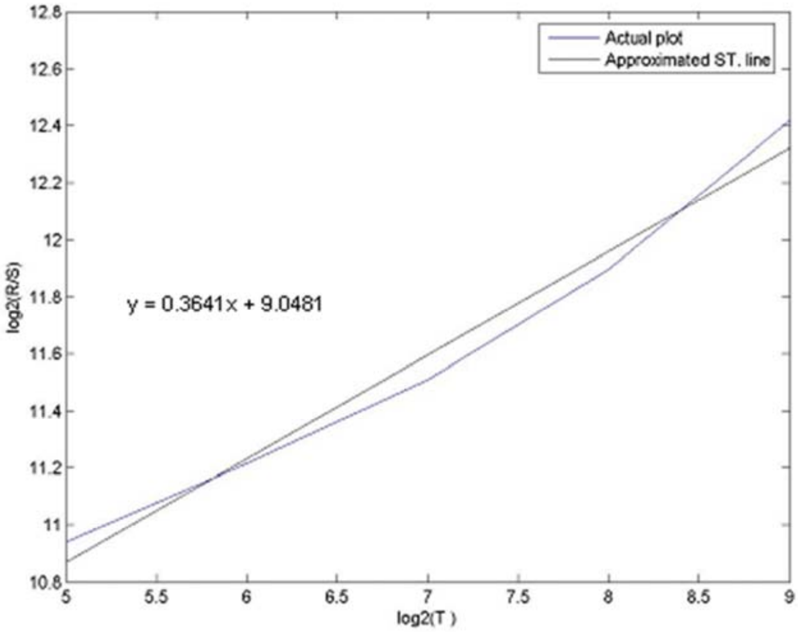

Figure 21. Actual and approximated straight line using N=2048 for’data115’.

Table 13. R/S Analysis using 4096 stimulated data for Data-115.

Slot size, T	$\log_2(T)$	$\log_2(R/S)$
512	9	12.5697
256	8	12.1587
128	7	11.8979
64	6	11.5438
32	5	11.2147

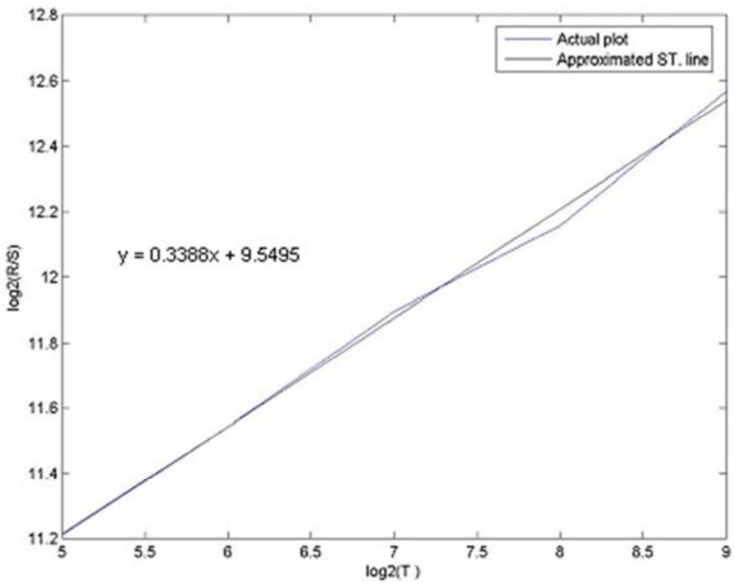

Figure 22. Actual and approximated straight line using N=4096 for *data115.*

Table 14. R/S Analysis using 1024 simulated data for Data-117.

Slot size, T	$log_2(T)$	$log_2(R/S)$
512	9	11.0151
256	8	10.9981
128	7	10.4695
64	6	9.9987
32	5	9.4855

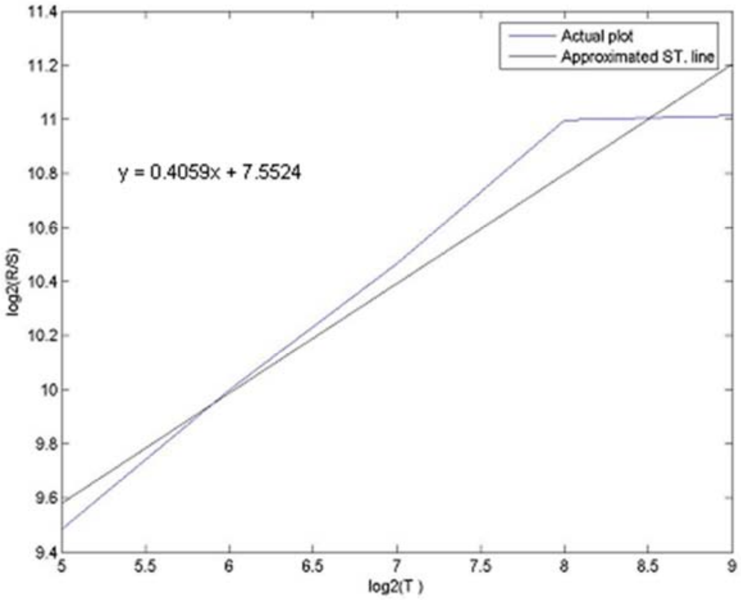

Figure 23. Actual and approximated straight line using N=1024 for *data117*.

Table 15. R/S Analysis using 2048simulated for Data-117.

Slot size, T	$log_2(T)$	$log_2(R/S)$
512	9	11.3151
256	8	10.9981
128	7	10.7695
64	6	10.2120
32	5	9.7685

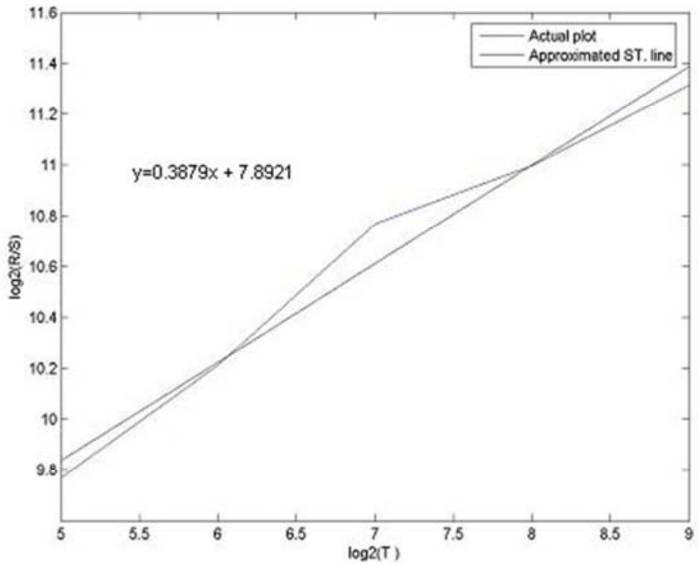

Figure 24. Actual and approximated straight line using N=2048 for *data117*.

Table 16. R/S Analysis using 4096 simulated data for Data-117.

Slot size, T	$log_2(T)$	$log_2(R/S)$
512	9	12.0094
256	8	11.7840
128	7	11.4543
64	6	10.9997
32	5	10.7596

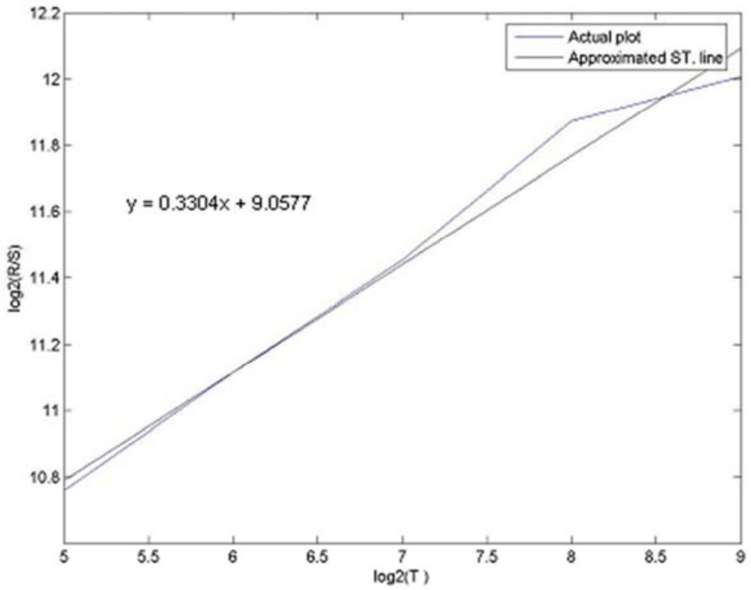

Figure 25. Actual and approximated straight line using N=4096 for *data117.*

Table 17. R/S Analysis using 1024 simulated data for Data-122.

Slot size, T	$log_2(T)$	$log_2(R/S)$
512	9	12.4374
256	8	12.1576
128	7	11.7642
64	6	11.5154
32	5	10.7596

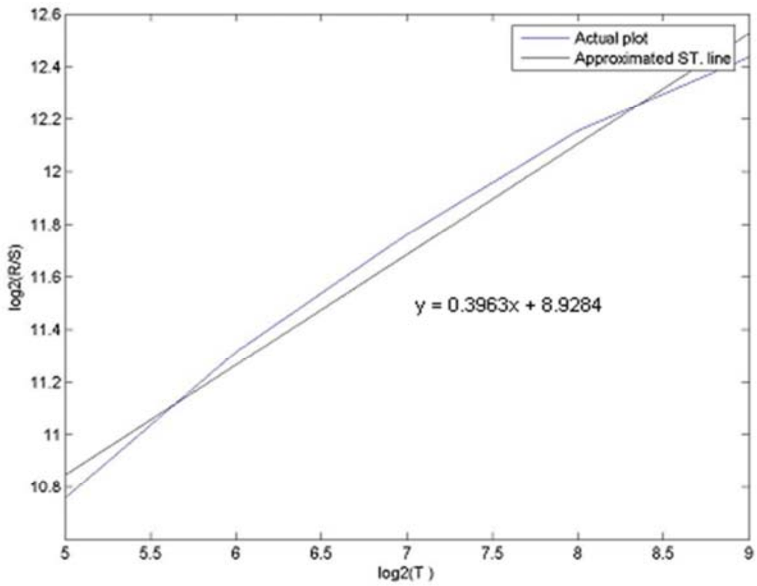

Figure 26. Actual and approximated straight line using N=1024 for *data122.*

Table 18. R/S Analysis using 2048 simulated data for Data-122.

Slot size, T	log_2(T)	log_2(R/S)
1024	10	13.1374
512	9	12.7576
256	8	12.2642
128	7	11.8954
64	6	11.5096
32	5	11.0215

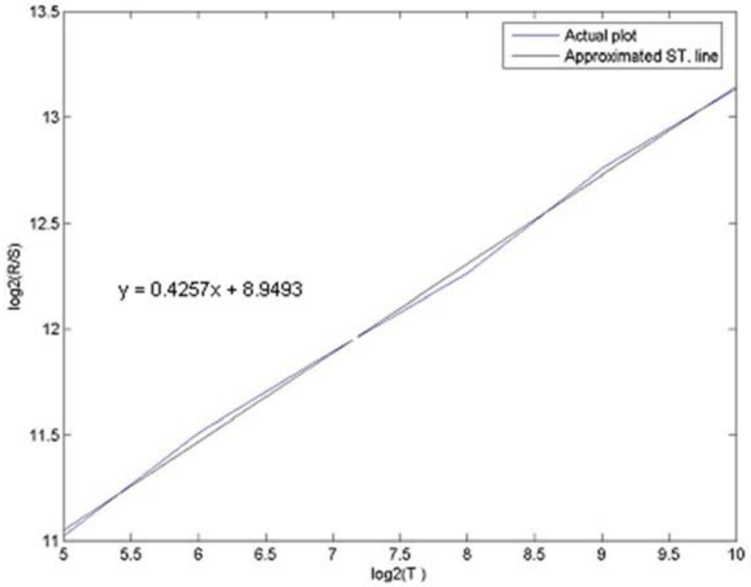

Figure 27. Actual and approximated straight line using N=2048 for *data122.*

Table 19. R/S Analysis using 4096 simulated data for Data-122.

Slot size, T	log_2(T)	log_2(R/S
1024	10	12.8864
512	9	12.5576
256	8	12.2642
128	7	11.8954
64	6	11.5096
32	5	10.9980

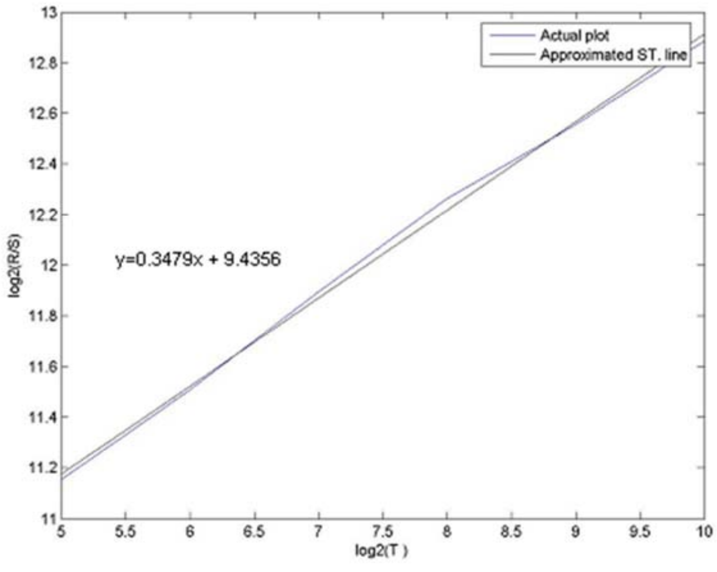

Figure 28. Actual and approximated straight line using N=4096 for data122.

c) Analysis

From the above graphs, the slope of the straight line is then used to calculate the fractal dimension.

In the R/S analysis, fractal dimension D=2-H (12)

Table 20. Range of Fractal Dimension of healthy persons obtained for different data using Rescaled Range method.

Data Number	Data Length, N	Fractal Dimension (D)
	1024	1.6558
'Data 115'	2048	1.6345
	4096	1.6612
	1024	1.6037
'Data 122'	2048	1.5743
	4096	1.6521
	1024	1.5941
'Data 117'	2048	1.6121
	4096	1.6626

From the above table, it is found that in Rescaled Range analysis there is no linear variation of fractal dimension with data length or closely spaced data series. The Standard Deviation is least when data length is 4096. The best result is obtained with data size, N=4096.

So, we can conclude that the range of fractal dimension for healthy person is 1.65 -1.67.

3. Result and Discussion

In this research, fractal dimensions are estimated for healthy person (normal data) for the case of three proposed methods.

Relative Dispersion Method: Range of Fractal Dimension of healthy persons is 1.31-1.41.

Power Spectral Density (PSD) Method: Range of Fractal Dimension of healthy persons is 1.7494 - 1.7989.

Rescaled Range (R/S) Method: Range of Fractal Dimension of healthy persons is 1.65-1.67.

Relative dispersion analysis is well suited for long signals. PSD method shows least biased results. In Rescaled range analysis, there is no linear variation of fractal dimension with data length or closely spaced data series.

We find a quick hint from the fact that, in general, data series is suited better by R/S analysis rather than RD analysis and PSD analysis. So, it has a better accuracy to calculate fractal dimension than other method.

4. Conclusions and Future Work

This paper aims to analysis heart rate variability (HRV) applying three different methods to calculate fractal dimension (FD) of instantaneous heart rate (IHR) derived from ECG. These non-linear techniques are applied to different raw data (RR intervals of ECG) that are derived from the sample ECG records from MIT-BIH database. The results found from this work are analysed to see the range of fractal dimension of healthy or normal persons according to bit rate in the examined ECG record. In future, there are so many opportunities to find fractal dimension of abnormal patients and how they vary between normal and abnormal. Box Counting Method is another efficient way to calculate fractal dimension. There also have some other classical techniques to determine heart rate analysis. Heart abnormalities will be detected by using several frequency domain methods such as cross entropy analysis, Support Vector Machine (SVM), Discrete Cosine Transform (DCT) and Wavelet Transform etc.

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