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Classification of ECG Signals Using Legendre Moments

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Abstract

During the last few decades, a huge amount of work is devoted to the study of classification techniques of Electrocardiogram (ECG) signals. In this paper, we present a robust algorithm based on shifted Legendre polynomials. These polynomials are successfully implemented to extract features from ECG signals. The ECG signals are first decomposed into its sub band using Legendre polynomials in specific time interval. The objective of the study is to classify two different types of ECG signals. We test two different classifier, KNN and simple logistic. A comparative study is presented on the behaviour and results of these classifiers. Experimental results are obtained for both classifiers and compared with some other results available in the open literature. The experimental results lead us to conclusion that the proposed method provides more accurate results as compare to other classification methods.

Keywords

ECG Signals, Simple Logistic Classifier, Legendre Polynomials, KNN Classifier

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1. Introduction

The electrocardiogram (ECG) is widely used as a diagnostic tool for determining different abnormality of the human heart. Due to the non-stationary nature of ECG signals, a good expert medical practitioner may also fail to diagnose different types of heart abnormality. In recent years, several methods have been developed for the detection and classification of the ECG signals. K. Balasundaram et al. [1] used wavelet analysis to classify three different types of abnormality, i.e. Ventricular Tachycardia (VT), Organized Ventricular Fibrillation (OVF) and Disorganized Ventricular Fibrillation (DVF). S. N. Yu et al. [2] used wavelet transformation and probabilistic neural network over small record from MIT-BIH database of 23 records, containing six different types of ECG signals. These records include Normal beats (N), left bundle branch block beats (LBBB), right bundle branch block beats (RBBB), paced beats (PB), arterial premature beat (APB), and premature ventricular contraction (PVC). M. Thomas et al. [3] developed a technique for classifying ECG signals based on using the coefficients of dual tree complex wavelet transform (DTCWT). M. Thomas also extracted 4 other features from QRS complex of each cycle. The database contains 48 files from different patients. M. Thomas classify several beats using artificial neural network (ANN) which includes normal (N), Paced beats (PB), LBBB, RBBB. S. Kadambe et al. [4] also worked on the classification of 3 types of ECG waves P and QRS complex, using VQ-based classifier to classify abnormal and normal P waves, QRS waves and T waves. R. J. Martis et al. [5] used principle components of

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discrete wavelet transform to extract features from ECG signals and used feed forward neural network (NN) to classify five different types of heart abnormalities i.e. Normal (N), RBBB, LBBB, APC and VPC. They used available data set of MIT-BIH arrhythmia database for classification. H. G. Hosseini et al. [6] used independent features of a compressed ECG data to classify six common waveforms. The author used multi layer perceptron to classify the extracted features from 10 ECG records of the MIT-BlH arrhythmia database. P. Chazal et al. [7] used manually extracted features from ECG signals to classify five different beats classes of ECG signals, i.e. normal beat, supra ventricular ectopic beat (SVEB), ventricular ectopic beat (VEB), fusion of normal and ventricular ectopic beats (VEB), or unknown beat types. Data is obtained from MIT-BIH arrhythmia database. A. Ebrahhimzadeh et al. [8] used Radial Basis Function (RBF) neural network to analyze ECG signals for cardiac arrhythmias diagnosis and classify five types of normal and abnormal heartbeats, LBBB, RBBB, APC and PVC. They used small data set of eight file of the MIT-BIH arrhythmia database. N. Emanet et al. [9] have proposed a technique to classify five heartbeat classes N, L, R, V and P using discrete wavelet transform (DWT) and Random Forest Algorithm. F. A. Afsar et al. [10] used wavelet domain analysis for feature extraction from ECG signals for the classification of six types of heart beats, which includes APB, Paced Beats (P), LBBB, RBBB, PVC and normal beats.

In the last few decades, the applications of polynomials in image and signal processing have attracted the attention of many researchers around the globe. For example, Dhannoon et al. [11] used polynomial basis for the compression and reconstruction of color images. H. Zhang et al. [12] proposed a new method to construct a set of blur invariants of grey scale image using the orthogonal Legendre moments. Legendre polynomials have gained high attention of much wider class of researcher due to which a collection of efficient results has been presented. K. M. Hosny et al. [13-17] effectively apply Legendre polynomial to compress, reconstruct, synthesis of grey scale images. Motivated by the results provided in the above references we analyze one dimension Legendre moments for the classification of ECG signals. In this article, we introduced Legendre polynomial which helps to identify features in normal and abnormal ECG signals. We validate the benefit on MIT-BIH database using MATLAB. The results obtains in our experiments is better from the already existence methods of feature extractions.

The rest of article is organized as: In section 2, we present some basic results from approximation theory, which are of basic importance in our analysis. In section 3, some computational aspects of Legendre polynomials are presented and efficient method is implemented for extraction of features from ECG signals. Section 4 is devoted to study of two different classifiers. In section 5, the proposed algorithm is implemented on data set of different ECG signals obtained from MIT-BIH arrhythmia database and some experimental results are obtained. In the same section we also compare our results with other results available in the literature. The last section, in which conclusion of the paper is presented is followed by acknowledgement of the paper.

2. Preliminaries

In this section, we summarize some known results and basic facts from approximation theory and Legendre polynomials, which are of basic importance in the sequel.

Consider a specific domain $[a, b]$, and let $C[a, b]$ be the space of all integral function well defined on the domain $[a, b]$. From the analysis presented in [19,20] every $f(x) \in C[a, b]$ have a series representation like

$$f(x) = \sum_{i=0}^{\propto} c_i \varphi_i(x), \tag{1}$$

Where c_i are some constants and $\varphi_i(x)$ are components of basis set, having the property of orthogonality and boundness. Different types of orthogonal polynomials acts as a basis sets. Some of the frequently used orthogonal polynomials are Legendre polynomials, Jacobi polynomails, Laguerre polynomails, Berenstein polynomials, Hermite polynomials and Zernike polynomials. In all these sets the simplest are Legendre polynomials; therefore we focus our self on this specific set of orthogonal polynomials. For details study on other polynomials and its application in different scientific disciplines we refer the reader to latest results by H. Khalil [19-22].

2.1. The Shifted Legendre Polynomials

The well known Legendre polynomials are defined on the domain [-1, 1], and are recursively defined as

$$P_{i+1}(x) = \frac{2i+1}{i+1} x P_i(x) - \frac{i}{i+1} P_{i-1}(x), \mathrm{i} = 0,1,2\ldots\ldots \tag{2}$$

where $P_0(x) = 1$, and $P_1(x) = x$. The transformation $t = \frac{(x+1)}{2}$, makes these polynomials applicable on the domain [0, 1] and the shifted Legendre polynomials obtained defined as

$$P_i(t) = \sum_{k=0}^{i} (\beth_{\mathrm{i,k}} \mathrm{t}^{\mathrm{k}}),\ i = 0,1,2 \ldots\ldots \tag{3}$$

$$\beth_{(i,k)} = \frac{(-1)^{i+k}(i+k)!}{(i-k)!(k!)^2} \tag{4}$$

These polynomials are orthogonal and the orthogonality condition is given as

$$\int_0^1 P_i(t)\, P_j(t) dx = \begin{cases} \frac{1}{2i+1} \; if \; i = j \\ 0, if \; i \neq j \end{cases} \quad (5)$$

By the use of orthogonality condition (5) any f (t) ∈ C ([0, 1]) can be written in the form of infinite series as (1), which can be truncated for practical use as

$$f(t) \approx \sum_{l=0}^{m} c_l P_i(t), \text{ where } c_l = (2l+1) \int_0^1 f(t) P_i(t) dt. \quad (6)$$

As $l \to \infty$ the partial sum become equal to the exact function. In vector notation, we can write

$$f(t) = C_M^T \Lambda_M^t(t). \quad (7)$$

Where

$$\Lambda_M^t(t) = [P_0(t), P_1(t), P_2(t) \ldots \ldots P_m(t)]^T, \quad (8)$$

and

$$C_M = [c_0 \; c_0 \; \ldots . c_i \; \ldots \ldots \; c_m \;]^T. \quad (9)$$

M = m + 1 is the scale level of the approximation.

2.2. Discretization of Legendre Polynomials

In this paper we use the discrete version of Legendre polynomials. The procedure for discretization of Legendre polynomials are given below. Let n be the size of a given ECG signal of a patient for some preselected time T. Then evaluate the Legendre polynomials on the domain [0, 1] and evaluate polynomials on the points $d_i, i = 0,1,2, \ldots . n-1$, where $d_i = \frac{i}{n-1}$, we get the discrete relation as

$$P_i^d[n] = P_i(d_j), j = 0,1,2 \ldots n-1 \quad (10)$$

The discreet function vector can then be seen as a n × M matrix given as

$$\Lambda_{n\times m}[M] = \left[P_0^d[n] \; P_1^d[n] \; \ldots \; P_i^d[n] \ldots P_m^d[n]\right]^T \quad (11)$$

The relation for orthogonality of discrete Legendre is the discrete analog of relation (5).

3. Feature Extraction

We can treat the ECG signal (which is assumed to be n discrete values) as a discrete function $f(d_j)$ defined on domain [0, 1]. In view of relation (5) we have approximation of the signal in terms of discrete Legendre polynomials. All we have to obtain is the coefficient vector as defined in (9). Remember we are using discrete Legendre polynomials, so the first question that arises is, why it is necessary to use discrete analog of Legendre polynomials. In answer to the question we will first study some computational aspects of Legendre polynomials.

3.1. Computational Aspects of Legendre Polynomials

The continuous polynomials can be well interrupted in MATLAB but only up to of order 25. As shown in Figure 1, the polynomials up to index 25 are smooth and bounded (Note that Legendre polynomials are absolutely bounded by unity). Now if we increase the scale level from 25 to 30 the orthogonal polynomials become more and more biased as shown in Figure 1.(a), (b), (c), (d). This occurs due to high underflow of information. However this drawback is removed by using the discretized version and using recurrence relation of Legendre polynomials as described in [18]. The loss of information is highly recovered using the recurrence relation and polynomials up to order 50 are recovered see for example Figure 2. The polynomials with scale level greater than 50 are shown in Figure 3, one can see that for n > 50 the Legendre polynomials are not correct because the polynomials become more and more discontinues and they loss the property of boundness.

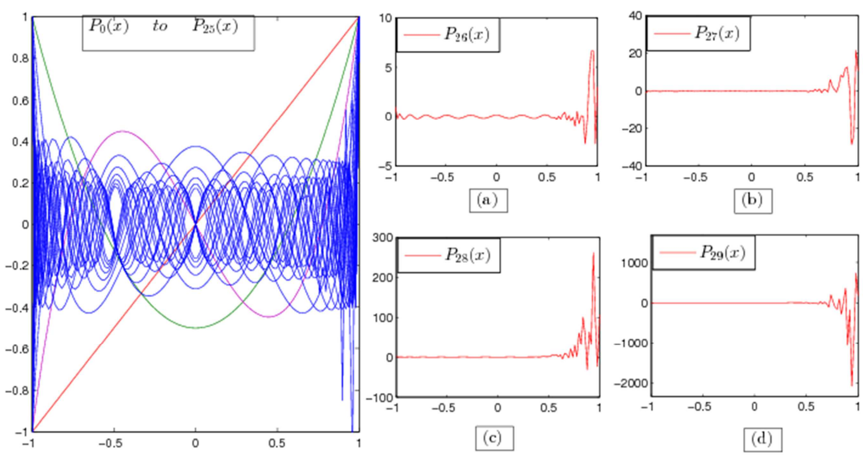

Figure 1. Analytical evaluation of Legendre polynomials using Matlab. (a) $P_{26}(x)$, (b) $P_{27}(x)$, (c) $P_{28}(x)$ and (d) $P_{29}(x)$.

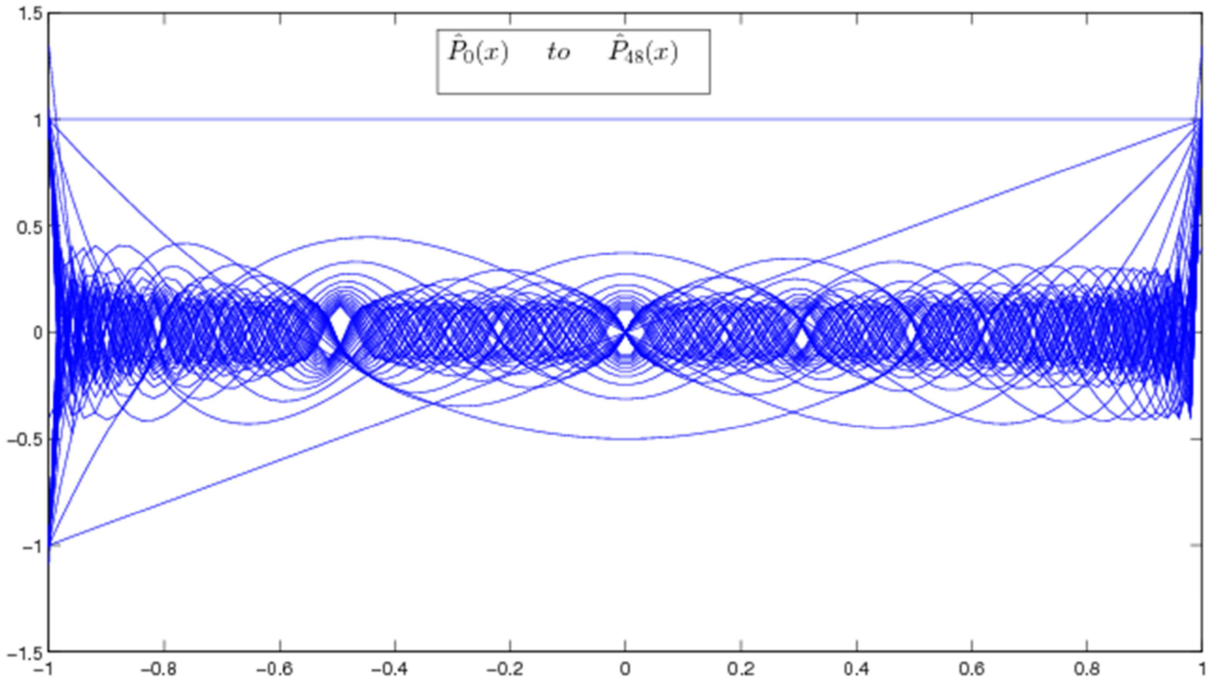

Figure 2. Analytical Evaluation of Discrete Legendre Polynomials.

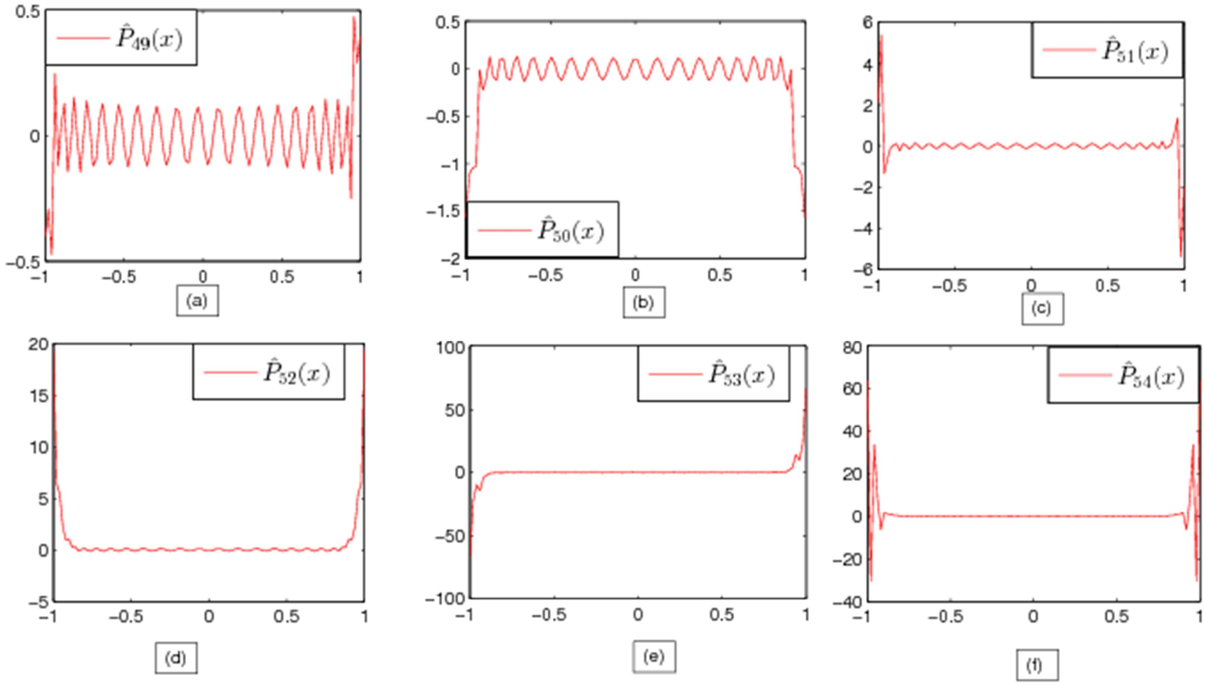

Figure 3. Analytical Evaluation of Discrete Legendre Polynomials.

3.2. Method of Feature Extraction

Consider $R_{[n]}$ be a given ECG signal, then the first m features set of $R_{[n]}$ can be a vector of m numbers. The coefficient vector $C[M]$ for the given signal can be calculated using the discretized version of relation (5) and (6). In matrix notation we can write $C[M]$ as

$$C[M] = R[n]\Lambda_{n\times m}[n]I_{M\times M} \tag{12}$$

Where $I_{M\times M}$ is square diagonal matrix with entries $C(i,i) = \frac{1}{(2i+1)(n-1)}$. To see the difference between normal and abnormal moments, we compute the moments of 20 ECG signals, in which 10 are normal and 10 are abnormal signals. The moments of these signals up to order 15 are shown in Figure 4. It can be easily noted that the normal and abnormal signals have different patterns. The difference between components of normal and abnormal components can be seen in Figure 4.

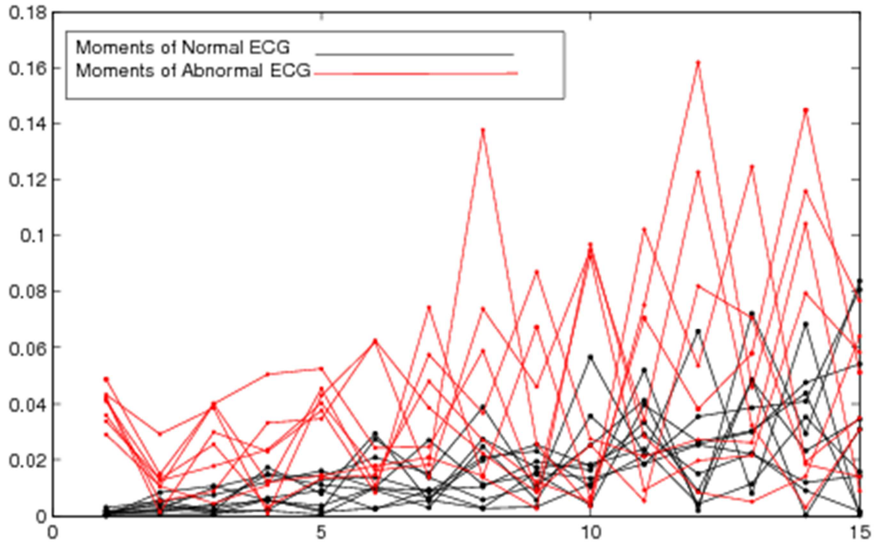

Figure 4. Normal and Abnormal moments of ECG signals.

4. Proposed Method

The entire methodology can be classified into two steps.

(i) Feature extraction

(ii) Classification.

In this work, we have focused on classification only and used the annotation file from MIT-BIH database in order to locate the binary classification of the ECG signals.

Feature extraction is the technique of extracting significant information from a signal. It is the most important step in pattern recognition application since each feature represents the given signal in lower dimension, thereby reducing the computational complexity and overcoming the problem of over fitting of the training samples resulting in good generalization to new samples. The selection of features extraction is also very important since a good classifier may fail to classify the ECG signals, if the features are not proper selected. Our aim is to check the features extracted using the method presented in previous section.

4.1. Classification

Object classification is an important task in computer vision applications, includes surveillance, automatic safety, and image retrieval. In the classification process, ideas and objects are recognized, differentiate and understood. In presented paper the classification is binary in nature (Normal ECG and Abnormal ECG). Classifier achieves this by making a classification decision based on the value of the linear combination of the features. Different Classifiers are available to classify the features from the normal and abnormal ECG. In this paper we use two different classifiers i.e. simple logistic and K-nearest neighbor for the classification of normal and abnormal ECG. A general introduction of these classifiers is given below.

4.2. Simple Logistic

For the classification of ECG into normal or abnormal, multi class classifier has been used, which is based on logistic regression. The logistic regression or logit regression is considered to be a type of probabilistic classification model mainly used for binary classification. The input variables to the classifier are called independent variables where as the output variables are called dependent variables. The classifier uses a function called logistic function for finding the probability that describes the outcome of the independent variables.

Logistic function

The logistic classifier uses logistic function for finding the probability of set of independent variables belonging to a particular class. It has values always between 0 and 1. Mathematically, logistic function can be expressed as

$$F(t) = \frac{e^t}{e^t+1} = \frac{1}{1+e^{-t}} \tag{13}$$

Where $F(t)$ is the Logistic function, t = function of independent variable, e = Exponential function. Suppose, we have independent variable x or combination of independent variables and t is its linear function, then the logistic function will be computed as

$$f(t) = \frac{1}{1+e^{-\beta_0+\beta_{1x}}}.$$

Where F (t) is the probability that the outcome equals a case,

β_0 = intercept of linear regression, β_1 is product of regression coefficient and independent variable. If there are more independent variables, then β_1 x can be expressed as

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_n x_n + \beta_m x_m.$$

Logit

The inverse of logistic function is known as logit represented by following equation.

$$g(x) = \ln \frac{f(x)}{1-f(x)} = \beta_0 + \beta_1 x_1.$$

Odds

Logistic regression is actually used for the prediction of the odds. The odds show that a particular event will occur i.e. a particular combination of independent variables will belong to a particular class. It can be mathematically represented as:

$$\frac{f(x)}{1-f(x)} = \beta_0 + \beta_1 x_1.$$

4.3. K-nearest Neighbor Classier

We also consider supervised learning algorithms known as K-nearest neighbor (KNN), a simplest classification technique. In this method, each feature is assign to a dimension to form a multi dimensional feature space [22]. However the performance of KNN may be better for optimal value of K [23]. KNN algorithm consists of two phases namely training phase and testing phase. In the training phase, the offset in each dimension is referred as feature vector. These feature vectors are used to train classier. These input data has labels which represent their class. In testing phase, the KNN classier is feeds with data and the algorithm generates a list of K nearest data values and assign a label to the data, which determine their class from which the data belong to. The KNN algorithm consists of following four steps.

• Step 1: Selection of suitable distance metric.

• Step 2: Train the classier. According to the equation $X = (x_i, y_i), i = 0,1 \dots n$, where x_i 's are training data set and $y_i' s$ are the class assigned to it and n represents amount of training pattern.

• Step 3: Test the classifier. The classifier is feeds with new data sets. The distance between the newly feed feature vector and already stored features is computed to identify of the class of these new data sets.

• Step 4: K nearest neighbors are chosen and asked to vote for the class of new data set.

5. Result and Discussion

In this study, 100 ECG records were selected from the MIT-BIH arrhythmia database for analysis and classification having equal proportion of normal and abnormal ECG signals. We train the classifier with 35 normal and 35 abnormal ECG signals. And the rest of ECG signals are tested to check the performance of proposed method. All the experiments were performed using feature set (FS1) containing 10 features from a given ECG signal.

The performance of both logistic and KNN classifier is observed. Classification performance is evaluated using four common metrics as found in the literature. These metrics are

$$Accuracy(Acc) = \frac{TP + TN}{TP + TN + FP + FN}.$$

$$Sevsitivity\ (Sec) = \frac{TP}{TP + FN}.$$

$$Specificity(Spc) = \frac{TN}{TN + FP}.$$

$$Positive\ predictivity\ (Ppr) = \frac{TP}{TP + FP}.$$

The confusion matrix of KNN and logistic classifier is displayed in Table 1. It is clear from Table 1 that 14 normal ECG and 12 abnormal ECG is correctly classified by KNN classifier. One normal ECG and three abnormal ECG are misclassified. However using logistic classifier, 15 normal ECG and 15 abnormal ECG signals are correctly classified. The accuracy, sensitivity, specificity and positive predictivity of normal, abnormal classes and average accuracy of both classifiers are given in Table 2. It is clearly indicated that for the current experiment the average accuracy of 85.44%, sensitivity of 86.66%, specificity of 84.22%, and positive predictivity of 87.32% is obtained using KNN classifier. In case of logistic classifier the accuracy is 100%.

Table 1. Confusion matrix of simple logistic and KNN classifiers.

Class	KNN Confusion matrix		Simple Logistic Confusion Matrix	
	Normal	Abnormal	Normal	Abnormal
Normal	14	1	15	0
Abnormal	3	12	0	15

Table 2. Comparing the performance of simple logistic and KNN classifiers.

Class	Performance matrix of KNN				Performance matrix of Simple Logistic			
	Acc%	Sen%	Spe%	Ppr%	Acc%	Sen%	Spe%	Ppr%
Normal	85.71	93.33	76.92	82.35	100	100	100	100
Abnormal	85.18	80	91.16	92.30	100	100	100	100
Avarage	85.44	86.86	84.82	87.32	100	100	100	100

6. Comparison Result of Proposed Method with Other Systems

In this section we compare the performance of the proposed method with other beats classification systems found in the literature. K. Balasundaram et al. [1] used continuous wavelet transform (CWT) and two level binary classifier. M. Thomas et al. [3] used dual tree complex wavelet transform (DTCWT). Vector Quantization (VQ)-based classifier is used in S. Kadambe et al [4]. In [5] Discrete Wavelet Transform (DWT) is used for the classification purpose. A combination of compressed feature and independent feature is used as a input to multilayered perceptron network. N. Emanet et al. [9] used Discrete Wavelet Transform (DWT) and random Forest algorithm. F. A. Afsar et al. [10] classify six classes of ECG using discrete Wavelet Transform (DWT) and simple k-Nearest Neighbor. The comparison results of our method with these referenced work is displayed in the Table 3. It is observed that the proposed method provide a highly accurate results by using Legendre moments and logistic classifiers.

Table 3. Comparison results of proposed method with other methods.

	Features	Classifiers	Classes	Accuracy
Proposed Method	Legendre Moments	KNN	2	84.44%
Proposed Method	Legendre Moments	Simple Logistic	2	100%
K. Balasundaram [1]	WT	LDA	2	93.7%
ShubhaKadambe [4]	WT	NN&VQ	3	94.00%
R. J. Martis [5]	DWT	PCA	5	99.9%
N. Emanet [9]	DWT	Random Forest Algorithm	5	99.8%
F. A. Afsar	DWT	KNN	5	99.5%

Note: We use Intel Core i3 with 2.40 processor and 2.00 GB of memory running under Widows 7 operating system for our analysis. MATLAB 7.6.0 (R2008a) is used for features extraction and related calculations. Weka 3.7.10 is used for training and classification.

7. Conclusion and Future Work

In this article, a novel technique for ECG classification based on Legendre polynomial, simple logistic and KNN is presented. For features extraction from ECG signals we have used Legendre polynomial while KNN and simple logistic classifier are used to classify normal and abnormal ECG signals. Results show about of 80.0% accuracy with use of KNN classifier and about 100% accuracy with the use of simple logistic classifier. It should be noted that by using only 10 features from ECG signals we obtain a very high accuracy. Therefore, it is concluded, that the proposed method is an excellent model for the computer-aided diagnosis of heart diseases based on ECG signals.

At present there are a lot of orthogonal polynomials, in this study only simplest class of orthogonal polynomials is implemented. It is expected that the proposed algorithm works even better by using some other classes of orthogonal polynomials like Berenstein, Jacobi or Laguerre. Our future work is related to find optimal class of orthogonal polynomials. The extension of the method to two dimensional case also falls within the domain of future work.

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