International Journal of Bioinformatics and Biomedical Engineering
Vol. 1, No. 2, 2015, pp. 153-161
http://www.aiscience.org/journal/ijbbe

A Study on Discrete Model of Three Species Syn Eco System with Unlimited Resources for the Second Species

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Abstract

In this paper, the three species syn eco-system comprises of a commensal (S_1), two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3, S_3 is a host of both S_1, S_2 and the second species has unlimited resources. The basic equations for this model constitute as three first order non-linear coupled ordinary difference equations. All possible equilibrium points are identified based on the model equations at two stages and criteria for their stability are discussed. Further the numerical solutions are computed for specific values of the various parameters and the initial conditions.

Keywords

Commensal, Equilibrium Point, Host, Oscillatory, Stable, Unstable

Received: July 7, 2015 / Accepted: August 8, 2015 / Published online: August 24, 2015

1. Introduction

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment sustain themselves on common resources. It is a common observations that the species of same nature can not flourish is isolation without any interaction with species of different kinds. Significant researches in the area of theoretical ecology have been discussed by Kot [1] and by [2]. Several ecologists and mathematicians contributed to the growth of this area of knowledge. Mathematical ecology can be broadly divided into two main sub-divisions, Autecology and Synecology, which are described in the treatises of Arumugam [3], Sharma [4]. Syn-ecology is an ecosystem comprised of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on.

Mathematical Modeling plays a vital role in providing insight into the mutual relationships (positive, negative) between the interacting species. The general concepts of Modeling in Biological Science have been initiated by several authors Ma [5], Murray [6], and Sze-Bi Hsu [7]. Recently the authors Papa Rao et al. [8], Shivareddy et al. [9], Srinivas [10] and Kumar et al. [11] discussed three species ecological models such as predation, completion and commensalism. Srinivas [12] studied the competitive ecosystem of two species and three species with limited and unlimited resources. Later, Narayan et al. [13] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Acharyulu et al. [14, 15] derived some productive results on various mathematical models of ecological Ammensalism with multifarious resources in the manifold directions. Further, Kumar [16] studied some mathematical models of ecological commensalism. The present author

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Prasad [17-22] investigated continuous and discrete models on the three species syn-ecosystems.

The present investigation is on an analytical study of a typical three species (S_1, S_2, S_3) syn-eco system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie, S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. Commensalism is a symbiotic interaction between two populations where one population (S_1) gets benefit from (S_2) while the other (S_2) is neither harmed nor benefited due to the interaction with (S_1). The benefited species (S_1) is called the commensal and the other, the helping one (S_2) is called the host species. A common example is an animal using a tree for shelter-tree (host) does not get any benefit from the animal (commensal). Some more real-life examples of commensalism are, i. The clownfish shelters among the tentacles of the sea anemone, while the sea anemone is not affected. ii. A flatworm attached to the horse crab and eating the crab's food, while the crab is not put to any disadvantage. iii. Sucker fish (echeneis) gets attached to the under surface of sharks by its sucker. This provides easy transport for new feeding grounds and also food pieces falling from the sharks prey, to Echeneis.

2. Basic Equations of the Model

2.1. Notation Adopted

$N_i(t)$: The population strength of S_i at time t, i = 1,2,3; t: Time instant ; a_i: Natural growth rate of S_i , i = 1, 2, 3; a_{ii}: Self inhibition coefficients of S_i, i = 1, 3; a_{12}, a_{13}: Interaction coefficients of S_1 due to S_2 and S_1 due to S_3; a_{23} : Interaction coefficient of S_2 due to S_3. Further the variables N_1, N_2, N_3 are non-negative and the model parameters a_1, a_2, a_3, a_{11}, a_{12}, a_{33}, a_{13}, a_{23} are assumed to be non-negative constants.

2.2. Basic Equations

Consider the growth of the species during the time interval $(t, t+1)$.

$$N_1(t+1) = N_1(t) + a_1 N_1(t) - a_{11} N_1^2(t) + a_{12} N_1(t) N_2(t) + a_{13} N_1(t) N_3(t) \quad (1)$$

$$N_2(t+1) = N_2(t) + a_2 N_2(t) + a_{23} N_2(t) N_3(t) \quad (2)$$

$$N_3(t+1) = N_3(t) + a_3 N_3(t) - a_{33} N_3^2(t) \quad (3)$$

The equations (1), (2) and (3) can be written in the nonlinear autonomous system of discrete equations as

$$N_1(t+1) = \alpha_1 N_1(t) - a_{11} N_1^2(t) + a_{12} N_1(t) N_2(t) + a_{13} N_1(t) N_3(t) \quad (4)$$

$$N_2(t+1) = \alpha_2 N_2(t) + a_{23} N_2(t) N_3(t) \quad (5)$$

$$N_3(t+1) = \alpha_3 N_3(t) - a_{33} N_3^2(t) \quad (6)$$

where $\alpha_i = (a_i + 1) \geq 1$, $i = 1,2,3$.

3. Equilibrium States

The equilibrium states for a discrete model are defined in terms of the period of no growth. i.e, $N_i(t+r) = N_i(t)$, $r = 1,2,3,...,$ where r is the period of the equilibrium state.

3.1. One Period Equilibrium States

$$N_i(t+1) = N_i(t) \ , \ i = 1,2,3 .$$

The system under investigation has five equilibrium states given by

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = 0$$

$$E_2 : \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \ when \ \alpha_3 > 1$$

$$E_3 : \bar{N}_1 = \frac{\alpha_1 - 1}{a_{11}}, \bar{N}_2 = 0 \ , \bar{N}_3 = 0, \ when \ \alpha_1 > 1$$

$$E_4 : \bar{N}_1 = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right], \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

$$when \ \alpha_1 > 1 \ and \ \alpha_3 > 1$$

$$E_5 : \bar{N}_1 = a_{13}\left(\frac{\alpha_3 - 1}{a_{11} a_{33}}\right), \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

$$when \ \alpha_1 = 1 \ and \ \alpha_3 > 1$$

3.2. The Stability Analysis of One Period Equilibrium States

3.2.1. The Stability of E_1

$N_i(t+r) = 0$, where r is an integer and i = 1, 2, 3.

Hence, E_1 (0, 0, 0) is stable.

3.2.2. The Stability of E_2

$$N_i(t+r) = 0, N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$$

where r is an integer and i = 1, 2.

Hence, E_2 is stable.

3.2.3. The Stability of E_3

$$N_1(t+r)=\frac{\alpha_1-1}{a_{11}}, N_i(t+r)=0,$$

where r is an integer and i = 2, 3.

Hence, E_3 is stable.

3.2.4. The Stability of E_4

$$N_1(t+r)=\frac{1}{a_{11}}\left[(\alpha_1-1)+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right)\right], N_2(t+r)=0,$$

$$N_3(t+r)=\frac{\alpha_3-1}{a_{33}}, \text{ where r is an integer.}$$

Hence, E_4 is stable.

3.2.5. The Stability of E_5

$$N_1(t+r)=a_{13}\left(\frac{\alpha_3-1}{a_{11}a_{33}}\right), N_2(t+r)=0, N_3(t+r)=\frac{\alpha_3-1}{a_{33}},$$

where r is an integer.

Hence, E_5 is stable.

At this stage all the five equilibrium states are stable.

3.3. Two Period Equilibrium States

$$N_i(t+2)=N_i(t)\ ,\ i=1,2,3$$

The system under investigation has twenty five equilibrium states given by

$$E_1: \bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=0$$

$$E_2: \bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=\frac{\alpha_3-1}{a_{33}}, \ when\ \alpha_3>1.$$

$$E_3: \bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$
when $\alpha_3>3$.

$$E_4: \bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$
when $\alpha_3>3$.

$$E_5: \bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=\frac{2}{a_{33}}, \text{ when } \alpha_3=3.$$

The states E_3 and E_4 coincide when $\alpha_3 = 3$ and do not exist when $\alpha_3 < 3$.

$$E_6: \bar{N}_1=\frac{\alpha_1-1}{a_{11}}, \bar{N}_2=0, \bar{N}_3=0, \ when\ \alpha_1>1.$$

$$E_7: \bar{N}_1=\frac{(\alpha_1+1)+\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}}, \bar{N}_2=0, \bar{N}_3=0,$$
when $\alpha_1>3$.

$$E_8: \bar{N}_1=\frac{(\alpha_1+1)-\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}}, \bar{N}_2=0, \bar{N}_3=0,$$
when $\alpha_1>3$.

$$E_9: \bar{N}_1=\frac{2}{a_{11}}, \bar{N}_2=0, \bar{N}_3=0, \text{ when } \alpha_1=3.$$

The states E_7 and E_8 coincide when $\alpha_1 = 3$ and do not exist when $\alpha_1 < 3$.

$$E_{10}: \bar{N}_1=\frac{\beta_1-1}{a_{11}}, \bar{N}_2=0, \bar{N}_3=\frac{\alpha_3-1}{a_{33}}, \text{ when } \beta_1,\alpha_3>1$$

$$\text{where } \beta_1=\alpha_1+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right).$$

$$E_{11}: \bar{N}_1=\frac{(\beta_1+1)+\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}}, \bar{N}_2=0, \bar{N}_3=\frac{\alpha_3-1}{a_{33}},$$
when $\beta_1>3$ and $\alpha_3>1$.

$$E_{12}: \bar{N}_1=\frac{(\beta_1+1)-\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}}, \bar{N}_2=0, \bar{N}_3=\frac{\alpha_3-1}{a_{33}},$$
when $\beta_1>3$ and $\alpha_3>1$.

$$E_{13}: \bar{N}_1=\frac{2}{a_{11}}, \bar{N}_2=0, \bar{N}_3=\frac{\alpha_3-1}{a_{33}}, \text{ when } \beta_1=3 \text{ and } \alpha_3>1.$$

The states E_{11} and E_{12} coincide when $\beta_1 = 3$ and do not exist when $\beta_1 < 3$.

$$E_{14}: \bar{N}_1=\frac{\gamma_1-1}{a_{11}}, \bar{N}_2=0, \bar{N}_3=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$
when $\gamma_1>1$ *and* $\alpha_3>3$,

$$\text{where } \gamma_1=\alpha_1+a_{13}\left[\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}\right].$$

$$E_{15}: \bar{N}_1=\frac{(\gamma_1+1)+\sqrt{(\gamma_1+1)(\gamma_1-3)}}{2a_{11}}, \bar{N}_2=0,$$

$$\bar{N}_3=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}, \text{when } \gamma_1,\alpha_3>3.$$

$$E_{16}: \bar{N}_1 = \frac{(\gamma_1+1)-\sqrt{(\gamma_1+1)(\gamma_1-3)}}{2a_{11}}, \bar{N}_2 = 0,$$

$$\bar{N}_3 = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}, \text{when } \gamma_1, \alpha_3 > 3.$$

$$E_{17}: \bar{N}_1 = \frac{2}{a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

when $\gamma_1 = 3$ and $\alpha_3 > 3$.

The states E_{15} and E_{16} coincide when $\gamma_1 = 3$ and do not exist when $\gamma_1 < 3$.

$$E_{18}: \bar{N}_1 = \frac{\mu_1 - 1}{a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

when $\mu_1 > 1$ *and* $\alpha_3 > 3$,

$$\text{where } \mu_1 = \alpha_1 + a_{13}\left[\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}\right].$$

$$E_{19}: \bar{N}_1 = \frac{(\mu_1+1)+\sqrt{(\mu_1+1)(\mu_1-3)}}{2a_{11}}, \bar{N}_2 = 0,$$

$$\bar{N}_3 = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}, \text{when } \mu_1, \alpha_3 > 3.$$

$$E_{20}: \bar{N}_1 = \frac{(\mu_1+1)-\sqrt{(\mu_1+1)(\mu_1-3)}}{2a_{11}}, \bar{N}_2 = 0,$$

$$\bar{N}_3 = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}, \text{when } \mu_1, \alpha_3 > 3.$$

$$E_{21}: \bar{N}_1 = \frac{2}{a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

when $\mu_1 = 3$ and $\alpha_3 > 3$.

The states E_{19} and E_{20} coincide when $\mu_1 = 3$ and do not exist when $\mu_1 < 3$.

$$E_{22}: \bar{N}_1 = \frac{\delta_1 - 1}{a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{2}{a_{33}}, \text{ when } \delta_1 > 1, \alpha_3 = 3,$$

$$\text{where } \delta_1 = \alpha_1 + \frac{2a_{13}}{a_{33}}.$$

$$E_{23}: \bar{N}_1 = \frac{(\delta_1+1)+\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{2}{a_{33}},$$

when $\delta_1 > 3, \alpha_3 = 3$.

$$E_{24}: \bar{N}_1 = \frac{(\delta_1+1)-\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{2}{a_{33}},$$

when $\delta_1 > 3, \alpha_3 = 3$.

$$E_{25}: \bar{N}_1 = \frac{2}{a_{11}}, \bar{N}_2 = 0, \bar{N}_3 = \frac{2}{a_{33}}, \text{ when } \delta_1 = 3, \alpha_3 = 3.$$

The states E_{23} and E_{24} coincide when $\delta_1 = 3$ and do not exist when $\delta_1 < 3$.

3.4. The Stability Analysis of Two Period Equilibrium States

The equilibrium states E_1, E_2 and E_6 are stable as established in 3.2. Now we will discuss the stability of other equilibrium states except these three states.

3.4.1. The Stability of E_3

$N_i(t+r) = 0$, where r is an integer and i = 1, 2.

$$N_3(t+2r) = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1}) = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_3 oscillates between

$$\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \text{ and } \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.2. The Stability of E_4

$N_i(t+r) = 0$, where r is an integer and i = 1, 2.

$$N_3(t+2r) = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1}) = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_4 oscillates between

$$\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \text{ and } \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.3. The Stability of E_5

$$N_i(t+r)=0, N_3(t+r)=\frac{2}{a_{33}},$$

where r is an integer and i = 1, 2.

Hence, E_5 is stable.

3.4.4. The Stability of E_7

$$N_1(t+2r)=\frac{(\alpha_1+1)+\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\alpha_1+1)-\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

where r is an integer.

$N_i(t+r)=0$, where r is an integer and i = 2, 3.

Hence, E_7 oscillates between

$$\frac{(\alpha_1+1)+\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}} \text{ and } \frac{(\alpha_1+1)-\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

when $\alpha_1 > 3$and is stable when $\alpha_1 = 3$.

3.4.5. The Stability of E_8

$$N_1(t+2r)=\frac{(\alpha_1+1)-\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\alpha_1+1)+\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

where r is an integer.

$N_i(t+r)=0$, where r is an integer and i = 2, 3.

Hence, E_8 oscillates between

$$\frac{(\alpha_1+1)-\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}} \text{ and } \frac{(\alpha_1+1)+\sqrt{(\alpha_1+1)(\alpha_1-3)}}{2a_{11}},$$

when $\alpha_1 > 3$and is stable when $\alpha_1 = 3$.

3.4.6. The Stability of E_9

$$N_1(t+r)=\frac{2}{a_{11}}, N_i(t+r)=0,$$

where r is an integer and i = 2, 3.

Hence, E_9 is stable.

3.4.7. The Stability of E_{10}

$$N_1(t+r)=\frac{\beta_1-1}{a_{11}}, N_2(t+r)=0, N_3(t+r)=\frac{\alpha_3-1}{a_{33}},$$

where r is an integer.

Hence, E_{10} is stable.

3.4.8. The Stability of E_{11}

$$N_1(t+2r)=\frac{(\beta_1+1)+\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\beta_1+1)-\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

$N_2(t+r)=0, N_3(t+r)=\frac{\alpha_3-1}{a_{33}}$, where r is an integer.

Hence, E_{11} oscillates between

$$\frac{(\beta_1+1)+\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}} \text{ and } \frac{(\beta_1+1)-\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

when $\beta_1 > 3$and is stable when $\beta_1 = 3$.

3.4.9. The Stability of E_{12}

$$N_1(t+2r)=\frac{(\beta_1+1)-\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\beta_1+1)+\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

$N_2(t+r)=0, N_3(t+r)=\frac{\alpha_3-1}{a_{33}}$, where r is an integer.

Hence, E_{12} oscillates between

$$\frac{(\beta_1+1)-\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}} \text{ and } \frac{(\beta_1+1)+\sqrt{(\beta_1+1)(\beta_1-3)}}{2a_{11}},$$

when $\beta_1 > 3$and is stable when $\beta_1 = 3$.

3.4.10. The Stability of E_{13}

$$N_1(t+r)=\frac{2}{a_{11}}, N_2(t+r)=0, N_3(t+r)=\frac{\alpha_3-1}{a_{33}},$$

where r is an integer.

Hence, E_{13} is stable.

3.4.11. The Stability of E_{14}

$N_1(t+r)\neq\frac{\gamma_1-1}{a_{11}}$, where r is an integer except 0 and 1

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{14} is unstable, when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.12. The Stability of E_{15}

$$N_1(t+2r)\neq\frac{(\gamma_1+1)+\sqrt{(\gamma_1+1)(\gamma_1-3)}}{2a_{11}},$$

where r is an integer except 0 and 1

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{15} is unstable, when $\alpha_3 > 3$and is oscillatory when $\alpha_3 = 3$.

3.4.13. The Stability of E_{16}

$$N_1(t+2r)\neq\frac{(\gamma_1+1)-\sqrt{(\gamma_1+1)(\gamma_1-3)}}{2a_{11}},$$

where r is an integer except 0 and 1.

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{16} is unstable, when $\alpha_3 > 3$and is oscillatory when $\alpha_3 = 3$.

3.4.14. The Stability of E_{17}

$N_1(t+r)\neq\frac{2}{a_{11}}$, where r is an integer except 0 and 1

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{17} is unstable, when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.15. The Stability of E_{18}

$N_1(t+r)\neq\frac{\mu_1-1}{a_{11}}$, where r is an integer except 0 and 1.

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{14} is unstable, when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.16. The Stability of E_{19}

$$N_1(t+2r)\neq\frac{(\mu_1+1)+\sqrt{(\mu_1+1)(\mu_1-3)}}{2a_{11}},$$

where r is an integer except 0 and 1.

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{19} is unstable, when $\alpha_3 > 3$and is oscillatory when $\alpha_3 = 3$.

3.4.17. The Stability of E_{20}

$$N_1(t+2r)\neq\frac{(\mu_1+1)-\sqrt{(\mu_1+1)(\mu_1-3)}}{2a_{11}},$$

where r is an integer except 0 and 1.

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{20} is unstable, when $\alpha_3 > 3$and is oscillatory when $\alpha_3 = 3$.

3.4.18. The Stability of E_{21}

$N_1(t+r) \neq \frac{2}{a_{11}}$, where r is an integer except 0 and 1.

$$N_2(t+r)=0,\ N_3(t+2r)=\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

$$N_3(t+\overline{2r+1})=\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}},$$

where r is an integer.

Hence, E_{21} is unstable, when $\alpha_3 > 3$and is stable when $\alpha_3 = 3$.

3.4.19. The Stability of E_{22}

$$N_1(t+r)=\frac{\delta_1-1}{a_{11}}, N_2(t+r)=0, N_3(t+r)=\frac{2}{a_{33}},$$

where r is an integer.

Hence, E_{22} is stable.

3.4.20. The Stability of E_{23}

$$N_1(t+2r)=\frac{(\delta_1+1)+\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\delta_1+1)-\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

$$N_2(t+r)=0, N_3(t+r)=\frac{2}{a_{33}},$$

where r is an integer.

Hence, E_{23} oscillates between

$$\frac{(\delta_1+1)+\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}} \text{ and } \frac{(\delta_1+1)-\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

when $\delta_1 > 3$and is stable when $\delta_1 = 3$.

3.4.21. The Stability of E_{24}

$$N_1(t+2r)=\frac{(\delta_1+1)-\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

$$N_1(t+\overline{2r+1})=\frac{(\delta_1+1)+\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

$$N_2(t+r)=0, N_3(t+r)=\frac{2}{a_{33}},$$

where r is an integer.

Hence, E_{24} oscillates between

$$\frac{(\delta_1+1)-\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}} \text{ and } \frac{(\delta_1+1)+\sqrt{(\delta_1+1)(\delta_1-3)}}{2a_{11}},$$

when $\delta_1 > 3$and is stable when $\delta_1 = 3$.

3.4.22. The Stability of E_{25}

$$N_1(t+r)=\frac{2}{a_{11}}, N_2(t+r)=0, N_3(t+r)=\frac{2}{a_{33}},$$

where r is an integer.

Hence, E_{25} is stable, when $\delta_3 = 3$and $\alpha_3 = 3$.

At this stage, in all twenty five equilibrium states, only the nine states E_1, E_2, E_5, E_6, E_9,E_{10}, E_{13}, E_{22}, E_{25} are stable and E_3, E_4, E_7, E_8, E_{11}, E_{12}, E_{23}, E_{24} are oscillatory and remaining all are unstable.

4. Numerical Examples

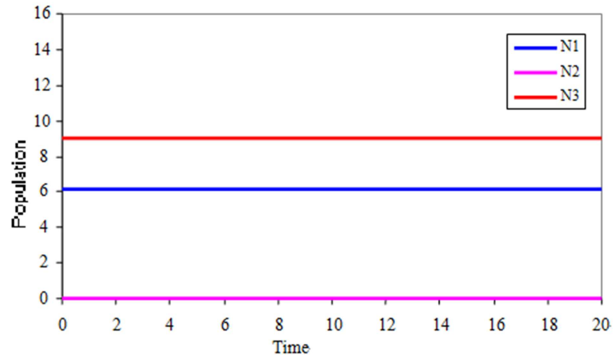

Figure 1. Variation of population against time for $a_1 = 1.5$, $a_{11} = 1$, $a_{13} = 0.4$, $a_2 = 1.05$, $a_{22} = 3.4$, $a_{23} = 4.53$, $a_3 = 2.7$, $a_{33} = 0.3$, $N_1(0) = 6.1$, $N_2(0) = 0$, $N_3(0) = 9$.

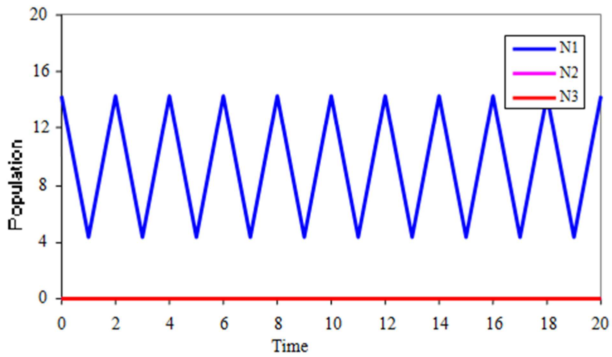

Figure 2. Variation of population against time for $a_1 = 3.6$, $a_{11} = 0.3$, $a_{13} = 2.6$, $a_2 = 4.7$, $a_{22} = 1.28$, $a_{23} = 3.33$, $a_3 = 1.23$, $a_{33} = 2.1$, $N_1(0) = 14.32$, $N_2(0) = 0$, $N_3(0) = 0$.

The numerical solutions of the discrete model equations computed for specific values of the various parameters and the initial conditions. The results are illustrated in Figures 1 to 4.

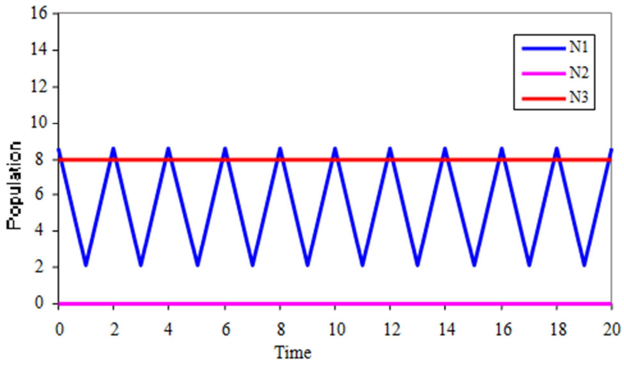

Figure 3. Variation of population against time for $a_1 = 1.5$, $a_{11} = 1.2$, $a_{13} = 0.4$, $a_2 = 2.11$, $a_{22} = 1.4$, $a_{23} = 0.50$, $a_3 = 1.6$, $a_{33} = 0.2$, $N_1(0) = 8.6$, $N_2(0) = 0$, $N_3(0) = 8$.

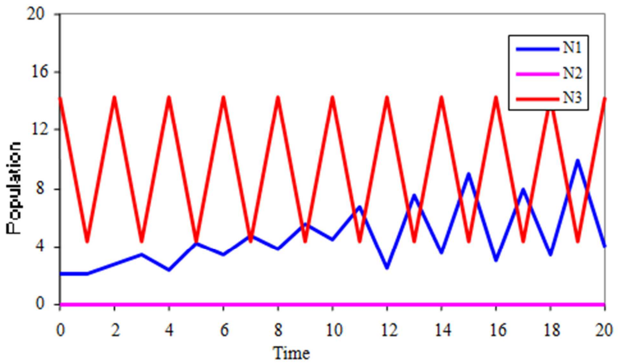

Figure 4. Variation of population against time for $a_1 = 2.6$, $a_{11} = 1.29$, $a_{13} = 0.01$, $a_2 = 0.7$, $a_{22} = 1.53$, $a_{23} = 0.85$, $a_3 = 3.6$, $a_{33} = 0.3$, $N_1(0) = 2.1$, $N_2(0) = 0$, $N_3(0) = 14.32$.

5. Discussion and Conclusions

The present paper deals with an investigation on a discrete model of three species syn eco-system with unlimited resources for the second species. The system comprises of a commensal (S_1) common to two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3, S_3 is a host of both S_1, S_2. All possible equilibrium points of the model are identified based on the model equations at two stages.

$$\text{Stage-I: } N_i(t+1) = N_i(t);\ i = 1, 2, 3.$$

$$\text{Stage-II: } N_i(t+2) = N_i(t);\ i = 1, 2, 3.$$

In Stage-I there are only five equilibrium points, while the Stage-II there would be twenty five equilibrium points. All the five equilibrium points in Stage-I are found to be stable while in stage-II only nine are stable. Further the numerical solutions for the discrete model equations are computed.

Acknowledgment

I thank to Professor (Retd), N.Ch. Pattabhi Ramacharyulu, Department of Mathematics, National Institute of Technology, Warangal (T.S.), India for his valuable suggestions and encouragement.

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